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Abhandlungen

A Wholistic Approach to Diversification Management: The Diversification Delta Strategy Applied to Non-Normal Return Distributions

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Abstract

In this paper we study a higher moment diversification measure, the so-called diversification delta (*Vermorken* et al. (2012)), in a dynamic portfolio optimization context. Particularly, we set up an investment strategy that dynamically maximizes the diversification delta for a given set of assets. Thus, we label the resulting optimized portfolio structure as Maximum Diversification Delta Portfolio (MDDP). Our out-of-sample empirical study reveals that considering crisis-periods, the MDDP is superior to popular investment strategies, such as Minimum-Variance-Portfolio, Risk-Parity-Portfolio and Equally-Weighted-Portfolio, in terms of risk adjusted returns, risk moments and certainty equivalents. However, in line with other diversification measures the MDDP is no longer superior in upward trending markets.

Ein ganzheitlicher Ansatz für das Diversifikationsmanagement: Die Diversifikationsdelta-Strategie angewandt auf nicht-normale Renditeverteilungen

Zusammenfassung

In der vorliegenden Studie untersuchen wir anhand des Diversifikationsdeltas (*Vermorken* et al. (2012)) die Effekte höherer Momente auf die Diversifikationsei-

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genschaften eines dynamisch optimierten Portfolios. In diesem Zusammenhang implementieren wir eine Investitionsstrategie, welche das Diversifikationsdelta für eine gegebene Asset Allokation dynamisch maximiert. Das jeweilige Resultat definieren wir als Maximum Diversification Delta Portfolio (MDDP). Unsere empirische Out-of-sample-Studie zeigt, dass das MDDP, insbesondere in Krisenszenarien, bessere Risiko-adjustierte Performanceergebnisse liefert als andere Investmentstrategien, wie z.B. Minimum-Varianz-Portfolios, Risk-Parity-Portfolios und gleichgewichtete Portfolios. Im Rahmen stetig steigender Kapitalmärkte zeigt sich, ähnlich wie bei anderen Diversifizierungsstrategien, keine bessere Performance.

Keywords: Portfolio Optimization, Diversification, Unsmoothing Returns, Hedge Funds, Financial Crisis

JEL Classification: G11, C61, G01

I. Introduction

In his seminal paper *Markowitz* (1952) provided the tenets of modern portfolio theory. One of the major results states that optimally diversified portfolios are superior in terms of risk adjusted performance when portfolio volatility is taken as the relevant risk measure. Precisely, in his analysis, *Markowitz* (1952) relies on expected means as reward measures and correlation properties and volatilities as risk measures of the underlying portfolio constituents. Although thereby only considering first and second moments of return distributions, diversification was identified as one of the most important drivers of risk adjusted performance regarding an investment strategy. Unfortunately, so far there exists no broadly accepted and satisfying methodology to adequately quantify and manage diversification (*Meucci* (2009)).

In recent research papers, quantifying diversification within a single measure has become popular. A number of different diversification measures were introduced. For example, PCA (Principal Component Analysis)-based diversification measures were presented by *Rudin/Morgan* (2006) and *Meucci* (2009). These measures quantify the number of independent components for a given set of assets (*Rudin/Morgan* (2006)) or the effective number of uncorrelated bets taken within a considered portfolio (*Meucci* (2009)). *Statman/Scheid* (2008) measure (the benefits of) diversification for two assets by the so called "forecasted return gap", which takes correlation and volatilities of the regarded assets in account. *Choueifaty/Coignard* (2008) and *Choueifaty* et al. (2011) introduced a simple diversification index, which is a ratio of the weighted average of volatilities divided by the portfolio volatility. By construction, this measure quantifies the value added, in terms of relative portfolio volatility reduction, by combining non-perfectly correlated assets.

However, regarding one important aspect, all these diversification measures fall short. They exclusively rely on the first two moments of return distributions, when measuring diversification. Considering return distributions that are non-normal, as it is the case for hedge fund returns for example, managing diversification by the above measures will not yield an optimal result with respect to higher risk moments (tail risks). This drawback was countered by *Vermorken* et al. (2012). They developed a diversification measure that evaluates the entire asset return distribution. This measure is called the diversification delta (DD) and is based on the differential Shannon entropy (*Shannon* (1948)), which quantifies the uncertainty of random variables taking in account the respective entire return distributions. In their empirical study, *Vermorken* et al. (2012) showed that a dynamically managed minimum variance portfolio (MVP) of non-normally distributed assets is characterized by superior DDs over time relative to an equally weighted portfolio with the same non-normally distributed assets.

The contribution of this article is as follows. In analogy to above mentioned studies, we take the DD of *Vermorken* et al. (2012) a step further and use it for direct management of portfolio diversification. In particular, we set up an investment strategy that dynamically maximizes the DD, resulting in a so-called Maximum DD Portfolio (MDDP), for a given set of assets. Similar to *Vermorken* et al. (2012) we examine the performance of MDDP relative to MVP, which solely relies on portfolio volatility as a diversification measure, and the equally weighted portfolio (1/N-P) benchmark, which performs no risk-based portfolio optimization at all. Nevertheless, even the simple 1/N-P is (mean-)variance-efficient under the strong assumption of equal (means,) variances and covariances among the considered set of assets. Furthermore, we use the popular Equal Risk Contribution Portfolio (ERCP) as an additional competitor to the MDDP. The ERCP was extensively discussed by *Maillard* et al. (2010) and is also known under the name Risk Parity Portfolio (*Qian* (2006)). Like MDDP and MVP, ERCP is a risk-based portfolio optimization method, since it relies solely on the covariance matrix for asset allocation optimization, and therefore qualifies as a fair competitor. In order to quantify the performance advantage of the MDDP with respect to higher moments relative to rival optimization approaches, we evaluate the skewness and the kurtosis of the implemented investment strategies, among others. Additionally, we calculate the certainty equivalent (CE), assuming the commonly used power utility function, of the resulting portfolio returns. By doing this, we quantify the attractiveness of the whole portfolio return distribution from a utility-based perspective. Finally, the CE-measure is used for bootstrap-based statistical inference analysis in the style of *Ledoit/Wolf* (2008).

To demonstrate the value added of MDDP in the presence of significant higher moments, we use hedge fund returns for the conducted empirical study. The use of hedge fund strategy return streams in this setting offers numerous advantages. First of all, the respective return time series are structurally far more independent from each other than it would be the case with any other investment class or structure. By using hedge fund returns, we are able to study the behavior of very different risk premia in the market and thereby not just relying on single return sources as it would be considering more traditional asset classes. In this respect *Lazanas/Staal* (2012) suggest that a diverse set of systematic risk premia is inherent in hedge fund returns. Therefore, value-adding diversification effects should be detectable more clearly in hedge fund returns than in returns of more traditional asset classes. Furthermore, it is widely accepted that hedge fund returns are non-normally distributed. Beside others, *Anand* et al. (2011) showed that hedge fund returns could be more precisely described by a skewed *t*-distribution. In addition, *Lambert* et al. (2012) demonstrated that exposures to higher-moment risk factors are important sources of hedge fund returns and therefore an integral component of the return generating process of hedge funds. Paying attention to non-normality of given return series in the process of portfolio formation, which is the case for the MDDP, can create substantial benefit for an investor especially with regard to tail-events.

The remainder of the article is structured as follows. In the next section, we discuss the dataset under consideration and describe the applied unsmoothing methodology, which is used to get rid of distorting serial correlations in regarded return time series. Following, we provide the mathematical foundation of the DD and set up the optimization problem leading to the MDDP. We then perform an out-of-sample empirical study, whereby the performance of the MDDP is analyzed relative to selected benchmark investment strategies. In the last section we conclude the main findings of this paper.

II. Dataset

As mentioned above, we use a diverse set of hedge fund (HF) return data. In particular, we choose six HFR[1] hedge fund sub-strategy indices: Distressed (Dstrss), Macro, Equity Hedge (Eq Hedge), Convertible Arbitrage (Cnvrt Arb), Merger Arbitrage (Mrg Arb) and Market Neutral (Mkt Ntr). They provide a broad set of hedge fund return styles reflecting the majority of existing hedge funds. Additionally, we assume that these hedge fund strategies are able to capture many of the available and existing risk premia in global capital markets. HFR provides a comprehensive set of various strategy indices, which have been calculated and published for a prolonged period. The relevant HFR return indices are considered on a daily frequency based on the underlying managed account structures of individual hedge fund managers (index constituents). The total dataset spans from April 1st, 2003 to April 24th, 2014 leading to 2785 (business-)daily return observations for each sub-strategy index.

