

## **Barro-Gordon Revisited: Reputational Equilibria in a New Keynesian Model**

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### **I. Introduction**

In order to fight the recessionary impacts of the financial crisis 2007–2009 central banks around the globe switched over to discretionary monetary policy. As the financial crisis seems to be overcome, monetary authorities have however to think about exit strategies and thus a way to credibly return to a commitment monetary policy. The topic of policy switching regimes and the resulting consequences for the credibility of central banks are already discussed in the famous study of *Barro/Gordon* (1983a, b). However, their framework is completely represented by a traditional Phillips curve, i. e. the authors do not consider any demand side effects which also played a crucial role in the subprime crisis. The authors moreover assume that the Central Bank can directly control for the inflation rate. More precisely, *Barro/Gordon* (1983a, b) assume that the policymaker controls an instrument which has a direct connection to the inflation rate – for instance, the money growth rate.<sup>1</sup>

The aim of this paper is not to depict the decision problem of a monetary authority under the circumstances of the financial crisis. Instead, this paper offers an approach which enables us to discuss the time-inconsistency problem à la *Kydland/Prescott* (1977) and *Barro/Gordon* (1983a, b) in a New Keynesian framework.<sup>2</sup> Within this framework we can solve the inconsistency problem and derive optimal time-consistent interest rate rules of Taylor-type. Thereby, the New Keynesian framework enables us to consider the demand side of the economy and to devi-

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<sup>1</sup> *Jarchow* ((2010), Chapter 5) also extends the *Kydland/Prescott-Barro/Gordon* approach for a demand side. However, this equation just determines the money growth rate. *Jarchow* ((2010), Chapter 5) moreover analyzes optimal monetary commitment and discretion strategies within this framework.

<sup>2</sup> See *Wohltmann/Krömer* (1989) for a comment on the different concepts of time-consistency in the economics literature.

ate from the assumption that the central bank can directly control for the inflation rate. Instead, the mechanism in New Keynesian models is as follows. (i) The central bank commits itself to follow an interest rate rule of Taylor-type. (ii) Private agents form inflation expectations. (iii) The central bank sets the interest rate and the households adjust their consumption expenditures according to the Euler consumption equation. (iv) Inflation is then determined by expected future inflation and the output realization via the New Keynesian Phillips curve.

There already exists a couple of studies which show that the crucial assumption made by *Barro/Gordon* (1983a,b) introducing the time-inconsistency problem – namely, that the central bank aims at an output gap target larger than zero – leads to an inflation bias in a New Keynesian framework (see amongst others *Clarida/Gali/Gertler* (1999)). However, an explicit derivation and the analysis of the resulting welfare consequences of the optimal monetary policy, including purely discretionary and inconsistent monetary policy as well as time-(in)consistent Taylor rules, are neglected in the literature.

Our main findings are as follows. We algebraically show that there exist Taylor rules which are superior to discretionary monetary policy. The central bank has however an incentive to deviate from its commitment in the absence of an appropriate punishment mechanism. By assuming that the central bank loses its reputation for one period when deviating once from its announcement as in *Barro/Gordon* (1983a,b), we moreover derive two areas of time-consistent Taylor rules. The optimal Taylor rule is included. This implies that in contrast to *Barro/Gordon* (1983a,b), the optimal time-consistent solution does not imply a net gain of inconsistent policy equal to zero but a negative net gain. A further difference to *Barro/Gordon* (1983a,b) is that the optimal time-consistent solution implies that inflation does not exceed its target level. Instead, both values coincide. Moreover, we find that optimal Taylor rules minimizing the social loss are independent of the Taylor rule coefficient on inflation. When restricting our analysis to empirically observed Taylor rule coefficients fulfilling the Taylor principle and a non-negative output gap coefficient, the optimal solution is a borderline solution,  $k_x = 0$ , but it still results in a negative net gain of inconsistent policy.

We moreover show that numerous estimated Taylor rules are time-inconsistent since the Taylor rule coefficient on inflation is too low. Hence, a policymaker must stronger react to changes in inflation in order to obtain a time-consistent commitment strategy. Finally, we additionally con-

sider a cost-push shock to the economy as it is widely considered in the monetary macroeconomics literature. In principle, all results remain unchanged. The only differences are that there does not exist an optimal time-consistent Taylor rule in explicit form anymore and that the area of time-consistent Taylor rules becomes graphically smaller. In contrast to *Barro/Gordon* (1983a,b), it is however still an optimal strategy not to choose a borderline solution implying a net gain of inconsistent policy equal to zero but a solution leading to a negative net gain.

The remainder is organized as follows. Section II. shortly describes the applied model. In Section III., we turn to monetary policy issues including the optimal discretionary monetary policy, simple Taylor rules, and the incentive to deviate from the announced policy rule. In Section IV., we derive the continuum of time-consistent Taylor rules and discuss optimal time-consistent rules. In Section V. and VI., we respectively investigate whether estimated Taylor rules are time-consistent and check our results for robustness by introducing a cost-push shock to the economy. The last section concludes.

## II. The Model

For the sake of simplicity, we apply a static approximation of the microfounded canonical New Keynesian model following *Bofinger/Meyer/Wollmershäuser* (2006).<sup>3</sup> The model can be represented by a three-dimensional equation system including an IS curve, a Phillips curve, and a monetary policy rule. The IS curve is given by

$$(1) \quad x = a - br$$

where  $x$  denotes the output gap which is defined as the deviation of output from its natural level.  $a$  is a constant.  $b$  represents the intertemporal elasticity of substitution.  $r$  is the real interest rate.

The second building block of the model is the static approximation of the New Keynesian Phillips curve

$$(2) \quad \pi = \pi^e + \delta x$$

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<sup>3</sup> *Bofinger/Meyer/Wollmershäuser* (2006) already highlight that their approach can be extended for implementing the Kydland/Prescott-Barro/Gordon approach. However, they only point out that an output gap target above zero as assumed in *Barro/Gordon* (1983a) leads to an inflation bias which was already shown by *Clarida/Gali/Gertler* (1999) within the dynamic New Keynesian model.

where  $\pi$  and  $\pi^e$  represent current and expected future inflation, respectively.  $\delta = (\eta + \sigma)(1 - \omega)(1 - \beta\omega)/\omega$  is the slope of the Phillips curve where  $\eta > 0$  and  $\sigma = 1/b > 0$  are the inverse of the Frisch elasticity of labor supply and the inverse of the intertemporal elasticity of substitution, respectively.  $\omega \in (0,1)$  denotes the Calvo parameter implying that a firm cannot adjust its price level with a probability of  $\omega$ .  $\beta \in (0,1)$  is the private discount factor.

