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The Low-Volatility Anomaly Revisited

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Abstract

The present study conducts two different strategies in order to exploit the low-volatility anomaly in the U.S., the European and the German equity market. The first strategy uses quadratic optimization to calculate optimal portfolio weights. The second strategy sorts stocks into portfolio quintiles based on past realized volatility. Our main findings show that both low-volatility strategies outperform the respective benchmark market portfolio. While the effect is strongest during bull-market periods, it gets weaker during periods of market downturns. Additional results show that in the U.S. market, the low-volatility anomaly can be explained by trading volume and operating profitability. In the German market, operating profitability and the dividend yield can explain the low-volatility effect while in the European market none of these characteristics play a role in explaining the effect. Overall, our findings provide evidence that the low-volatility anomaly still is a robust phenomenon that is inherent in mature capital markets.

Keywords: Low-Volatility Anomaly, Portfolio Optimization, Risk-Return Tradeoff

JEL Classification: G1

I. Introduction

The low-volatility effect describes the long-term average outperformance of low-volatility portfolios relative to the market portfolio along with the relative underperformance of high-volatility portfolios versus low-volatility portfolios on a risk-adjusted basis (see e.g. *Blitz/van Vliet* 2007). The observed phenomenon contradicts rational economic theory that higher risk should be compen-

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sated with higher expected returns, and therefore, it has attracted considerable interest of financial practitioners and academics.[1, 2]

There are different ways to implement low-volatility strategies. For example, *Denoiseux* (2014) uses Exchange Traded Funds (ETFs) and a minimum-variance approach to create a low-volatility portfolio via simulation. His main finding shows that the ETF-portfolio significantly outperforms the benchmark market portfolio. Another way of constructing the low-volatility portfolio is to minimize the variance of a market index using portfolio optimization of *Markowitz* (1952). For example, *Haugen/Baker* (1991), *Kleeberg* (1993), *Clarke* et al. (2006) and *Wagner/Wolpers* (2008) find evidence that investing in a value-weighted market portfolio is not efficient as an optimized portfolio earns a higher return without adding more risk in terms of volatility. Another less sophisticated but more simple and feasible approach to construct a low-volatility portfolio is to rank stocks according to past volatility and sort them into quantiles, deciles, or halves (see for example *Blitz/van Vliet* 2007, *Baker/Haugen* 2012, *Dutt/Humpherey-Jenner* 2013 and *Blitz* et al. 2019). In contrast to those studies, the present study applies both approaches, minimum-variance optimization and portfolio sorting, in order to test whether the low-volatility anomaly is still alive in mature markets.[3]

The low-volatility strategy has become a popular and widely accepted investment style and has attracted increased investor attention in the late-2000s (see e.g. *van Vliet* (2018) and *Blitz* et al. (2019)). Our investigations cover the recent

[1] The early study of *Haugen/Heins* (1975) finds that over the long run, portfolios with lower variance in monthly returns yield higher average returns than high variance portfolios. *Haugen/Baker* (1991) create a minimum-variance portfolio that has a higher expected return relative to the market portfolio. Later on, *Blitz/van Vliet* (2007) find that simple historical volatilities result in even stronger evidence of the low-volatility anomaly, compared to CAPM betas. While most of empirical research on the low-volatility anomaly focuses on the U.S. market (see for example *Haugen/Baker* (1991), *Clarke* et al. (2006) and *Baker* et al. (2011), amongst others), international evidence is provided by *Blitz/van Vliet* (2007). They show that the anomaly is robust in the Global, the European and the Japanese stock market. Results for the German equity market are provided by *Kleeberg* (1993) while *Dutt/Humpherey-Jenner* (2013) provide evidence that the anomaly exists in emerging and developed markets around the world.

[2] The present article focuses on risk as measured in terms of volatility. Another strand of literature examines the so-called "beta-anomaly", which describes the related phenomenon that high-beta stocks underperform low-beta stocks on a risk-adjusted basis. The beta-anomaly is subject to empirical investigations (see e.g. *Frazzini/Pederson* (2014) and *Cederburg/O'Doherty* (2016)) and theoretical considerations (see e.g. *Buchner/Wagner* (2015)).