In table 1 we analyze short-term (i.e. up to five lags) autocorrelation functions of the respective total return series and their respective statistical significance as hedge fund returns are prone to exhibit significant

Table 1

Autocorrelations for the Considered Reported Hedge Fund Returns up to Five Lags

(*P*-values of the Ljung-Box-Test (*Ljung/Box* (1978)) regarding the statistical significance of autocorrelations are stated in brackets under the respective values.)

Lag Number	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr
1	0.1377 *(0.0001)*	0.1035 *(0.0001)*	0.1475 *(0.0001)*	0.1806 *(0.0001)*	-0.0086 *(0.6558)*	0.0747 *(0.0001)*
2	0.1481 *(0.0001)*	0.0294 *(0.0001)*	0.0246 *(0.0001)*	0.2759 *(0.0001)*	-0.0168 *(0.6222)*	-0.0331 *(0.0001)*
3	0.1231 *(0.0001)*	0.0364 *(0.0001)*	0.0348 *(0.0001)*	0.2773 *(0.0001)*	0.0147 *(0.6762)*	-0.0357 *(0.0001)*
4	0.0934 *(0.0001)*	0.0442 *(0.0001)*	0.0201 *(0.0001)*	0.2526 *(0.0001)*	-0.0668 *(0.0094)*	-0.0090 *(0.0002)*
5	0.0962 *(0.0001)*	-0.0043 *(0.0001)*	-0.0157 *(0.0001)*	0.3105 *(0.0001)*	-0.0751 *(0.0001)*	-0.0510 *(0.0001)*

[1] Hedge Fund Research; https://www.hedgefundresearch.com/.

autocorrelations (*Di Cesare* et al. (2011)). These autocorrelations are for the most part caused by illiquidity, i.e. low frequency of price determination, of the underlying investments and/or legal manipulations, e.g. by using slowly adapting valuation models, of reported returns by hedge fund managers (*Getmansky/Makarov* (2004)). When the measurement frequency is high, which is the case for daily returns, the above issues create some degree of artificial persistence (serial correlation) in measured return series.

As expected, table 1 reveals that especially Distressed and Convertible Arbitrage strategies, which are known for heavily investing in illiquid assets, exhibit high serial correlations. Additionally, since the respective p-values are for the most part close to zero, all but Mrg Arb sub-strategy returns exhibit statistically significant serial correlation values under a confidence level of 99 %. This persistence in style indices returns obfuscates the true riskiness of the underlying data generating process. In order to eliminate the distortive effects of autocorrelations on calculated risk (co-)moments, we follow *Glawischnig/Seidl* (2013) and remove autocorrelations in the considered in-sample dataset prior to portfolio optimization. The elimination of autocorrelated structures in a given dataset is called unsmoothing, as we assume that the original autocorrelated dataset is the result of a smoothing process. Unsmoothing the given dataset should lead to a more solid set of data reflecting the "true" inherent risk within each strategy. This should also lead to more robust results within the MDDP optimization process. Following, we describe how the regarded dataset is unsmoothed (adjusted).

For the adjustment of reported returns to remove their respective autocorrelations we use the algorithm proposed by *Geltner* (1991,1993) and extended by *Okunev/White* (2003) to allow for higher-order serial correlation adjustments. Under this adjustment algorithm, as mentioned above, we assume the following return smoothing process:

$$r_{0,t} = (1-\alpha) r_{m,t} + \sum_{i=1}^{m} \beta_i r_{0,t-i}, \qquad \textit{with} \quad \alpha = \sum_{i=1}^{m} \beta_i, \tag{1}$$

where $r_{0,t}$ is the reported return at time t with zero (unsmoothing-)adjustments and $r_{m,t}$ is the true underlying return at time t created by making m (unsmoothing-)adjustments to reported returns. Simply stated, we assume that the reported return is a combination of the current true underlying return and previous reported returns. Considering the above mentioned explanation for high serial autocorrelations in hedge fund return series, the assumed smoothing process is reasonable. Particularly, under the assumed smoothing process current reported returns contain a substantial part of past reported returns. This can be induced by portfolio assets that are illiquid (i.e. low price determination frequency) or marked to model using a slowly adapting valuation algorithm. Given the above smoothing process, the applied unsmoothing algorithm aims to reset a serial correlation value for a specific serial lag to zero by the following general solution adjustment:

$$r_{m,t} = \frac{r_{m-1,t} - c_m r_{m-1,t-m}}{1 - c_m}, \tag{2}$$

with

$$c_m = \frac{\left(1 + \alpha_{m-1,2m}\right) \pm \sqrt{\left(1 + \alpha_{m-1,2m}\right)^2 - 4\alpha_{m-1,m}^2}}{2\alpha_{m-1,m}}, \tag{3}$$

where $\alpha_{m,n}$ is the serial correlation of order n for a return series after m adjustments. Specifically, we start (so $m = 1$) by calculating the first- and second-order serial correlations for the considered unadjusted return series, meaning we firstly determine $\alpha_{0,1}$ and $\alpha_{0,2}$. These α-values are used to quantify the c-parameter, which is in turn applied to equation 2 performing the first adjustment on the return series. This first adjustment eliminates first-order autocorrelation in the considered return series implying that afterwards the return series is considered to be once adjusted. Following, the above described routine is performed for $m = 2$ using the once adjusted return series and so on. The procedure halts when the specified amount of maximum adjustments ($m_{\max}$) is reached. Finally, note that the discussed general unsmoothing solution only applies if $\alpha_{m-1,m}^2 \leq \frac{\left(1 + \alpha_{m-1,2m}\right)^2}{4}$. For reasons of consistency, we unsmooth each of the reported return series of the considered six sub-strategy indices, whereby we focus on autocorrelations up to the fifth serial correlation lag[2] meaning that $m_{\max} = 5$.

[2] As extensively discussed by *Okunev/White* (2003), eliminating the m-th serial correlation will render all autocorrelations of order $< m$ slightly unequal to zero. To contain this imprecision, the described unsmoothing algorithm can be performed several times, setting m to zero each time. In this paper the described entire unsmoothing algorithm is run twice on each return time series.

In table 2 we present summary statistics for the entire dataset before and after adjustments. Firstly, due to the fact that our dataset entails the subprime financial crisis period, most style indices exhibit a relatively low annualized mean return. In fact, Convertible Arbitrage and Market Neutral even exhibit negative mean returns, since they were affected most negatively by the turbulence surrounding the financial crisis. Secondly, in the smoothed as well as in the unsmoothed case Merger Arbitrage is most attractive for a Markowitz-Investor among regarded sub-strategies, as it provides the highest return for a unit of risk in the form of σ_{ann}. To put it differently, among considered sub-strategies Merger Arbitrage is characterized by the highest Sharpe Ratio, which we calculate under the assumption of a zero risk free interest rate. Further, we demonstrate in table 2 that all sub-strategies exhibit statistically significant deviations from non-normality. In particular, all p-values of the conducted Jarque-Bera-Test signal highly significant non-normality for the smoothed and unsmoothed case. As expected, hedge fund style indices with significant serial correlations exhibit higher standard deviations after unsmoothing implying that the underlying (unreported) data generating process is riskier than suggested by reported (managed) returns. In the case of Convertible Arbitrage, which suffers substantially from serial correlations, volatility almost doubles after unsmoothing. Interestingly, hedge fund style indices that partly display negative higher order serial correlations (Merger Arbitrage and Market Neutral) are slightly less riskier after the adjustment. In these cases the smoothing of hedge fund returns results in an overstatement of riskiness associated with the underlying return generating process. Finally, the effect of unsmoothing on higher risk moments is not unidirectional as some style indices exhibit an improvement with respect to higher risk moments while other style indices are characterized by a respective deterioration.[3]

In table 3 we evaluate the influence of unsmoothing on co-dependencies of sub-strategy returns. In this regard we make the same observation as *Glawischnig/Seidl* (2013). Precisely, in contrast to the standard deviation, unsmoothing reduces throughout the average (∅()) correlation

[3] As analytically demonstrated by *Getmansky* et al. (2004), smoothing returns doesn't impact their respective means meaning that theoretically mean returns in both parts of table 2 should be identical. The observed slight differences in annualized mean returns are caused by marginal rounding inaccuracies of our software. These rounding inaccuracies are amplified by performing the whole unsmoothing procedure twice and by annualizing daily mean returns.