### III. Monetary Policy – The One Period Game

In the following, we will discuss different types of policy regimes, namely the optimal discretionary monetary policy, *D*, the commitment regime à la Taylor, *TR*, and the regime under inconsistent policy, *IP*. Independently of the assumed type of monetary policy, the central bank seeks to minimize a social loss function.

As shown by *Galí* ((2008), Chapter 4) and *Woodford* ((2003), Chapter 6), the second order approximation of the households' utility function delivers a quadratic loss function which represents flexible inflation targeting in the spirit of *Svensson* (1999). The static approximation of this function is given by

$$(3) \quad V = (\pi - \pi^T)^2 + \lambda x^2$$

where  $\pi^T$  represents the target inflation rate and  $\lambda = \delta/\theta \in (0,1)$  is the central bank's preference parameter on stabilizing the output gap.  $\theta > 1$  denotes the intratemporal elasticity between goods.

Following *Barro/Gordon* (1983a,b), we additionally assume that the central bank's target of the output gap is positive, i.e.  $x^T > 0$ . An economic rationale is that e.g. monopolistic distortions or taxes keep potential output below its efficient level (see *Clarida/Galí/Gertler* (1999)). Then the social loss is given by

$$(4) \quad V = (\pi - \pi^T)^2 + \lambda(x - x^T)^2$$

An alternative approach to include the problem of time-inconsistency into the model is to assume an asymmetric loss function (see *Cukierman/Gerlach* (2003), *Nobay/Peel* (2003), or *Ruge-Murcia* (2003)). There is empirical evidence for both approaches (see for instance *Ireland* (1999) and *Gerlach* (2003)). However, there is no micro-foundation for such a loss

function, at all. We moreover want to remain as close as possible to the Kydland/Prescott-Barro/Gordon approach.

### 1. Discretionary Monetary Policy

In this section, we will derive the optimal discretionary monetary policy. In this regime, the expected inflation rate is taken as given for the central bank since the monetary authority applies a sequential optimization. Therefore, it is unable to make credible announcements concerning the design of monetary policy that could influence private expectations.

The central bank minimizes the social loss (4) subjected to the Phillips curve (2).<sup>4</sup> Inserting the Phillips curve (2) in the social loss function (4) and optimizing the resulting equation with respect to the output gap yields the following first order condition:

$$(5) \quad 2\delta(\pi^e + \delta x - \pi^T) + 2\lambda(x - x^T) = 0$$

Inserting (5) in the Phillips curve and taking rational expectations yields the expected inflation rate under discretionary monetary policy.

$$(6) \quad \pi^e|_D = \pi^T + \frac{\lambda}{\delta}x^T = \pi|_D$$

Since we do not consider any shocks in the economy, expected inflation aligns current inflation, i.e.  $\pi|_D = \pi^e|_D$ .<sup>5</sup> Further note that (expected) inflation is above the central bank's target level when the monetary authority aims at a positive output gap since  $\lambda, \delta > 0$ . This implies that inflation only coincides with its target level when the central bank's preferences would represent strict inflation targeting ( $\lambda = 0$ ). This is a very intuitive result since in this case the central bank is not concerned about the output gap, at all. The central bank's target level of the output gap represents an inflation bias in the solution of inflation which pushes inflation above its target level.

In the absence of shocks the Phillips curve (2) and  $\pi|_D = \pi^e|_D$  imply an output gap equal to zero

$$(7) \quad x|_D = 0$$

<sup>4</sup> Note that the IS curve is not a binding restriction in this case.

<sup>5</sup> In Section VI., we shortly discuss whether and how our results change when considering shocks in the economy.

The solution of the output gap is zero and thus independent of the corresponding target level,  $x^T$ . It moreover coincides with the discretionary solution in the case where the central bank does not target a positive output gap, i. e.  $x|_D^{x^T > 0} = x|_D^{x^T = 0} = 0$ . Since under rational expectations the model structure including the loss function is known by private agents, the intention of the central bank to push the output gap above its natural level fails. Instead, the solution of output remains unchanged and that of inflation is ‚biased‘. Consequently, the central bank cannot simultaneously stabilize inflation and the output gap. This implies that a positive output gap target introduces a trade-off for monetary policy.

When combining (6) and (7), discretionary monetary policy can be expressed as a targeting rule (see *Svensson (1999)*) given by

$$(8) \quad x|_D - x^T = -\frac{\delta}{\lambda} [\pi|_D - \pi^T]$$

implying a negative relationship between the stabilization of inflation and the output gap at the respective target level.

Finally, the social loss under discretionary monetary policy can be derived by inserting the solutions of the output gap (7) and inflation (6) in the welfare function (4):

$$(9) \quad V|_D = \left[ \left( \frac{\lambda}{\delta} \right)^2 + \lambda \right] (x^T)^2$$

The loss is strictly positive when assuming  $x^T \neq 0$ .

## 2. Simple Rules

In this section, we will derive the social loss when the central bank credibly commits itself to follow a simple monetary policy rule of Taylor-type. Since the commitment is credible in this case, the central bank influences private expectations in this policy regime.

The Taylor rule is commonly represented as

$$(10) \quad i = i^T + k_\pi(\pi - \pi^T) + k_x(x - x^T)$$

where  $k_x$  and  $k_\pi$  are the elasticities of the nominal interest rate,  $i$ , with respect to the deviation of the output gap and the inflation rate from their respective target level. In the following, we will refer to them as

Taylor rule coefficients. The real interest rate which is the argument of the IS curve (1) is then obtained from the nominal interest rate via the well-known Fisher equation.

$i^T$  is the central bank's target level of the nominal interest rate which follows

$$(11) \quad i^T = r^T + \pi^T$$

The corresponding target level of the real interest rate,  $r^T$ , follows from the IS equation and is given by

$$(12) \quad r^T = \frac{1}{b}(a - x^T)$$

Note that the target level of the real interest rate coincides with its natural level,  $r^n = a/b$ , in the borderline case  $x^T = 0$ .

Combining the Taylor rule (10) with (11), (12), the IS curve (1), and the Phillips curve (2) and taking expectations, yields the expected inflation rate under the monetary policy regime  $TR$

$$(13) \quad \pi^e|_{TR} = \pi^T + \frac{1 + bk_x}{b(k_\pi - 1)}x^T = \pi|_{TR}$$

which again coincides with the current inflation rate due to the absence of shocks and the assumption of rational expectations.

As long as  $-\infty < k_\pi < \infty$  and  $k_x \neq -1/b$ , (expected) inflation does not align its target level when the monetary authority seeks to achieve a positive target level of the output gap. More precisely, it holds that

$$(14) \quad \begin{aligned} \pi|_{TR} = \pi^e|_{TR} > \pi^T & \quad \text{if} \quad \begin{cases} k_\pi > 1 \wedge k_x > -1/b \\ k_\pi < 1 \wedge k_x < -1/b \end{cases} \\ \pi|_{TR} = \pi^e|_{TR} \leq \pi^T & \quad \text{otherwise} \end{aligned}$$

for  $k_\pi \neq 1$ . The inflation rate thus exceeds its target level if  $k_\pi$  and  $k_x$  are both sufficiently large or small. Otherwise, the target level of inflation exceeds expected inflation.