[3] In our empirical framework, we apply the traditional approach of portfolio optimization according to *Markowitz* (1952). Another more sophisticated approach is proposed by *Frahm/Memmel* (2010), for example. They develop shrinkage estimators for minimum-variance portfolios that are able to dominate the traditional estimator with respect to out-of-sample variance of the portfolio return.

period from 2000 to 2018 and provide evidence that the anomaly has not been arbitraged away and still prevails in the U.S. market as well as in the European and the German stock market after the establishment of low-volatility strategies. Further, our empirical results contribute to existing literature in the following ways. In line with *Soe* (2012), for example, the findings show that the low-volatility (LV) quintile and the minimum-variance (MV) portfolio are equally effective in minimizing long-term average portfolio volatility. *Walkshäusl* (2014) further state that LV and MV yield similar CAPM alphas in developed markets. However, our results show that the MV portfolio may outperform the LV approach by generating higher average returns and higher *Fama/French* (1993) alphas. Furthermore, we find differences in the robustness of the low-volatility anomaly across different states of the economy. Both strategies, MV and LV, generate positive and statistically significant abnormal returns in bull-markets. In bear-markets, the low-volatility portfolios still show an average outperformance relative to the benchmark market portfolio, however, the effect gets weaker.

To deliver a potential explanation for the low-volatility effect, *Clarke* et al. (2006) argue that stocks included in low-volatility portfolios tend to have higher book-to-market ratios and a lower market capitalization. Hence, the higher expected returns of low-volatility stocks might therefore be due to value and size premia. The recent study of *Perras/Wagner* (2019) suggests that higher expected returns in association with low volatility may be attributable to illiquidity effects. Further, *Blitz/van Vliet* (2007) and *Ang* et al. (2006) further control for leverage, trading volume, turnover, bid-ask-spreads and co-skewness. Our results show that the low-volatility anomaly disappears in the U.S. market after controlling for trading volume and operating profitability. In the German market, the anomaly vanishes after controlling for operating profitability and the dividend yield. Interestingly, the anomaly holds in the European market after controlling for these characteristics.

An alternative explanation for the occurrence of the low-volatility effect on capital markets is provided by *Li* et al. (2014). They suggest that low-volatility portfolios need to be rebalanced rather frequently in order to achieve positive excess returns. The high rebalancing frequency implies higher transaction costs that may compensate any gains generated by the low-volatility strategy which, overall, might explain why the anomaly is a long-term and robust phenomenon. In contrast, our results show that the risk-adjusted returns of low-volatility strategies generated by a semi-annually portfolio rebalancing frequency are very similar to the risk-adjusted returns of low-volatility strategies that apply monthly portfolio rebalancing. This indicates that market frictions such as transaction costs cannot serve as a valid explanation for the robust existence of the low-volatility effect. The latter result is consistent to the recent finding of *van Vliet* (2018) who shows that an efficient low-volatility trading strategy does not need anomalously high trading levels.

The remainder of this paper is organized as follows. Section II. presents the data and the methodology that is applied to construct low-volatility portfolios, Section III. reports the main empirical results and identifies potential drivers of the low-volatility anomaly. Section VI. concludes.

II. Data and Methodology

Our empirical investigations focus on the U.S., the European and the German equity market. The U.S. stock market data covers all stocks of the S&P 500 Index. For the European market, we rely on the STOXX Europe 600 stocks and for the German market we use stocks of the HDAX that contains 101 stocks of the DAX, the MDAX and the TECDAX. Quote prices are in local currency and obtained from Thomson Datastream.

In our analysis, we use the daily closing price, $P_{i,t}$, of stock i at day t to calculate the continuously compounded return, R_{it}:

(1) $$R_{i,t} = \ln(P_{i,t}) - \ln(P_{i,t-1})$$

We use two approaches to study the low-volatility anomaly. The first approach is based on *Markowitz* (1952) and includes minimizing the portfolio variance σ_p^2:

(2) $$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{i,j} \rightarrow MIN!$$

s. t.

(3) $$\forall i \quad 0 \leq w_i \leq 1,$$

(4) $$\sum_{i=1}^{N} w_i = 1,$$

where $i \neq j$. The portfolio weights of stocks i and j are denoted by w_i and w_j. The variance of stock i is denoted by σ_i^2 and $\sigma_{i,j}$ represents the covariance of stocks i and j. Constraint (3) prohibits short-selling (see e.g. *Chan* et al. 1999) and Constraint (4) ensures that the portfolio is fully invested.

