Credit and Capital Markets, Volume 52, Issue 4, pp. 573–617
Scientific Papers

Optimal Sticky Prices Under Rational Inattention*

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Abstract

This paper presents a model in which price setting firms decide what to pay attention to, subject to a constraint on information flow. When idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate conditions. When we calibrate the model to match the large average absolute size of price changes observed in micro data, prices react fast and by large amounts to idiosyncratic shocks, but only slowly and by small amounts to nominal shocks. Nominal shocks have strong and persistent real effects.

An optimizing trader will process those prices of most importance to his decision problem most frequently and carefully, those of less importance less so, and most prices not at all. Of the many sources of risk of importance to him, the business cycle and aggregate behavior generally is, for most agents, of no special importance, and there is no reason for traders to specialize their own information systems for diagnosing general movements correctly.

– Robert E. Lucas (1977, 21)

Keywords: Rational Inattention: Sticky Prices, Real Effects of Nominal Shocks

JEL Classification: D21, D83, E31, E52

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* First printed in The American Economic Review (June, 2009), pp. 769–803 (published by The American Economic Association).

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For helpful comments, we thank Mark Gertler, three anonymous referees, Klaus Adam, Gianluca Benigno, Michael Burda, Larry Christiano, Giancarlo Corsetti, Wouter Den Haan, Martin Eichenbaum, Andrea Gerali, Bob Gordon, Oleksiy Kryvtsov, John Leahy, Emi Nakamura, Aviv Nevo, Alessandro Pavan, Giorgio Primiceri, Federico Ravenna, Chris Sims, Frank Smets, Harald Uhlig, Laura Veldkamp, David Vestin, Michael Woodford, and seminar participants at various institutions. This research was supported by the Deutsche Forschungsgemeinschaft through the Collaborative Research Center 649 *Economic Risk*. The views expressed in this paper are solely those of the authors and do not necessarily reflect the views of the European Central Bank

DOI https://doi.org/10.3790/ccm.52.4.573 | Generated on 2025-04-05 01:52:02
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Edmund S. Phelps (1970) proposed the idea that real effects of monetary policy are due to imperfect information. Lucas (1972) formalized this idea by assuming that agents observe the current state of monetary policy with a delay. The Lucas model has been criticized on the grounds that information concerning monetary policy is published with little delay. However, Christopher A. Sims (2003) points out that if agents cannot attend perfectly to all available information, there is a difference between publicly available information and the information actually reflected in agents' decisions. We think that a convincing model of real effects of monetary policy due to imperfect information must have two features. First, information concerning the current state of monetary policy must be publicly available. Second, it must be optimal for agents to pay little attention to this information. This paper develops a model with both features. The model helps explain micro and macro evidence on prices.

In the model, price setting firms decide what to pay attention to. Firms' inability to attend perfectly to all available information is modeled as a constraint on information flow, as in Sims (2003). Firms can change prices every period at no cost. The profit-maximizing price depends on the price level, real aggregate demand, and an idiosyncratic state variable reflecting firm-specific cost or demand conditions. Firms face a trade-off between paying attention to aggregate conditions and paying attention to idiosyncratic conditions. We close the model by specifying exogenous stochastic processes for nominal aggregate demand and for the idiosyncratic state variables reflecting firm-specific conditions.

The model makes the following predictions. Firms adjust prices in every period, but nonetheless impulse responses of prices to shocks are sticky – dampened and delayed relative to the impulse responses under perfect information. The extent of stickiness in a particular impulse response depends on the amount of attention allocated to that type of shock. When idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate conditions. Prices then respond strongly and quickly to idiosyncratic shocks, but only weakly and slowly to aggregate shocks. In addition, there are feedback effects because firms track endogenous variables (the price level and real aggregate demand). When other firms pay limited attention to aggregate conditions, the price level responds less to a nominal shock than under perfect information. If prices are strategic complements, this implies that each firm has even less incentive to attend to aggregate conditions. The price level responds even less to a nominal shock, and so on.

We calibrate the stochastic process for nominal aggregate demand using US macro data. We calibrate the stochastic process for the idiosyncratic state variables so as to match the average absolute size of price changes in US micro data. *Klenow/Kryvtsov* (2008) study micro data that the Bureau of Labor Statistics col-

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lects to compute the consumer price index.[1] They find that half of all nonhousing consumer prices last fewer than 3.7 months and, conditional on the occurrence of a price change, the average absolute size of the price change is about 10 percent. To match the large average absolute size of price changes in the data, idiosyncratic volatility in the model has to be one order of magnitude larger than aggregate volatility. This implies that firms allocate almost all attention to idiosyncratic conditions. Therefore, prices respond strongly and quickly to idiosyncratic shocks, but only weakly and slowly to nominal shocks. Nominal shocks have strong and persistent real effects. The model can explain why the price level responds slowly to monetary policy shocks, despite the fact that individual prices change fairly frequently and by large amounts.[2] The model can also explain the empirical finding of *Boivin/Giannoni/Mihov* (2009) that sectoral prices respond quickly to sector-specific shocks and slowly to monetary policy shocks.

We use the model to study how the optimal allocation of attention and the dynamics of prices depend on the firms' environment. When the variance of nominal aggregate demand increases, firms shift attention toward aggregate conditions and away from idiosyncratic conditions. Since firms allocate more attention to aggregate conditions, a given nominal shock has smaller real effects. However, the reallocation of attention is not large enough to compensate fully for the fact that the size of nominal shocks has increased. Firms make larger mistakes in tracking aggregate conditions, and therefore output volatility increases. In addition, since firms allocate less attention to idiosyncratic conditions, firms also make larger mistakes in tracking idiosyncratic conditions. The prediction that real volatility always increases when nominal shocks become larger differs markedly from the Lucas model.[3] At the same time, our model is consistent with the empirical finding of *Lucas* (1973) that the Phillips curve becomes steeper as the variance of nominal aggregate demand increases.

The model has some shortcomings. First, it cannot explain why prices remain fixed for some time. In the model, prices change in every period. It may be that reality is a combination of a menu cost model and the model presented here. One could add a menu cost. Adding a menu cost is likely to increase the real effects of nominal shocks even further.[4] Second, in some models of price setting,

[1] See also *Bils/Klenow* (2004), and *Nakamura/Steinsson* (2008a).

[2] A number of different schemes for identifying monetary policy shocks yield the result that the price level responds slowly to monetary policy shocks. See, for example, *Christiano/Eichenbaum/Evans* (1999), *Leeper/Sims/Zha* (1996), and *Uhlig* (2005).

[3] In the Lucas model, as the variance of nominal aggregate demand increases, prices become more precise signals of nominal aggregate demand. Therefore, as the variance of nominal aggregate demand goes to infinity, real volatility goes to zero.

[4] For menu cost models calibrated to micro data on prices, see, for example, *Gertler/Leahy* (2006), *Golosov/Lucas* (2007), *Midrigan* (2007), and *Nakamura/Steinsson* (2008b).

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the optimal decision is so simple that it may be unclear why firms make mistakes at all. We think that, in reality, setting the profit-maximizing price is complicated. In this paper, we start from the premise that setting the profit-maximizing price is complicated, and we study the implications. We focus on the tension between attending to aggregate conditions and attending to idiosyncratic conditions.[5] Third, it is difficult to calibrate the parameter that bounds the information flow. We do not provide independent evidence on the right value for this parameter. We choose a value for the parameter such that firms set prices that are close to the profit-maximizing prices. We think this is realistic.

This paper builds on *Sims* (1998, 2003). Sims argues that agents cannot attend perfectly to all available information. He proposes modeling agents' limited attention as a constraint on information flow. The firms' attention problem in our model is, after a log-quadratic approximation of the profit function, similar to the quadratic tracking problem with an information flow constraint studied in Section 4 of Sims (2003). One difference is that firms in our model face a tradeoff between tracking aggregate conditions and tracking idiosyncratic conditions. Another difference is that firms in our model track endogenous variables. This introduces the feedback effects.

This paper is also related to the recent literature on real effects of monetary policy due to imperfect information. Michael Woodford (2003a) studies a model in which firms observe nominal aggregate demand with exogenous idiosyncratic noise. Woodford assumes that firms pay little attention to aggregate conditions. We identify the circumstances under which firms find it optimal to pay little attention to aggregate conditions, and we study how the optimal allocation of attention and the dynamics of prices depend on the firms' environment. *Mankiw/Reis* (2002) develop a different model in which information disseminates slowly. They assume that in every period a fraction of firms obtains perfect information concerning all cur- rent and past disturbances, while all other firms continue to set prices based on old information. *Reis* (2006) shows that a model with a fixed cost of obtaining perfect information can provide a microfoundation for this kind of slow information diffusion. Note that in *Mankiw/Reis* (2002) and in *Reis* (2006), prices respond with equal speed to all disturbances. In our model, prices respond quickly to some shocks and slowly to other shocks.

The rest of the paper is organized as follows. Section I. introduces tools that we use to quantify information flow. Section II. presents the model. Section III.

[5] *Zbaracki et al.* (2004) provide some evidence in support of the view that setting the profit-maximizing price is complicated. They study price adjustment practices of a large US manufacturing firm. They find that price adjustment costs comprise 1.2 percent of the firm's revenue and 20.3 percent of its net margin. Furthermore, they find that the managerial costs of price adjustment ("thinking costs") are much larger than the physical costs of price adjustment ("menu costs").

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derives the firms' price setting behavior for a given allocation of attention. Section IV. solves a special case of the model analytically. Thereafter, we return to the model in its general form. In Section V. we study the firms' attention problem in detail. In Section VI. we compute the rational expectations equilibrium for a variety of different parameter values. Section VII. contains extensions and discusses shortcomings. Section VIII concludes.

I. Quantifying Information Flow

In this section, we present tools from information theory that we use to quantify information flow.[6] The basic idea of information theory is to quantify information as reduction in uncertainty, where uncertainty is measured by entropy. The entropy of a random variable X that has a normal distribution with variance σ^2 is

$$H(X) = \frac{1}{2}\log_2(2\pi e\sigma^2).$$

In the univariate normal case, entropy is a function of the variance. The entropy of a random vec- tor $\mathbf{X} = (X_1, \ldots, X_T)$ that has a multivariate normal distribution with covariance matrix $\mathbf{\Omega}$ is

$$H(\mathbf{X}) = \frac{1}{2}\log_2\left[\,(2\pi e)^T \det\mathbf{\Omega}\right]. \tag{1}$$

In the multivariate normal case, entropy is a function of the number of random variables and their covariance matrix. Entropy as a measure of uncertainty has appealing properties. For example, the entropy of a random vector of given dimension and with given variances is largest when the random variables are independent. Furthermore, when the random variables are independent, the entropy of the random vector equals the sum of the entropies of the individual random variables.

In information theory, conditional uncertainty is measured by conditional entropy. When $\mathbf{X} = (X_1, \ldots, X_T)$ and $\mathbf{Y} = (Y_1, \ldots, Y_T)$ have a multivariate normal distribution, the conditional entropy of $\mathbf{X}$ given $\mathbf{Y}$ is

$$H(\mathbf{X} \mid \mathbf{Y}) = \frac{1}{2}\log_2\left[\,(2\pi e)^T \det\mathbf{\Omega}_{\mathrm{X}\mid\mathrm{Y}}\right], \tag{2}$$

where $\mathbf{\Omega}_{\mathrm{X}\mid\mathrm{Y}}$ is the conditional covariance matrix of $\mathbf{X}$ given $\mathbf{Y}$.

Equipped with measures of uncertainty and conditional uncertainty, one can quantify the amount of information that one random vector contains about an-

[6] See *Cover/Thomas* (1991) for a detailed presentation of information theory.

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other random vector as the difference between unconditional uncertainty and conditional uncertainty. For example, the amount of information that $\mathbf{Y} = (Y_1, \ldots, Y_T)$ contains about $\mathbf{X} = (X_1, \ldots, X_T)$ is

$$I(\mathbf{X}; \mathbf{Y}) = H(\mathbf{X}) - H(\mathbf{X} \mid \mathbf{Y}). \tag{3}$$

This measure of information is called mutual information and turns out to be symmetric: $I(\mathbf{X}; \mathbf{Y}) = I(\mathbf{Y}; \mathbf{X})$.

Similarly, one can quantify the information flow between stochastic processes as the average per-period amount of information that one process contains about another process. Let $X_1, \ldots, X_T$ and $Y_1, \ldots, Y_T$ denote the first T elements of the processes $\{X_t\}$ and $\{Y_t\}$. The information flow between the processes $\{X_t\}$ and $\{Y_t\}$ can be defined as

$$\mathrm{I}\left(\{X_t\}; \{Y_t\}\right) = \lim_{n \to \infty} \frac{1}{T}\ I(X_1, \ldots, X_T; Y_1, \ldots, Y_T), \tag{4}$$

where $\{X_t\}$ and $\{Y_t\}$ may be vector processes. We will use the definition of information flow (4) to state a constraint on the per-period amount of information that a decision maker can absorb.

