Sovereign Debt, the Blessing Aspects and the Implications for the Euro Area

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Summary: The Euro area has a unique monetary authority that governs money creation, but several individual-countries' sovereign debts that differ in terms of safety. We analyse: i) the interactions between the financial and real sector in such an environment; ii) the role of government bonds as liquidity instruments; iii) whether and how the correlation structure of the sovereign-bonds' market values affects the portfolio composition of liquidity instruments and prices, and the scope for a debt management policy at the Euro area level.

Zusammenfassung: Der Euroraum hat eine einheitliche Währungsbehörde, die die Geldschöpfung regelt, aber die einzelnen Eurostaaten begeben jeweils ihre eigenen Staatschulden. Folglich ist auch die Sicherheit der Staatsanleihen sehr unterschiedlich. Vor diesem Hintergrund wird analysiert: i) die Wechselwirkungen zwischen dem finanziellen und dem realer Sektor; ii) die Rolle von Staatsanleihen als Liquiditätsinstrumente; iii) ob und wie sich die Korrelation der Marktwerte der Staatsanleihen auf die Zusammensetzung von liquiden Instrumenten im Portfolio und die Preise derselben auswirkt und den Spielraum für eine Schuldenmanagementpolitik im Euroraum einschränkt.

→ JEL classifications: E44, H63, G18
→ Keywords: Safe assets, liquidity, sovereign debt management, sovereign debt spreads

1 Introduction

Siemens, Europe's largest engineering group, is setting up its own bank.

As well as broadening its sources of funding, it would allow the company to deposit cash at the Bundesbank, Germany's central bank.

"Frankly, we'd be happy to get no interest rate just to know our cash is completely safe", a person close to the company says. (FT September 2, 2010)

Berkshire Hathaway disclosed its liquidity instruments:

80% of $ 10.8 billion on non-US Gov. Debt are from Germany, UK, Canada, Australia and the Netherlands

Comment (Guy LeBas chief fixed income strategist):

"*If a firm is looking at government debt as a source of potential liquidity, then it's extremely important to remain in these bulletproof nations"* (Bloomberg 27. 02. 2012)

Why don't these firms, i. e. the industrial firm Siemens and the financial firm Berkshire Hathaway simply use bank deposits as a liquidity instrument? The key observation is that deposits are bank liabilities and their safety is constrained by bank income pledgeability and/or by deposit insurance. The latter being limited to "small" amounts make bank deposits money/liquidity instruments only for households.

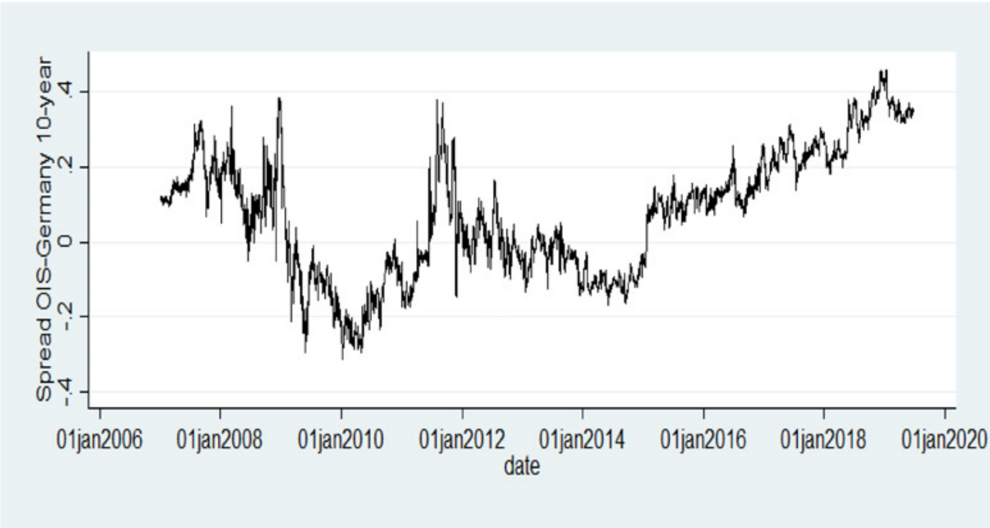
Siemens and Berkshire Hathaway are just an example of the so called "institutional cash pools", i. e. the large, centrally managed cash balances of global corporations and institutional investors. Institutional cash pools have become increasingly prominent since the 1990s as a byproduct of globalization (see Pozsar, 2011, and with specific reference to financial/banking corporations Bruno and Shin, 2011). As documented by Pozsar (2011), over 90% of institutional cash pools are subject to written cash investment policies which govern the investment styles and fiduciary responsibilities of their managers. In order of priority, the objectives of these policies are: (i) safety of principal; (ii) liquidity; and (iii) yield. The empirical evidence clearly shows that institutional cash pools' preferred habitat is not deposits, but insured deposit alternatives: Government insured securities (government debt securities) and privately insured money market instruments, such as REPOs and asset backed commercial paper – where collateral provides safety and substitutes for government guarantee (Gorton, 2010; Stein, 2011; Krishnamurthy and Vissing-Jorgensen, 2012; Singh and Stella, 2012)

To sum up, the demand for money/liquidity instruments of corporations, a large sector of the economy and most active player in the real investment process and in the money/credit market, is not satisfied by M2 types of money, but rather directly or indirectly (via REPO arrangements) by securities that meet the requirement of safety of principal and liquidity.

Moreover, liquidity is an input of firms' investment process. Liquidity can possibly be offered by banks, and indeed liquidity provision is the key function of banks, but banks also need liquid assets to perform this function. This makes (safe) sovereign debt the pillar of a monetary area. In the USA, treasury debt securities are named “deposit”: Treasuries are the M2 equivalent for financial and non-financial firms. Krishnamurthy and Vissing-Jorgensen (2012) document that US Treasury bonds enjoy a collateral/liquidity-safety premium of about 0.7 % per year on average. Figure 1 plots the difference between the safe rate of interest (measured by the Overnight Index Swap) and the German government debt yield. It provides an indication of the current collateral/liquidity-safety premium enjoyed by safe sovereign debt securities in the Euro Area.

Figure 1

Collateral Premium (OIS – German Bund Yield Spread)



However, while the Euro Area has a unique monetary authority that governs money creation, not all sovereign debt securities are equally safe. Indeed, there are several individual-countries’ sovereign debts that differ in terms of safety. This paper analyses the interactions between the financial and the real sector in such an environment.

We borrow from the literature (Holmstrom and Tirole, 1998, 2011; and from reality) the idea that firms and financial institutions are best viewed as ongoing entities whose project completion requires renewed injections of resources.[1] Limited pledgeability of project outcomes constrains the amount of outside finance that can be raised and gives rise to the need of hoarding liquid/safe assets to cope with adverse shocks and/or to take future investment opportunities. In such an environment, the value of an asset to a firm is determined by the resources it gives access to when resources are most valuable – i.e., when projects need completion and/or further investment-opportunities materialize. Firms are willing to pay a premium for liquidity/safety: the liquidity/

1 Corporations’ concern for refinancing is emphasized in various contexts by the finance literature – Thakor, Hong and Greenbaum (1981), and Froot, Scharfstein, and Stein (1993), among others.

safety's benefits amount to the option value of exercising future investment opportunities that would not be taken otherwise.