We use the past three years of daily returns to calculate the optimal portfolio weights of MV portfolio according to Equation (2). The MV portfolio is then rebalanced in a monthly frequency and the rolling estimation procedure is repeated until the end of the sample period in 2018.

In a second approach, we divide the stocks into quintile portfolios based on their past standard deviation. This allows to compare the expected portfolio returns across the quintiles to those of the market portfolio. We start by calculating the standard deviation for the first three years of daily returns for each stock in the market index. Next, those stocks are sorted into equal-weighted quintile portfolios according to the past standard deviation. We hold each portfolio for the following month before rebalancing. The process is repeated until the end of the sample period in 2018 resulting in a time series of monthly portfolio returns for each quintile.

Our analysis reports several statistics for the market portfolio, the MV portfolio and the quintile portfolios. Particularly, we calculate the geometric annualized returns in order to compare the cross-section of returns and the annualized standard deviations of monthly returns. Further, we calculate Sharpe ratios for the different approaches and markets. The geometric annualized return $\overline{R_p}$ for the respective portfolio p is calculated as follows:

$$\overline{R_p} = \sqrt[T]{\prod_{t=1}^{T}\left(1+R_{p,t}\right)^{t}} - 1. \tag{5}$$

Here, $R_{p,t}$ denotes the monthly return of portfolio p, at period t. The annualized standard deviation $\overline{\sigma_p}$ of the monthly returns is calculated as:

$$\overline{\sigma_p} = \sqrt{\sigma_p^2} * \sqrt{T} \tag{6}$$

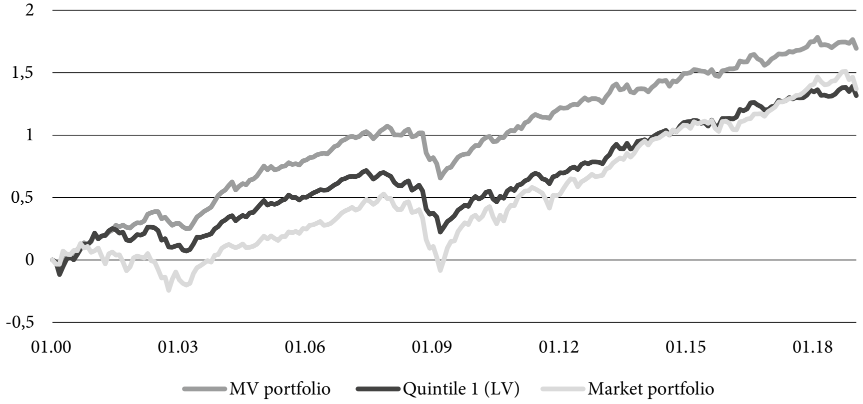
Here, σ_p^2 denotes the variance of the respective portfolio p over the total number of periods T. Next, we calculate the Sharpe ratio SR_p, where the excess return $\overline{R_p} - R_f$ is divided by the annualized standard deviation $\overline{\sigma_p}$ of portfolio p:

$$SR_p = \frac{\bar{R}_p - R_f}{\bar{\sigma}_p} \tag{7}$$

III. Empirical Results

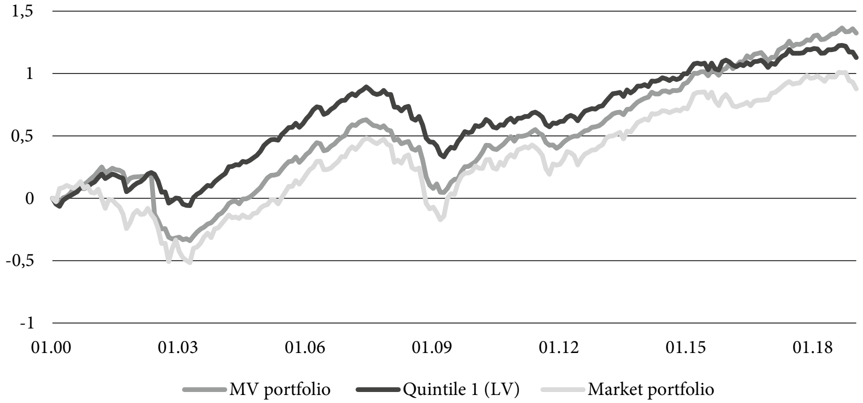
1. Is the Low-Volatility Effect Still Alive?