When combining equation (13) and the Phillips curve (2), we obtain the solution path of the output gap

$$(15) \quad x|_{TR} = x|_D = 0$$

As in the case of the discretionary monetary policy, the solution of the output gap is zero and thus independent of its target level.<sup>6</sup> Hence, equation (15) also represents the solution of the borderline case  $x^T = 0$  where the central bank targets a closed output gap. By contrast, the solution of inflation is biased since  $\pi|_{TR} \neq \pi^T$  if  $x^T > 0$ . Considering a positive target level of the output gap thus leads to higher inflation while the resulting output gap remains unchanged.

In order to obtain the social loss under the policy regime  $TR$  for arbitrary coefficients  $k_\pi$  and  $k_x$  (except for  $k_\pi = 1$ ), we finally insert the solutions of the output gap and inflation in the welfare function (4):

$$(16) \quad V|_{TR} = \left[ \left( \frac{1 + bk_x}{b(k_\pi - 1)} \right)^2 + \lambda \right] (x^T)^2$$

The loss under  $TR$  is strictly positive. Note that  $V|_{TR}(k_\pi = 1)$  is not defined.

It is moreover straightforward to see that the social loss  $V|_{TR}$  is minimized if  $k_\pi$  either tends to (plus or minus) infinity or if  $k_x = -1/b$ . According to equation (13), both solutions result in  $\pi|_{TR} = \pi^T$  and  $x|_{TR} = 0$ . This implies that the central bank can only stabilize inflation at its target level. Note however that under discretionary monetary policy, the central bank can neither stabilize inflation nor the output gap at their respective target levels.

See Figure 1 for a graphical illustration of (16). In contrast to  $V|_D$ , the social loss (16) is not bounded from above. More precisely,  $V|_{TR}$  tends to infinity if  $k_\pi \rightarrow 1$  or  $k_x \rightarrow \pm \infty$ .

When comparing the loss under  $TR$  with its discretionary counterpart, we obtain the well-known result that there exist Taylor rules which result in a smaller loss than in the regime  $D$ :<sup>7</sup>

$$(17) \quad V|_D - V|_{TR} = \left[ \left( \frac{\lambda}{\delta} \right)^2 - \left( \frac{1 + bk_x}{b(k_\pi - 1)} \right)^2 \right] (x^T)^2 > 0$$

<sup>6</sup> Note that this result changes when allowing the central bank to re-optimize after their announcement, i.e. in the regime of inconsistent monetary policy (see Section III.3.).

<sup>7</sup> For the corresponding analysis of this topic within a dynamic New Keynesian model see amongst others *Rudebusch/Svensson* (1999), *Woodford* (1999), and *Clarida/Gali/Gertler* (1999). See also *Dennis* (2010) for an insightful discussion of this topic.



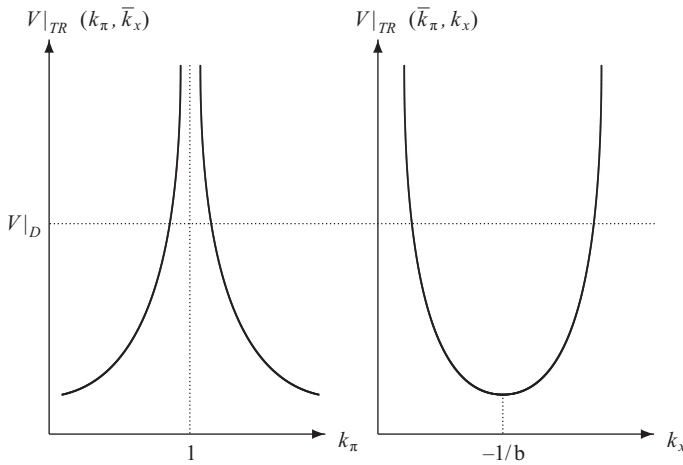


Figure 1: Comparing  $V|_D$  and  $V|_{TR}$

if and only if

$$(18) \quad \left(\frac{\lambda}{\delta}\right)^2 > \left(\frac{1 + bk_x}{b(k_\pi - 1)}\right)^2 \Leftrightarrow |k_\pi - 1| > \frac{\delta}{\lambda b} |1 + bk_x|$$

This implies

$$(19) \quad V|_D > V|_{TR} \quad \text{if} \quad \begin{cases} k_\pi > \max\left\{1 + \frac{\delta}{\lambda b}(1 + bk_x), 1 - \frac{\delta}{\lambda b}(1 + bk_x)\right\} \\ k_\pi < \min\left\{1 + \frac{\delta}{\lambda b}(1 + bk_x), 1 - \frac{\delta}{\lambda b}(1 + bk_x)\right\} \end{cases}$$

$$(20) \quad V|_D = V|_{TR} \quad \text{if} \quad k_\pi = 1 + \frac{\delta}{\lambda b}(1 + bk_x) \quad \vee \quad k_\pi = 1 - \frac{\delta}{\lambda b}(1 + bk_x)$$

For a graphical illustration, see Figure 2. The grey areas assign  $k_x/k_\pi$ -combinations where the loss in regime  $TR$  is larger than under discretionary monetary policy while the white areas denote  $k_x/k_\pi$ -combinations where the opposite holds true, i.e. the regime  $TR$  is favorable. On the borderlines, the social losses in both regimes coincide, i.e.  $V|_D = V|_{TR}$ .

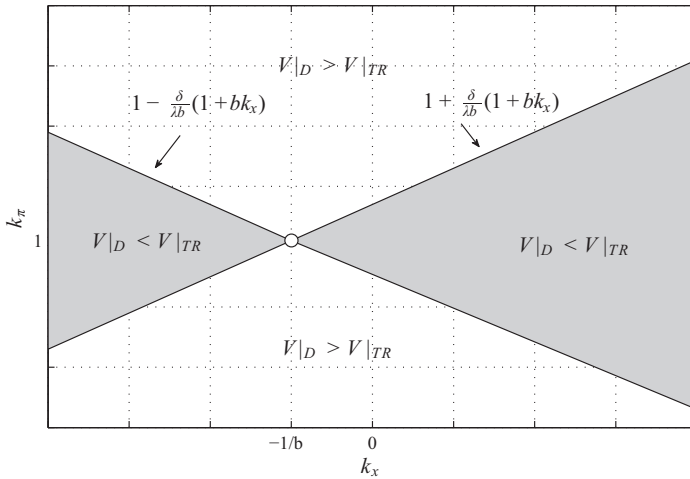


Figure 2: Graphical Illustration of Inequality (19)

### 3. Inconsistent Policy

In this section, we will show that the central bank has an incentive to deviate from the announced Taylor rule and thus renege on their commitment if the monetary authority is faced with a purely static one-period optimization approach.