Firms may insure against liquidity needs by securing credit lines from financial institutions; that is, they can contract with a bank for the right to draw specific amounts of cash by a given date. Thanks to these arrangements, liquidity to corporations is provided by the bank, while the burden of liquidity hoarding is on the bank. The bank needs to hold a sufficient amount of liquid assets in order to fund the take-downs that its clients/firms are entitled to make under a credit line/loan commitment. Liquidity provision is the key activity of banks – the largest share of commercial and industrial loans are take-downs under loan commitments – credit lines (Bhattacharya and Thakor, 1993; Strahan, 2008). During the financial crisis, banks holding assets with low market liquidity (e. g. mortgage-backed securities, and asset backed securities) increased their holdings of liquid assets and lowered their liquidity provision to firms – new commitments to lend shrunk (Cornett et al., 2011).

The intimate relation between banks' liquidity provision and liquid assets holdings makes the availability of safe/liquid assets at the center of the credit/investment process. We focus on outside liquidity, and with reference to the euro area, we allow for a range of sovereign bonds that differ with regard to the volatility of their market values (Section 2). We analyse the resulting equilibrium asset prices, collateral/liquidity-safety premia, liquidity instrument holdings and real investment (Section 3). We show that credit expansion, real investment and return on capital are increasing functions of the amount of safe/liquid assets, the reverse holds for collateral/liquidity premia and bond spreads. Safe/liquid assets' availability is determined by the amount of sovereign bonds outstanding and crucially by the volatility of their market values. An increase in market-value volatility of a bond induces a substitution away from that bond and the macro effect of depleting the amount of assets that are eligible for satisfying liquidity needs, and for sufficiently high volatility, the bond loses the status of liquid asset (it's excluded from asset holdings for liquidity purposes).

We then examine how the correlation structure of the bond market values affects the optimal composition of liquidity instrument holdings and prices, the optimal debt management policies and whether these policies can be implemented at the nation-state level or do instead require a supranational institution (Section 4).

We enlighten two key points. A risky bond can be viewed as a combination of different securities, one that by virtue of sufficient collateral (e. g. tax revenue) pays a safe/guaranteed outcome for sure and a risky one that in good (up-turn) states pays what is left in excess of the safe/guaranteed outcome, if any. For a liquidity seeking institution the safe component is highly valuable, the risky component exposes the holder to the risk of curtailing real investment and exposes banks to the risk of failing in providing insurance against liquidity needs.[2] Unbundling the security package, in the finance jargon "tranching", eliminates the inefficient real-investment cut and produces positive effects. The second observation is that the greater the amount of safe liquid assets, the greater the level of real investment. With negative correlation of bonds' crisis states, pooling the risky sovereign debts and then applying tranching so as to insulate the safe component allows for an aggregate amount of safe assets greater than that that would be obtained by tranching the sovereign debts at

2 This complements the analysis in Chiesa (2019) where the inefficiency arising from risky bond holdings results from the idle money balances held in non-crisis states.

the nation-state level – that is, the optimal debt management policy cannot be performed at the single country level.[3]

Related Literature

The role of government debt, and more generally outside liquidity, in facilitating the intertemporal allocation of resources has been emphasized by several papers and relies on contractual frictions that limit the enforceability of claims on future income (Woodford, 1990; Kiyotaki and Moore, 2018; Holmstrom and Tirole, 1998, 2011). The most closely related paper is Bolton and Jeanne (2011) that analyses the role for government debt securities as collateral for borrowing. The key assumption is the asynchronicity between resource availability and real investment opportunities (as in Woodford, 1990, and Kiyotaki and Moore, 2018). Non pledgeability of investment returns prohibits unsecured (non-collateralized) borrowing, real investment undertaking relies on transferring resources into the future by investing resource endowments in government bonds that can be used as collateral for secured (collateralized) borrowing. The safer the government debt, the greater the amount of secured borrowing that can be raised and the investment that can be attained. In a multi country world, safe debt is a public good and selfish governments will supply a socially sub-optimal amount of safe debt. Our paper differs in several respects, by contrast to Bolton and Jeanne, we focus on the joint determination of firms'/banks' composition of liquid asset portfolios, real investment/credit-lines provision, collateral/liquidity premia and bond spreads. We consider firms and financial institutions as ongoing entities whose investment projects' completion requires renewed injections of resources. The larger the amount of assets that are eligible for satisfying liquidity needs, the larger is the scale of investment, and the lower are the liquidity premia and bond spreads. An increase in market-value volatility of a bond induces a macro effect of depleting the amount of assets that are eligible for satisfying liquidity needs, and a substitution away from that bond. For sufficiently high volatility, the bond loses the status of liquid asset (it's excluded from asset holdings for liquidity purposes). By contrast, in Bolton and Jeanne a volatile/risky bond always enters in a portfolio, since it allows increasing the investment size in the (risky-debt) no-default state. They analyze various forms of fiscal integration that can mitigate the incentives to undersupply safe debt and find that they reduce the welfare of the country that provides the "safe-haven" asset. We focus on debt management, and whether optimal debt management policies require a supranational institution.

In the euro area banks rely on bonds issued by their own countries; the "diabolic loop" between sovereign risk and bank risk. To break this loop, Brunnermeier et al. (2017) advocate sovereign-bond securities, the most senior of which would play the role of safe asset (European Safe Bonds). Lane and Langfield (2018) highlight how sovereign bond-backed securities could help to enhance financial stability in the euro area. We show that safe assets allow for insurance against liquidity needs and are a key determinant of real investment.

Finally, this paper complements Chiesa (2019), in that risky bonds expose the holder to the risk of curtailing real investment and banks to the risk of failing in providing insurance against liquidity needs.

3 The importance of secured/collateralized government debt for a sounder euro area monetary system is emphasized by Nyborg (2011). The euro-nomics group (2011) and Brunnermeier et al. (2017) point out the vital importance of a European safe asset for the long run survival of the euro-zone and call for the creation of European Safe Bonds, where safeness is provided by pooling the sovereign bonds and then tranching the pooled debt so as to create a security whose safeness is ensured by sufficient collateral.

DOI https://doi.org/10.3790/vjh.89.1.9

2 The Model

There are three periods $t = 0, 1, 2$. Agents (final investors) are risk neutral and value consumption streams according to

$$U(c_0, c_1, c_2) = E(c_0 + c_1 + c_2) \tag{A1}$$

that is, agents' intertemporal marginal rate of substitution (IMRS) is equal to 1. Each agent receives a resource endowment at each date, and this is sufficiently large to ensure that resource scarcity does not limit the investment scale, this will be constrained by contractual frictions (limited pledgeability). Specifically we shall assume that cash flows can be diverted and hence cannot be pledged (cannot be seized by outsiders/creditors). This will limit the ability to borrow and provides a role for the ex-ante planning of liquid assets' holdings.