This section presents preliminary results regarding the existence of the low-volatility anomaly. Figure 1, 2 and 3 show the development of cumulated returns of the two low-volatility approaches relative to the U.S., the European and the German market portfolio.



Note: The figure shows the cumulated returns of the minimum-variance (MV) Portfolio, the LV quintile and the value-weighted market portfolio of the U.S market. The sample period ranges from January 2000 to December 2018.

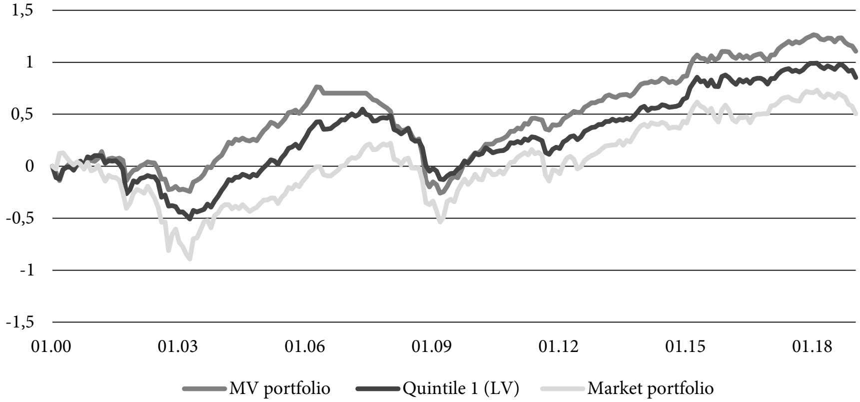
Figure 1: Cumulated Returns, U.S. Market



Note: The figure shows cumulated returns of the minimum-variance (MV) Portfolio, the LV quintile and the market portfolio of the European market. The sample period ranges from January 2000 to December 2018.

Figure 2: Cumulated Returns, European Market

Figure 1 shows cumulative returns over the entire sample period from January 2000 until December 2018 for the MV and the LV portfolio and the U.S. market portfolio. Clearly, the MV portfolio outperforms the market portfolio over the entire sample period. While the LV portfolio also shows overperformance relative to the market portfolio, the effect reversed in the most recent years. Further, the stock market crash associated with the global financial crisis in 2008 is less pronounced for the LV and the MV portfolio. Figure 2 and Figure 3 show cu-



Note: The figure shows cumulated returns of the minimum-variance (MV) Portfolio, the LV quintile and the market portfolio of the German market. The sample period ranges from January 2000 to December 2018.

Figure 3: Cumulated Returns, German Market

mulative returns for the European and German equity market. It is apparent that the LV and the MV portfolio also yield higher cumulative returns than the market portfolio. In these two markets, the low-volatility effect remains stable and robust over the entire sample period.

Table 1 reports the descriptive statistics of average and Fama-French risk-adjusted returns on the MV and LV portfolio.[4] Panel A of Table 1 reports results for the U.S. market. The annualized return of the market portfolio averages 6.39 % while the corresponding risk of the market portfolio (measured by annualized standard deviation) is 13.99 %. We further observe the Sharpe ratio for the market by dividing the annualized excess returns by annualized standard deviation. Considering an average T-bill rate of 1.6 % over the sample period, the market Sharpe ratio yields a value of 0.335. The second row of Panel A reports the results for the MV portfolio. The strategy generates an annualized standard deviation of 10.13 % that is substantially lower than that of the market portfolio. Further, the strategy generates a highly significant average annualized return of 8.73 % (with a corresponding t-value of 3.72) which reveals the remarkable outperformance relative to the market portfolio. Consistent to this finding, the Sharpe ratio of the MV portfolio is substantially higher than the

[4] We thank an anonymous referee for the suggestion to test whether low-volatility portfolios significantly overperform on a risk-adjusted basis. To obtain risk-adjusted returns, the portfolio returns are regressed on the market, size and value factor of *Fama/French* (1993). Data on the European and U.S. factors is kindly provided by Kenneth R. French on his personal website. Due to data limitations, European factor data is also used to estimate Fama-French alphas for the German equity market.