Table 2

Summary Statistics for Smoothed (Reported) and Unsmoothed (Adjusted) Return Time Series for the Entire Sample Period (April 1st, 2003 to April 24th, 2014)

Smoothed Return Series

	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr
μ_{ann}	0.29 %	1.19 %	1.47 %	–1.75 %	4.16 %	–0.14 %
σ_{ann}	3.78 %	6.49 %	6.46 %	6.55 %	4.54 %	4.08 %
Skewness	–3.1284	–0.9724	–0.8027	–4.3232	1.7521	–0.1132
Kurtosis	44.9184	7.1918	5.5033	50.8676	71.0171	17.3122
SR_{ann}	0.0778	0.1840	0.2270	–	0.9173	–
J-B-Test	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010

Unsmoothed Return Time Series

	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr
μ_{ann}	0.28 %	1.21 %	1.60 %	–1.54 %	4.17 %	–0.29 %
σ_{ann}	5.51 %	7.78 %	7.73 %	12.42 %	3.79 %	3.89 %
Skewness	–3.0973	–0.8437	–0.8332	–2.9045	2.0628	–0.1005
Kurtosis	47.8976	6.7201	6.4993	36.2185	74.6284	17.1408
SR_{ann}	0.0509	0.1562	0.2076	–	1.1025	–
J-B-Test	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010

Legend: μ_{ann} = annualized mean; σ_{ann} = annualized standard deviation; SR_{ann} = annualized Sharpe Ratio; J-B-Test = Jarque-Bera-Test, p-value.

coefficient[4], as shown by the last two rows in table 3. Nevertheless, in portfolio context, higher standard deviations will overcompensate the risk reduction induced by lower average co-dependencies (*Chopra/Ziemba* (1993)) increasing overall portfolio risk in the unsmoothing case. For reasons stated above, the in-sample period in our empirical study is based on unsmoothed sub-strategy returns. In the following section, we outline the DD of *Vermorken* et al. (2012) and show how an investment strategy, which dynamically maximizes the DD, can be set up.

[4] Since the correlation coefficient between two time series is defined as their respective covariance divided by the product of each standard deviation, a *ceteris paribus* increase in standard deviations will lead to a reduction in the correlation coefficient.

Table 3

Correlation Analysis of Smoothed (Reported) and Unsmoothed (Adjusted) Time Series for the Entire Sample Period (April 1st, 2003 to April 24th, 2014)

(The bottom/upper triangle comprises correlation values for smoothed/unsmoothed returns. In the last two rows we calculate the average correlation values for each type of time series.)

Smoothed vs. Unsmoothed Time Series

	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr
Dstrss	–	0.0848	0.0797	0.146	-0.0142	0.0848
Macro	0.0977	–	0.2362	–0.0077	0.0812	0.1509
Eq Hedge	0.1175	0.2448	–	0.0382	0.4923	0.1521
Cnvrt Arb	0.2006	0.0001	0.0868	–	-0.0336	-0.0075
Mrg Arb	0.0142	0.0875	0.5098	-0.0003	–	0.0989
Mkt Ntr	0.1014	0.1555	0.1568	-0.0212	0.0871	–
∅ *(smoothed)*	0.1063	0.1171	0.2231	0.0532	0.1396	0.0959
∅ *(unsmoothed)*	0.0762	0.1091	0.1997	0.0271	0.1249	0.0958

III. Methodology

1. The Maximum Diversification Delta Portfolio

The DD of *Vermorken* et al. (2012) is grounded on the differential Shannon entropy (*Shannon* (1948)). For a random variable X, taking values $x \in X, X = \mathbb{R}$, the entropy H is defined as

(4)
$$H = -\int_x f(x) \log f(x) dx,$$

where $f(x)$ is the probability density function of X. Following *Vermorken* et al. (2012), we estimate the entropy of a given random variable with a nonparametric method known as k-d partitioning, which was introduced by *Stowell/Plumbley* (2009). The entropy measures the uncertainty of a given distribution irrespective of its location. As demonstrated by *Vermorken* et al. (2012), higher probability of tail events for a regarded return distribution will affect its entropy, whereas variance will be largely unaffected. Therefore, additional to measuring the second risk moment, the entropy also detects (penalizes) undesired high level of concentration around the tails, i.e. higher (even) risk moments, of the considered asset returns distribution.

Having defined the differential Shannon entropy, we now turn to the DD, which is based on this entropy measure. Let $X_1, X_2, \ldots, X_N$ be the time series vectors of risky assets and $\boldsymbol{w} = (w_1, w_2, \ldots, w_N)$ the respective portfolio weights vector, where $\sum_{i=1}^{N} w_N = 1$. With this in mind, the DD as a function of $\boldsymbol{w}$ can be defined as follows:

$$DD(\boldsymbol{w}) = \frac{\exp\left(\sum_{i=1}^{N} w_i H(X_i)\right) - exp\left(H\left(\sum_{i=1}^{N} w_i X_i\right)\right)}{\exp\left(\sum_{i=1}^{N} w_i H(X_i)\right)}, \text{ with } i = 1,\ldots,N$$

(5)

$$= \frac{exp\left(\overline{H(X)}\right) - exp\left(H(X_P)\right)}{exp\left(\overline{H(X)}\right)}$$

DD is the ratio of the weighted average entropy of asset return time series $\overline{H(X)}$ minus the entropy of the respective portfolio return time series $H(X_P)$, $X_P = \sum_{i=1}^{N} w_i X_i$. To put it differently, the DD measures the relative reduction in uncertainty, when forming a portfolio for a given set of assets, taking in account the entire asset return distributions. In this context, a reduced uncertainty of a considered portfolio return distribution equals an increased (improved) diversification of this portfolio. Furthermore, *Vermorken* et al. (2012) demonstrated that the DD worsens/improves with increasing/decreasing average correlation of the underlying assets and worsens/improves with decreasing/increasing amount of underlying portfolio constituents, thus, exhibiting desired characteristics of a diversification measure.

As mentioned above, one of the contributions of this paper is directly managing diversification of a portfolio with regard to the DD. For this purpose, we set up the following optimization problem, which leads to the MDDP (Maximum Diversification Delta Portfolio):

(6) $$\max_{\boldsymbol{w}} DD(\boldsymbol{w})$$

(7) $$s.t.: w_i \geq 0, \quad i = 1,\ldots,N$$

(8) $$\sum_{i=1}^{N} w_i = 1$$

In contrast to *Vermorken* et al. (2012), we not only quantify DDs of alternative investment strategies, but in addition define a new investment strategy that directly maximizes the DD for a given set of assets and their respective historical return series. Hence, in this regard we follow *Vermorken* et al. (2012) and perform an empirical DD estimation. For the sake of simplicity, we omit an estimation of the DD based on predicted discrete multivariate return distributions in our analysis and leave it to future research. Furthermore, the MDDP optimization problem comprises a constrained nonlinear maximization. Thus, the above optimization problem should be prone to the local minima problem. To contain this problem, we used the popular Matlab™ -routine *fmincon* for optimization. In particular, the initial optimization was performed 1000 times applying random starting solutions and selecting the optimization result (weights) characterized by the highest objective function value (DD). The optimization for the subsequent in-sample period uses optimized weights of the preceding in-sample period and additionally performs 100 random optimizations. Analogously, the best optimization result is selected and so on. This optimization methodology proved robust with respect to local minima and path dependence problems.