Assets

We assume that the storage technology (holding cash under the mattress) is prohibitively costly, and that purchasing power can be transferred into the future by investing in securities; primarily, sovereign debt securities. For simplicity, we assume that there are two individual-countries' sovereign debts: G government debt and I government debt.

The unit price of i sovereign debt at date 0 is denoted q_i , and the total amount of i sovereign debt is denoted B^i, $i = G, I$.

The G bonds are safe. G's unit value at date 1 is $V_G = 1$ for sure. The I bonds are "risky". At date 1 with probability τ a crisis state occurs and the I bond value drops from the non-crisis states' value α^I to $\underline{\alpha}^I < \alpha^I$. The I bond's expected unit value is

$$V_I \equiv \alpha^I - \tau\Delta^I = x \leq 1$$

$$\Delta^I \equiv \alpha^I - \underline{\alpha}^I \tag{A2}$$

For any given expected value x, the greater the expected crisis loss, $\tau\Delta^I$, the greater the volatility and, as we will see, the less suitable the I bond is as liquidity instrument.

We define the **collateral** premium on government bond i as the excess payment made at date 0 for this bond relatively to its date 1 expected value, that is $q_i - V_i$, $i = G, I$.

At an equilibrium:

$$q_I \geq V_I \equiv x, \ q_G \geq V_G \equiv 1$$

$q_I < x$, $q_G < 1$ are ruled out by $(A1)$ and $(A2)$. That is, **collateral** premia cannot be negative.

Firms/Banks

There are N firms/banks. They are risk-neutral and evaluate consumption streams according to $(A1)$. A firm/bank i, $i = 1, ..N$, has initial net worth A_i at date 0 and no endowment in future periods. For simplicity, all firms/banks face the same investment opportunity that requires injections of resources at date 0 and at date 1 and delivers returns at date 2. Specifically, at date 0, i chooses the size of investment I_i; this defines the amount of resources to be invested by i at date 0. At date 1, the amount S per unit of investment must be injected for the project to generate returns. At the final date 2, the return y per unit of completed investment obtains. We assume:

$$y > 1 + S \tag{A3}$$

That is, investment projects are positive in net present value (by (A3)). However only the amount b per unit of completed project can be seized by lenders/investors, and $b < 1 + S$. The limited pledgeability of project returns coupled with the reinvestment need at date 1 requires liquidity holdings of amount S per unit of investment.

The above is a reduced form of two possible models. One simply refers to a firm that faces a constant-to-scale real investment opportunity which requires one unit of resources at date 0, and S at date 1, per unit of investment. For a given investment size I_i chosen at date 0, the reinvestment needs at date 1 amount to SI_i, and reinvestment (project completion) will be feasible only if $i's$ liquidity holdings at date 1 do not fall below SI_i. An alternative, and prominent, case refers to i being a bank endowed with net worth (capital) A_i which faces a continuum of borrowers/firms. Each borrower has an investment opportunity that requires one unit at date 0 and S additional units at date 1. The bank at date 0 chooses the size of its credit-lines' portfolio I_i, where a credit line allows a bank's borrower to withdraw one unit at date 0 and S units at date 1. At date 1 the bank will face withdrawals of total amount SI_i and will be able to satisfy borrowers' liquidity needs (i.e, the credit lines withdrawals) only if its liquidity holdings do not fall below SI_i. If i is a bank, then I_i defines the amount of credit extended by i; that is, the scale of investment originated by i.

We shall concentrate on scarcity of safe assets, and assume:

$$B^G < \frac{S \sum A_i}{1 - b + S} \tag{A4}$$

where Σ denotes the summation from 1 to N.

To simplify the analysis we also assume:

$$\mathrm{y} - \mathrm{b} > \frac{\mathrm{S} \sum_{\mathrm{i}=1}^{\mathrm{N}} A_i}{\mathrm{B}^{\mathrm{G}}} \tag{A5}$$

Assumption (A5) will imply that at equilibrium all firms/banks are active (invest) – that is, conditionally upon N firms/banks being active, the return on capital exceeds the Intertemporal Marginal Rate of Substitution (which is 1, by (A1)). Henceforth, $\sum$ denotes the summation from 1 to N.

2.1 Liquidity Demand and Investment Choice

Limited pledgeability of real investment returns implies that liquidity must be planned in advance. At date 0, firm/bank i chooses its liquid assets portfolio: the amount of G government bonds, L_i^G, and the amount of I government bonds, L_i^I. Suppose that the liquidity portfolio includes the I bonds, that is $L_i^I > 0$, then the resources available in the I bond's non-crisis state amount to $L_i^G + \alpha L_i^I$, whereas in the I crisis state these will be reduced by the loss on I bond holdings $\Delta^I L_i^I$. It can be shown that conditional on using the I bonds for liquidity purposes, the equilibrium is either one where the I bonds are held in a quantity sufficient to complete the project also in the I bond crisis state, or one where the portfolio outcome satisfies the resource requirement of project completion in the non-crisis state, i. e. $L^G + \alpha L^I = SI$. In the former case, the inefficiency arising from I bond holdings are the idle money balances in the I bond's non-crisis state as examined in Chiesa (2019). A sufficiently large crisis state loss Δ^I and/or the lack of pledgeability of financial assets rule out this former case, and if the I bonds are used for liquidity purposes then the project will be completed entirely only in the I non-crisis state:

$$L_i^G + \alpha^I L_i^I = SI_i \tag{LC}$$

In the crisis state, the liquidity available amounts to $L_i^G + \underline{\alpha}^I L_i^I \equiv SI_i - \Delta^I L_i^I$, and the completed project, I_i^c will be curtailed by $\frac{\Delta^I}{S} L_i^I$. For any given investment size I_i, the state contingent completed project's size is as in Table 1.

Table 1

State-contingent Completed Project's Size

I bond's state	Completed project's size I_i^c
Non-crisis	$I_i^c = I_i$
Crisis	$I_i^c = I_i - \frac{\Delta^I}{S} L_i^I$

Investors/lenders' payoff is constrained by return pledgeability, b per unit of completed project. For a given investment size I_i, and I bonds holdings L_i^I investors' expected payoff is $(1-\tau)bI_i + \tau b\left(I_i - \frac{\Delta}{S} L_i^I\right)$, or equivalently, $b\left[I_i - \tau\frac{\Delta^I}{S} L_i^I\right]$. Accordingly, for investors to supply funds to bank/firm i the following participation constraint must hold:

$$I_i + q_G L_i^G + q_I L_i^I - A_i \leq b\left(I_i - \tau\frac{\Delta^I}{S} L_i^I\right) \tag{PC}$$