A simple example may be helpful. If $\{X_t, Y_t\}$ is a bivariate Gaussian white noise process, the information flow between the process $\{X_t\}$ and the process $\{Y_t\}$ equals

$$I(\{Xt\}\ ; \{Yt\}) = \frac{1}{2} log\, 2\left(\frac{1}{1 - p_{X,Y}^2}\right), \tag{5}$$

where $\rho_{X,Y}$ is the correlation coefficient between X_t and Y_t.[7] This example illustrates that information flow is invariant to scaling of the variables and is bounded below by zero.

II. Model

1. Description of the Economy

There is a continuum of firms indexed by $i \in [0, 1]$. Time is discrete and indexed by t.

[7] See Appendix A in *Maćkowiak/Wiederholt* (2007) for proof.

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Firm i sells good i. Every period $t = 1, 2, \ldots$, the firm sets the price of the good, P_{it}, so as to maximize the expected discounted sum of profits

$$\mathrm{E}_{it}\left[\sum_{\tau=t}^{\infty}\beta^{\tau-t}\pi\left(P_{i\tau}, P_{\tau}, Y_{\tau}, Z_{i\tau}\right)\right], \tag{6}$$

where E_{it} is the expectation operator conditioned on information of firm i in period t, $\beta \in (0,1)$ is a discount factor, and $\pi(P_{it}, P_t, Y_t, Z_{it})$ are real profits in period t. Real profits depend on the price set by the firm, P_{it}, the price level, P_t, real aggregate demand, Y_t, and an idiosyncratic state variable reflecting firm-specific cost or demand conditions, Z_{it}. We assume that the function π is twice continuously differentiable and homogenous of degree zero in its first two arguments, i.e., real profits depend only on the relative price P_{it}/P_t. We also assume that π is a single-peaked function of P_{it} for given P_t, Y_t, and Z_{it}. These assumptions are satisfied, for example, by a standard model of monopolistic competition.

Prices are physically fully flexible, i.e., firms can change prices at no cost in every period. Firms take as given the stochastic processes for the price level, $\{P_t\}$, for real aggregate demand, $\{Y_t\}$, and for the idiosyncratic state variables, $\{Z_{it}\}$. These assumptions imply that the price setting problem is a static problem:

$$\max_{P_{it}} E_{it}\left[\pi\left(P_{it}, P_t, Y_t, Z_{it}\right)\right]. \tag{7}$$

We specify the aggregate environment of firms by postulating an exogenous stochastic process for nominal aggregate demand.[8] Let

$$Q_t = P_t Y_t \tag{8}$$

denote nominal aggregate demand, and let $q_t = \ln Q_t - \ln \bar{Q}$ denote the log-deviation of nominal aggregate demand from its nonstochastic value.[9] We assume that q_t follows a stationary Gaussian process with mean zero and absolutely summable autocovariances. The price level is defined by

$$\ln P_t = \int_0^1 \ln P_{it}\, di. \tag{9}$$

[8] This approach is common in the literature. For example, *Lucas* (1972), *Woodford* (2003a), *Mankiw/Reis* (2002), and *Reis* (2006) also postulate an exogenous stochastic process for nominal aggregate demand.

[9] For simplicity, the nonstochastic value is constant. One could introduce a deterministic trend. This would not affect results.

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One obtains the same equation in a standard model of monopolistic competition after a log-linearization. We specify the idiosyncratic environment of firms by postulating an exogenous stochastic process for the idiosyncratic state variables. Let $z_{it} = \ln Z_{it} - \ln \overline{Z}$ denote the log-deviation of idiosyncratic state variable i from its nonstochastic value. We assume that the z_{it}, $i \in [0, 1]$, follow a common stationary Gaussian process with mean zero and absolutely summable autocovariances. Furthermore, we assume that the processes $\{z_{it}\}$, $i \in [0,1]$, are pairwise independent and independent of $\{q_t\}$. It follows from Theorem 2 in Uhlig (1996) that

$$\int_0^1 z_{it}\, di = 0. \tag{10}$$

Next we formalize the idea that agents cannot attend perfectly to all available information. Following Sims (2003), we model limited attention as a constraint on information flow. Let s_{it} denote the signal that decision maker i receives in period t. The signal formalizes the new information that the decision maker uses in period t. The signal can be vector valued. Let $s_i^t = \{s^1{}_i, s_{i2},\ldots, s_{it}\}$ denote the sequence of all signals that the decision maker has received up to period t. This sequence formalizes all the information that the decision maker uses in period t. We introduce the following constraint on information flow:

$$\mathrm{I}\,(\{P_t, Z_{it}\}; \{s_{it}\}) \leq \kappa. \tag{11}$$

The operator I defined in Section I measures the information flow between economic conditions (summarized by P_t and Z_{it}) and the signal s_{it}. The information flow constraint (11) states that the average per-period amount of information that the sequence of signals contains about the sequence of economic conditions cannot exceed the parameter κ. Thus the decision maker can absorb only a limited amount of information per period.

We model the idea that decision makers can process only a limited amount of information per period due to limited cognitive ability. We formalize this idea as a constraint on the information flow between economic conditions (summarized by P_t and Z_{it}) and the signal s_{it}. There are several alternative formulations of the information flow constraint that yield the same equilibrium. First, instead of including the price level in the information flow constraint, we could have included any other aggregate variable in the information flow constraint. We prove below that this yields the same equilibrium. The reason is that all aggregate variables are driven by the same innovations – the innovations to nominal aggregate demand. Second, instead of restricting the information flow between economic conditions and the signal, we could have directly restricted the information flow between economic conditions and the price setting behavior. We

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show below that these two formulations yield the same equilibrium. The only reason we decided to think of price setting behavior as based on signals is that it facilitates a comparison of our model with the large literature on models with an exogenous information structure.[10] We think that information flow is a good reduced-form description of the mental resources required to take good decisions:

(i) When the information flow is large (κ is high), the price setting behavior is close to the profit-maximizing pricing behavior;

(ii) When the decision maker allocates a large fraction of the information flow (her attention) to one variable, mistakes in the response to that variable become small; and

(iii) The decision maker needs to allocate more information flow to a variable with high variance or low serial correlation (for a given variance) to make small mistakes in the response to that variable.

We let first choose the allocation of attention. Formally, in period zero, the decision maker in firm i solves

(12) $$\max_{\{s_{it}\}\in\Gamma} E\left[\sum_{t=1}^{\infty}\beta^t \pi\left(P_{it}^*, P_t, Y_t, Z_{it}\right)\right]$$

subject to the information flow constraint (11), where

(13) $$P_{it}^* = \arg\max_{P_{it}} E\ \left[\pi\left(P_{it}, P_t, Y_t, Z_{it}\right) \mid s_i^t\right].$$

The decision maker chooses the stochastic process for the signal so as to maximize the expected discounted sum of profits. She has to respect the information flow constraint (11), and takes into account how the signal process affects the price setting behavior (13). For example, the decision maker knows that if she pays no attention to idiosyncratic conditions, she will not respond to changes in idiosyncratic conditions.[11]

The decision maker can choose the stochastic process for the signal from the set Γ, the set of all signal processes that have the following four properties. First, the signal that decision maker i receives in period t contains no information about future innovations to nominal aggregate demand and future innovations

[10] In addition to *Lucas* (1972), *Woodford* (2003a), and *Mankiw/Reis* (2002), see, for example, the literature on forecasting the forecasts of others (e.g., *Townsend* (1983)), the literature on the social value of information (e.g., *Morris/Shin* (2002)), and the literature on global games (e.g., *Morris/Shin* (2003)).

[11] Here, we assume that the decision maker chooses the signal process once and for all. In Section VIIC we let the decision maker reconsider the choice of the signal process.

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to the idiosyncratic state variable, i. e., the signal contains no information about shocks that nature has not drawn yet. Second, the signal follows a stationary Gaussian process:

$$\{s_{it}, p_t, q_t, z_{it}\} \text{ is a stationary Gaussian process,} \tag{14}$$

where p_t denotes the log-deviation of the price level from its value at the solution of the nonstochastic version of the model. We relax the Gaussianity assumption in Section VIIA. There, we show that Gaussian signals are optimal when the objective function is quadratic, and we also study the optimal form of uncertainty when the objective function is not quadratic. Third, the signal can be partitioned into one subvector that contains only information about aggregate conditions and another subvector that contains information only about idiosyncratic conditions:

$$\mathbf{s_{it}} = (s_{1it}, s_{2it}), \tag{15}$$

Where

$$\{s_{1it}, p_t, q_t\} \text{ and } \{s_{2it}, z_{it}\} \text{ are independent.} \tag{16}$$

This assumption formalizes the idea that paying attention to aggregate conditions and paying attention to idiosyncratic conditions are separate activities. For example, attending to the price level, or to the current state of monetary policy, is a separate activity from attending to firm-specific productivity. We relax this assumption in Section VIIB where we also discuss it in detail. Fourth, all noise in signals is idiosyncratic. This assumption accords well with the idea that the friction is the decision makers' limited attention rather than the availability of information.[12]

Finally, we make a simplifying assumption. We assume that each firm receives a long sequence of signals in period one:

$$\mathbf{s_i^1} = \{\mathbf{s_{i-\infty}}, \ldots, \mathbf{s_{i1}}\}. \tag{17}$$

This assumption implies that the price set by each firm follows a stationary process. This simplifies the analysis.[13]

[12] Conditions (14) and (16) can be satisfied only when $\{p_t, q_t\}$ is a stationary Gaussian process and $\{p_t, q_t\}$ and $\{z_{it}\}$ are independent. We will verify that this is true in equilibrium.

[13] One can show that receiving a long sequence of signals in period one does not change the information flow in (11).

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2. Equilibrium

An equilibrium of the model are stochastic processes for the signals, $\{\mathbf{s}_{it}\}$, the prices, $\{P_{it}\}$, the price level, $\{P_t\}$, and real aggregate demand, $\{Y_t\}$, such that:

(i) Given $\{P_t\}$, $\{Y_t\}$, and $\{Z_{it}\}$, each firm $i \in [0,1]$ chooses the stochastic process for the signal optimally in period $t = 0$ and sets the price for its good according to equation (13) in the following periods; and

(ii) In every period $t = 1, 2, \ldots$ and in every state of nature, the price level is given by (9) and real aggregate demand is given by (8).

III. Price Setting

In this section, we derive the price setting behavior for a given allocation of attention. Thereafter, we study the optimal allocation of attention. We work with a log-quadratic approximation of the profit function around the nonstochastic solution of the model. This yields: (i) a log-linear equation for the profit-maximizing price; and (ii) a log-quadratic equation for the loss in profits due to a suboptimal price. We start by deriving the nonstochastic solution of the model. Suppose that $Q_t = \bar{Q}$ for all t and $Z_{it} = \bar{Z}$ for all i, t. The price set by firm i in period t is then given by

$$\pi_1(P_{it}, P_t, Y_t, \bar{Z}) = 0,$$

where π_1 denotes the derivative of the profit function with respect to its first argument. Since all firms set the same price, in equilibrium,

$$\pi_1(P_t, P_t, Y_t, \bar{Z}) = 0.$$

Since π is homogenous of degree zero in its first two arguments, π_1 is homogenous of degree minus one. Multiplying the last equation by $P_t > 0$ yields

$$\pi_1(1, 1, Y_t, \bar{Z}) = 0.$$

This equation characterizes equilibrium real aggregate demand, denoted $\bar{Y}$.[14] The equilibrium price level equals

$$\bar{P} = \frac{\bar{Q}}{\bar{Y}}.$$

[14] We assume that the equation has a unique solution.

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Next we compute a log-quadratic approximation of the profit function around the nonstochastic solution of the model. Let $\hat{\pi}$ denote the profit function expressed in terms of log-deviations: $\hat{\pi}(p_{it}, p_t, y_t, z_{it}) = \pi(\bar{P}e^{p_{it}}, \bar{P}e^{p_t}, \bar{Y}e^{y_t}, \bar{Z}e^{z_{it}})$, where a small letter denotes the log-deviation of the variable from its value at the nonstochastic solution (e.g., $p_{it} = \ln P_{it} - \ln \bar{P}$). Let $\tilde{\pi}$ denote the second-order Taylor approximation of the function $\hat{\pi}$ at the origin:

$$\text{(18)} \quad \tilde{\pi}(p_{it}, p_t, y_t, z_{it}) = \hat{\pi}_1 p_{it} + \frac{\hat{\pi}_{11}}{2} p_{it}^2 + \hat{\pi}_{12} p_{it} p_t + \hat{\pi}_{13} p_{it} y_t + \hat{\pi}_{14} p_{it} z_{it}$$

$$+ \text{ terms independent of } p_{it},$$

where $\hat{\pi}_1$, for example, denotes the derivative of $\hat{\pi}$ with respect to its first argument evaluated at the origin. It is straightforward to show that $\hat{\pi}_1 = 0$, $\hat{\pi}_{11} < 0$, and $\hat{\pi}_{12} = -\hat{\pi}_{11}$.