If the central bank credibly announces to follow a specifically calibrated Taylor rule, expected inflation is tied at a given level according to (13). However, the central bank can then achieve a welfare gain by re-optimizing in a discretionary manner. In this case, the monetary authority will not implement the announced policy rule. We will refer to this policy regime as inconsistent monetary policy, *IP*.

The maximization problem of the central bank under *IP* is given by

$$\begin{aligned}
 \max_{x, \pi} \quad & L = (\pi - \pi^T)^2 + \lambda(x - x^T)^2 \\
 \text{s.t.} \quad & \pi = \pi^e + \delta x \\
 & \pi^e = \pi^e|_{TR}
 \end{aligned}
 \tag{21}$$

As in the discretionary case, the first order condition with respect to the output gap is given by

$$x|_{IP} = -\frac{\delta}{\lambda + \delta^2} [\pi^e|_{TR} - \pi^T] + \frac{\lambda}{\lambda + \delta^2} x^T
 \tag{22}$$

By inserting (13) in (22), we obtain the solution of the output gap under the inconsistent policy regime,  $IP$ .

$$(23) \quad x|_{IP} = \frac{1}{\lambda + \delta^2} \left[ \lambda - \frac{\delta(1 + bk_x)}{b(k_\pi - 1)} \right] x^T$$

In contrast to the purely discretionary monetary policy and the regime under commitment to a Taylor rule, the solution of the output gap (23) now depends on its target level and consequently is not equal to zero in general. This implies that under rational expectations the central bank's intention to push output above its natural level can only be achieved via inconsistent monetary policy.<sup>8</sup>

However, it moreover follows from (23) that Taylor rules fulfilling  $k_\pi = 1 + \frac{\delta}{\lambda b}(1 + bk_x)$  result in  $x|_{IP} = x|_D = x|_{TR} = 0$ . As shown before, this expression additionally represents a combination of  $k_\pi$  and  $k_x$  which results in  $V|_D = V|_{TR}$ .

The solution of inflation is obtained from (13) and (23) via the Phillips curve (2)

$$(24) \quad \pi|_{IP} = \pi^T + \frac{\lambda}{\lambda + \delta^2} \frac{\alpha - \delta b}{b(k_\pi - 1)} x^T$$

where  $\alpha \equiv 1 + b(k_x + \delta k_\pi)$ . Since the monetary authority deviates from its credible announcements, current and expected inflation only coincide in this policy regime if  $k_\pi = 1 - \frac{\delta}{\lambda b}(1 + bk_x) \Leftrightarrow \pi|_{IP} = \pi^e_{TR}$ . As shown in equation (20), this expression moreover represents a combination of  $k_\pi$  and  $k_x$  which result in  $V|_D = V|_{TR}$ . In line with our previous finding under discretionary monetary policy, inflation would coincide with its target level in the regime  $IP$  if the central bank follows strict inflation targeting. Moreover,  $\pi|_{IP} = \pi^T$  holds if  $\alpha = \delta b \Leftrightarrow k_\pi = 1 - \frac{1}{\delta b}(1 + bk_x)$  where  $k_x \neq -1/b$  resulting in  $x|_{IP} = x^T$  according to (23). This implies that only under inconsistent monetary policy it is possible to stabilize both inflation and the output gap at their respective target levels.

The combination of (23) and (24) necessarily yields the same targeting rule as in the discretionary case (cf. equation (8)):

$$(25) \quad x|_{IP} - x^T = -\frac{\delta}{\lambda} [\pi|_{IP} - \pi^T]$$

<sup>8</sup> Remark:  $x|_{IP} = x^T$  if  $k_\pi = 1 - (1 + bk_x)/(\delta b)$  where  $k_x \neq -1/b$ .

The social loss under inconsistent monetary policy can finally be obtained by inserting (23) and (24) in (4).

$$(26) \quad V|_{IP} = \frac{\lambda}{\lambda + \delta^2} \left[ \frac{\alpha - \delta b}{b(k_\pi - 1)} \right]^2 (x^T)^2 \geq 0$$

By definition,  $V|_{IP} \leq V|_{TR}$  must hold. It follows that

$$(27) \quad V|_{TR} - V|_{IP} = \frac{(x^T)^2}{\lambda + \delta^2} \left\{ \frac{(1 + bk_x)\delta}{b(k_\pi - 1)} - \lambda \right\}^2 \geq 0$$

#### IV. Time-Consistent Taylor Rules

In this section, we will derive a continuum of time-consistent simple rules. This is done by assuming a long-run planning horizon of the monetary authority as in *Barro/Gordon* (1983a, b).

As shown in the previous section, the central bank has an incentive to re-optimize, if it can credibly announce to follow a commitment strategy. If its announcements are not credible, private expectations are given for the central bank and the monetary authority should follow a discretionary monetary policy. By assuming that the central bank loses its reputation, if it deviates once from its announcement, i.e. if the central bank switches over to the regime *IP*, one can find both a continuum of time-consistent and time-inconsistent simple rules. More precisely, we assume a punishment interval of one period implying that the central bank loses its reputation for exactly one period when renegeing on their commitment once.<sup>9</sup> The announcements of the central bank will then no longer be credible such that private agents will form their expectations as in the discretionary case.

In this framework à la *Barro/Gordon* (1983a, b), the central bank is faced with a simple cost-benefit calculation where the benefit,  $B$ , is the welfare gain resulting from the inconsistent policy in comparison with the implementation of the announced Taylor rule,  $V|_{TR} - V|_{IP}$ . The cost,  $C$ , is the discounted next period welfare loss resulting from the sacrifice in

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<sup>9</sup> Alternatively, one can analyze the case where the central bank loses its reputation for all times when renegeing once. However, the qualitative results remain unchanged.

the central bank's reputation,  $V|_D - V|_{TR}$ . The net gain,  $N$ , of the inconsistent policy is then given by<sup>10</sup>

$$(28) \quad N = B - C = (V|_{TR} - V|_{IP}) - \frac{1}{1+z} (V|_D - V|_{TR})$$

By setting  $N = 0$  and inserting (17) and (27), we obtain

$$(29) \quad [(1 + bk_x)\delta - \lambda b(k_\pi - 1)] \left[ \frac{(1 + bk_x)\delta - \lambda b(k_\pi - 1)}{(\lambda + \delta^2)b^2(k_\pi - 1)^2} + \frac{(1 + bk_x)\delta + \lambda b(k_\pi - 1)}{(1 + z)b^2(k_\pi - 1)^2\delta^2} \right] = 0$$

Equation (29) is satisfied if the first term in the product of the l.h.s. of (29) is equal to zero, i.e.

$$(30) \quad k_\pi^* = 1 + \frac{\delta}{\lambda b}(1 + bk_x)$$

Obviously,  $k_\pi^*$  is an increasing function in  $k_x$ . Moreover, these combinations of Taylor rule coefficients result in  $V|_D = V|_{TR}$  (cf. equation (19)) since

$$(31) \quad V|_{TR}(k_\pi^*) = \left( \left[ \frac{\lambda}{\delta} \right]^2 + \lambda \right) (x^T)^2 = V|_D$$

Hence, all  $k_x/k_\pi$ -combinations fulfilling condition (30) result in the same social loss.