Note that the MDDP is primary a risk-based portfolio selection methodology as it relies solely on risk moments of given return distributions for portfolio optimization. That's why, with regard to its original definition the MDDP only fits investors who are not interested in integrating mean estimators into their portfolio selection process. Alternatively, this issue can be solved by integrating the DD in a reward-risk-framework leading to DD-efficient portfolios for a given expected portfolio return constraint. To be specific, DD-efficient portfolios in the spirit of *Markowitz* (1952) can be determined by maximizing DD for a required portfolio return $\left(\mu_P^*\right)$ and in this way construct the mean-DD-efficient frontier. In this regard, we can directly find the investor-specific portfolio on the DD-efficient frontier by maximizing the following reward-risk-trade-off:

(9) $$\max_{\boldsymbol{w}} \lambda \cdot \mu_P(\boldsymbol{w}) + (1-\lambda) \cdot DD(\boldsymbol{w}), \qquad \lambda = [0,1],$$

where $\mu_P(\boldsymbol{w})$ denotes the expected portfolio return, which is a product of expected asset returns and the respective asset weights. λ represents the investor-specific reward-risk-trade-off parameter. For small λ values, diversification (i.e. risk reduction) is the main concern. On the other side, when λ approaches one, the investor focuses increasingly on porfolio's expected return. As we are interested in the empirical characteristics of the (global) MDDP, our empirical study solely concentrates on the risk-based optimization problem described by equation 6. The analysis of empirical characteristics of DD-efficient portfolios considering expected portfolio returns is left to future research. Further, in order to keep our optimization routine tractable, we introduce a long-only and a full-investment constraint when constructing the MDDP (see constraint 7 and constraint 8)

2. *Competing Portfolio Optimization Methods*

The performance of MDDP is compared with three alternative portfolio selection approaches. Since the MDDP is basically a risk-based investment strategy, adequate competing portfolio optimization methods should also be risk-based. In this regard, the first competitor is the simple 1/N-P. Assuming a very crude covariance matrix estimate, namely equal variance and equal covariances for all assets, the 1/N-P represents a risk-efficient portfolio structure. *DeMigue* et al. (2009) demonstrated via an out-of-sample study the superiority of the 1/N-P over established optimization methods in the presence of significant estimation errors. Moreover, we implement the classical MVP by solving the following optimization problem:

$$\begin{aligned} &\min_{\boldsymbol{w}} \sigma_P \\ s.t.:\ & w_i \geq 0, \quad i = 1,\dots,N \\ &\sum_{i=1}^{N} w_i = 1 \end{aligned} \tag{10}$$

Whereby σ_P represents the portfolio volatility and is more precisely defined as: $\sigma_P = \sqrt{\boldsymbol{w}'\boldsymbol{V}\boldsymbol{w}}$. In this equation $\boldsymbol{w}$ and $\boldsymbol{V}$ stand for the column weights vector and the empirically estimated covariance matrix, respectively. Referring to the study of *DeMiguel* et al. (2009), *Kritzman* et al. (2010) proved the out-of-sample superiority of MVP and other well established portfolio optimization methods over the simple 1/N-P. To be specific, *Kritzman* et al. (2010) replicated the investigation of *DeMiguel* et al. (2009), using significantly longer estimation (in-sample) periods of 10 to 20 years and thereby containing estimation errors. Given these results, *Kritzman* et al. (2010) concluded that an optimized asset allocation is strongly preferable to a simple 1/N-strategy if the degree of estimation errors is at most moderate.

Finally, we implement the popular ERCP (Equal Risk Contribution Portfolio) as our last competing portfolio optimization method. Basically, the ERCP is a risk-based extension of the 1/N-principle. Simply stated, the ERCP seeks a portfolio structure that equates absolute contributions of underlying assets to portfolio volatility. In order to provide an analytical foundation of the ERCP-concept, we need to start with the definition of a marginal risk contribution (MRC) of a considered i-th asset to portfolio volatility:[5]

$$MRC_i = \frac{\delta \sigma_P}{\delta w_i} = \frac{w_i \sigma_i^2 + \sum_{j \neq i} w_j \sigma_{ij}}{\sigma_P}, \tag{11}$$

whereby σ_{ij} is the covariance between the i-th and j-th asset. The MRC_i is analytically defined as the partial derivative of portfolio volatility with respect to the weight of the i-th asset (w_i). Simply speaking, the MRC_i specifies by how much portfolio volatility changes when the weight of the i-th asset is infinitesimally varied. In order to calculate MRC for all portfolio constitutes, equation 11 needs to be solved for each asset. Alternatively, the column vector of MRC ($\boldsymbol{MRC}$) with respect to all considered portfolio assets is easily obtained by:

$$\boldsymbol{MRC} = \frac{\boldsymbol{Vw}}{\sigma_P} \tag{12}$$

The calculated MRC is the starting point for the calculation of the absolute risk contribution (ARC). As its name implies, the ARC defines how much a regarded asset contributes to portfolio volatility in absolute terms. Analytically, the ARC of the i-th asset is given by:

$$ARC_i = w_i \cdot MRC_i \tag{13}$$

Note that consequently the entire portfolio volatility can be split in ARC of the underlying assets, meaning that the following relation must hold:

$$\sigma_P = \sum_i ARC_i \tag{14}$$

Given the ARC, the ERCP is aiming for an asset allocation that equates all ARC. This can be achieved by minimizing the squared sum of all ARC-differences. For this purpose, *Maillard* et al. (2010) defined the following optimization problem:

$$\min_{\boldsymbol{w}} \sum_i \sum_j \left(ARC_i - ARC_j \right)^2 \tag{15}$$

[5] The following analytical foundation of the ERCP was provided by *Maillard* et al. (2010). Additionally, compact discussions on the ERCP are given by *Poddig/Unger* (2012) and *Baitinger* (2014).

Due to the quadratic structure of this optimization problem, its minimum will lie at zero. In this paper we dynamically solve this optimization problem for each in-sample period, in order to construct the ERCP investment strategy. For the sake of consistency, constraints 7 and 8 also apply to the ERCP-optimization. In this context, the short-sale constraint is anyway mostly satisfied by the ERCP as most assets are characterized by a significant positive ARC to portfolio volatility. In analogy to the MDDP, the ERCP is primary a risk-based portfolio optimization method and therefore qualifies as fair competitor. Moreover, the ERCP is very popular in professional asset management domains as can be witnessed by numerous professional papers on this topic, see for example *Qian* (2006), *Allen* (2010) and *Levell* (2010). Finally, note that all of the competing non-naive portfolio selection methods (MVP+ERCP) solely rely on the second (co-)moment ((co-)variance) of return distributions for a risk-based portfolio optimization. In contrast to these investment strategies, the MDDP additionally takes in account higher risk moments of return distributions and should therefore lead to portfolio structures that exhibit superior crisis-period performance. Following, we employ the MDDP in the context of an empirical out-of-sample study and evaluate its performance relative to the 1/N-P, MVP and ERCP.

IV. Empirical Study

1. General Setup

As described above, our in-sample dataset consists of unsmoothed daily returns for six hedge fund style indices. In order to pay attention to the fact that in a realistic setting the highest possible rebalancing frequency of a fund of hedge funds would be on a monthly basis, we optimize the asset allocation for various rebalancing frequencies starting at a monthly and ending up in a semi-annual rebalancing/optimization. Further, we employ a rolling window estimation approach, in order to create investment strategies that flexibly react to possible changes in the underlying data generating process. As we also intend to analyze the impact of variations in the length of the in-sample period on the performance of implemented investment strategies, we perform our empirical study using in-sample periods of 6 to 36 months, whereby we increase the in-sample-period length by 6-months steps. For the sake of brevity, we only present performance statistics resulting from a 6- and 36-months in-sample period.[6] Note that even though we consider our dataset from a monthly perspective, the optimization is performed on the underlying (business-)daily HF returns. Thus, the 6-months and the 36-months in-sample period comprises on average around 126 ($6 \cdot 21 = 126$) and 756 ($36 \cdot 21 = 756$) data points, respectively. Finally, due to reasons of consistency, the out-of-sample period is determined on the basis of the maximum in-sample period length of 36-months and hence spans from April 3rd, 2006 to April 24th, 2014 resulting in around 97 months[7] of daily HF returns.