We rule out bond short-selling by imposing

$$L_i^G \geq 0,\ \ L_i^I \geq 0 \tag{NNC}$$

The firm/bank's expected profits Π_i are given by the non-pledgeable income $R \equiv y - b$ per unit of completed project times the size of the completed project:

$$\Pi_i = R\left[I_i - \tau\frac{\Delta^I}{S}L_i^I\right]$$

Firm/bank's optimization problem amounts to choose the real investment size I_i, and bond portfolio $(L_i^c,\ L_i^I)$ so as to maximize expected profits s.t. the investor's participation constraint, the liquidity/reinvestment constraint and non-short selling constraint:

$$max_{I,L^G,L^I}\left\{\Pi_i \equiv R\left[I_i - \tau\frac{\Delta^I}{S}L_i^I\right]\right\}$$

st

$(PC), (LC), (NNC)$

Substituting (LC) into the investor participation constraint (which at an optimum holds at equality) gives the investment size:

$$I_i = \frac{A_i - L_i^I(q_I + b\tau\frac{\Delta^I}{S} - \alpha^I q_G)}{1 - b + q_G S} \tag{1}$$

The firm/bank's expected profits $\Pi_i \equiv R\left[I_i - \tau\frac{\Delta^I}{S}L_i^I\right]$ are then given by:

$$\Pi_i = \rho\left[A_i - \left(q_I - xq_G + \tau\frac{\Delta^I}{S}\right)L_i^I\right] \tag{2}$$

$$\rho \equiv \frac{R}{1 - b + q_G S}$$

ρ is the return per unit of capital devoted to real investment, increasing in the pledgeable income b (feasible leverage), and decreasing in the price of the safe asset q_G. At equilibrium, $\rho \geq 1$ (by (A1))

Let $\widehat{q}_I\ (q_G)$ be given by:

$$\frac{\partial\Pi_i}{\partial L_i^I} \equiv -\rho(\widehat{q}_I - xq_G + \tau\frac{\Delta^I}{S}) = 0$$

that is

$$\widehat{q}_I\ (q_G)\ = xq_G - \tau\frac{\Delta^I}{S} \tag{3}$$

Lemma 1: *The I bonds are used for liquidity purposes, $L_i^I > 0$, if and only if $q_I \leq\ \widehat{q}_I\ (q_G)$, and for $q_I <\ \widehat{q}_I\ (q_G)$, the G bonds are excluded from liquidity holdings.*

This follows because

$$q_I > \widehat{q}_I\ (q_G) \rightarrow \frac{\partial \Pi_i}{\partial L_i^I} < 0 : (L_i^I = 0, L_i^G = SI_i)$$

$$q_I < \widehat{q}_I\ (q_G) \rightarrow \frac{\partial \Pi_i}{\partial L_i^I} > 0 : (L_i^I = \frac{SI_i}{\alpha^I}, L_i^G = 0)$$

$$q_I = \widehat{q}_I\ (q_G) \rightarrow \frac{\partial \Pi_i}{\partial L_i^I} = 0 : (L_i^I > 0, L_i^G = SI_i - \alpha^I L_i^I > 0)$$

For the I bond to be used as liquidity instrument it must be that $q_I \leq \widehat{q}_I\ (q_G)$, and $\widehat{q}_I\ (q_G) < q_G$ (by $x \leq 1$, and Δ^I >0). That is, the price that liquidity seekers are willing to pay for the I bond is lower than that of the safe G bond and the more so the greater the I bond's volatility. The key point is that the greater the I bond's volatility, the greater the expected crisis loss, $\tau\Delta^I$, the greater then the amount of investment that will be curtailed. And therefore, for any given investment size I_i installed at date 0, the greater the investment return loss.

It follows from Lemma 1 that if the volatile I bonds are used as liquidity instruments, i. e., if $q_I \leq \widehat{q}_I\ (q_G)$, then necessarily they sell at a discount with respect to the safe G bonds, and the greater the G bond price q_G, the greater the discount. Bond prices are bounded below by the expected value of their terminal value – that is, $q_I \geq x$, $q_G \geq 1$ (by $(A1)$, $(A2)$). Then, for sufficiently high levels of volatility such that $\widehat{q}_I\ (q_G) < x$, the I bonds will be excluded from liquid asset holdings, $L_i^I = 0$, and held by "buy and hold" investors.

3 Equilibrium: Aggregate Investment, Bond Prices and Spreads

We first observe that the safe G bonds are used for liquidity purposes:

Lemma 2: *At an equilibrium: i) $q_I \geq \widehat{q}_I\ (q_G)$ – the G bonds are demanded for liquidity purpose, $L_i^G > 0$, $\forall_i$; ii) if $q_I > \widehat{q}_I(q_G)$, then $q_I = x$.*

Proof: i) By contradiction: Suppose $q_I < \widehat{q}_I(q_G)$ and therefore $L_i^G = 0, \forall i$, i. e., the G bonds are held entirely by "buy and hold investors". Then $q_G = 1$, and $\widehat{q}_I(q_G = 1) = x - \tau\frac{\Delta^I}{S} < x$ which contradicts $q_I < \widehat{q}_I(q_G)$ since $q_I \geq x$ (by $(A1)$, $(A2)$). Part ii) follows because for $q_I > \widehat{q}_I(q_G)$, $L_i^I = 0$, $\forall i$, i. e., the I bonds are held entirely by "buy and hold investors" which implies that the I bond equilibrium price equals the I bond's expected unit value, i. e. $q_I = x$ (by $(A1)$, $(A2)$).

Moreover, the G bonds carry a strictly positive collateral premium:

Lemma 3 *At equilibrium the collateral premium on the G bonds is strictly positive – the amount of G bonds outstanding is absorbed entirely by the demand for liquidity.*

Proof: At an equilibrium $q_I \geq \widehat{q}_I(q_G)$ (by Lemma 2), then it amounts to proving that for $q_I \geq \widehat{q}_I(q_G)$, $q_G > 1$. This follows because:

DOI https://doi.org/10.3790/vjh.89.1.9

a) if at equilibrium $q_I = \widehat{q}_I(q_G)$, then $q_G = \frac{\tau A^I}{xS} + \frac{q_I}{x} > 1$, by $q_I \geq x$.

b) if at equilibrium $q_I > \widehat{q}_I(q_G)$, then $L_i^I = 0, I_i = \frac{A_i}{1-b+q_G S}$. That is, liquidity is provided exclusively by the G bonds and the aggregate demand for G bonds is $\sum \; L_i^G = S \sum \; I_i \equiv \frac{S\sum \; A_i}{1-b+q_G S}$. Clearly, the aggregate demand for G bonds, that is $\sum \; L_i^G \equiv \frac{S\sum \; A_i}{1-b+q_G S}$, is decreasing in q_G, and if $q_G = 1$, then demand exceeds supply, i.e. $\sum \; L_i^G \equiv \frac{S\sum \; A_i}{1-b+S} > B^G$ (by (A4)). It then follows that the G bonds' market clearing, $\sum \; L_i^G = B^G$, implies that $q_G > 1$.