After the log-quadratic approximation of the profit function, the price set by firm i in period t is given by

$$\text{(19)} \quad p_{it}^* = E[p_{it}^{\lozenge} \mid \mathbf{s}_\mathbf{i}^\mathbf{t}],$$

where $p^{\lozenge}$ denotes the profit-maximizing price of good i in period t:

$$\text{(20)} \quad p_{it}^{\lozenge} = p_t + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} y_t + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} z_{it}.$$

The price set by the firm equals the conditional expectation of the profit-maximizing price, which is log-linear in the price level, real aggregate demand, and the idiosyncratic state variable. The ratio $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$ determines the sensitivity of the profit-maximizing price to real aggregate demand. In the terminology of Lawrence Ball and David Romer (1990), a low value of $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$ corresponds to a high degree of real rigidity. The ratio $(\hat{\pi}_{14}/|\hat{\pi}_{11}|)$ determines the sensitivity of the profit-maximizing price to idiosyncratic conditions.

In the next sections, we will use the following convenient notation. Let $\Delta_t = p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|) y_t$ denote the profit-maximizing response to aggregate conditions. Furthermore, let $\hat{\Delta}_{it} = E[\Delta_t \mid \mathbf{s}_\mathbf{i}^\mathbf{t}]$ and $\hat{z}_{it} = E[z_{it} \mid \mathbf{s}_\mathbf{i}^\mathbf{t}]$ denote the conditional expectations of Δ_t and z_{it}. One can then write the pricing equations (19)–(20) as

$$\text{(21)} \quad p_{it}^* = \hat{\Delta}_{it} + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} \hat{z}_{it},$$

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$$p_{it}^{\diamond} = \Delta_t + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}\hat{z}_{it}. \tag{22}$$

Whenever the price set by the firm (19) differs from the profit-maximizing price (20), there is a loss in profits due to a suboptimal price. The period t loss in profits due to a suboptimal price equals

$$\tilde{\pi}(p_{it}^{\diamond}, p_t, y_t, z_{it}) - \tilde{\pi}(p_{it}^{*}, p_t, y_t, z_{it}) = \frac{|\hat{\pi}_{11}|}{2}(p_{it}^{\diamond} - p_{it}^{*})^2. \tag{23}$$

(See Appendix A.) The allocation of attention will affect the price (19), and thereby the profit loss (23).

If firms face no information flow constraint, all firms set the profit-maximizing price. Computing the integral over all i of the profit-maximizing price (20) and using equations (8)–(10) yields the following equation for the price level:

$$p_{it}^{\diamond} = \left(1 - \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}\right) p_t + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} q_t. \tag{24}$$

The fixed point of this mapping is the equilibrium price level in the absence of an information flow constraint. Assuming $\hat{\pi}_{13} \neq 0$, the unique fixed point is

$$p_{it}^{\diamond} = q_t. \tag{25}$$

Hence, if firms face no information flow constraint, the price level moves one-for-one with nominal aggregate demand.

IV. Analytical Solution when Exogenous Processes Are White Noise

Next we study the optimal allocation of attention and we derive the rational expectations equilibrium of the model. When q_t and z_{it} follow white noise processes, the model can be solved analytically. In this section, we illustrate the main mechanisms of the model with the help of this simple example. Afterward, we solve the model under more realistic assumptions concerning the exogenous processes.

In this section, we assume that q_t follows a white noise process with variance $\sigma_q^2 > 0$, and all the z_{it}, $i \in [0, 1]$, follow a white noise process with variance $\sigma_z^2 > 0$. We guess that the equilibrium price level is a log-linear function of nominal aggregate demand:

$$p_t = \alpha\, q_t. \tag{26}$$

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The guess will be verified. The guess implies that the profit-maximizing response to aggregate conditions is given by

(27) $$\Delta_t = [\alpha + \frac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}(1-\alpha)]\ q_t \, .$$

For ease of exposition, we restrict the firms' choice of signals to signals of the form "true state plus white noise error":

(28) $$s_{1it} = \Delta_{t+} \varepsilon_{it} \ ,$$

(29) $$s_{2it} = z_{it} + \psi_{it} \ ,$$

where $\{\varepsilon_{it}\}$ and $\{\psi_{it}\}$ are idiosyncratic Gaussian white noise processes that are mutually independent and independent of $\{q_t\}$ and $\{z_{it}\}$. Here we use a result that we prove in Section V: when the variables being tracked follow white noise processes, signals of the form "true state plus white noise error" are optimal. See Propositions 3 and 4.[15]

Since the price level and the idiosyncratic state variables follow white noise processes, and signals have the form (28)–(29), the information flow constraint (11) becomes

(30) $$\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2}+1\right)}_{\kappa_1}+\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_z^2}{\sigma_\psi^2}+1\right)}_{\kappa_2}\leq\kappa.$$

(See Appendix B.) Here κ_1 denotes the information flow concerning aggregate conditions, and κ_2 denotes the information flow concerning idiosyncratic conditions. When the information flow constraint is binding, firms face a trade-off: attending more carefully to aggregate conditions (increasing κ_1) requires attending less carefully to idiosyncratic conditions (reducing κ_2).

A given allocation of attention (a pair κ_1 and κ_2 with $\kappa_1 + \kappa_2 \leq \kappa$) is associated with the following signal-to-noise ratios:

(31) $$\frac{\sigma_\Delta^2}{\sigma_\varepsilon^2} = 2^{2\kappa_1} - 1,$$

[15] It does not matter that the signal concerning aggregate conditions is a signal concerning Δ_t. Since Δ_t, p_t, and y_t are all linear functions of q_t, one can make signal (28) a signal concerning nominal aggregate demand, the price level, or real aggregate demand simply by multiplying the signal with a constant. This yields a new signal that is associated with the same information flow, the same conditional expectation of Δ_t, and the same price setting behavior.

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$$\frac{\sigma_z^2}{\sigma_\psi^2} = 2^{2\kappa_2} - 1. \tag{32}$$

Signals (28)–(29) with signal-to-noise ratios (31)–(32) imply the following pricing behavior:

$$\begin{aligned} p_{it}^* &= \frac{\sigma_\Delta^2}{\sigma_\Delta^2 + \sigma_\varepsilon^2} s_{1it} + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|} \frac{\sigma_z^2}{\sigma_z^2 + \sigma_\psi^2} s_{2it} \\ &= \left(1 - 2^{-2\kappa_1}\right)\left(\Delta_t + \varepsilon_{it}\right) + \frac{\hat{\pi}_{14}}{|\hat{\pi}_{11}|}\left(1 - 2^{-2\kappa_2}\right)\left(+z_{it} + \psi_{it}\right) \end{aligned} \tag{33}$$

This pricing behavior is associated with the following expected discounted sum of losses in profits due to suboptimal prices:

$$\begin{aligned} & \mathrm{E}\left[\sum_{t=1}^{\infty} \beta^t \left\{\tilde{\pi}\left(p_{it}^{\diamondsuit}, p_t, y_t, z_{it}\right) - \tilde{\pi}\left(p_{it}^*, p_t, y_t, z_{it}\right)\right\}\right] \\ &= \sum_{t=1}^{\infty} \beta^t \frac{|\hat{\pi}_{11}|}{2} E\left[\left(p_{it}^{\diamondsuit} - p_{it}^*\right)^2\right] \\ &= \frac{\beta}{1-\beta} \frac{|\hat{\pi}_{11}|}{2}\left[2^{-2\kappa_1}\ \sigma_\Delta^2 + \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 2^{-2\kappa_2}\ \sigma_z^2\right]. \end{aligned} \tag{34}$$

The first equality follows from (23). The second equality follows from (22) and (31)–(33).

When a firm chooses the allocation of attention (a pair κ_1 and κ_2 with $\kappa_1 + \kappa_2 \leq \kappa$), the firm trades off losses in profits due to imperfect tracking of aggregate conditions and losses in profits due to imperfect tracking of idiosyncratic conditions. The optimal allocation of attention is the solution to

$$\min_{\kappa_1 \in [0,\kappa]} \frac{\beta}{1-\beta} \frac{|\tilde{\pi}_{11}|}{2}\left[2^{-2\kappa_1}\ \sigma_\Delta^2 + \left(\frac{\tilde{\pi}_{14}}{\tilde{\pi}_{11}}\right)^2 2^{-2(\kappa-\kappa_1)}\ \sigma_z^2\right]. \tag{35}$$

Assuming $\hat{\pi}_{14} \neq 0$, the unique solution to the firm's attention problem is $if\ x \in \left[2^{-2k}, 2^{2k}\right] if\ x \geq 2^{2k}$

$$\kappa_1^* = \begin{cases} \kappa & if\ x \geq 2^{2k} \\ \frac{1}{2}\kappa + \frac{1}{4}\log_2(x) & if\ x \in \left[2^{-2k},\ 2^{2k}\right] \\ 0 & if\ x \leq 2^{-2k} \end{cases} \tag{36}$$

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where $x = \sigma_\Delta^2 / \left[\left(\tilde{\pi}_{14} / \tilde{\pi}_{11} \right)^2 \sigma_z^2 \right]$. The attention allocated to aggregate conditions, κ_1^*, is increasing in x – the variance of the profit-maximizing price due to aggregate shocks divided by the variance of the profit-maximizing price due to idiosyncratic shocks. (See equation (22).) The implications are straightforward. When idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate conditions. The price (33) then responds strongly to idiosyncratic shocks, but only weakly to aggregate shocks. This can explain why individual prices move around a lot and, at the same time, why prices respond little to nominal shocks.

Computing the integral over all *i* of the price (33) yields the price level under rational inattention

$$p_t^* = \left(1 - 2^{-2\kappa_1^*}\right) \Delta_t \,. \tag{37}$$

The equilibrium price level under rational inattention is the fixed point of the mapping between the guess (26) and the actual law of motion (37). Assuming $\hat{\pi}_{13} > 0$, the unique fixed point is

$$p_t^* = \begin{cases} \dfrac{\left(2^{2\kappa} - 1\right) \dfrac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}}{1 + \left(2^{2\kappa} - 1\right) \dfrac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|}} q_t & \text{if } \lambda \geq 2^{-k} + \left(2^{\kappa} - 2^{-\kappa}\right) \dfrac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} \\ (1 - 2)^{-\kappa} \lambda - 1) q_t & \text{if } \lambda \in \left[2^{-\kappa}, 2^{-\kappa} + \left(2^{\kappa} - 2^{-\kappa}\right) \dfrac{\hat{\pi}_{13}}{|\hat{\pi}_{11}|} \right] \\ 0 & \text{if } \lambda \leq 2^{-\kappa} \end{cases} \tag{38}$$

Where $\lambda = (\tilde{\pi}_{13} \ \sigma_q \ / \ |\tilde{\pi}_{14}| \sigma_z)$.[16] The response of the price level to a nominal shock is increasing in λ. When idiosyncratic conditions are more important or more variable than aggregate conditions, firms focus on idiosyncratic conditions and pay little attention to aggregate conditions. Individual prices then respond weakly to changes in aggregate conditions, and therefore the price level responds weakly to nominal shocks. In addition, there are feedback effects because the profit-maximizing price depends on the prices set by other firms. When other firms pay limited attention to aggregate conditions, the price level responds less to a nominal shock than under perfect information, while real aggregate demand responds more to a nominal shock than under perfect information. If prices are strategic complements, i.e., $(\hat{\pi}_{13} / |\hat{\pi}_{11}|) < 1$, the first effect

[16] The derivation of equation (38) is in the Technical Appendix of *Maćkowiak/Wiederholt* (2007).

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dominates in the sense that $\Delta_{t=}\ p_t + \left(\hat{\pi}_{13} / \left|\hat{\pi}_{11}\right|\right)\left(q_t - p_t\right)$ responds less to a nominal shock than under perfect information. Thus, σ_Δ^2 falls. Hence, if prices are strategic complements, the fact that other firms pay limited attention to aggregate conditions implies that each firm finds it optimal to pay even less attention to aggregate conditions.[17] The price level responds even less to a nominal shock, and so on. The feedback effects are the stronger the smaller is $(\hat{\pi}_{13} / |\hat{\pi}_{11}|)$, i.e., the higher is the degree of real rigidity.

When λ is very small, firms allocate no attention to aggregate conditions and the price level equals its value at the nonstochastic solution of the model. By contrast, when λ is very large, firms allocate all attention to aggregate conditions. Note that there is always a unique linear rational expectations equilibrium.[18]

V. The Firms' Attention Problem

We now return to the model in its general form. We show how to solve the model when q_t and z_{it} follow arbitrary stationary Gaussian processes. In this section, we derive two results: (i) the firms' attention problem can be stated as a problem of choosing directly conditional expectations, rather than signals; and (ii) the firms' attention problem can be solved analytically when the variables being tracked follow first-order autoregressive processes. We will employ the first result in the next section to compute the rational expectations equilibrium of the model for a variety of parameter values. We will employ the second result in this section to solve the model analytically in another special case. We use this special case to illustrate how persistence affects the optimal allocation of attention.