2. *Average Holdings and Turnover Analysis*

In this section we investigate holdings and turnover data of implemented investment strategies for the considered out-of-sample period. Table 4 shows the respective results assuming a rolling estimation period of six months and various rebalancing frequencies. For the sake of clarity, we solely focus on a monthly (1m = 1 month), quarterly (3m) and semi-annually (6m) rebalancing frequency. Starting with the average holdings weights, all optimization based methods minimize their exposure to the Equity Hedge strategy. With regard to the summary statistics of return time series from table 2, this observation is somewhat surprising since the Equity Hedge strategy exhibits the second highest Sharpe Ratio. On the other side, the correlation analysis in table 3 makes clear that Equity Hedge is in fact a relatively unattractive asset from a diversification point of view. Specifically, Equity Hedge is characterized by the highest average correlation with other assets. Additionally, it has a relatively high volatility. In sum, all these negative risk characteristics lead to its avoidance by risk-based portfolio selection approaches. Furthermore, table 4 reveals that with regard to average holdings, the MVP-strategy is somewhat concentrated making it less attractive for practical purposes.

The last two columns of table 4 explore turnover characteristics of the respective investment strategies. As expected the 1/N-P possesses the lowest total and average turnover volumes. Interestingly, the MDDP displays the most unattractive turnover characteristics, followed by the

[6] Detailed study results can be requested from the authors.

[7] When downloading the HF returns the month April 2014 was not over. For reasons of simplicity, we assume that April 24th, 2014 is the last trading day of this month and hence the respective month is complete.

Table 4

Average Holdings and Turnover Data for Various Rebalancing Frequencies Using a 6-Months In-Sample Period

(Out-of-sample period: April 3rd, 2006 to April 24th, 2014)

Rebalancing Frequency = 1m

Optimization Method	Average Holdings						Total Turnover	Turnover/Trade
	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr		
MDDP	20.60 %	13.79 %	8.59 %	21.74 %	17.65 %	17.62 %	5523.56 %	56.94 %
ERCP	18.45 %	12.50 %	7.79 %	20.31 %	22.87 %	18.08 %	1433.53 %	14.78 %
MVP	17.78 %	9.34 %	2.10 %	24.30 %	28.71 %	17.77 %	1680.76 %	17.33 %
1/N-P	16.66 %	16.67 %	16.66 %	16.65 %	16.69 %	16.66 %	124.74 %	1.29 %

Rebalancing Frequency = 3m

Optimization Method	Average Holdings						Total Turnover	Turnover/Trade
	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr		
MDDP	19.23 %	12.04 %	9.71 %	21.21 %	21.15 %	16.66 %	2016.64 %	63.02 %
ERCP	18.22 %	11.62 %	8.22 %	20.30 %	24.32 %	17.32 %	968.50 %	30.27 %
MVP	19.07 %	8.40 %	2.16 %	22.38 %	31.59 %	16.40 %	1351.07 %	42.22 %
1/N-P	16.65 %	16.67 %	16.66 %	16.63 %	16.74 %	16.66 %	81.96 %	2.56 %

Rebalancing Frequency = 6m

Optimization Method	Average Holdings						Total Turnover	Turnover/Trade
	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr		
MDDP	17.82 %	14.08 %	8.31 %	25.22 %	17.80 %	16.77 %	1240.60 %	77.54 %
ERCP	18.84 %	12.92 %	7.74 %	21.50 %	22.26 %	16.74 %	647.78 %	40.49 %
MVP	22.71 %	12.00 %	1.76 %	24.02 %	26.21 %	13.31 %	1087.22 %	67.95 %
1/N-P	16.53 %	16.74 %	16.67 %	16.49 %	16.85 %	16.72 %	62.52 %	3.91 %

MVP-strategy, especially when a rebalancing frequency of 1 month is applied. In this respect, a rebalancing frequency of at least 3 months is preferable as it significantly reduces total turnover and hence transaction related costs. Regarding the average turnover per rebalancing activity (trade), it increases with the decrease of rebalancing frequency. This finding grounds on two facts. Firstly, decreasing the rebalancing frequency equates to an increase in the holding period. This in turns increases the impact of different asset value developments on the portfolio struc-

Table 5

Average Holdings and Turnover Data for Various Rebalancing Frequencies Using a 36-Months In-Sample Period

(Out-of-sample period: April 3rd, 2006 to April 24th, 2014)

Rebalancing Frequency = 1

Optimization Method	Average Holdings						Total Turnover	Turnover/Trade
	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr		
MDDP	19.80 %	17.79 %	9.04 %	20.65 %	11.64 %	21.07 %	1821.94 %	18.78 %
ERCP	17.98 %	12.65 %	9.12 %	16.47 %	21.66 %	22.13 %	278.95 %	2.88 %
MVP	16.71 %	7.66 %	0.29 %	16.08 %	33.22 %	26.03 %	337.08 %	3.48 %
1/N-P	16.66 %	16.67 %	16.66 %	16.65 %	16.69 %	16.66 %	124.74 %	1.29 %

Rebalancing Frequency = 3

Optimization Method	Average Holdings						Total Turnover	Turnover/Trade
	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr		
MDDP	21.12 %	16.88 %	9.44 %	20.18 %	11.07 %	21.30 %	879.77 %	27.49 %
ERCP	18.03 %	12.53 %	9.17 %	16.28 %	21.77 %	22.22 %	203.46 %	6.36 %
MVP	18.80 %	6.08 %	0.46 %	13.63 %	34.27 %	26.76 %	240.62 %	7.52 %
1/N-P	16.65 %	16.67 %	16.66 %	16.63 %	16.74 %	16.66 %	81.96 %	2.56 %

Rebalancing Frequency = 6

Optimization Method	Average Holdings						Total Turnover	Turnover/Trade
	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr		
MDDP	20.70 %	16.79 %	8.83 %	19.82 %	12.11 %	21.75 %	474.23 %	29.64 %
ERCP	17.92 %	12.47 %	9.23 %	16.24 %	21.83 %	22.31 %	167.31 %	10.46 %
MVP	16.23 %	6.01 %	0.58 %	14.66 %	32.50 %	30.01 %	167.12 %	10.44 %
1/N-P	16.53 %	16.74 %	16.67 %	16.49 %	16.85 %	16.72 %	62.52 %	3.91 %

ture meaning that relative portfolio weights on average change more over time when the holding period is extended. Secondly, a lower rebalancing frequency in combination with a rolling estimation scheme increases the data differences between considered in-sample periods and their preceding counterparts. This fact also induces additional turnover volumes.

In table 5 we repeat the above study using a longer in-sample period of 36 months. With regard to average asset holdings the same observations and conclusions as in table 4 apply to the 36-months estimation period. Additionally, table 5 reveals that total turnover as well as average turnover per trade is greatly reduced when using a longer in-sample period. Using longer in-sample periods decreases data differences between timely adjunct estimation samples. From a theoretical point of view, this fact should tendentially reduce turnover volumes. Summing up, the MDDP exhibits the most unattractive turnover characteristics. One possible explanation for this observation is the reliance of the MDDP-approach on the entire multivariate return distribution and by this its partial sensitivity to distributional tails (tail-events) which by definition occur in an irregular fashion. On the other side, the respective turnover volumes can be greatly decreased by reducing rebalancing frequencies or/and increasing the length of the estimation sample. In addition, turnover volumes can be contained by introducing a volume penalty parameter in the respective objective function. This modification of the MDDP is left to future research.

3. *Performance Analysis Using a 14-Months Crisis-Period*

As our out-of-sample data spans from 3rd, 2006 to April 24th, 2014 and therefore covers the turbulent financial crisis period, we use this opportunity to evaluate the performance of implemented investment strategies with respect to different subperiods. In particular, we define a disjoint crisis- and non-crisis-period. We choose the following methodology for construction of the crisis-period: Firstly, we calculate for each HF style index and month the respective volatility which is based on the underlying daily returns. In a second step, we calculate a simple average of volatilities resulting from step one over the six HF style indices. Finally, those out-of-sample months, which are characterized by top x-percentage volatilities with regard to the volatility vector from step two, are declared crisis-period. Alternatively, a crisis-period could be conditioned on the Lehman Brothers insolvency. We prefer the volatility-based approach as it leads to an extremely adverse crisis-period.[8] In this regard, the non-crisis-period is just the disjoint counterpart of the crisis-period. For the first empirical study we define out-of-sample months, which belong to

[8] Furthermore, we additionally performed our complete empirical study using a crisis-period determination, which is conditioned on the Lehman Brothers insolvency. The main results of this study correspond with results from a volatility-based determination of the crisis-period. The respective detailed study results can be requested from the authors.

the top 15 % volatility group, as the crisis-period. Given the fact that the out-of-sample period comprises 97 months, the crisis-period consists of 14 months[9]. Moreover, we start by implementing a rolling in-sample period of six months. This estimation period will be subsequently extended to 36 months. Finally, for the sake of brevity the following discussions solely focus on results for an assumed rebalancing frequency of 3 months.[10]

In table 6 we present summary statistics for the six HF style indices with respect to the defined subperiods. Table 6 demonstrates that even though volatility is a two-sided risk measure, the volatility-based approach yields a rather extreme crisis-period, especially in terms of the first two distributional moments. Specifically, compared to the non-crisis-period, the crisis period is characterized by extremely negative mean returns and twice as much volatility. Further, all three subperiods, namely crisis-, non-crisis- and total-oos-period, which is just the disjoint union of the two preceding subperiods, exhibit significant non-normality. This fact, should favor the MDDP-approach over methodologies that neglect higher risk moments altogether, like the MVP and the ERCP.