Let q_G^*, q_I^* denote the G bond and the I bond equilibrium price respectively. It follows from Lemma 2 that

$$q_I^* \; = max(x, \widehat{q}_I(q_G^*)) \tag{4}$$

Figure 2

I Bond Equilibrium Price: $q_I^* = \max[x, \widehat{q}_I(q_G^*)]$

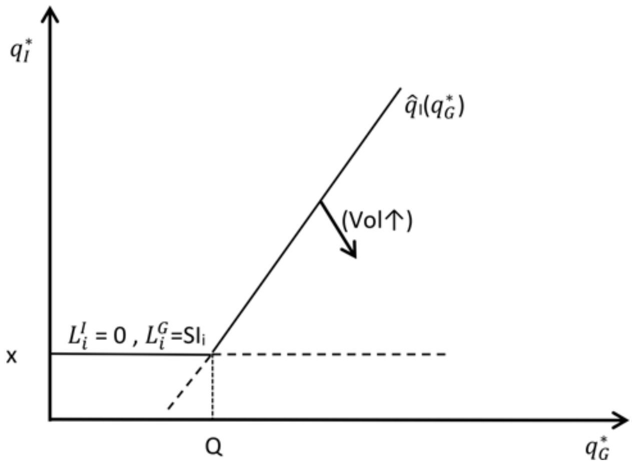


Figure 2 illustrates the results of Lemma 1 and Lemma 2: The I (G) bonds are used as liquidity instruments on and below (above) the $\widehat{q}_I(q_G)$ line (by Lemma 1). The equilibrium lies on the curve defined by the union of the x line and the $\widehat{q}_I(q_G^*)$ line (by Lemma 2). If the equilibrium lies on the x line, that is for $\widehat{q}_I(q_G^*) < x$, then $L_i^I = 0, \forall i$; the I bonds are excluded from liquidity instruments and are held entirely by "buy and hold investors". If it lies on the $\widehat{q}_I(q_G^*)$ line, that is for $\widehat{q}_I(q_G^*) \geq x$, then the I bonds are used as liquidity instrument.

In any case, the amount of G bonds outstanding is absorbed entirely by the demand for liquidity, that is the G-bonds market clears for $\sum L_i^G = B^G$ (by Lemma 2).

Let Q be defined by $\widehat{q}_I(q_G = Q) = x$, that is

$$Q = 1 + \tau \frac{\Delta^I}{xS} \tag{5}$$

Q is the intersection of the $\widehat{q}_I(q_G)$ line with the x line (see Fig. 1). Note that $Q > 1$, increasing in the I bond's volatility (increasing in the probability of crisis τ and the crisis loss Δ^I).

As shown in Fig. 1 if $Q \leq q_G^*$, then $\widehat{q}_I\left(q_G^*\right) \geq x$. The I bonds are used as liquidity instruments and carry a collateral premium, i. e. $q_I^* > x$, iff $Q < q_G^*$. For $Q > q_G^*$, liquidity is provided exclusively by the G bonds: $L_i^I = 0$, $\forall i$, and $q_I^* = x$.

As the I bond volatility increases, the region where I bonds have no liquidity status expands: as $\tau\Delta^I$ increases, the $\widehat{q}_I(q_G)$ line shifts downwards, Q increases, and the region $Q > q_G^*$ expands.

Depending on the amount of safe assets relative to aggregate capital and the volatility of the I bond, the equilibrium will belong to one of three possible equilibrium regimes that differ according to whether and the extent to which the I bonds are used as liquidity instruments.

Proposition 1 *If the volatility of the I bond value is sufficiently small so that*

$$\frac{\tau\Delta^I}{x} < \frac{S\sum A_i}{B^G + xB^I} - (1 + S - b), \tag{C$_1$}$$

then the outstanding volumes of both G and I bonds are absorbed entirely by the demand for liquidity instruments, and bond prices are:

$$q_G^* = \underline{q}_G \equiv \frac{\sum A_i}{(B^G + B^I x)} - \frac{1-b}{S} > 1$$

$$q_I^* = \widehat{q}_I\left(\underline{q}_G\right) \equiv x\underline{q}_G - \tau\frac{\Delta^I}{S} > x$$

Aggregate investment put in place is $\sum I = \frac{B^G + \alpha^I B^I}{S}$, *completed investment is procyclical.*

Proof: Appendix A

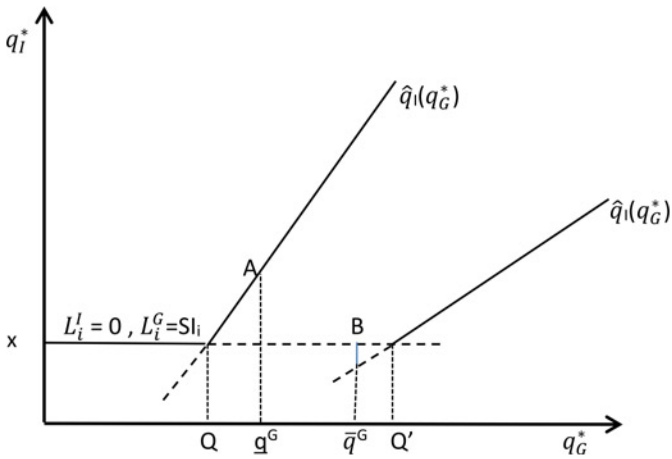
Equilibrium regimes are summarized in Table 2 below and are illustrated in Figure 3.

For sufficiently low I bond's volatility, if condition (C$_1$) holds, the equilibrium is point A in Figure 3: the G bond market clearing price $q_G^* = \underline{q}_G$ exceeds Q, the intersection of the $\widehat{q}_I(q_G)$ line with the x line, and therefore $q_I^* = \widehat{q}_I(q_G^*) > x$. Bond spread, $\underline{q}_G - \widehat{q}_I\left(\underline{q}_G\right)$, is increasing in the volatility of the I bonds, and in the collateral/liquidity premium $q_G^* - 1$ (i. e. the scarcer are safe assets). In the I bond's non-crisis state, projects are completed entirely. In the crisis state, each firm cuts off $\frac{\Delta^I}{S} L_i^I$ investment units, and hence in the aggregate completed projects amount to $\frac{B^G + \alpha^I B^I}{S} - \frac{\Delta^I}{S} B^I$, or

DOI https://doi.org/10.3790/vjh.89.1.9

Figure 3

Bond Markets Equilibrium Regimes



equivalently $\frac{B^G+\alpha^I B^I}{S}$. Therefore the expected value of the resources generated by aggregate investment amounts to $y\left[\frac{B^G+xB^I}{S}\right]$ and the expected value of the aggregate investment's surplus is

$$\left(\frac{B^G+xB^I}{S}\right)[y-(1+S)]-\frac{\tau\Delta^I}{S}B^I.$$

If the riskiness (volatility) of the I bond is sufficiently high that condition (C_I) fails to hold, then the I bonds are either used only partially, $\sum L_i^I < B^I$, or not at all $L_i^I = 0, \forall i$, the equilibrium is point B in Figure 3. The I bond does not enjoy a collateral premium, $q_I^* = x$, and the aggregate investment put in place and completed is lower.