In this section, we guess that the equilibrium price level follows a stationary Gaussian process that is driven only by the innovations to nominal aggregate demand:

[17] In other words, strategic complementarity in price setting leads to strategic complementarity in price setters' allocation of attention. *Hellwig/Veldkamp* (2009) obtain a similar result. They find that strategic complementarity in actions leads to strategic complementarity in information acquisition.

[18] There are similarities between this model and the setup studied in the literature on the social value of information. The price set by a firm is a linear function of the conditional expectation of nominal aggregate demand (an exogenous aggregate variable) and the conditional expectation of the price level (the average action of other firms). We solve for a linear equilibrium by making a guess concerning the price level and by verifying the guess. This resembles the solution procedure in Section IC in *Morris/Shin* (2002), and in *Angeletos/Pavan* (2007). Note that the price set by a firm can be expressed as a weighted average of first-order beliefs about q_t and p_t or as a weighted average of higher-order beliefs about q_t. Computing higher-order beliefs can be useful to show that the linear equilibrium is the unique equilibrium (see Section ID in *Morris/Shin* (2002)) and to interpret equilibrium (see Section 3 in *Woodford* (2003a)).

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$$p_t = \sum_{l=0}^{\infty} \alpha_l \nu_{t-l}, \tag{39}$$

where ν_t denotes the period t innovation to nominal aggregate demand. The sequence $\{\alpha_l\}_{l=0}^{\infty}$ is absolutely summable and ν_t follows a Gaussian white noise process. The guess (39) will be verified in the next section. The firms' attention problem is given by (11)–(13). Lemma 1 will allow us to simplify the objective.

LEMMA 1: *If the profit function is given by (18) and (39) holds, then*

$$\mathrm{E}\left[\sum_{t=1}^{\infty} \beta^t \pi\left(P_{it}^*, P_t, Y_t, Z_{it}\right)\right] = E\left[\sum_{t=1}^{\infty}\left(P_{it}^{\diamond}, P_t, Y_t, Z_{it}\right)\right] - \frac{\beta}{1-\beta}\frac{\left|\hat{\pi}_{11}\right|}{2} E\left[\left(p_{it}^{\diamond} - p_{it}^*\right)^2\right], \tag{40}$$

where $p_{it}^{\diamond}$ is the profit-maximizing price.

PROOF:

See Appendix C.

Expected profits equal expected profits at the profit-maximizing behavior minus expected losses in profits due to suboptimal prices. When the profit function is given by (18), the loss in profits due to a suboptimal price is given by (23). Using the stationarity of the prices (21)–(22) yields Lemma 1. Furthermore, using the independence assumption (16) yields the result that the mean squared error in pricing equals the mean squared error in the response to aggregate conditions plus the mean squared error in the response to idiosyncratic conditions:

$$E\left[\left(p_{it}^{\diamond} - p_{it}^*\right)^2\right] = E\left[\left(\Delta_t - \widehat{\Delta_{it}}\right)^2\right] + \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it} - \hat{z}_{it}\right)^2\right]. \tag{41}$$

Lemma 2 will allow us to state the information flow constraint in terms of conditional expectations rather than signals.

LEMMA 2: *If (39) holds, then*

$$I\left(\{P_t, Z_{it}\}; \{\mathbf{s_{it}}\}\right) = I\left(\{p_t\}; \{s_{1it}\}\right) + I\left(\{z_{it}\}; \{s_{2it}\}\right) \tag{42}$$

$$\geq I\left(\{p_t\}; \left\{\widehat{\Delta_{it}}\right\}\right) + I\left(\{z_{it}\}; \left\{\widehat{z_{it}}\right\}\right) \tag{43}$$

$$= I\left(\{\Delta_t\}; \left\{\widehat{\Delta_{it}}\right\}\right) + I\left(\{z_{it}\}; \left\{\widehat{z_{it}}\right\}\right). \tag{44}$$

Furthermore, if s_{1lit} and s_{2it} sare scalars, inequality (43) holds with equality.

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PROOF:

See Appendix D in *Maćkowiak/Wiederholt* (2007).

The first equality in Lemma 2 states that the total information flow equals the information flow concerning aggregate conditions plus the information flow concerning idiosyncratic conditions. This follows from the independence assumption (16). The weak inequality in Lemma 2 states that the signals contain weakly more information than the conditional expectations computed from the signals. The weak inequality holds with equality when the signals s_{1it} and s_{2it} are scalars. Finally, the last equality in Lemma 2 states that the conditional expectation $\widehat{\Delta_{it}}$ contains the same amount of information about p_t and Δ_t. The reason is that all aggregate variables (and linear combinations of them) are driven by the same innovations – the innovations to nominal aggregate demand.

Equipped with Lemma 1, Lemma 2, and equation (41), we can solve the firms' attention problem by the two-step procedure given in Proposition 1.

PROPOSITION 1: *If the profit function is given by* (18) *and* (39) *holds, a signal process obtained by the following two-step procedure solves the firms' attention problem* (11)–(13).

1. *Solve*

$$\min_{\{\hat{\Delta}_{it},\hat{z}_{it}\}} \left\{ E\left[\left(\Delta_t - \hat{\Delta}_t\right)^2\right] + \left(\frac{\hat{\pi}_{14}}{\hat{\pi}_{11}}\right)^2 E\left[\left(z_{it} - \hat{z}_{it}\right)^2\right] \right\} \tag{45}$$

subject to

$$I\left(\{\Delta_t\};\{\hat{\Delta}_{it}\}\right) + I\left(\{z_{it}\};\{\hat{z}_{it}\}\right) \leq \quad , \tag{46}$$

$$\left\{\Delta_t, \hat{\Delta}_{it}, z_{it}, \hat{z}_{it}\right\} \textit{ is a stationary Gaussian process}, \tag{47}$$

$$\left\{\Delta_t, \hat{\Delta}_{it}\right\} \text{ and } \left\{z_{it}, \hat{z}_{it}\right\} \textit{ are independent.} \tag{48}$$

Denote the solution by $\left\{\hat{\Delta}^*_{it}, \hat{z}^*_{it}\right\}$.

2. *Show that a bivariate signal process* $\{s_{1it}, s_{2it}\} \in \Gamma$ *exists with the property*

$$\hat{\Delta}^*_{it} ,= E\left[\Delta_t \middle| s^t_{1i}\right], \tag{49}$$

$$\hat{z}^*_{it} = E\left[z_{it} \middle| s^t_{2i}\right]. \tag{50}$$

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PROOF:

See Appendix D.

In step one, we solve directly for the expectations $\hat{\Delta}_{it}$ and $\hat{z}_{it}$, subject to a constraint on information flow. Recall that the price set by firm i in period t is given by equation (21). Thus, step one amounts to solving directly for the optimal pricing behavior, subject to a constraint on information flow between the profit-maximizing pricing behavior (given by Δ_t and z_{it}) and the actual pricing behavior (given by $\hat{\Delta}_{it}$ and $\hat{z}_{it}$. Objective (45) is a monotonic transformation of objective (12). (See Lemma 1 and equation (41).) Information flow constraint (46) is actually weaker than information flow constraint (11). (See Lemma 2.) In step two, we have to show that there exist signals that yield the expectations obtained in step one as conditional expectations of Δ_t and z_{it}, given s_{1it} and s_{2it}, respectively. The signals s_{1it} and s_{2it} are required to be scalars because the weak inequality (43) then holds with equality and, therefore, the fact that the expectations satisfy (46) implies that the signals satisfy (11).

(Again, see Lemma 2.) The advantage of solving the firms' attention problem by the two-step procedure given in Proposition 1 is that the problem in step one resembles the problem studied in Section 4 of *Sims* (2003). Furthermore, the problem in step two turns out to have a trivial solution. *Sims* (2003) studies a problem of the form

$$\min_{\mu,b,c} E\left[\left(X_t - Y_t\right)^2\right] \tag{51}$$

subject to

$$I\left(\{X_t\};\{Y_t\}\right) \leq \kappa_j\,, \tag{52}$$

$$X_t = \sum_{l=0}^{\infty} \alpha_l u_{t-l}\,, \tag{53}$$

$$Y_t = \mu + \sum_{l=0}^{\infty} b_l u_{t-l} + \sum_{l=0}^{\infty} c_l \in_{t-l}, \tag{54}$$

where the sequences $\{a_l\}_{l=0}^{\infty}$, $\{b_l\}_{l=0}^{\infty}$ and $\{c_l\}_{l=0}^{\infty}$ are absolutely summable and u_t and $\in_t$ follow independent Gaussian white noise processes with unit variance. Here, the decision maker chooses a process for Y_t to track X_t, subject to a constraint on the information flow between $\{X_t\}$ and $\{Y_t\}$ and the restriction that (X_t, Y_t) has to follow a stationary Gaussian process. Setting $X_t = \Delta_t$, $Y_t = \hat{\Delta}_{it}$, and $\kappa_j = \kappa_1$ yields one of the subproblems that has to be solved in step one of Proposition 1. Setting $X_t = z_{it}$, $Y_t = \hat{z}_{it}$, and $\kappa_j = \kappa_2$ yields the other subproblem that has

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to be solved in step one of Proposition 1. There are two differences between the firms' attention problem and the problem studied in Section 4 of Sims (2003). First, the decision maker who has to set a price faces a multidimensional tracking problem. She has to choose the attention allocated to aggregate conditions, κ_1, and the attention allocated to idiosyncratic conditions, κ_2. Furthermore, the decision maker tracks an endogenous variable – the profit-maximizing response to aggregate conditions, $\Delta_t = p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)y_t$. This introduces the feedback effects.

In the next section, we implement the solution procedure given in Proposition 1 numerically.

In the rest of this section, we present analytical results.

PROPOSITION 2: *A solution to problem* (51)–(54) *satisfies:*

$$E[X_t - Y_t] = 0, \tag{55}$$

and, for all $k = 0, 1, 2, \ldots$,

$$E[(X_t - Y_t)Y_{t-k}] = 0. \tag{56}$$

PROOF:

See Appendix E.

It follows from Proposition 2 that step two of Proposition 1 has a trivial solution, $s_{1it} = \hat{\Delta}^*_{it}$ and $s_{2it} = \hat{z}^*_{it}$. We will show below that step two also has less trivial solutions. That step two has many solutions should not be surprising because there are many signals that yield the same conditional expectations.

Turning to step one of Proposition 1, we now show that this problem can be solved analytically when Δ_t and zit follow first-order autoregressive processes. The reason is that problem (51)–(54) can be solved analytically in the AR(1) case.

PROPOSITION 3: *If*

$$X_t = \rho X_{t-1} + au_t \tag{57}$$

with $\rho \in [0, 1)$, *the following process is a solution to problem* (51)–(54):[19]

$$Y_t^* = \sum_{l=0}^{\infty}\left[\rho^l - \frac{1}{2^{2\kappa_j}}\left(\frac{\rho}{2^{2\kappa_j}}\right)^l\right]au_{t-l} + \sum_{l=0}^{\infty}\sqrt{\frac{1}{2^{2\kappa_j}}\frac{2^{2\kappa_j}-1}{2^{2\kappa_j}-\rho^2}}\left(\frac{\rho}{2^{2\kappa_j}}\right)^l a \in_{t-1}. \tag{58}$$

[19] If $\rho = 0$, we use the convention $0^0 = 1$.

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The value of the objective at the solution equals

$$E\left[\left(X_t - Y_t^*\right)^2\right] = \frac{a^2}{2^{2\kappa_j} - \rho^2}. \tag{59}$$

PROOF

See Appendix G in *Maćkowiak/Wiederholt* (2007).

Proposition 3 reveals several properties of the solution in the AR(1) case. First, the response to an innovation in X_t is either hump-shaped or monotonically decreasing, because the impulse response function is a difference between two exponentially decaying series. See the term in square brackets in (58).[20] Second, one can write the value of the objective at the solution as

$$E\left[\left(X_t - Y_t^*\right)^2\right] = \sigma_X^2 \frac{1-\rho^2}{2^{2\kappa_j} - \rho^2}, \tag{60}$$

where σ_X^2 denotes the variance of X_t. It follows that the mean squared error is increasing in the variance of the variable being tracked, decreasing in the persistence of the variable being tracked (holding constant the variance of the variable being tracked), and decreasing in information flow. Third, differentiating (60) with respect to κ_j yields that the marginal value of information flow is increasing in the variance of the variable being tracked, increasing or decreasing in the persistence of the variable being tracked (holding constant the variance of the variable being tracked), and decreasing in information flow.

Now it is straightforward to derive the optimal allocation of attention when Δ_t and z_{it} follow first-order autoregressive processes. In the case of an interior solution, the optimal allocation of attention has the property that the marginal value of information flow concerning aggregate conditions equal the marginal value of information flow concerning idiosyncratic conditions. Equating the two and using $\kappa_1 + \kappa_1 = \kappa$ yields

$$\kappa_1^* = \frac{1}{2}\log_2\left(\frac{\frac{\sigma_\Delta \left|\hat{\pi}_{11}\right|}{\sigma_z \left|\hat{\pi}_{14}\right|}\sqrt{1-\rho_\Delta^2}\; 2^{\kappa} + \sqrt{1-\rho_z^2}\; \rho_\Delta^2}{\frac{\sigma_\Delta \left|\hat{\pi}_{11}\right|}{\sigma_z \left|\hat{\pi}_{14}\right|}\sqrt{1-\rho_z^2}\; \rho_z^2\, 2^{-\kappa} + \sqrt{1-\rho_z^2}}\right). \tag{61}$$

Equations (58) and (61) give the solution to step one of Proposition 1 in the AR(1) case. We have already shown that step two of Proposition 1 has a trivial

[20] The response is hump-shaped if κ_j is less than $-[1/(2\ln 2)]\ \ln[(1-\rho)/\rho]$, and monotonically decreasing otherwise.