Using the above subperiod division we evaluate the performance of the respective optimization methods in table 6. Besides the "traditional" performance measures, we quantify the certainty equivalent (CE) of the considered return distributions of implemented investment strategies assuming the following power utility function (e.g. as in *Danthine/Donaldson* (2005)):

$$U(Y) = \frac{Y^{1-\gamma}}{1-\gamma}, \qquad \gamma > 1, \tag{16}$$

where γ is the risk aversion parameter and Y the considered wealth level. Given an investor with this utility function and γ-parameter, we find a certain annualized return $\left(CE(\gamma)_{ann}\right)$ at which the investor is indifferent between this return and the uncertain payoff-structure (lottery) of the considered investment strategy. To put it more specifically, firstly, we calculate the expected utility for the considered out-of-sample investment strategy returns assuming equal likelihood of occurrence. After that, we find a certain return equivalent that offers the same amount of utility. The advantage of the CE relative to more conventional risk-adjusted per-

[9] $97 \cdot 0.15 = 14.55$. We rounded the number downwards.

[10] Detailed study results, assuming alternative rebalancing frequencies and estimation periods, can be requested from the authors.

Table 6

Out-Of-Sample Period Summary Statistics for the Underlying Hedge Fund Style Indices Using a Crisis-Period that Comprises 14 High-Volatility Months

(Out-of-sample period: April 3rd, 2006 to April 24th, 2014)

Crisis-Period

	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr	Average
μ_{ann}	–33.91 %	–14.57 %	–29.69 %	–51.83 %	–1.35 %	–4.08 %	–22.57 %
σ_{ann}	6.94 %	11.44 %	12.07 %	15.68 %	10.97 %	7.93 %	10.84 %
Skewness	–3.4991	–0.9058	–0.4069	–2.1590	1.3464	–0.1273	–0.9586
Kurtosis	29.3551	5.9934	4.5777	12.1311	20.1483	12.0320	14.0396
SR_{ann}	–	–	–	–	–	–	–
J-B-Test	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010

Non-Crisis-Period

	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr	Average
μ_{ann}	4.12 %	1.64 %	5.33 %	10.35 %	4.83 %	0.65 %	4.49 %
σ_{ann}	3.24 %	5.69 %	5.67 %	4.19 %	3.01 %	3.59 %	4.23 %
Skewness	–0.5230	–0.6154	–0.3451	0.4194	–0.6313	0.1122	–0.2639
Kurtosis	13.8304	8.2158	4.4326	6.3139	8.0741	4.5861	7.5755
SR_{ann}	1.2705	0.2889	0.9403	2.4696	1.6029	0.1814	1.0602
J-B-Test	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010

Total-OOS-Period

	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr	Average
μ_{ann}	–2.60 %	–0.92 %	–0.74 %	–2.29 %	3.89 %	–0.06 %	–0.45 %
σ_{ann}	4.13 %	6.86 %	7.05 %	7.38 %	5.04 %	4.50 %	5.83 %
Skewness	–3.2512	–1.0452	–0.7705	–4.1183	1.7956	–0.1170	–1.2511
Kurtosis	45.3018	10.5118	7.9814	45.6420	66.6137	18.6948	32.4576
SR_{ann}	–	–	–	–	0.7725	–	–
J-B-Test	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010

formance measures (e.g. Sharpe Ratio) is that it provides a risk-adjusted return measure, whereby risk is defined in terms of all risk moments and not just with respect to volatility. Further, by increasing the risk aversion parameter we can put greater emphasis on higher risk moments. Finally, the popular Sharpe Ratio is not reasonably defined for negative mean returns. On the other hand, the CE is defined for any return distribution irrespective of its location, i.e. mean parameter. That's why, we choose the CE for our statistical inference analysis.

As the recent inference analysis literature prefers the more robust bootstrap-based methods (see for example *Vinod/Morey* (1999), *Nankervis* (2005), *Ledoit/Wolf* (2008) and *Scherer* (2009)), we implement a bootstrap-based hypothesis testing methodology in the style of *Ledoit/Wolf* (2008). Specifically, we test the following null hypothesis:

$$H_0\colon CE(\gamma)_{ann}^{MDDP} - CE(\gamma)_{ann}^{alternative} = 0, \tag{17}$$

whereby the first term is always the CE of the MDDP for a considered γ-parameter and the second term is the CE of an alternative competing investment strategy, i.e. ERCP, MVP or 1/N-P. Broadly speaking, the implemented bootstrap-based methodology doesn't assume a distribution of the test statistic, it rather constructs the respective distribution from empirical data via bootstrapping. Further, in analogy to *Ledoit/Wolf* (2008) we studentize the bootstrapped test statistic, as it yields more precise asymptotic properties.[11] Table 7 presents the p-values of this test in brackets under the respective CE value.

Analyzing the crisis-period study results in table 7 reveals a superior performance of the MDDP, especially with respect to the mean and the variance. To be more precise, the MDDP exhibits the highest mean return and the second lowest volatility. Moreover, looking at the CE and the respective p-values, the outperformance of the MDDP is throughout statistically significant. In this context the MDDP is statistically significant superior to the ERCP, MVP and 1/N-P assuming a significance level of 1 %, 10 % and 5 %, respectively. Looking at the non-crisis period somewhat reverses previous findings. Particularly, in terms of mean return the MDDP only surpasses the MVP and exhibits the second highest volatility. The respective CE measures and p-values reveal that the performance of the first three investment strategies lies in a narrow corridor, resulting in the fact that the corresponding performance differences are not statistically significant. On the other side, the simple 1/N-P is statistically significant superior to MDDP at a significance level of 5 %. This superior

[11] For reasons of brevity, we mostly skip the analytical details of this approach. A compressed discussion on this bootstrap-based hypothesis testing methodology is provided by *Baitinger* (2014).

Table 7

Out-Of-Sample Performance Results Using a 6-Months In-Sample Period, a 3-Months Rebalancing Frequency and a Crisis-Period that Comprises 14 High-Volatility Months

(Out-of-sample period: April 3rd, 2006 to April 24th, 2014)

Crisis-Period

	μ_{ann}	σ_{ann}	Skewness	Kurtosis	SR_{ann}	$CE(1)_{ann}$	$CE(2)_{ann}$	$CE(10)_{ann}$
MDDP	–18.53 %	5.10 %	0.2102	18.8229	–	–18.74 %	–19.06 %	–19.58 %
ERCP	–21.90 %	4.94 %	–0.5180	10.5721	–	–22.09 %	–22.38 %	–22.85 %
						(0.0080)	*(0.0085)*	*(0.0094)*
MVP	–22.11 %	5.66 %	–0.0548	13.0332	–	–22.36 %	–22.73 %	–23.35 %
						(0.0643)	*(0.0617)*	*(0.0538)*
1/N-P	–23.44 %	5.48 %	–0.9106	5.7530	–	–23.67 %	–24.02 %	–24.60 %
						(0.0237)	*(0.0207)*	*(0.0185)*