According to the definition of the net gain of inconsistent policy (28) and  $N = 0$ , it follows that  $V|_{TR}(k_\pi^*) = V|_{IP}(k_\pi^*)$  if  $V|_{TR}(k_\pi^*) = V|_D$ . Consequently, the gain and cost of inconsistent policy are both equal to zero if  $k_\pi = k_\pi^*$ .

The second solution of (29) is given by

$$(32) \quad k_\pi^{**} = 1 + \gamma \frac{\delta}{\lambda b}(1 + bk_x)$$

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<sup>10</sup> By assuming that the central bank loses its reputation for all times when deviating once from the announcement, the total gain resulting from  $IP$  would be given by

$$\begin{aligned} N' &= B - C' = (V|_{TR} - V|_{IP}) - \sum_{i=1}^{\infty} \left( \frac{1}{1+z'} \right)^i (V|_D - V|_{TR}) \\ \Leftrightarrow N' &= (V|_{TR} - V|_{IP}) - \frac{1}{z'} (V|_D - V|_{TR}) \end{aligned}$$

where

$$(33) \quad \gamma \equiv \frac{(1+z)\delta^2 + (\lambda + \delta^2)}{(1+z)\delta^2 - (\lambda + \delta^2)}$$

Note that

$$(34) \quad \gamma < -1 \Leftrightarrow z < \tilde{z} = \frac{\lambda + \delta^2}{\delta^2} - 1 = \frac{1}{\theta\delta}$$

Assuming  $\gamma < -1$ ,  $k_\pi^{**}$  is a decreasing function in  $k_x$ .<sup>11</sup>

Consequently, it holds that the net gain of inconsistent monetary policy is negative if either  $k_\pi$  is larger than  $\max\{k_\pi^*, k_\pi^{**}\}$  or smaller than  $\min\{k_\pi^*, k_\pi^{**}\}$

$$(35) \quad N < 0 \quad \text{if} \quad \begin{cases} k_\pi > \max\{k_\pi^*, k_\pi^{**}\} \\ k_\pi < \min\{k_\pi^*, k_\pi^{**}\} \end{cases}$$

Obviously, the two borderline solutions  $k_\pi^*$  and  $k_\pi^{**}$  would coincide if  $\gamma = 1$ . However, it follows from the definition of  $\gamma$  (equation (33)):

$$\gamma = 1 \Leftrightarrow \lambda + \delta^2 = 0 \Leftrightarrow \theta\delta = -1$$

which is a contradiction since  $\theta > 1$  and  $\delta > 0$ . Since  $k_\pi^* \neq k_\pi^{**}$  there always exist two time-consistent areas of Taylor rule coefficients whose borderlines are determined by  $k_\pi^*$  and  $k_\pi^{**}$  defined in equations (30) and (32), respectively.

In contrast to the first borderline solution of (29), both the gain and cost of inconsistent policy are positive if the central bank applies a Taylor rule fulfilling  $k_\pi^{**}$ :

$$(36) \quad V|_{TR}(k_\pi^{**}) - V|_{IP}(k_\pi^{**}) = \frac{\lambda^2}{\lambda + \delta^2} \frac{(1-\gamma)^2}{\gamma^2} (x^T)^2 > 0$$

$$(37) \quad V|_D - V|_{TR}(k_\pi^{**}) = \frac{\lambda^2}{\delta^2} \frac{(\gamma^2 - 1)}{\gamma^2} (x^T)^2 > 0$$

since  $\gamma^2 > 1$ . Then  $V|_D > V|_{TR}(k_\pi^{**}) > V|_{IP}(k_\pi^{**})$  while  $V|_D = V|_{TR}(k_\pi^*) = V|_{IP}(k_\pi^*)$ . Consequently, it follows that  $V|_{TR}(k_\pi^*) > V|_{TR}(k_\pi^{**})$ . Hence, the second borderline solution  $k_\pi^{**}$  determines time-consistent Taylor rules

<sup>11</sup> In Section V, we provide a numerical example. There, we will show that a standard calibration results in  $\tilde{z} \approx 0.97$  which would imply an implausible high time preference rate so that condition (34) is economically plausible.

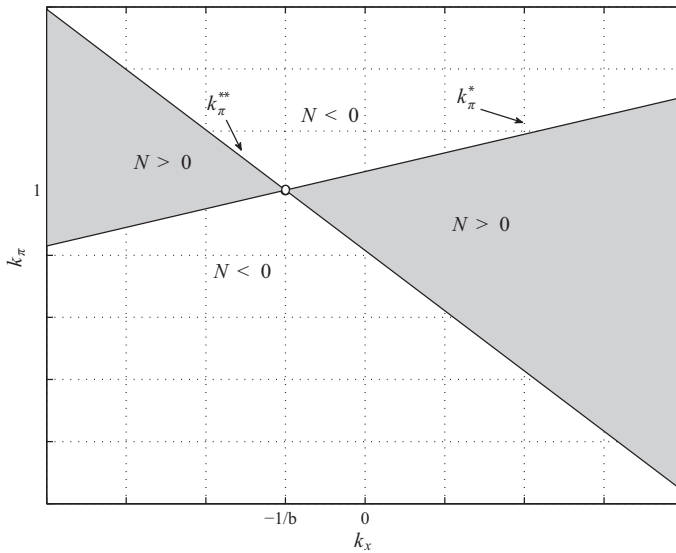


Figure 3: Time-Consistent and Time-Inconsistent Taylor Rules

which generate a smaller loss when compared to  $k_x^*$ . Taylor rules fulfilling  $k_x^{**}$  are thus favorable. However, this solution is not optimal in the class of time-consistent Taylor rules.

For a graphical illustration of the time-consistent and time-inconsistent simple rules, see Figure 3. The light grey areas assign time-inconsistent combinations of  $k_\pi$  and  $k_x$  while the white areas assign time-consistent Taylor rules. Note that  $N(k_\pi = 1)$  is not defined since  $V|_{IP}(k_\pi = 1)$  and  $V|_{TR}(k_\pi = 1)$  are not defined.