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solution. It turns out that in the AR(1) case, signals of the form "true state plus white noise error" also have the property (49)–(50).

PROPOSITION 4: *If* X_t *is given by* (57) *and* Y_t^* *is given by* (58), *then the signal*

$$S_t = X_t + \sqrt{\frac{2^{2\kappa_j}}{(2^{2\kappa_j} - 1)(2^{2\kappa_j} - \rho^2)}} a \in_t \tag{62}$$

has the property

$$Y_t^* = E[X_t \mid S^t]. \tag{63}$$

PROOF

See Appendix H in *Maćkowiak/Wiederholt* (2007).

We employed Propositions 3 and 4 in Section IV when we restricted the firms' choice of signals to signals of the form "true state plus white noise error." We will now employ Propositions 3 and 4 to solve the model analytically in another special case. If q_t and z_{it} follow first-order autoregressive processes and $(\hat{\pi}_{13} / |\hat{\pi}_{11}|) = 1$, then Δ_t and z_{it} follow first-order autoregressive processes. The equilibrium of the model is then given by equations (21), (58), (61), and (62). Note that the equilibrium allocation of attention depends in a complicated way on persistence. In particular, reducing the persistence of a variable (holding constant its variance) may increase or decrease the attention allocated to that variable. The reason is the following. Decreasing the persistence of a variable (holding constant its variance) always reduces the quality of tracking that variable for a given allocation of attention, but the marginal value of reallocating attention to that variable may go up or down.

There are several alternative formulations of the firms' attention problem that yield the same solution. First, it does not matter which aggregate variable appears in the information flow constraint (11), because equation (44) holds for any aggregate variable (not only for p_t).[21] The reason is that in this model all aggregate variables are driven by the same innovations – the innovations to nominal aggregate demand. Second, since in step one of Proposition 1 we restrict the information content of the price setting behavior directly, we could, from the start, have restricted the information content of the price setting behavior, instead of the information content of the signal.

[21] See proof of Lemma 2.

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VI. Numerical Solutions

In this section we compute the rational expectations equilibrium of the model for a variety of different parameter values. We solve the model numerically, because for most parameter values, the profit-maximizing response to aggregate conditions, $\Delta_t = p_t + (\hat{\pi}_{13} / |\hat{\pi}_{11}|)\ (q_t - p_t)$, in equilibrium does not follow a first-order autoregressive process. Therefore, we cannot apply Proposition 3. We compute the solution as follows. First, we make a guess concerning the process for the price level. Second, we solve the firms' attention problem by the two-step procedure given in Proposition 1. In step one we solve directly for the optimal pricing behavior, subject to a constraint on information flow between the profit-maximizing pricing behavior and the actual pricing behavior.[22] In step two we show that there exist scalar signals that yield this pricing behavior as the conditional expectation of the profit-maximizing pricing behavior. Third, we compute individual prices from equation (21), and the price level from equation (9). We compare the process for the price level that we obtain to our guess, and we update the guess until a fixed point is reached.

1. The Benchmark Economy

In order to solve the model numerically, we have to specify the exogenous processes for nominal aggregate demand and for the idiosyncratic state variables. We also have to specify the parameters $(\hat{\pi}_{13} / |\hat{\pi}_{11}|)$, $(\hat{\pi}_{14} / |\hat{\pi}_{11}|)$ and κ.

We calibrate the stochastic process for nominal aggregate demand using quarterly US nominal GNP data from 1959:I to 2004:I.[23] We take the natural log of the data and we detrend the data by fitting a second-order polynomial in time. We estimate the equation $q_t = \rho\, q_{t-1} + \nu_t$, where q_t is the deviation of the log of nominal GNP from its fitted trend. The estimate of ρ that we obtain is, after rounding off, 0.95 and the standard deviation of the error term is 0.01. The moving average representation of the estimated process is $q_t = \sum_{l=0}^{\infty} \rho^l \nu_{t-l}$. Since with geometric decay, shocks die out after a very large number of periods, and since computing time increases fast with the number of lags, we approximate the estimated process by a process with linear decay that dies out after 20 periods.[24]

---

[22] This is a constrained minimization problem. The first-order conditions are given in Appendix I in *Maćkowiak/Wiederholt* (2007). The first-order conditions are a nonlinear system of equations that we solve numerically.

[23] The source is the National Income and Product Accounts of the United States.

[24] For the benchmark parameter values, we also solved the model with geometric decay and 80 lags. While computing time was many times larger, the results were little affected.

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We calibrate the stochastic process for the idiosyncratic state variables so as to match the average absolute size of price changes in US micro data. *Klenow/Kryvtsov* (2008) find that the median price changes every 3.7 months and that, conditional on the occurrence of a price change, the average absolute size of the price change is 11.5 percent. Klenow and Kryvtsov also find that, when they exclude sale-related price changes, the average absolute size of price changes falls to 9.7 percent. We choose the standard deviation of z_{it} such that the average absolute size of price changes in our model equals 9.7 percent under perfect information. This yields a standard deviation of z_{it} that is 11.8 times the standard deviation of q_t. It is unclear whether one should exclude sale-related price changes. We decided to match 9.7 percent instead of 11.5 percent because this yields a smaller standard deviation of z_{it} and, therefore, less attention will be allocated to idiosyncratic conditions.[25] In the benchmark economy, we abstract from the fact that, in the data, prices remain fixed for longer than a quarter, whereas in our model prices change every quarter. Later we take into account that this difference in price duration may affect the estimated size of idiosyncratic shocks given an observed size of price changes. Finally, in the benchmark economy, we assume the same decay in the z_{it} process as in the q_t process.

The ratio $(\hat{\pi}_{14} / |\hat{\pi}_{11}|)$ determining the sensitivity of the profit-maximizing price to the idiosyncratic state variable has the same effects on equilibrium as the variance of the idiosyncratic state variable. Therefore, we normalize $(\hat{\pi}_{14} / |\hat{\pi}_{11}|)$ to one and we choose only the variance of the idiosyncratic state variable.

The ratio $(\hat{\pi}_{13} / |\hat{\pi}_{11}|)$ determining the sensitivity of the profit-maximizing price to real aggregate demand is a standard parameter in models with monopolistic competition. Woodford (2003b, ch. 3) recommends a value between 0.1 and 0.15. In the benchmark economy, we set $(\hat{\pi}_{13} / |\hat{\pi}_{11}|) = 0.15$ and later we show how changes in $(\hat{\pi}_{13} / |\hat{\pi}_{11}|)$ affect the solution.

We choose the parameter that bounds the information flow such that firms set prices that are close to the profit-maximizing prices. Based on this reasoning, we set $\kappa = 3$ bits.[26] The following calculations illustrate $\kappa = 3$ bits. Allocating 1, 2, and 3 bits of information flow to the problem of tracking a Gaussian white noise process yields a ratio of posterior variance to prior variance of 1/4, 1/16, and 1/64, respectively. Tracking an autocorrelated process is easier. Allocating 1, 2, and 3 bits of information flow to the problem of tracking a Gaussian AR(1) process with $\rho = 0.95$ yields a ratio of posterior variance to prior variance of 1/32, 1/155, and 1/647, respectively. These numbers follow from equation (60). These numbers imply that, with $\kappa = 3$ bits, the information flow is large enough

[25] For the same reason, we decided to choose the standard deviation of z_{it} such that the average absolute size of price changes equals 9.7 percent under perfect information instead of under rational inattention. This again yields a smaller standard deviation of z_{it}.

[26] Information flow is measured in bits (see *Sims* (2003)).

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to track both aggregate and idiosyncratic conditions well. Therefore decision makers set prices that are close to the profit-maximizing prices. Thus losses in profits due to suboptimal pricing are small and the marginal value of information flow is low. Hence, decision makers have little incentive to increase the information flow.

To set the parameter that bounds the information flow, one cannot query oneself about the information processing capacity of humans in the real world and endow decision makers in the model with the same capacity. The reason is that economic models are drastic simplifications of the real world. For example, in our model, decision makers take no decision apart from the price setting decision and they need only track one firm-specific variable. One has to choose the parameter that bounds the information flow taking into account the simplicity of the model. We choose the parameter such that firms in the model do very well.

Figures 1 and 2 summarize the results for the benchmark economy. Firms allocate 4 percent of their attention to aggregate conditions and 96 percent of their

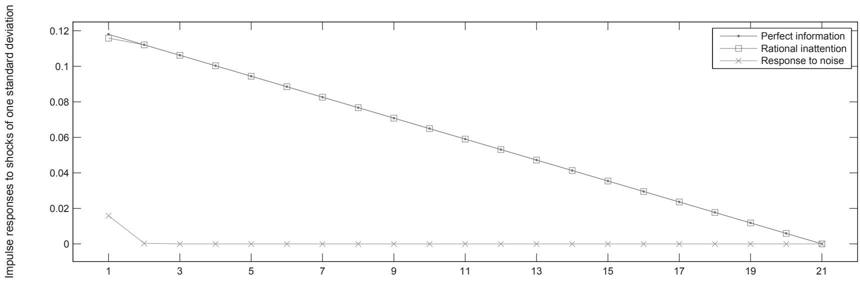

Figure 1: Impulse Responses of an Individual Price to an Innovation in the Idiosyncratic State Variable, Benchmark Economy

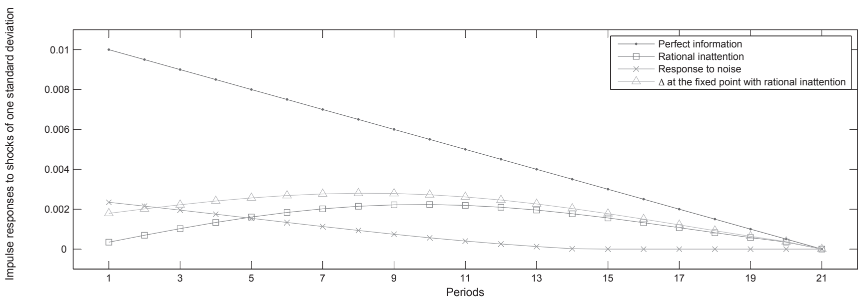

Figure 2: Impulse Responses of an Individual Price to an Innovation in Nominal Aggregate Demand, Benchmark Economy

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attention to idiosyncratic conditions. The average absolute size of price changes equals 9.6 percent. The optimal allocation of attention implies the following price setting behavior. Figure 1 shows the impulse response of an individual price to an innovation in the idiosyncratic state variable. The response to an idiosyncratic shock under rational inattention (the line with squares) is almost as strong and quick as the response under perfect information (the line with points). The line with crosses is the impulse response of an individual price to noise in the signal concerning idiosyncratic conditions.

Figure 2 shows the impulse response of an individual price to an innovation in nominal aggregate demand. The response to a nominal shock under rational inattention (the line with squares) is dampened and delayed relative to the response under perfect information (the line with points). The line with crosses in Figure 2 is the impulse response of an individual price to noise in the signal concerning aggregate conditions. Since the effect of idiosyncratic noise washes out in the aggregate, the line with squares is also the impulse response of the price level to an innovation in nominal aggregate demand. Under rational inattention, the price level responds weakly and slowly to a nominal shock. The reasons are as follows. First, to match the large average absolute size of price changes in the data, idiosyncratic volatility in the model has to be one order of magnitude larger than aggregate volatility. Therefore, firms allocate almost all attention to idiosyncratic conditions, implying that prices respond weakly and slowly to changes in aggregate conditions. Second, the profit-maximizing response to aggregate conditions, $\Delta_t = p_t + (\hat{\pi}_{13}/|\hat{\pi}_{11}|)(q_t - p_t)$, is endogenous. It depends on the price setting behavior of other firms. If all firms set the profit-maximizing price, the price level moves one-for-one with nominal aggregate demand, and Δ_t equals q_t. By contrast, when firms face an information flow constraint, the price level moves less than one-for-one with nominal aggregate demand, and Δ_t differs from q_t. The impulse response of Δ_t to an innovation in nominal aggregate demand at the rational inattention fixed point is given by the line with triangles in Figure 2. Note that firms track the profit- maximizing response to a nominal shock fairly well. Nevertheless, the rational inattention fixed point is far away from the perfect information fixed point. This is because of the feedback effects. Rational inattention of other firms changes both the profit-maximizing response to a nominal shock and the optimal allocation of attention. If all other firms were to set the profit-maximizing price, Δ_t would equal q_t and firms would find it optimal to allocate about three times as much attention to aggregate conditions.