Non-Crisis-Period

	μ_{ann}	σ_{ann}	Skewness	Kurtosis	SR_{ann}	$CE(1)_{ann}$	$CE(2)_{ann}$	$CE(10)_{ann}$
MDDP	3.31 %	2.10 %	–0.2045	4.4521	1.5748	3.27 %	3.20 %	3.09 %
ERCP	3.39 %	1.95 %	–0.4540	4.8755	1.7427	3.35 %	3.29 %	3.19 %
						(0.8110)	*(0.7874)*	*(0.7420)*
MVP	2.97 %	2.08 %	–0.6725	7.1811	1.4268	2.92 %	2.86 %	2.75 %
						(0.5275)	*(0.5345)*	*(0.5350)*
1/N-P	4.50 %	2.24 %	–0.4344	4.5969	2.0069	4.45 %	4.37 %	4.24 %
						(0.0116)	*(0.0121)*	*(0.0156)*

Total-OOS-Period

	μ_{ann}	σ_{ann}	Skewness	Kurtosis	SR_{ann}	$CE(1)_{ann}$	$CE(2)_{ann}$	$CE(10)_{ann}$
MDDP	–0.23 %	2.81 %	–0.4526	31.5457	–0.0815	–0.31 %	–0.43 %	–0.62 %
ERCP	–0.78 %	2.68 %	–1.3569	20.9299	–0.2917	–0.85 %	–0.96 %	–1.14 %
						(0.1264)	*(0.1285)*	*(0.1525)*
MVP	–1.17 %	2.96 %	–0.9781	27.7738	–0.3938	–1.25 %	–1.38 %	–1.60 %
						(0.0945)	*(0.0924)*	*(0.0893)*
1/N-P	–0.17 %	3.03 %	–1.6295	12.8275	–0.0553	–0.26 %	–0.40 %	–0.63 %
						(0.9295)	*(0.9624)*	*(0.9880)*

How to read the p-values in brakets: For example, consider the $CE(10)_{ann}$ for the ERCP-strategy in the upper third of the table, i.e. 22.09 %. The p-value underneath this number belongs to the test of the following null hypothesis:
H_0: $CE(1)_{ann}^{MDDP} - CE(1)_{ann}^{MRCP} = 0$. The respective bootstrapped p-value of 0.0080 indicates that the difference between the CEs is statistically significant at a significance level of 1 %, meaning that the CE of the MDDP strategy, given an assumed risk aversion of $\gamma = 1$, is statistically significant higher.

Table 8

Out-Of-Sample Performance Results Using a 36-Months In-Sample Period, a 3-Months Rebalancing Frequency and a Crisis-Period that Comprises 14 High-Volatility Months

(Out-of-sample period: April 3rd, 2006 to April 24th, 2014)

	μ_{ann}	σ_{ann}	Skewness	Kurtosis	SR_{ann}	$CE(1)_{ann}$	$CE(2)_{ann}$	$CE(10)_{ann}$
Crisis-Period								
MDDP	–29.05 %	5.46 %	–1.5717	9.3588	–	–29.26 %	–29.58 %	–30.12 %
ERCP	–25.94 %	5.40 %	–1.1827	7.7214	–	–26.16 %	–26.48 %	–27.03 %
						(0.0014)	*(0.0001)*	*(0.0011)*
MVP	–26.36 %	5.74 %	–1.2485	8.6455	–	–26.60 %	–26.97 %	–27.59 %
						(0.0191)	*(0.0172)*	*(0.0202)*
1/N-P	–23.44 %	5.48 %	–0.9106	5.7530	–	–23.67 %	–24.02 %	–24.60 %
						(0.0033)	*(0.0030)*	*(0.0033)*
Non-Crisis-Period								
MDDP	3.37 %	2.27 %	–0.1982	4.4439	1.4846	3.32 %	3.24 %	3.10 %
ERCP	3.48 %	2.09 %	–0.3255	4.4055	1.6677	3.44 %	3.37 %	3.26 %
						(0.7017)	*(0.6667)*	*(0.6239)*
MVP	3.11 %	2.05 %	–0.3550	5.8968	1.5167	3.07 %	3.01 %	2.90 %
						(0.6358)	*(0.6419)*	*(0.6816)*
1/N-P	4.50 %	2.24 %	–0.4344	4.5969	2.0069	4.45 %	4.37 %	4.24 %
						(0.0090)	*(0.0086)*	*(0.0101)*
Total-OOS-Period								
MDDP	–2.19 %	3.08 %	–2.1502	19.3303	–0.7096	–2.28 %	–2.42 %	–2.66 %
ERCP	–1.48 %	2.93 %	–2.0123	17.9588	–0.5055	–1.56 %	–1.69 %	–1.91 %
						(0.0218)	*(0.0158)*	*(0.0155)*
MVP	–1.86 %	3.00 %	–2.2642	22.4129	–0.6197	–1.95 %	–2.08 %	–2.31 %
						(0.4755)	*(0.4659)*	*(0.4494)*
1/N-P	–0.17 %	3.03 %	–1.6295	12.8275	–0.0553	–0.26 %	–0.40 %	–0.63 %
						(0.0002)	*(0.0002)*	*(0.0002)*

performance of the 1/N-P in the non-crisis period results in its superiority in the total out-of-sample period (crisis-, + non-crisis-period) closely tracked by the MDDP, as the performance difference between 1/N-P and MDDP is statistically indistinguishable. Summing up, the MDDP demonstrates a clear outperformance relative to competing strategies in the crisis-period, whereas its performance in the non-crisis-period is somewhat mediocre. Further, with respect to the entire out-of-sample the 1/N-P exhibits a close lead over MDDP. On the other hand, with regard to our data sample the 1/N-P clearly prooved inadequate in a stress phase.

Following, we repeat the above calculations assuming an in-sample-period of 36 months. The results are shown in table 8. Interestingly, an extension of the in-sample period strongly deteriorates the performance of all optimization-based methods, whereby the decline in performance of the MDDP is most pronounced. The MDDP even statistically significantly underperforms competing optimization methodologies in the crisis-period. A possible explanation for this phenomenon lies in a decreased flexibility ("speed of reaction") of optimization based investment strategies to sudden stress scenarios when the in-sample size is increased.

4. Performance Analysis Using a 24-Months Crisis-Period

In this part of the article we repeat the empirical study from the above section with an extended crisis-period. By doing this, we aim to analyze the sensitivity of our previous results with respects to changed data partition. Particularly, we define out-of-sample months, which belong to the top 25 % volatility group, as the crisis-period resulting in (rounded) 24 high-volatility months.

Table 9 presents summary statistics for each subperiod and HF style index. Compared to the 14-months crisis period, the 24-months crisis period is less severe as it doesn't focus on the most adverse months. Further, it is on average characterized by a more favorable non-crisis period.

Table 9

Out-Of-Sample Period Summary Statistics for the Underlying Hedge Fund Style Indices Using a Crisis-Period that Comprises 24 High-Volatility Months

(Out-of-sample period: April 3rd, 2006 to April 24th, 2014)

Crisis-Period

	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr	Average
μ_{ann}	–25.92 %	–13.31 %	–21.84 %	–31.30 %	–1.51 %	–3.20 %	–16.18 %
σ_{ann}	6.19 %	10.15 %	10.46 %	12.81 %	8.92 %	6.71 %	9.21 %
Skewness	–3.3215	–1.0346	–0.5212	–2.6971	1.4095	–0.1527	–1.0529
Kurtosis	29.5710	7.1466	5.2930	17.9665	27.3093	14.0501	16.8894
SR_{ann}	–	–	–	–	–	–	–
J-B-Test	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010

Non-Crisis-Period

	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr	Average
μ_{ann}	6.70 %	3.59 %	7.49 %	9.87 %	5.76 %	1.01 %	5.74 %
σ_{ann}	2.95 %	5.29 %	5.36 %	3.98 %	2.71 %	3.45 %	3.96 %
Skewness	0.3512	–0.1658	–0.2451	0.1709	–0.0900	0.1917	0.0355
Kurtosis	14.4649	5.4843	4.1847	5.1534	5.4154	4.7163	6.5699
SR_{ann}	2.2731	0.6790	1.3972	2.4822	2.1263	0.2926	1.4502
J-B-Test	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010