As shown in Section III, the social loss under TR (16) does not have a real optimal solution for  $k_\pi$  but for  $k_x$  which is given by  $k_x^{opt.} = -1/b$  for arbitrary  $k_\pi$ .  $k_x^{opt.}$  results in  $\pi^e|_{TR} = \pi|_{TR} = \pi^T$  (cf. Section III.2.) implying that the monetary authority successfully achieves its inflation target. Under this specific Taylor rule, inflation is thus not biased by the introduction of a positive output gap target. Hence, the social loss only results from the inability of the central bank to achieve its output gap target,  $x^T > 0$ .

$k_x = k_x^{opt.}$  implies a social loss given by

$$(38) \quad V|_{TR}(k_x^{opt.}) = \lambda(x^T)^2 < V|_{TR}(k_x^{**}) < V|_{TR}(k_x^*)$$

As shown by *Bullard/Mitra* (2002), the following condition is moreover necessary to ensure that the dynamic version of the New Keynesian model has a unique and stable equilibrium (see also *Walsh* ((2010), Chapter 5))

$$(39) \quad \delta(k_\pi - 1) + (1 - \beta)k_x > 0 \Leftrightarrow k_\pi > 1 - \frac{1 - \beta}{\delta}k_x$$

The stability of the whole system thus crucially depends on the Taylor rule coefficients. In the following, we will thus assume that the Taylor principle,  $k_\pi > 1$ , and the condition  $k_x \geq 0$  hold. These assumptions are commonly applied in the literature and find support in numerous empirical studies.<sup>12</sup> Then the stability condition (39) is obviously satisfied. However,  $k_x \geq 0$  implies that the optimal solution  $k_x^{opt.} = -1/b < 0$  is not feasible anymore. Further note that  $k_\pi^{**}$  is also not feasible anymore since  $k_\pi^{**} \leq 1$  if  $k_x \geq 0$ . Hence, the conditions  $k_\pi > 1$  and  $k_x \geq 0$  cause only the borderline solution of  $N = 0$  to remain which generates the greater loss since  $V|_{TR}(k_\pi^* > 1) > V|_{TR}(k_\pi^{**} > 1)$ . Moreover,  $k_\pi > 1$  and  $k_x \geq 0$  imply  $\pi > \pi^T$  (cf. equation (13)).

The relevant time-consistent area of Taylor rule coefficients is obtained by combining (35) and (39), and  $k_x \geq 0$ . In particular, condition (39) rules out the second time-consistent area where  $k_\pi < 1$ . For a graphical illustration, see Figure 4.<sup>13</sup> In Figure 4, the relevant combinations of  $k_\pi$  and  $k_x$  resulting in time-consistency are assigned with the dark grey area. As in Figure 3, the light grey areas assign inconsistent Taylor rules.

Finally, we have to determine the  $k_x/k_\pi$ -combination within the relevant time-consistent area which yields the lowest social loss. When restricting our analysis to  $k_\pi > 1$  and  $k_x \geq 0$ , it directly follows from equation (16) that the social loss in the regime *TR* decreases if  $k_\pi$  increases and  $k_x$  decreases. Due to the assumed lower bound of  $k_x$ , the lowest social loss is obtained if  $k_x = 0$ , i.e. on the left boundary of the relevant time-consistent area in Figure 4 where  $V|_{TR}$  strictly declines if  $k_\pi$  increases. In Figure 4, this is indicated with the black arrow.

We have shown that there exist two areas of  $k_x/k_\pi$ -combinations resulting in time-consistent Taylor rules. Moreover, we find optimal Taylor rules with  $k_x^{opt.} = -1/b$  for arbitrary  $k_\pi$  resulting in  $\pi = \pi^T$ ,  $x = 0$ , and

<sup>12</sup> See amongst others *Smets/Wouters* (2007, 2003), *Taylor* (1999, 1993), and *Clarida/Gali/Gertler* (2000).

<sup>13</sup> Note that (39) has a very small but negative slope since the private discount factor,  $\beta$ , is typically assumed to be very close to unity.



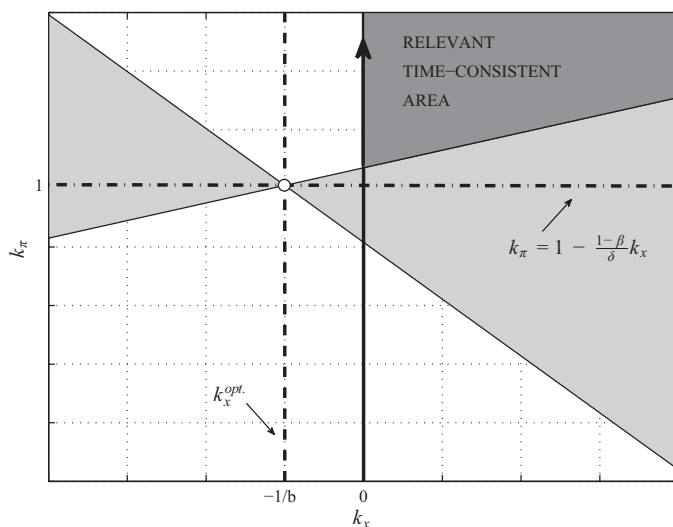


Figure 4: The Relevant Area of Time-Consistent Taylor Rules

$V|_{TR} = \lambda x^T$ . These optimal Taylor rules are within the time-consistent areas implying  $N < 0$ . Hence, there exist two important differences when compared to *Barro/Gordon* (1983a, b). First, in their framework the optimal time-consistent solution implies  $\pi > \pi^T$ . Second, their optimal monetary policy rule results in a borderline solution where  $N = 0$ .

In our framework, we only obtain an optimal borderline solution when restricting the coefficients to  $k_\pi > 1$  and  $k_x \geq 0$ . Then the optimal solution is on the left border of the relevant time-consistent area, i. e.  $k_x = 0$ , and results in  $\pi > \pi^T$ . However, these optimal Taylor rules still do not imply  $N = 0$  but a negative net gain of inconsistent policy. Hence, they are strictly time-consistent.

## V. A Numerical Example

In this section, we will investigate whether different estimated Taylor rules are time-consistent.

As standard in the literature, we assume both the inverse of intertemporal elasticity of substitution and the inverse of the Frisch elasticity of labor supply to be equal to one. Moreover, we assume the private discount factor,  $\beta$ , to be 0.99 implying a long-run interest rate of about 4%.