The impulse response of y_t to an innovation in nominal aggregate demand equals the difference between q_t (the line with points in Figure 2) and p_t (the line with squares in Figure 2). It is apparent that nominal shocks have strong and persistent real effects.

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Figures 3 and 4 show simulated price series. Figure 3 shows a sequence of prices set by an individual firm under rational inattention (crosses) and the sequence of profit-maximizing prices (diamonds). Since we have chosen a high value for κ, firms track the profit-maximizing price very well. For an individual firm, the ratio of posterior variance to prior variance of the profit-maximizing price is $^1/_{300}$.[27] Therefore, losses in profits due to suboptimal pricing are small and the marginal value of information flow is low.[28] Figure 4 shows sequences of aggregate price levels. The equilibrium price level under rational inattention (crosses) differs markedly from the equilibrium price level under perfect information (diamonds). The reason is the optimal allocation of attention in combination with the feedback effects. To illustrate that firms make fairly small mistakes in tracking the equilibrium price level, Figure 4 also shows an individual firm's conditional expectation of the price level at the rational inattention fixed point (points).

The early New Keynesian literature emphasized that changes in real activity can be an order of magnitude larger than losses of individual firms. See, for example, *Akerlof/Yellen* (1985). We obtain a similar result. The rational inattention equilibrium is far away from the perfect information equilibrium, despite the fact that losses in profits due to suboptimal pricing are small.

In the benchmark economy, prices respond strongly and quickly to idiosyncratic shocks, but only weakly and slowly to nominal shocks. The model can explain why the price level responds slowly to monetary policy shocks, despite the fact that individual prices change fairly frequently and by large amounts. The model can also explain the empirical finding of *Boivin/Giannoni/Mihov* (2009) that sectoral prices respond quickly to sector-specific shocks and slowly to monetary policy shocks.

We now turn to examining how changes in parameter values affect the optimal allocation of attention and the dynamics of prices and output.

---

[27] Formally, $E[(p_{it}^{\diamond} - p^{*})^2]/E[(p_{it}^{\diamond})^2]$ equals $^1/_{300}$.

[28] The mean squared error in the response to aggregate conditions is $E[(\Delta_t - \hat{\Delta}_{it})^2] = 0.00004$. The mean squared error in the response to idiosyncratic conditions is $(\hat{\pi}_{14}/\hat{\pi}_{11})^2 E[(z_{it} - \hat{z}_{it})^2] = 0.00026$. To compute the expected per-period loss in profits due to suboptimal price setting, we need to multiply these numbers by the concavity of the profit function divided by two, $(|\hat{\pi}_{11}|/2)$. See equations (23) and (41). Using either of the two examples given in Section VIIB with the parameter values given below equation (65) yields $|\hat{\pi}_{11}| = 15\,\bar{Y}$. We then arrive at an expected per-period loss 11 in profits due to imperfect tracking of aggregate conditions equal to $0.0003\,\bar{Y}$ and an expected per-period loss in profits due to imperfect tracking of idiosyncratic conditions equal to $0.0019\,\bar{Y}$. The marginal value of information flow equals $0.0027\,\bar{Y}$.

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2. Varying Parameter Values

When the variance of nominal aggregate demand increases, firms shift attention toward aggregate conditions and away from idiosyncratic conditions. Since firms allocate more attention to aggregate conditions, a given nominal shock has smaller real effects. However, the reallocation of attention is not large enough to compensate fully for the fact that the size of nominal shocks has increased. Firms make larger mistakes in tracking aggregate conditions, and therefore output volatility increases. In addition, since firms allocate less attention to idiosyncratic conditions, firms also make larger mistakes in tracking idiosyncratic conditions. The prediction that real volatility always increases when the variance of nominal aggregate demand increases differs markedly from the Lucas model. At the same time, our model is consistent with the empirical finding of Lucas (1973) that the Phillips curve becomes steeper as the variance of nominal aggregate demand increases.

When the variance of the idiosyncratic state variables increases, firms shift attention toward idiosyncratic conditions and away from aggregate conditions.

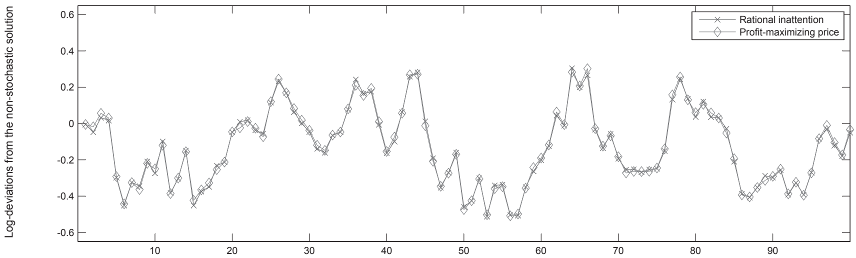

Figure 3: Simulated Price Set by an Individual Firm in the Benchmark Economy

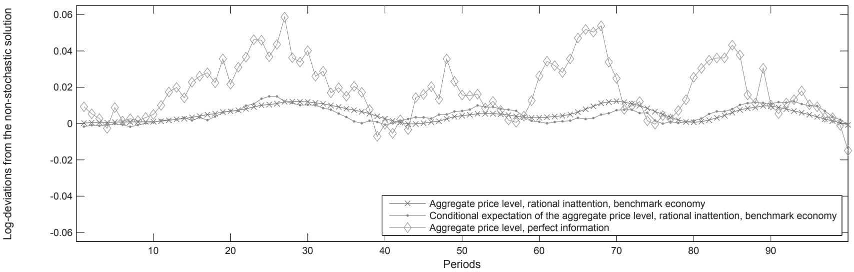

Figure 4: Simulated Aggregate Price Level

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Therefore, the response of the price level to a nominal shock becomes more dampened and delayed.[29]

The predictions described above continue to hold in a model with an endogenous κ. Suppose that firms can choose the information flow facing an increasing, strictly convex cost function. Consider again the effects of increasing the variance of nominal aggregate demand. Since the marginal value of information flow about aggregate conditions increases, firms choose a higher κ and therefore the marginal cost of information flow increases. This implies that the marginal value of information flow about aggregate and idiosyncratic conditions has to increase. Firms track aggregate and idiosyncratic conditions less well.

Figure 5 illustrates how the ratio ($\hat{\pi}_{13} / |\hat{\pi}_{11}|$) affects the solution. When the profit-maximizing price is less sensitive to real aggregate demand (i.e., when ($\hat{\pi}_{13} / |\hat{\pi}_{11}|$) is lower), the response of the price level to a nominal shock is more dampened and delayed. The reason is that the feedback effects are stronger. Note that the feedback effects are important for generating strong and persistent real effects.

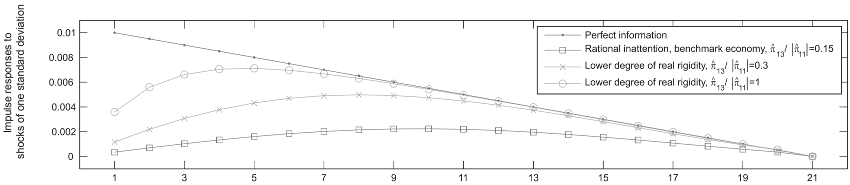

Figure 5: Impulse Responses of the Aggregate Price Level to an Innovation in Nominal Aggregate Demand

Figure 6 illustrates how changes in κ affect the solution. With $\kappa = 3$, nominal shocks have real effects for 20 quarters. With $\kappa = 4$ and $\kappa = 5$, nominal shocks have real effects for about 8 and 5 quarters, respectively. Thus, the model's prediction that nominal shocks have real effects is robust to changes in the value of κ. The values $\kappa = 3, 4, 5$ imply a ratio of posterior variance to prior variance of the profit-maximizing price of 1/300, 1/750, and 1/1,750, respectively, and expected per-period losses in profits (as a fraction of steady-state output)

[29] In *Maćkowiak/Wiederholt* (2007), we also investigate how persistence of the exogenous processes q_t and z_{it} affects the equilibrium. We find that reducing the persistence of a process (holding the variance of the process constant) increases profit losses due to suboptimal pricing, because the firms' tracking problem becomes more difficult. Concerning the allocation of attention, we find that reducing the persistence of a process may increase or decrease the attention allocated to that process. For the benchmark economy, reducing the persistence of z_{it} increases the attention allocated to idiosyncratic conditions.

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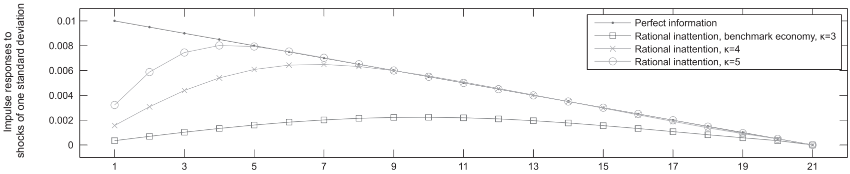

Figure 6: Impulse Responses of the Aggregate Price Level to an Innovation in Nominal Aggregate Demand

of 1/5 of 1 percent, 1/12 of 1 percent, and 1/25 of 1 percent, respectively. We find numbers in this range reasonable. Note that this model generates large real effects with small profit losses, despite the fact that there are large idiosyncratic shocks. This is because prices respond almost perfectly to idiosyncratic shocks.

We have chosen the variance of the idiosyncratic state variables so as to match the average absolute size of price changes in US micro data. So far, we have abstracted from the fact that, in the data, prices remain fixed for longer than a quarter, whereas in our model prices change every quarter. Now we investigate how this difference in price duration may affect the estimated variance of idiosyncratic shocks, given an observed size of price changes. In *Maćkowiak/Wiederholt* (2007), we consider the following simple model. Firms can adjust prices every T periods as in John B. Taylor (1980), firms have perfect information, and the profit-maximizing price follows a Gaussian random walk. In this simple model, increasing the price duration from three months to T months raises the expected absolute price adjustment by a factor of $\sqrt{T/3}$. Furthermore, when the profit-maximizing price follows a stationary process instead of a random walk, increasing the price duration from three months to T months raises the expected absolute price adjustment by *less* than a factor of $\sqrt{T/3}$.

Motivated by these observations, we computed the equilibrium of our model to match an average absolute size of price changes of 6.3 percent because 0.063 $\sqrt{7.2/3} = 0.097$, where 7.2 months is the median price duration excluding sale-related price changes and 9.7 percent is the average absolute size of price changes excluding sale-related price changes reported in *Klenow/Kryvtsov* (2008). Real effects of nominal shocks decrease but remain sizable (see Figure 7). Note that here we use the median price duration excluding sale-related price changes (7.2 months instead of 3.7 months), we use the average absolute size of price changes excluding sale-related price changes (9.7 percent instead of 11.5 percent), and we ignore that, in our model, the profit maximizing price follows a stationary process instead of a random walk.

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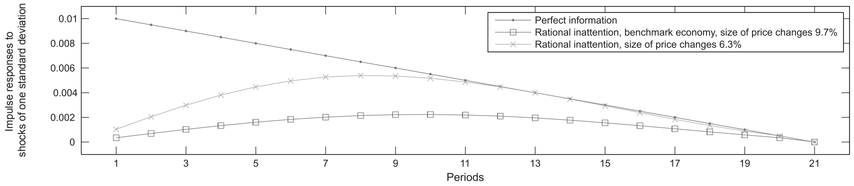

Figure 7: Impulse Responses of the Aggregate Price Level to an Innovation in Nominal Aggregate Demand

3. Optimal Signals

We always verify that scalar signals with property (49)–(50) exist. These are optimal signals. One pair of optimal signals are the conditional expectations themselves, $s_{1it} = \hat{\Delta}^*_{it}$ and $s_{2it} = \hat{z}^*_{it}$. This follows from Proposition 2. As a matter of fact there are many optimal signals. The reason is that there are many signal processes that yield the same conditional expectations.

Given an optimal signal, it is easy to construct a new optimal signal. For example, one can multiply the signal with a constant. More generally, applying a one-sided linear filter (with absolutely summable coefficients) to a scalar signal that is an element of the set Γ yields a new scalar signal that is an element of the set Γ. Furthermore, many such filters do not change the conditional expectation computed from the signal.[30] In this case, if the initial signal is optimal, the new signal is also optimal. An optimal signal about aggregate conditions can be a signal concerning the price level, real aggregate demand, nominal aggregate demand, or a linear combination of those variables. For the equilibrium, it does not matter whether firms pay attention to the price level or to real aggregate demand. What matters for the equilibrium is the attention allocated to aggregate conditions.

VII. Extensions and Shortcomings

1. The Gaussianity Assumption

So far we have assumed that signals follow a Gaussian process. In *Maćkowiak/Wiederholt* (2007), we drop the Gaussianity assumption. First, we show that Gaussian signals are optimal when the objective in the firms' attention problem is quadratic and the variables being tracked follow a Gaussian process. Hence, after the log-quadratic approximation of the profit function, Gaussian signals

[30] For this purpose, assumption (17) is important.

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are optimal and dropping the Gaussianity assumption has no effect on the equilibrium of the model.