Table 2

Summary of Results

	I bond volatility *Low*	I bond volatility *Medium*	I bond volatility *High*
I bonds used as liquidity instruments	entire amount B^I	partially ($< B^I$)	*none*
I bond price q_I^*	$q_I^* > x$	x	x
G bond price q_G^*	$\underline{q}_G$	$\underline{q}_G < q_G^* < \overline{q}_G$	$\overline{q}_G$
Bond Spread s^*	$\underline{s}$	$\underline{s} < s^* < \bar{s}$	$\bar{s}$
Investment $\sum I$ (completed level)	(expected value) $\frac{B^G+xB^I}{S}$	in between	$\frac{B^G}{S}$
Aggregate Surplus	$\left(\frac{B^G+xB^I}{S}\right)[y-(1+S)]-\frac{\tau\Delta^I}{S}B^I$	in between	$\left(\frac{B^G}{S}\right)[y-(1+S)]$

where: $q_I^* \equiv \max\left[x, \widehat{q}_I\left(q_G^*\right)\right]$ is the equilibrium value of the I bond; $s^* \equiv q_G^* - q_I^*$ is the equilibrium value of the bond spread, ; $\underline{s} = \underline{q}_G - \widehat{q}_I\left(\underline{q}_G\right)$; $\overline{s} = \overline{q}_G - x$, where $\overline{q}_G$ is the G bond market clearing condition when liquidity is provided exclusively by the G bonds; that is $L_i^I = 0,\ \forall i$, and $\overline{q}_G$: $S\sum I_i(\overline{q}_G) \equiv B^G$, with $I_i(\overline{q}_G)$ given by (1).

The first column of Table 2 summarizes the equilibrium attained when condition (C_1) holds: I bond's volatility is sufficiently small so that the I bonds are used as liquidity instruments, $\sum L_i^I = B^I$, and the aggregate investment installed attains the maximum. However, there are inefficiencies in that the investment completion is curtailed in the I debt crisis state. The second and the third columns summarize the equilibria attained if condition (C_1) fails to hold: I bond's volatility is sufficiently great so that the amount of I bonds used as liquidity instruments is lower than the outstanding amount B^I, that is $\sum L_i^I < B^I$, and aggregate investment falls below the level that would attain if the entire amount of I bonds outstanding would be used for liquidity purposes.

Credit expansion, real investment and return on capital are increasing functions of the amount of safe assets, the reverse holds for collateral/safety premia and bond spreads. Liquid assets' availability is determined by the amount of sovereign bonds outstanding and crucially by the volatility of their market values. An increase in volatility of the I bonds (an increase in $\tau\Delta^I$) produces the macro effect of depleting the aggregate amount of liquid assets with a corresponding increase of the collateral/safety premium $q_G^* - 1$, and an increase in the opportunity cost of using the I bonds for liquidity purposes (the threshold $\widehat{q}_I(q_G)$ lowers). As volatility increases, equilibrium shifts from the first column to the second column and for sufficiently high volatility, that is for sufficiently great inefficient investment cut induced by I bond holdings, the I bonds lose the status of liquid assets – the equilibrium is defined by the third column.

4 Multiple Risky Assets and Debt Management Policies

We extend the analysis by incorporating an additional risky sovereign bond. This allows to examine how the correlation structure of the bond market values affects the optimal composition of liquidity instrument holdings and prices. It provides the inputs for the analysis of debt management policies, and whether optimal policies do or do not require supranational institutions.

Let's enrich the framework by adding a risky bond, named *P*; that is there are two risky bonds, I and P. For simplicity, assume that the two bonds have the same expected value, $x^P = x^I \equiv x$. The *i* bond's expected crisis loss is

$$\tau^i\Delta^i \equiv \alpha^i - x,\ \ i = I, P,$$

and let that of the P bond to be the greatest, $\tau^P\Delta^P > \tau^I\Delta^I$ (i. e., $\alpha^P > \alpha^I$). That is, the P bond, if used for liquidity purposes, entails the greatest amount of investment curtail. And therefore, for any given investment size I_i determined at date 0, the greatest investment return loss.

Consider two possible cases:

1. perfect positive correlation of crisis states, that is the two bonds' crisis states coincide

2. perfect negative correlation of crisis states, i. e., when the I bond is in a crisis state, the P bond is in a non-crisis state and viceversa.

In case 1, lack of diversification, the two bonds will be ranked according to their expected crisis loss. Since $\tau^I \Delta^I < \tau^P \Delta^P$, the I bond is ranked first and if an equilibrium is one where the P bond is used as liquidity instrument, then necessarily the entire amount of the I bond outstanding is used for liquidity purposes. The I bond will carry a collateral premium, and priced above the P bond. Indeed by Lemma 2:

$$q_i^* = \max(x, \widehat{q}_I(q_G)), \ i = I, P$$

$$\widehat{q}_I(q_G) = xq_G - \tau^i \frac{\Delta^i}{S}, \ i = I, P \text{ (by (3))}$$

and

$$\widehat{q}_I(q_G) > \widehat{q_P}(q_G) \text{ (by } \tau^I \Delta^I < \tau^P \Delta^P).$$

which implies that the riskiest P bond has the largest yield spread relatively to the safe G bond.

Consider now case 2 – perfect negative correlation of crisis states. Consider a security composed of one unit of I bond and one unit of P bond. Let's name this security b. One unit of b will have expected value $x^b \equiv 2x$, two possible outcomes, the "non-crisis" outcome $\alpha^b = \max\left(\underline{\alpha}^I + \alpha^P, \underline{\alpha}^P + \alpha^I\right)$ and the "crisis" outcome, $\underline{\alpha}^b = \min\left(\underline{\alpha}^I + \alpha^P, \underline{\alpha}^P + \alpha^I\right)$, and the expected crisis loss $\tau^b \Delta^b \equiv \alpha^b - x^b$.

Combining the two risky securities allows for risk diversification, it minimizes the expected crisis loss entailed by the two components:

$$\tau^b \Delta^b < \tau^I \Delta^I + \tau^P \Delta^P$$

because $\tau^b \Delta^b \equiv \alpha^b - x^b$, $\tau^i \Delta^i \equiv \alpha^i - x$, $i = I, P$ and:

$$\alpha^b - x^b < \left(\alpha^I - x\right) + \left(\alpha^P - x\right).$$

Suppose security b is placed in the market. Then, by Lemma 2, the market price of this security will be q_b^* :

$$q_b^* = \max(x^b, \widehat{q}_b(q_G))$$

$$\widehat{q}_b(q_G) \equiv x^b q_G - \tau^b \frac{\Delta^b}{S}$$

and

$$\widehat{q}_b(q_G) > \widehat{q}_I(q_G) + \widehat{q_P}(q_G)$$

that is the security obtained by combining the risky bonds is priced above the sum of the prices that the I and P bonds would have if they could not be combined. The price difference $\widehat{q}_b(q_G) - (\widehat{q}_I(q_G) + \widehat{q_P}(q_G))$ captures the benefits of diversification.

Suppose for simplicity that the volumes of the I and P bonds outstanding are the same, and that the number of securities obtained by combining the two bonds is still B^I and the equilibrium analysis is exactly the same as in Section 3, once $\tau^I \Delta^I$ is replaced by $\tau^b \Delta^b$, and x by x^b . Let Q_b :

$$Q_b = 1 + \tau^b \frac{\Delta^b}{x^b S}$$

then $\widehat{q}_b(q_G = Q_b) \equiv x^b$; that is, Q_b is the intersection of the $\widehat{q}_b(q_G)$ line with the x^b line. If parameter values are such that $Q_b < q_G^* \equiv \underline{q}_G$, that is if condition (C_1) holds, then $\widehat{q}_b(q_G^*) > x^b$, the security b is used as liquidity instrument – i.e. both risky bonds enter the safe-liquid assets' portfolio. Moreover, the volatility of b is lower than that of the riskiest component, $Q_b < Q_P$ (by $\tau^b \Delta^b < \tau^P \Delta^P$). That is, diversification provides liquidity status also to the riskiest P bond. The availability of safe-liquid assets and real investment expand.