*Table 1*  
**Estimated Taylor Rules and Time-Consistency**

	$k_x^{est.}$	$k_x^{est.}$	$N$	$k_x  _{N=0}^{est.}$
<i>SW</i> (2007)	2.03	0.08	0.03	15.70
<i>SW</i> (2007)*	2.03	0.08	0.01	7.48
<i>CGG</i> (2000)	2.15	0.93	0.04	12.58
<i>Taylor</i> (1999)	2.53	0.76	0.02	11.56
<i>Taylor</i> (1993)	1.50	0.50	0.13	10.00

\* Beside the Taylor rule coefficients *Smets/Wouters* (2007) additionally estimate the structural parameters  $\beta = 0.99$ ,  $\omega = 0.65$ ,  $\sigma = 1.39$ ,  $\eta = 1.92$  and set  $\theta = 10$ . Otherwise, we stick to our baseline calibration.

The intratemporal elasticity of substitution between goods,  $\theta$ , is set to 6. Moreover, we assume that firms adjust their price level every three quarters on average. This implies a Calvo parameter,  $\omega$ , equal to 0.75. The output gap target is set to 0.1 implying that the potential output level is 10% higher than the current one. Finally, we assume that the central bank's time preference rate,  $z$ , equals the long-run interest rate of 4%. This calibration implies  $\tilde{z} = 0.97 \Leftrightarrow \gamma = -3.24 < -1$  such that condition (34) holds.

Applying this calibration, we now want to calculate the net gain of inconsistent monetary policy for estimated Taylor rules. Table 1 shows the results for the estimated Taylor rule coefficients of *Smets/Wouters* (2007),<sup>14</sup> *Clarida/Gali/Gertler* (2000),<sup>15</sup> *Taylor* (1999),<sup>16</sup> and *Taylor* (1993).

Table 1 indicates that all these estimated Taylor rule coefficients,  $k_x^{est.}$  and  $k_x^{est.}$ , imply time-inconsistent monetary policy rules since they result

<sup>14</sup> Since *Smets/Wouters* (2007) additionally estimate the structural parameters, we test two specifications. First, we apply our baseline calibration and only apply the estimated values for the Taylor rule coefficients. Second, we apply the complete estimated parameter set of *Smets/Wouters* (2007), i.e.  $\beta = 0.99$ ,  $\omega = 0.95$ ,  $\sigma = 1.39$ ,  $\eta = 1.92$ , and  $\theta = 10$ .

<sup>15</sup> More precisely, we choose the estimated values for the Volcker-Greenspan era (1979:1996) of the baseline estimation.

<sup>16</sup> More precisely, we choose the estimated values of the latest period (1987:1997) since this is the period which is the closest to the estimation period of *Clarida/Gali/Gertler* (2000).

in a positive net gain of inconsistent monetary policy. Table 1 moreover shows the  $k_\pi$ -coefficients which result in  $N = 0$  for given  $k_x = k_x^{est}$ .<sup>17</sup> Economically, this implies that the central bank must stronger react to changes in inflation to deliver a time-consistent Taylor rule.

## VI. Extensions

In this section, we investigate whether our results qualitatively change when additionally considering a cost-push shock to the economy. This assumption implies a Phillips curve given by

$$(40) \quad \pi = \pi^e + \delta x + \varepsilon$$

where  $\varepsilon$  is a cost-push shock with  $E(\varepsilon|I) = 0$ . In monetary economics, a cost-push shock is typically assumed to introduce a trade-off for monetary policy between stabilizing output and inflation.<sup>18</sup> Following *Barro/Gordon* (1983a,b), we assume that private expectations,  $E$ , about inflation are formed before the shock occurs. This implies that when forming expectations about inflation, the shock,  $\varepsilon$ , is not included in the information set of private agents,  $I$  (see also *Walsh* (2010), Chapter 8, *Lohmann* (1992), or *Persson/Tabellini* (1990)).

The solution of the model including a cost-push shock for inflation, the output gap, and the social loss in the regimes,  $D$ ,  $TR$ , and  $IP$  are shown in Table 2.<sup>19</sup>

Naturally, the solutions simplify to those obtained in Section III and IV if the cost-push shock is not existent, i. e. if  $\varepsilon = 0$ . The main differences to the previously analyzed case without a shock are as follows. First, expected and current inflation do not coincide since the cost-push shock is not included in the private information set. Second, the output gap is not equal to zero in the regime  $D$  and  $TR$  if  $\varepsilon > 0$ . This implies that the intention of the monetary authority to push output above its natural level does not only fail but the output gap also becomes negative in the presence of the shock.

<sup>17</sup> Naturally, one can conduct the same experiment for  $k_x$  for given  $k_\pi = k_\pi^{est}$ . However, the resulting values for  $k_x$  turn out to be negative.

<sup>18</sup> Note that so far we have abstracted from this assumption since the positive output gap target already introduces an equivalent trade-off.

<sup>19</sup> The explicit derivations for solutions in the presence of a cost-push shock are available from the authors upon request.

*Table 2*  
**Solutions in the Presence of a Cost-Push Shock,  $\varepsilon$**

<i>Regime D</i>	
Inflation:	$\pi _D = \pi^T + \frac{\lambda}{\delta} x^T + \frac{\lambda}{\lambda + \delta^2} \varepsilon \neq \pi^e _D = \pi^T + \frac{\lambda}{\delta} x^T \text{ if } \varepsilon \neq 0$
Output:	$x _D = -\frac{\delta}{\delta^2 + \lambda} \varepsilon$
Social Loss:	$V _D = \frac{\lambda}{\lambda + \delta^2} \left[ \frac{\delta^2 + \lambda}{\delta} x^T + \varepsilon \right]^2$
<i>Regime TR</i>	
Inflation:	$\pi _{TR} = \pi^T + \frac{1 + bk_x}{b(k_\pi - 1)} x^T + \frac{1 + bk_\pi}{\alpha} \varepsilon$ $\neq \pi^e _{TR} = \pi^T + \frac{1 + bk_x}{b(k_\pi - 1)} x^T \text{ if } \varepsilon \neq 0$
Output:	$x _{TR} = -\frac{bk_\pi}{\alpha} \varepsilon$
Social Loss:	$V _{TR} = (1 + bk_x)^2 \left[ \frac{1}{b(k_\pi - 1)} x^T + \frac{1}{\alpha} \varepsilon \right]^2 + \lambda \left[ x^T + \frac{bk_\pi}{\alpha} \varepsilon \right]^2$
<i>Regime IP</i>	
Inflation:	$\pi _{IP} = \pi^T + \frac{\lambda}{\lambda + \delta^2} \left[ \frac{\alpha - \delta b}{b(k_\pi - 1)} x^T + \varepsilon \right]$ $\neq \pi^e _{IP} = \pi^e _{TR} = \pi^T + \frac{1 + bk_x}{b(k_\pi - 1)} x^T \text{ if } \varepsilon \neq 0$
Output:	$x _{IP} = \frac{1}{\lambda + \delta^2} \left[ \lambda - \frac{\delta(1 + bk_x)}{b(k_\pi - 1)} \right] x^T - \frac{\delta}{\lambda + \delta^2} \varepsilon$
Social Loss:	$V _{IP} = \frac{\lambda}{\lambda + \delta^2} \left[ \frac{\alpha - \delta b}{b(k_\pi - 1)} x^T + \varepsilon \right]^2$

Unfortunately, the solution of the system is now too complex to derive time-consistent or optimal Taylor rules analytically. Therefore, we check our previous results numerically by applying the calibration of Section V. Additionally, we normalize the shock impact to unity.