Total-OOS-Period

	Dstrss	Macro	Eq Hedge	Cnvrt Arb	Mrg Arb	Mkt Ntr	Average
μ_{ann}	–2.60 %	–0.92 %	–0.74 %	–2.29 %	3.89 %	–0.06 %	–0.45 %
σ_{ann}	4.13 %	6.86 %	7.05 %	7.38 %	5.04 %	4.50 %	5.83 %
Skewness	–3.2512	–1.0452	–0.7705	–4.1183	1.7956	–0.1170	–1.2511
Kurtosis	45.3018	10.5118	7.9814	45.6420	66.6137	18.6948	32.4576
SR_{ann}	–	–	–	–	0.7725	–	–
J-B-Test	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010

The out-of-sample results in table 10 for a 6-months in-sample period are by and large in line with above findings. To be specific, the MDDP statistically significantly outperforms alternative investment strategies in the extended crisis period and performs relatively mediocre in the shortened non-crisis-period making the MDDP the second best performer for the entire out-of-sample period. Summarizing, the results at hand confirm so far that the analogous findings from table 10 are not caused by a specific data partition setup. Similarly, the performance data in ta-

Table 10

Out-Of-Sample Performance Results Using a 6-Months In-Sample Period and a Crisis-Period that Comprises 24 High-Volatility Months

(Out-of-sample period: April 3rd, 2006 to April 24th, 2014)

Crisis-Period								
	μ_{ann}	σ_{ann}	Skewness	Kurtosis	SR_{ann}	$CE(1)_{ann}$	$CE(2)_{ann}$	$CE(10)_{ann}$
MDDP	–13.20 %	4.26 %	0.0463	22.8801	–3.0994	–13.35 %	–13.59 %	–13.98 %
ERCP	–15.56 %	4.16 %	–0.7677	12.8315	–3.7378	–15.70 %	–15.92 %	–16.29 %
						(0.0103)	*(0.0123)*	*(0.0114)*
MVP	–15.69 %	4.78 %	–0.3180	15.4377	–3.2849	–15.89 %	–16.18 %	–16.66 %
						(0.0831)	*(0.0719)*	*(0.0669)*
1/N–P	–16.00 %	4.61 %	–1.2004	7.5228	–3.4739	–16.18 %	–16.45 %	–16.90 %
						(0.0683)	*(0.0616)*	*(0.0551)*
Non-Crisis-Period								
	μ_{ann}	σ_{ann}	Skewness	Kurtosis	SR_{ann}	$CE(1)_{ann}$	$CE(2)_{ann}$	$CE(10)_{ann}$
MDDP	4.51 %	2.03 %	–0.2393	4.7300	2.2223	4.47 %	4.40 %	4.29 %
ERCP	4.69 %	1.84 %	–0.3238	4.7570	2.5561	4.66 %	4.60 %	4.52 %
						(0.5674)	*(0.5539)*	*(0.5074)*
MVP	4.21 %	1.91 %	–0.4033	6.3127	2.2020	4.17 %	4.11 %	4.02 %
						(0.5834)	*(0.6067)*	*(0.6079)*
1/N-P	5.74 %	2.16 %	–0.2935	4.3540	2.6640	5.69 %	5.62 %	5.50 %
						(0.0084)	*(0.0081)*	*(0.0075)*
Total-OSS-Period								
	μ_{ann}	σ_{ann}	Skewness	Kurtosis	SR_{ann}	$CE(1)_{ann}$	$CE(2)_{ann}$	$CE(10)_{ann}$
MDDP	–0.23 %	2.81 %	–0.4526	31.5457	–0.0815	–0.31 %	–0.43 %	–0.62 %
ERCP	–0.78 %	2.68 %	–1.3569	20.9299	–0.2917	–0.85 %	–0.96 %	–1.14 %
						(0.1213)	*(0.1344)*	*(0.1384)*
MVP	–1.17 %	2.96 %	–0.9781	27.7738	–0.3938	–1.25 %	–1.38 %	–1.60 %
						(0.0982)	*(0.0950)*	*(0.0887)*
1/N-P	–0.17 %	3.03 %	–1.6295	12.8275	–0.0553	–0.26 %	–0.40 %	–0.63 %
						(0.9314)	*(0.9635)*	*(0.9980)*

ble 11 also complies with previous findings as the performance of all optimization based methods drastically deteriorates with an increase of the in-sample period. On one hand, the extension of the estimation period should improve the performance of optimization based investment strategies as it reduces estimation uncertainty. On the other hand, the same extension of the in-sample period creates a slowly adapting optimization methodology which especially reacts inadequately to sudden stress scenarios like the subprime financial crisis. The latter effect seems to dominate in the regarded dataset.

V. Conclusion

In this article we propose a new approach to portfolio diversification management. Specifically, we show how the DD of *Vermorken* et al. (2012), which represents a higher moment diversification measure, can be incorporated within a portfolio optimization leading to the MDDP. In order to demonstrate the advantage of the additional consideration of higher moments within portfolio formation, we used hedge fund style returns in the empirical study, as they exhibit significant non-normalities. Given the fact that hedge fund returns are prone to exhibit serial correlation, which distort the true riskiness of the underlying data generating process, we unsmooth the dataset prior to our empirical study. In the empirical study we show that with regard to a stress period the MDDP asset allocation significantly outperforms alternative investment strategies given an in-sample period of six months. Strikingly, in our setting, increasing the estimation period to 36 months leads to a distinct performance deterioration of optimization-based methods. This observation can be traced back to reduced robustness of optimization-based investment strategies with respect to sudden stress scenarios when the estimation period is extended. Further, we find that the MDDP has its greatest merits in volatile crisis scenarios. Finally, the conducted turnover analysis reveals that the MDDP produces relatively high turnover volumes. However, this problematic issue can be easily tackled by increasing the holding period or/and by introducing a turnover penalty term in the respective optimization problem.

Table 11

Out-Of-Sample Performance Results Using a 36-Months In-Sample Period and a Crisis-Period that Comprises 24 High-Volatility Months

(Out-of-sample period: April 3rd, 2006 to April 24th, 2014)

	μ_{ann}	σ_{ann}	Skewness	Kurtosis	SR_{ann}	$CE(1)_{ann}$	$CE(2)_{ann}$	$CE(10)_{ann}$
Crisis-Period								
MDDP	–20.58 %	4.55 %	–1.9304	12.6631	–4.5200	–20.75 %	–21.00 %	–21.42 %
ERCP	–18.31 %	4.51 %	–1.5127	10.2572	–4.0636	–18.48 %	–18.73 %	–19.15 %
						(0.0013)	*(0.0021)*	*(0.0015)*
MVP	–18.52 %	4.82 %	–1.5815	11.2456	–3.8452	–18.71 %	–19.00 %	–19.48 %
						(0.0314)	*(0.0341)*	*(0.0421)*
1/N-P	–16.00 %	4.61 %	–1.2004	7.5228	–3.4739	–16.18 %	–16.45 %	–16.90 %
						(0.0011)	*(0.0013)*	*(0.0002)*
Non-Crisis-Period								
MDDP	4.84 %	2.23 %	–0.1528	4.5648	2.1710	4.79 %	4.71 %	4.58 %
ERCP	4.87 %	2.01 %	–0.1593	4.1456	2.4267	4.82 %	4.76 %	4.66 %
						(0.9119)	*(0.8736)*	*(0.8163)*
MVP	4.42 %	1.91 %	0.1078	4.5143	2.3075	4.38 %	4.32 %	4.22 %
						(0.4204)	*(0.4564)*	*(0.4879)*
1/N-P	5.74 %	2.16 %	–0.2935	4.3540	2.6640	5.69 %	5.62 %	5.50 %
						(0.0423)	*(0.0366)*	*(0.0353)*
Total-OSS-Period								
MDDP	–2.19 %	3.08 %	–2.1502	19.3303	–0.7096	–2.28 %	–2.42 %	–2.66 %
ERCP	–1.48 %	2.93 %	–2.0123	17.9588	–0.5055	–1.56 %	–1.69 %	–1.91 %
						(0.0204)	*(0.0143)*	*(0.0166)*
MVP	–1.86 %	3.00 %	–2.2642	22.4129	–0.6197	–1.95 %	–2.08 %	–2.31 %
						(0.4790)	*(0.4659)*	*(0.4442)*
1/N-P	–0.17 %	3.03 %	–1.6295	12.8275	–0.0553	–0.26 %	–0.40 %	–0.63 %
						(0.0002)	*(0.0003)*	*(0.0001)*

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