An interesting issue is the characterization of the equilibrium when the b security is not traded as such; the securities that are traded are the P and the I bonds. Portfolio composition and real investment will be the same as above:

Lemma 4 *If $Q_b < q_G^* \equiv \underline{q}_G$, then necessarily both risky bonds are combined in the liquid asset portfolios*

Proof. By contradiction, suppose $Q_b < q_G^* = \underline{q}_G$ (i.e., condition (C_1) holds) and either no risky bond is used for liquidity purposes, that is both bonds are held by buy-and-hold investors and therefore $q_I^* = q_P^* = x$, or only the i risky bond is used, i.e. $q_i^* = \widehat{q}_i(q_G^*) > x,\ q_j^* = x,\ j \neq i$. Then combining the two bonds would cost $q_I^* + q_P^* < \widehat{q}_I(q_G^*) + \widehat{q_P}(q_G^*)$. And since $\widehat{q}_I(q_G^*) + \widehat{q_P}(q_G^*) < \widehat{q}_b(q_G^*)$, the demand for the G bond would be nil (by Lemma 1), which cannot possibly be an equilibrium (by Lemma 2).

But what about bond prices? The I and P bond prices satisfy

$$q_I^* + q_P^* = \widehat{q}_b(q_G^*) \tag{6}$$

$$q_I^* \geq \max(\widehat{q}_I(q_G^*), x) \tag{7}$$

$$q_P^* \geq \max(\widehat{q_P}(q_G^*), x) \tag{8}$$

If $Q_b < q_G^* \equiv \underline{q}_G$, in equilibrium both risky bonds are combined in the liquid asset portfolios (Lemma 4). Necessarily, bond prices q_i^*, $i = I, P$, are such that the i bond enters the liquidity portfolio only if combined with the $j \neq i$ bond – conditions $(7) - (8)$ hold. And since the cost of combining the two bonds amounts to the sum of the two bond prices, equality (6) must hold (by Lemma 2).

DOI https://doi.org/10.3790/vjh.89.1.9

Since $\widehat{q}_b(q_G) > \widehat{q}_I(q_G) + \widehat{q_P}(q_G)$, there exist bond prices q_I^*, q_P^* that satisfy $(6) - (8)$, and indeed there is a continuum of values that satisfy $(6) - (8)$, which means multiplicity of equilibria. Such a multiplicity could possibly be eliminated by a supranational institution that coordinates bond issuances and bond pooling.

4.1 Debt Management

The correlation structure of the bond market values determines the optimal debt management policies, and whether these can be implemented at the country level or do instead require a supranational institution.

Two key observations. The first one is that bond yield volatility leads to inefficiency in that investment completion is curtailed in debt crisis states. As observed in Chiesa (2019), a risky bond can be viewed as a bundle of two securities, a safe one that pays the crisis-state outcome for sure and a risky one that pays nothing in the crisis state, and what is left in excess of the crisis-state outcome in non-crisis states. For a liquidity seeking institution the safe component is highly valuable, the risky component entails the risk of curtailing investment. Unbundling the security package, in the finance jargon “tranching”, eliminates the inefficient real-investment cut and produces positive effects. The second observation is that the greater the amount of safe liquid assets, the greater the level of real investment. With negative correlation of crisis states, pooling the risky sovereign debts and then applying tranching so as to insulate the safe component allows for an aggregate amount of safe assets greater than that that would be obtained by tranching the sovereign debts at the nation-state level – the optimal debt management policy cannot be performed at the single country level.

The positive effects produced by tranching can be easily appreciated by examining the base model of a single risky debt. A risky bond, in our case the I bond, can be viewed as a bundle of two securities: a safe one that pays $\underline{\alpha}^I$ for sure, and a risky one that pays 0 in the crisis state, and $\Delta^I = \alpha^I - \underline{\alpha}^I$ in non-crisis states. Unbundling the security package so as to insulate the safe component eliminates the inefficient real-investment cut. It amounts to tranching the debt so as to create a security whose safeness is ensured by sufficient collateral (real assets and tax revenue). Specifically, the former I debt is replaced by $\underline{\alpha}^I B^I$ safe securities that pay one unit with strict priority, and B^I “risky” securities that make the holders residual claimants. These are tailored for “buy and hold” investors, and will be priced q_r :

$$q_r = x - \underline{\alpha}^I \qquad (by\ A1)$$

The safe security is a liquidity instrument, perfect substitute of the safe G bond, and as such priced q_G^*. The revenue per unit of the former security bundle (the former I bond) is now $q^u = q_r + \underline{\alpha}^I q_G^*$, that is

$$q^u = x + \underline{\alpha}^I \left(q_G^* - 1\right). \tag{9}$$

The spread between the liquidity instruments is nil, and aggregate investment/credit is determined by the supply of safe/liquid assets, $B_G + \underline{\alpha}^I B^I$,

$$\sum I_i = \frac{B^G + \underline{\alpha}^I B^I}{S}.$$

That is, whenever in the absence of tranching the former I bonds are not used entirely as liquidity instruments (condition (C_1) fails to hold) aggregate investment increases and the risky issuer's cost of debt lowers. If condition (C_1) holds, unbundling/tranching lowers the aggregate level of investment projects put in place but it eliminates the inefficiency arising from the investment cut and the real resources' loss in debt crises. Moreover, with unbundling the equilibrium price for liquidity, q_G^*, increases – which means that sovereign G's cost of debt lowers. The key is that unbundling the I bond security package so as to insulate the safe component eliminates the crisis state investment cut that an I bond bundle entails; with unbundling/tranching the real return on investment expands. Therefore, for any given price of liquidity, q_G , aggregate investment/credit is greater with unbundling than with bundling. Since the demand for liquid assets is proportional to investment/credit, the price q_G^* that clears the bond market is greater with unbundling than with bundling.

Proposition 2: *Unbundling the safe component of the risky sovereign bond from the state contingent one eliminates inefficient real investment completion cuts. It lowers sovereign cost of debt and increases real investment when condition (C_1) fails to hold.*

Tranching is the key to safety, its positive effects are summarized above. With several risky bonds, the correlation structure of the bond market values determines whether optimal debt management requires a supranational institution. With positive correlation of crisis states, the solution that would be obtained by tranching the risky debts at the nation-state level cannot be improved upon. In our example of two risky bonds, I, P, if bonds' crisis states are positively correlated insulating each bond's safe component, $\underline{\alpha}^i, i = I, P$, allows for the total amount of safe assets, $B^G + \sum \underline{\alpha}^i B^i$, and the associated level of real investment, $\sum I_i = \frac{B^G + \sum \underline{\alpha}^i B^i}{S}$, to attain the maximum.