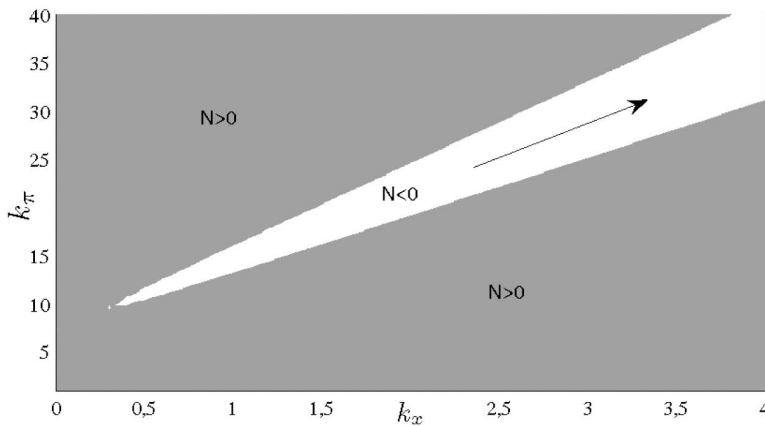


Figure 5: Numerically Obtained Time-Consistent and Time-Inconsistent Areas in the Presence of a Cost-Push Shock ( $\varepsilon > 0$ )

Figure 5 shows the resulting areas of time-consistent and time-inconsistent Taylor rules for  $k_\pi > 1$  and  $k_x \geq 0$ . It indicates that in contrast to the case without a shock (cf. Figure 3 or 4) there exists an upper bound for  $k_\pi$  given  $k_x$  to generate a time-consistent Taylor rule. The area of time-consistent rules thus becomes graphically smaller. Economically, this is a plausible result since the information asymmetry naturally leads to an incentive for the monetary authority to switch over to inconsistent policy. Moreover, it is optimal that both Taylor rule coefficients tend to infinity in the presence of the cost-push shock. In Figure 5, this is indicated with a black arrow. Hence, there does not exist an optimal Taylor rule in explicit form anymore.

## VII. Conclusion

We incorporate the time-inconsistency problem à la *Barro/Gordon* (1983a,b) in a static approximation of the standard New Keynesian model by assuming that the central bank aims at an output gap target larger than zero. This enables us to analyze time-consistent interest rate rules of Taylor-type in a model with a demand side. By contrast, *Barro/Gordon* (1983a,b) only consider the supply side of an economy via a Phillips curve and assume that the monetary authority can directly control for the inflation rate.

We algebraically show that there exist Taylor rules which are superior to discretionary monetary policy. The central bank has however an incentive to deviate from its commitment in the absence of a punishment mechanism. When assuming that the central bank loses its reputation for one period when deviating once from its announcement as in *Barro/Gordon* (1983a,b), we derive two areas of time-consistent Taylor rules. The optimal Taylor rule is included. This implies that in contrast to *Barro/Gordon* (1983a,b), the optimal solution does not imply a net gain of inconsistent policy equal to zero but a negative net gain. A further difference to *Barro/Gordon* (1983a,b) is that the optimal time-consistent solution implies that inflation does not exceed its target level. Instead, both values coincide. Moreover, we find that optimal Taylor rules minimizing the social loss are independent of the Taylor rule coefficient on inflation. When restricting our analysis to empirically observed Taylor rule coefficients fulfilling the Taylor principle and a non-negative output gap coefficient, the optimal solution is a borderline solution,  $k_x = 0$ . However, this solution still implies a negative net gain of inconsistent policy.

Moreover, we show that numerous estimated Taylor rules are time-inconsistent since the Taylor rule coefficient on inflation is too low. A policymaker thus has to react stronger to changes in inflation in order to obtain a time-consistent commitment strategy. Finally, we additionally consider a cost-push shock to the economy as it is widely considered in the monetary macroeconomics literature. In principle, all results remain unchanged. The only differences are that there does not exist an optimal time-consistent Taylor rule in explicit form anymore and that there exists an upper bound for the Taylor rule coefficient on inflation given the other coefficient is fixed. In contrast to *Barro/Gordon* (1983a,b), it is however still an optimal strategy not to choose a borderline solution implying a net gain of inconsistent policy equal to zero but a solution leading to a negative net gain.

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## Summary

### **Barro-Gordon Revisited: Reputational Equilibria in a New Keynesian Model**

The aim of this paper is to solve the inconsistency problem à la Barro/Gordon within a New Keynesian model and to derive time-consistent interest rate rules of Taylor-type. We find a multiplicity of time-consistent rules. In contrast to the famous Kydland/Prescott-Barro/Gordon approach, implementing a monetary rule where the cost and benefit resulting from inconsistent policy coincide – which implies a net gain of inconsistent policy behavior equal to zero – is not optimal. Instead, the solution can be improved by moving into the time-consistent area where the net gain of inconsistent policy is negative. When additionally considering a cost-push shock, the area of time-consistent simple rules of Taylor type becomes graphically smaller. Finally, we find that numerous estimated Taylor rules are time-inconsistent since the empirically observed coefficient on inflation is too low. (JEL E52, E58, E30)

## Zusammenfassung

### **Reputationsgleichgewichte in einem Neukeynesianischen Modellrahmen**

In diesem Beitrag untersuchen wir das Inkonsistenzproblem à la Barro/Gordon in einem Neukeynesianischen Modellrahmen, um zeitkonsistente Zinsregeln vom Taylor-Typ zu bestimmen. Unsere Analyse zeigt, dass eine Vielzahl von zeitkonsistenten Regeln existiert, jedoch weisen diese im Optimum nicht wie im berühmten Kyland/Prescott-Barro/Gordon-Ansatz die Eigenschaft auf, dass die Kosten einer inkonsistenten Zinspolitik gerade mit dem zugehörigen Nutzen zusammenfallen. Stattdessen ergibt sich eine Wohlfahrtsverbesserung, wenn der Nettogewinn der inkonsistenten Politik negativ wird. Berücksichtigt man zusätzlich einen Kostenschock, wird das Kontinuum an zeitlich stabilen Taylor-Regeln grafisch gesehen kleiner. In einem letzten Schritt überprüfen wir, ob eine Reihe von bereits geschätzten Taylor-Regeln die Eigenschaft der Zeitkonsistenz aufweist. Es zeigt sich, dass dies nicht der Fall ist, da der geschätzte Koeffizient für Inflation zu klein ist.