Second, we study the optimal form of uncertainty without the log-quadratic approximation of the profit function. Once we work without the log-quadratic approximation of the profit function, we must specify a particular profit function. Therefore, we specify a particular profit function.

We then study the attention problem of an individual firm assuming (i) that q_t and z_{it} follow white noise processes; and (ii) that the price level is given by $p_t = \alpha q_t$. We continue to assume that the responses to aggregate and idiosyncratic conditions are independent. Let p_{it}^A and p_{it}^I denote the price responses to aggregate and idiosyncratic conditions, respectively. We solve for the optimal joint distribution of $(p_{it}^A, p_{it}^I \;\; q_t, z_{it})$ by discretizing the distribution and by maximizing expected profits subject to the information flow constraint. We obtain the following results: (i) the attention allocated to aggregate conditions is the same with and without the log-quadratic approximation of the profit function; and (ii) without the log-quadratic approximation of the profit function, the optimal joint distribution of $(p_{it}^A, p_{it}^I \;\; q_t, z_{it})$ has some non-Gaussian features, but, for our choice of κ, the departures from Gaussianity are small. Recall that, for our choice of κ, the marginal value of information flow is low.

By contrast, when we decrease κ substantially (i.e., when we raise the marginal value of information flow significantly), the optimal joint distribution of $(p_{it}^A, p_{it}^I \;\; q_t, z_{it})$ looks very different from a Gaussian distribution.

These results resemble the findings in Sims (2006). With a quadratic objective, Gaussian signals are optimal. With a nonquadratic objective, Gaussian uncertainty is a good approximation when the marginal value of information flow is low, while Gaussian uncertainty is a bad approximation when the marginal value of information flow is high.[31]

2. The Independence Assumption

Up to this point, we have assumed that paying attention to aggregate conditions and paying attention to idiosyncratic conditions are separate activities. We now relax the independence assumption (15)–(16), and we discuss the assumption in detail.

When we drop the independence assumption, the decision maker in a firm can pay attention to any linear combination of aggregate conditions and idiosyncratic conditions. In this case, one can show that, after the log-quadratic approximation of the profit function, it is optimal to receive a signal of the form

[31] See *Sims* (2006, p. 161).

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"profit-maximizing price plus noise." The reasons are as follows. First, the profit loss (23) depends only on the distance between the actual price (21) and the profit-maximizing price (22). Furthermore, the profit-maximizing price (22) is a particular linear combination of aggregate and idiosyncratic conditions. Thus, the decision maker cares only about this linear combination of aggregate and idiosyncratic conditions. Second, receiving a signal equal to the sum of two signals is weakly less information flow than receiving a vector containing the two separate signals.

However, we think this version of the model misses an important feature of reality. Recall that the only source of noise in the model is the decision maker's limited attention. Thus, assuming that the decision maker can receive a signal of the form "profit-maximizing price plus noise" amounts to assuming that the decision maker can attend directly to the profit-maximizing price (or to a variable that reveals perfectly the profit-maximizing price). This may be an appealing model in some economic contexts. We think that, in most economic contexts, decision makers cannot attend directly to the optimal decision (or to a variable that reveals perfectly the optimal decision). In most economic contexts, decision making is about first paying attention to a variety of variables and then combining these different pieces of information in a single decision. This observation can also explain why the task of decision making is typically allocated to highly skilled individuals and why taking good decisions requires so much time and effort.

The independence assumption is the simplest way of modeling the idea that decision making is about first paying attention to a variety of variables, and then combining these different pieces of information in a single decision. That some pieces of information contain information about aggregate conditions only, and other pieces of information contain information about idiosyncratic conditions only, is not crucial for our results. Instead, it is crucial for our results that decision makers cannot attend directly to the optimal decision (or to a variable that reveals perfectly the optimal decision). To make this point, we consider two examples in which decision makers can attend to output sold by the firm ("sales"). In Example 1, sales reveal information about aggregate conditions only. The aim of this example is to show that attending to sales does not necessarily violate the independence assumption. In Example 2, sales reveal information about both aggregate conditions and idiosyncratic conditions. We show that prices still respond strongly to idiosyncratic shocks, and weakly to aggregate shocks.

Example 1: Assume that demand for good i equals $d\left(P_{it}/P_t\right)Y_t$, output of firm i equals $f\left(L_{it}\right)Z_{it}$ where L_{it} is labor input and Z_{it} is firm-specific productivity, and the real wage equals $w\left(Y_t\right)$. The functions d, f, and w are twice continuously differentiable with $d' < 0, f' > 0, f'' < 0$, and $w' > 0$. To ensure the existence and

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uniqueness of a solution to the price setting problem, assume that d has the following properties: the price elasticity of demand is greater than one in absolute value, and the desired markup is nondecreasing in market share. After the log-quadratic approximation of the profit function around the nonstochastic solution of the model, the profit-maximizing price is given by

$$p_{it}^{\diamond} = p_t + \frac{\omega_w + \omega_{mc,y}}{1-\left(\omega_{\mu,y} + \omega_{mc,y}\right)\omega_{y,p}} y_t + \frac{\omega_{mc,z}}{1-\left(\omega_{\mu,y} + \omega_{mc,y}\right)\omega_{y,p}} z_{it}, \tag{64}$$

where the ω are elasticities at the nonstochastic solution of the model, $\omega_w > 0$ is the elasticity of the real wage with respect to aggregate output, $\omega_{mc,y} > 0$ is the elasticity of real marginal cost with respect to the firm's output, $\omega_{mc,z} < 0$ is the elasticity of real marginal cost with respect to firm-specific productivity, $\omega_{y,p} < 0$ is the price elasticity of demand, and $\omega_{\mu,y} \geq 0$ is the elasticity of the desired markup with respect to market share. Attending to aggregate conditions here can mean attending to the price level, P_t, or to aggregate output, Y_t, but attending to aggregate conditions can also mean attending to sales, $d\left(P_{it} / P_t\right)Y_t$, because the decision maker knows her own decision (the price of the good) and, apart from P_{it}, sales depend on aggregate conditions only.

Attending to idiosyncratic conditions here means attending to firm-specific productivity. This example shows that attending to sales does not necessarily violate the independence assumption. Furthermore, we think it is a good description of reality that price setters in a firm first pay attention to sales and to firm-specific productivity, and then combine these different pieces of information in a price setting decision.

The previous example is special in the sense that all idiosyncratic shocks are idiosyncratic productivity shocks. Next, consider an example with idiosyncratic demand shocks.

Example 2: Demand for good i equals $d\left(P_{it} / P_t\right)Y_t Z_{it}$ and output of firm i equals $f\left(L_{it}\right)$. All other assumptions are the same as in Example 1. After the log-quadratic approximation of the profit function, the profit-maximizing price is again given by equation (64), but with $\omega_{mc,z} = \omega_{mc,y}$.

Attending to sales now violates the independence assumption because sales reveal information about both aggregate conditions and idiosyncratic conditions.[32] Therefore, one could argue that if idiosyncratic shocks are idiosyncratic demand shocks and decision makers can attend to sales, results may differ from the results presented in Sections IV to VI. To address this criticism, we solve the

[32] Attending to idiosyncratic conditions now means attending to good-specific demand (e.g., attending to a marketing report about the relative position of good i in its market).

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model in the white noise case, assuming that idiosyncratic shocks are idiosyncratic demand shocks and decision makers can choose from among the following signals:

$$\begin{pmatrix} s_{1it} \\ s_{2it} \\ s_{3it} \end{pmatrix} = \begin{pmatrix} q_t \\ z_{it} \\ -\omega_{y,p} p_t + y_t + z_{it} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1it} \\ \varepsilon_{2it} \\ \varepsilon_{3it} \end{pmatrix} \tag{65}$$

where ε_{1it}, ε_{2it} and ε_{3it} follow independent Gaussian white noise processes. The first, second, and third signals formalize the idea of attending to aggregate conditions, attending to idiosyncratic conditions, and attending to sales, respectively.[33] We solve this version of the model numerically. We set $\omega_w = 0.25$, $\omega_{mc,y} = 0.5$, $\omega_{y,p} = -4$, $\omega_{\mu,y} = 0.5$, $\sigma_q = 0.01$, $\sigma_z = 0.85$, and $\kappa = 3$.[34] We find that firms allocate all of their attention to sales, and prices still respond strongly to idiosyncratic shocks and weakly to aggregate shocks.[35] When we increase κ from three to seven, firms also pay some attention directly to aggregate conditions, but prices still respond more strongly to idiosyncratic shocks than to aggregate shocks. Only as κ goes to infinity do prices respond to the same extent to idiosyncratic as to aggregate shocks. To understand these results, note that one can write the profit-maximizing price (64) as

$$p_{it}^{\lozenge} = \frac{\omega_{mc,z}}{1-\left(\omega_{\mu,y}+\omega_{mc,y}\right)\omega_{y,p}} \left[\left(\frac{1}{\omega_{mc,y}} - \frac{\omega_{\mu,y}}{\omega_{mc,y}}\,\omega_{y,p} - \omega_{y,p}\right) p_t + \left(\frac{\omega_w}{\omega_{mc,y}} + 1\right) y_t + z_{it}\right]. \tag{66}$$

Comparing the expression for sales in (65) to the expression in square brackets in the equation for the profit-maximizing price (66) reveals that sales have a smaller *relative* weight on aggregate conditions than the profit-maximizing price. Formally, the coefficients on p_t and y_t in the expression for sales in (65) are both smaller than the corresponding coefficients in the expression in square brackets in (66), while the coefficient on z_{it} is the same in the two expres-

[33] The price of the good, p_{it}, does not appear in the third signal because the decision maker knows her own decision.

[34] These elasticities yield $(\hat{\pi}_{13}/|\hat{\pi}_{11}|) = 0.15$, which is the value of $(\hat{\pi}_{13}/|\hat{\pi}_{11}|)$ in the benchmark economy. We set $\sigma_q = 0.01$ because, in the benchmark economy, the standard deviation of the innovation in nominal aggregate demand equals 1 percent. We set $\sigma_z = 0.85$ because this value yields an average absolute size of price changes of about 10 percent. This value for σ_z is larger than the standard deviation of the innovation in the idiosyncratic state variables in the benchmark economy, because here $(\hat{\pi}_{14}/|\hat{\pi}_{11}|)$ is smaller than one, whereas in the benchmark economy we normalized $(\hat{\pi}_{14}/|\hat{\pi}_{11}|)$ to one.

[35] At the rational inattention fixed point, individual prices underreact to aggregate shocks by 50 percent and react almost perfectly to idiosyncratic shocks. Furthermore, $p_t = (1/7)\ q_t$.

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sions.[36] This is because the profit-maximizing price depends on aggregate conditions not only through sales, but also through the price level, the real wage, and the desired markup. Thus, compared with the profit-maximizing price, sales are a variable that is "biased" toward idiosyncratic demand shocks.

Furthermore, if a decision maker can attend to a variable that is "biased" toward idiosyncratic shocks or another variable that is "biased" toward aggregate shocks, then the decision maker pays more attention to the former variable so long as idiosyncratic shocks cause more variation in the profit-maximizing price than aggregate shocks. As a result, prices respond strongly to idiosyncratic shocks, but only weakly to aggregate shocks. This is precisely the mechanism emphasized in Sections IV to VI, and the mechanism continues to operate here.[37] This example shows that the assumption that some pieces of information contain information about aggregate conditions only, and other pieces of information contain information about idiosyncratic conditions only, is not crucial for our results. Instead, it is crucial for our results that decision makers cannot attend directly to the profit-maximizing price (or to a variable that reveals perfectly the profit-maximizing price).

Situations in which a price setter can attend directly to a variable that reveals perfectly the profit-maximizing price are likely to be very special. We doubt that those situations arise in reality. They may arise as special cases in some models. For example, in a model of monopolistic competition with a constant price elasticity of demand, the profit-maximizing price equals a constant markup times nominal marginal cost. Hence, one could argue that, if the price elasticity of demand is constant and if, in addition, decision makers can attend directly to nominal marginal cost, then decision makers can attend to a variable that reveals perfectly the profit-maximizing price. However, it is important to keep in mind the following two observations. First, with decreasing returns (and in recent versions of the New Keynesian model it is often assumed that there are decreasing returns because capital is fixed in the short run and there are decreasing returns to labor), it is unclear how a price setter can attend directly to nominal marginal cost before setting the price. With an upward sloping marginal cost schedule, nominal marginal cost depends on the firm's output, which de-

[36] This result holds under the general restrictions on the functions d, f, and w given at the beginning of this subsection.

[37] We also solved an example with both idiosyncratic productivity shocks and idiosyncratic demand shocks. In that example, we split the idiosyncratic variation in prices equally between the two types of idiosyncratic shocks. Keeping all other parameters the same as in the earlier example in this paragraph, we found that firms allocate half of their attention to idiosyncratic productivity shocks, half of their attention to sales, and no attention directly to aggregate conditions. Prices respond strongly to idiosyncratic shocks (both idiosyncratic productivity shocks and idiosyncratic demand shocks), but only weakly to aggregate shocks.