If sovereign bonds' crisis states are negatively correlated, risk-diversification becomes a further tool to safety.

Consider a supranational institution that pools the two risky sovereign bonds, the I and P bonds, so as to replicate the b security examined above, and then it engages in tranching; that is, it insulates the safe component $\underline{\alpha}^b$ of the pooled risky assets. Since $\underline{\alpha}^b = \min\left(\underline{\alpha}^I + \alpha^P, \underline{\alpha}^P + \alpha^I\right) > \underline{\alpha}^I + \underline{\alpha}^P$, the total amount of safe assets and real investment are maximised and cannot be replicated by tranching the risky debts at the nation-state level.

Pooling (risk diversification) and then tranching (unbundling the safe component from the state contingent one) magnifies the positive effects summarized in Proposition 2. Thanks to unbundling/tranching, the revenue per unit of the b security raises to $q^u = x^b + \underline{\alpha}^b\left(q_G^* - 1\right)$, unambiguously greater than that that would be attained with tranching at the nation-state level – that is, the risky bond issuers' joint costs of debt lowers. How to distribute these benefits between the risky sovereigns is an open question.

DOI https://doi.org/10.3790/vjh.89.1.9

5 Conclusions

The intimate relation between banks' liquidity provision and liquid assets holdings makes the availability of safe/liquid assets at the center of the credit/investment process. Financial frictions, and specifically, limited income pledgeability constrains the amount of inside liquidity and provides a key role for outside liquidity, namely sovereign debt. With reference to the euro area, we have allowed for a range of sovereign bonds that differ with regard to the volatility of their market values, and analysed the resulting equilibrium asset prices, collateral/liquidity-safety premia, liquidity instrument holdings and real investment. We have shown that credit expansion, real investment and return on capital are increasing functions of the amount of safe/liquid assets, the reverse holds for liquidity/collateral premia and bond spreads. Safe/liquid assets' availability is determined by the amount of sovereign bonds outstanding and crucially by the volatility of their market values. An increase in market-value volatility of a bond induces a substitution away from that bond and the macro effect of depleting the amount of assets that are eligible for satisfying liquidity needs, and for sufficiently high volatility, the bond loses the status of liquid asset (it's excluded from asset holdings for liquidity purposes).

We have then examined how the correlation structure of the bond market values affects the optimal composition of liquidity instrument holdings and prices, the optimal debt management policies and whether these policies can be implemented at the nation-state level or do instead require a supranational institution.

We have enlightened two key points. A risky bond can be viewed as a combination of different securities, a safe one that by virtue of sufficient collateral (e.g. tax revenue) pays a safe/guaranteed outcome for sure and a risky one that in good (up-turn) states pays what is left in excess of the safe/guaranteed outcome, if any. For a liquidity seeking institution the safe component is highly valuable, the risky component exposes the holder to the risk of curtailing real investment and exposes banks to the risk of failing in providing insurance against liquidity needs. Unbundling the security package, in the finance jargon "tranching", eliminates the inefficient real-investment cut and produces positive effects. The second observation is that the greater the amount of safe liquid assets, the greater the level of real investment. With negative correlation of crisis states, pooling the risky sovereign debts and then applying tranching so as to insulate the safe component allows for an aggregate amount of safe assets greater than that that would be obtained by tranching the sovereign debts at the nation-state level – that is, the optimal debt management policy cannot be performed at the single country level. The risky bond issuers' joint costs of debt lowers. How to distribute these benefits between the risky sovereigns is however an open question that deserves further study.

Current research points out that the nowadays *seignorage* consists in the ability of reputable institutions to raise financing with securities that enjoy collateral/liquidity-safety premia (0.7% per year on average for US Treasury bonds, the same order of magnitude of traditional *seignorage*, Krishnamurthy & Vissing-Jorgensen, 2012; Greenwood and Vayanos, 2014; Greenwood, Hanson and Stein, 2015; Greenwood et al. 2014). While in emerging countries, central banks may possibly have greater reputation than Governments and may possibly be the only institutions that can enjoy *seignorage*, in non-emerging countries, Governments' ability to issue securities that enjoy collateral premia allows to limit their cost of funding and, under distortionary taxes, the burden of taxation. An indication of the current collateral premium enjoyed by safe sovereign debt securities in the Euro area is provided in Figure 1 – it depicts the difference between the safe

DOI https://doi.org/10.3790/vjh.89.1.9

rate of interest (measured by the Overnight Index Swap) and the German government debt yield. Our model suggests that an expansion of safe debt would be welfare improving.

APPENDIX A

Proof of Proposition 1

Consider an equilibrium where the entire amount of the I bonds outstanding is used for liquidity purposes, i.e. $\sum L_i^I = B^I$, and $q_I^* = \widehat{q}_I(q_G) > x$. The demand for the G bond is $L_i^G = SI_i - \alpha^I L_i^I$ (by (LC)) and the bond markets clear for $\sum L_i^G \equiv S\sum I_i - \alpha^I \sum L_i^I = B^G$, and $\sum L_i^I = B^I$, where

$$\sum I_i = \frac{\sum A_i - B^I\left(q_I + b\tau\frac{\Delta^I}{S} - \alpha q_G\right)}{1 - b + q_G S} \qquad (by\ (1))$$

$$q_I = \widehat{q}_I(q_G) \equiv x q_G - \tau\frac{\Delta^I}{S} \qquad (by\ (3))$$

The bond market clearing condition is then $S\sum I_i = B^G + \alpha^I B^I$, that is

$$\frac{\sum A - B^I(q_I + b\tau\frac{\Delta^I}{S} - \alpha^I q_G)}{1 - b + q_G S} = \frac{B^G + \alpha^I B^I}{S} \tag{A1}$$

and market-clearing bond prices are:

$$q_G^* = \underline{q}_G$$

$$\underline{q}_G \equiv \frac{\sum A_i}{(B^G + B^I x)} - \frac{1-b}{S} \qquad (by\ (A1))$$

$$q_I^* = \widehat{q}_I\left(\underline{q}_G\right) \equiv x\underline{q}_G - \tau\frac{\Delta^I}{S} \qquad (by\ (3))$$

Firm/bank i's expected profits:

$$\Pi_i = \rho\left[A_i - \left(q_I^* - x q_G^* + \tau\frac{\Delta^I}{S}\right)L_i^I\right]$$

$$\equiv \left[\frac{R}{1 - b + \underline{q}_G S}\right]A_i \quad by \quad q_I^* = \widehat{q}_I(q_G^*)$$

DOI https://doi.org/10.3790/vjh.89.1.9

This is an equilibrium if $\widehat{q}_I\left(\underline{q}_G\right) > x$, that is, if $\underline{q}_G > Q \equiv 1 + \tau\frac{\Delta^I}{xS}$ which holds if and only if:

$$\frac{\tau\Delta^I}{x} < \frac{S\sum A_i}{B^G + xB^I} - (1 + S - b). \tag{C$_I$}$$

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DOI https://doi.org/10.3790/vjh.89.1.9