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pends on the price of the good.[38] Second, a constant price elasticity of demand is a very special case. In addition, it seems to be a poor description of reality.

Industry studies find that the price elasticity of demand is not constant.[39] Not surprisingly, models with a constant price elasticity of demand are hardly ever used in the empirical industrial organization literature.

3. Reconsidering the Allocation of Attention

Suppose that, in a period $t > 0$, a decision maker can reconsider the allocation of attention. Formally, the decision maker again solves problem (11)–(13), but the first period is period $t + 1$ and the decision maker has received signals up to period t. After the log-quadratic approximation of the profit function, the decision maker's objective depends only on conditional variances.

Furthermore, the realization of the signal process up to period t affects conditional means, but does not affect conditional variances. In a Gaussian environment, conditional variances are independent of realizations. In fact, due to the stationarity assumption in (14) and assumption (17), conditional variances are constant over time. Hence, in period zero, the decision maker anticipates correctly the conditional variances in all following periods and has no incentive to reoptimize in period t.

4. Shortcomings

The model has some shortcomings. It cannot explain why prices remain fixed for some time. In the model, prices change in every period. It may be that reality is a combination of a menu cost model and the model presented here. One could add a menu cost, which is likely to increase the real effects of nominal shocks even further. For a given allocation of attention, the menu cost will make the response of the price level to a nominal shock even more dampened and delayed. If prices are strategic complements, this implies that firms shift attention away from aggregate conditions. In addition, firms may also shift attention toward idiosyncratic conditions and away from aggregate conditions, because changes in idiosyncratic conditions are more likely to move the price outside the inaction band. These observations suggest that there may be interesting interactions between a menu cost and rational inattention.

[38] As a matter of fact, the same argument applies to sales.

[39] See, for example, *Goldberg* (1995) who studies the US automobile industry. Goldberg finds that the price elasticity of demand changes as the price of the good changes, and proposes this finding as an explanation for incomplete exchange rate pass-through.

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In this paper, we try to make progress in modeling how agents take decisions in complex environments. In this respect, we think that the model has two shortcomings. First, we do not spell out all factors that make the price setting decision complicated. We assume a general profit function. We summarize the market-specific factors by the idiosyncratic state variable. We choose a value for the information flow parameter such that firms take good but not perfect decisions. We focus on the tension between attending to aggregate conditions and attending to idiosyncratic conditions. In many models of price setting used in macroeconomics, the optimal decision is so simple that it may be unclear why firms make mistakes at all. For example, in a model with monopolistic competition, constant price elasticity of demand, and linear technology in homogeneous labor, the profit-maximizing price equals a constant markup times the nominal wage divided by labor productivity. We think that, in reality, setting the profit-maximizing price is substantially more complicated, e.g., the desired markup may vary, there may be decreasing returns, there may be different types of labor, there may be various other inputs, the interaction with competitors may be complex, the interaction with customers may be complex, etc. In the future, it could be desirable to spell out all factors that make the price setting decision complicated.

Second, rational inattention captures some (but certainly not all) aspects of decision making in complex environments. Rational inattention captures the idea that taking good decisions is more complicated when firms operate in a more volatile and less persistent environment. Rational inattention does not capture the idea that the size of mistakes also depends on how complex the actual computation is that leads to the decision. The latter aspect of decision making has been emphasized by *Gabaix/Laibson* (2000).

VIII. Conclusion

This paper proposes the following explanation for real effects of monetary policy shocks. Idiosyncratic conditions are more variable or more important than aggregate conditions. Therefore, price setters pay more attention to idiosyncratic conditions than to aggregate conditions. This implies that prices respond strongly and quickly to idiosyncratic shocks, but only weakly and slowly to nominal shocks. Moreover, if prices are strategic complements, the fact that other firms pay limited attention to aggregate conditions reduces the incentive for an individual firm to pay attention to aggregate conditions.

The model can explain why the price level responds slowly to monetary policy shocks, despite the fact that individual prices change fairly frequently and by large amounts. The model can also explain why sectoral prices respond quickly to sector-specific shocks and slowly to monetary policy shocks. Furthermore,

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the model generates large real effects with small profit losses, despite the fact that there are large idiosyncratic shocks.

It matters how we model price stickiness. Rational inattention suggests different lessons for monetary policy than standard sticky price models. It suggests that stabilizing monetary policy is good because it allows the private sector to focus on market-specific conditions. Rational inattention also suggests that the optimal allocation of attention will change as monetary policy changes.

It will be interesting to develop a dynamic stochastic general equilibrium model with rational inattention on the side of firms and households and to compare the predictions to, for example, *Christiano/Eichenbaum/Evans* (2005) and *Smets/Wouters* (2003). Furthermore, it will be interesting to study the interactions between a menu cost and rational inattention. In addition, it seems promising to apply this modeling approach to other areas in economics where it has been noted that idiosyncratic uncertainty dominates aggregate uncertainty.[40] Finally, it will be interesting to compare the predictions of the model to micro and macro data.

Appendix A: Profit Loss

$$\begin{aligned}
\tilde{\boldsymbol{\pi}}\left(p_{it}^{\diamondsuit}, p_t, y_t, z_{it}\right) - \tilde{\boldsymbol{\pi}}\left(p_{it}^{*}, p_t, y_t, z_{it}\right) &= \hat{\boldsymbol{\pi}}_1\left(p_{it}^{\diamondsuit} - p_{it}^{*}\right) + \frac{\hat{\boldsymbol{\pi}}_{11}}{2}\left(p_{it}^{\diamondsuit}\right)^2 - \frac{\hat{\boldsymbol{\pi}}_{11}}{2}\left(p_{it}^{*}\right)^2 \\
&\quad + \left(\hat{\pi}_{12} p_t + \hat{\pi}_{13} y_t + \hat{\pi}_{14} z_{it}\right)\left(p_{it}^{\diamondsuit} - p_{it}^{*}\right) \\
&= -\frac{|\hat{\pi}_{11}|}{2}\left(p_{it}^{\diamondsuit}\right)^2 + \frac{|\hat{\pi}_{11}|}{2}\left(p_{it}^{*}\right)^2 \\
&\quad + |\hat{\pi}_{11}| p_{it}^{\diamondsuit}\left(p_{it}^{\diamondsuit} - p_{it}^{*}\right) \\
&= \frac{|\hat{\pi}_{11}|}{2}\left(p_{it}^{\diamondsuit} - p_{it}^{*}\right)^2 .
\end{aligned}$$

The first equality follows from equation (18). The second equality follows from $\hat{\pi}_1 = 0$, $\hat{\pi}_{11} < 0$, $\hat{\pi}_{12} = -\hat{\pi}_{11}$, and equation (20).

Appendix B: Information Flow Constraint in the White Noise Case

The independence assumption (16) implies that

$$I\left(\{P_t, Z_{it}\};\{\mathbf{s}_{\mathbf{it}}\}\right) = I\left(\{p_t\};\{s_{1it}\}\right) + I\left(\{z_{it}\};\{s_{2it}\}\right).$$

[40] See, for example, *Pischke* (1995).

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(See Lemma 2.) The fact that both $\{p_t, s_{1it}\}$ and $\{z_{it}, s_{2it}\}$ are bivariate Gaussian white noise processes implies that

$$I(\{p_t\};\{s_{1it}\}) + I(\{z_{it}\};\{s_{2it}\}) = \frac{1}{2}\log_2\left(\frac{1}{1-\rho^2_{p,s_{1i}}}\right) + \frac{1}{2}\log_2\left(\frac{1}{1-\rho^2_{z_i,s_{2i}}}\right)$$

(See equation (5).) Using equations (26)–(28) yields

$$\rho^2_{p,s_{1i}} = \frac{\sigma^2_\Delta}{\sigma^2_\Delta + \sigma^2_\varepsilon}.$$

Using equation (29) yields

$$\rho^2_{z_i,s_{2i}} = \frac{\sigma^2_z}{\sigma^2_z + \sigma^2_\psi}.$$

Combining results yields

$$I(\{P_t, Z_{it}\};\{\mathbf{s_{it}}\}) = \frac{1}{2}\log_2\left(\frac{\sigma^2_\Delta}{\sigma^2_\varepsilon}+1\right) + \frac{1}{2}\log_2\left(\frac{\sigma^2_z}{\sigma^2_\psi}+1\right).$$

Appendix C: Proof of Lemma 1

First, if the profit function is given by (18), then

$$E\left[\sum_{t=1}^{\infty}\beta^t \pi\left(P^{\Diamond}_{it}, P_t, Y_t, Z_{it}\right)\right] - E\left[\sum_{t=1}^{\infty}\beta^t \pi\left(P^{*}_{it}, P_t, Y_t, Z_{it}\right)\right]$$

$$= E\left[\sum_{t=1}^{\infty}\beta^t \tilde{\pi}\left(p^{\Diamond}_{it}, p_t, y_t, z_{it}\right)\right] - E\left[\sum_{t=1}^{\infty}\beta^t \tilde{\pi}\left(p^{*}_{it}, p_t, y_t, z_{it}\right)\right]$$

$$= E\left[\sum_{t=1}^{\infty}\beta^t \frac{|\hat{\pi}_{11}|}{2}\left(p^{\Diamond}_{it} - p^{*}_{it}\right)^2\right],$$

where the second equality follows from equation (23). Second, if the profit function is given by (18), the price setting behavior is given by equations (19)–(20). The equation $y_t = q_t - p_t$, the Gaussianity assumption (14), and the guess (39) imply that ($p^{\Diamond}_{it}$, $\mathbf{s_{it}}$) follows a stationary Gaussian process. Furthermore, assumption (17) implies that a long sequence of signals is available in every period. It follows that

$$= E\left[p^{\Diamond}_{it} \mid \mathbf{s^t_i}\right] = \mu + \gamma(\mathbf{L})\ \mathbf{s_{it}}\ ),$$

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where μ is a constant and $\boldsymbol{\gamma}(\mathbf{L})$ is an infinite order vector lag polynomial. Thus, $p_{it}^{\Diamond} - p_{it}^{*}$ follows a stationary process, implying that $E[(p_{it}^{\Diamond} - p_{it}^{*})^2]$ is independent of t. Combining results yields equation (40).

Appendix D: Proof of Proportion 1

First, if the profit function is given by (18) and if (39) holds, objective (45) is a monotonic transformation of objective (12). (See Lemma 1 and equation (41).) Therefore, one can use either objective to evaluate decisions. Second, information flow constraint (11) implies (46). (See Lemma 2.) Furthermore, the definition of the set Γ, $\Delta_t = p_t + (\hat{\pi}_{13} / |\hat{\pi}_{11}|)(q_t - p_t)$ and assumption (17) imply (47)–(48). It follows that expected profits at a solution to problem (11)–(13) cannot be strictly larger than expected profits at a solution to problem (45)–(48).

Third, suppose that a bivariate signal process $\{s_{1it}, s_{2it}\} \in \Gamma$ exists with property (49)–(50). Since the signals s_{1it} and s_{2it} are scalars, the weak inequality (43) holds with equality. The fact that $\{\hat{\Delta}_{it}^{*}, \hat{z}_{it}^{*}\}$ satisfies (46) then implies that the signal process $\{s_{1it}, s_{2it}\}$ satisfies the information flow constraint (11). Furthermore, the fact that $\{\hat{\Delta}_{it}^{*}, \hat{z}_{it}^{*}\}$ is a solution to problem (45)–(48) implies that these signals must be a solution to problem (11)–(13).

Appendix E: Proof of Proposition 2

First, the mean of Y_t affects objective (51), but does not affect the information flow in (52). Therefore, a solution to problem (51)–(54) has to satisfy

$$E[X_t - Y_t] = 0.$$

Second, a solution to problem (51)–(54) has to satisfy, for all $k = 0, 1, 2, \ldots$,

$$E[(X_t - Y_t) Y_{t-k}] = 0.$$

Take a process $\{Y_t^{'}\}$ that does not have this property. Formally, for some $k \in \{0, 1, 2, \ldots\}$,

$$E[(X_t - Y_t^{'}) Y_{t-k}^{'}] \neq 0.$$

Define a new process $\{Y_t^{'}\}$ by the following equation:

$$Y_t^{''} = Y_t^{'} + \gamma\, Y_{t-k}^{'},$$

where γ is the projection coefficient in the linear projection of $X_{t,} - Y_t^{'}$ on $Y_{t-k}^{'}$. The new process has the property

$$I(\{X_{t,}\};\{Y_t^{''}\}) = I(\{X_{t,}\};\{Y_t^{'}\}),$$

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because applying a linear filter does not change the information flow. (See proof of Lemma 2.) Furthermore, the new process is of the form (54). Finally, it is easy to verify that the new process has the property

$$E\left[(X_t - Y_t'')^2\right] < E\left[(X_t - Y_t')^2\right].$$

Hence, the process $\{Y_t'\}$ cannot be a solution to problem (51)–(54).

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DOI https://doi.org/10.3790/ccm.52.4.573 | Generated on 2025-04-05 01:52:02
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