Table 1

Return Statistics

	Return	FF-α	Standard Deviation	Sharpe Ratio
Panel A: U.S. market				
Market portfolio	6.39 %** [2.16]		13.99 %	0.335
MV portfolio	8.73 %*** [3.72]	0.51 %*** [3.41]	10.13 %	0.691
Quintile 1 (LV)	6.50 %*** [2.69]	0.33 %*** [2.65]	10.88 %	0.441
Quintile 2	6.27 %** [2.14]	0.23 %* [1.94]	13.82 %	0.331
Quintile 3	6.11 %* [1.80]	0.12 % [0.94]	17.22 %	0.256
Quintile 4	5.65 % [1.55]	0.04 % [0.31]	19.16 %	0.207
Quintile 5 (HV)	5.69 % [1.43]	0.01 % [0.09]	26.42 %	0.151
Panel B: European market				
Market portfolio	3.42 % [1.20]		15.50 %	0.102
MV portfolio	6.27 %* [1.87]	0.44 %** [2.14]	12.55 %	0.350
Quintile 1 (LV)	5.48 %** [2.12]	0.35 %** [2.32]	10.62 %	0.340
Quintile 2	4.03 % [1.27]	0.15 % [0.82]	13.90 %	0.157
Quintile 3	2.45 % [0.89]	0.01 % [0.03]	16.40 %	0.038
Quintile 4	2.48 % [0.88]	–0.01 % [–0.05]	20.08 %	0.033
Quintile 5 (HV)	–0.46 % [0.42]	–0.20 % [–0.83]	26.02 %	–0.086

	Return	FF-α	Standard Deviation	Sharpe Ratio
Panel C: German market				
Market portfolio	0.43 % [–0.17]		20.74 %	–0.066
MV portfolio	4.70 % [1.43]	0.26 % [0.98]	15.31 %	0.185
Quintile 1 (LV)	3.33 % [1.20]	0.26 % [1.09]	15.23 %	0.098
Quintile 2	0.33 % [0.48]	–0.05 % [–0.20]	19.99 %	–0.073
Quintile 3	3.51 % [1.10]	0.18 % [0.65]	23.43 %	0.071
Quintile 4	5.46 % [1.42]	0.35 % [1.29]	23.15 %	0.155
Quintile 5 (HV)	–0.82 % [0.49]	–0.12 % [–0.36]	30.18 %	–0.086

Notes: The table summarizes the annualized return statistics of the U.S., the European and the German market over the sample period from January 2000 to December 2018. The geometrical mean returns (Eq. (5)), the annualized volatility of the monthly returns (Eq. (6)) and the Sharpe ratio (Eq. (7)) are calculated over the sample period from January 2000 to December 2018. Heteroscedasticity and autocorrelation (HAC) robust t-statistics are reported in brackets. Superscripts *, ** and *** denote statistical significance (at least) at the 10 %, 5 % and 1 % level.

market Sharpe ratio. The latter finding of a higher Sharpe ratio is due to both, a higher average excess return generated by the strategy and a lower average standard deviation. The remaining rows of Panel A report results on the quintile portfolios sorted by volatility. In general, the effect appears to be weaker for the LV strategy than for the MV strategy as the annualized return of the LV quintile (6.50 %) is only slightly higher than the annualized market return. According to that, the Sharpe ratio of the LV quintile is 0.441 while the higher Sharpe ratio to the market portfolio seems to be mainly driven by the lower annualized standard deviation of 10.88 %. However, the statistics on the quintile portfolio returns provide further evidence on the low-volatility effect. The annualized return does not only show a decreasing pattern from quintile portfolio 1 (LV quintile portfolio) to portfolio 5 (high volatility quintile portfolio), the statistical significance is also vanishing along with an increase in average volatility. To reveal that our main findings hold in terms of risk-adjusted returns, we report estimated *Fama/French* (1993) alphas (FF-α) in the second column of Table 2. The results show positive and high abnormal returns of the MV and the LV portfolio that are both significant at least at the 1 % level. Further, we still observe a decreasing pattern in abnormal returns as well as a vanishing statistical significance from the LV portfolio (quintile 1) to the high volatility portfolio (quintile 5).

Panel B of Table 1 reports the results for the European market which, in general, correspond to our findings on the U.S. market. The market portfolio averages an annualized return 3.42 % over the 19-year sample period with an annualized standard deviation of 15.50 %. The Sharpe ratio of the market is 0.102 (considering a sample average of EURIBOR-rate of 1.8 %). The reported annualized return of the MV portfolio is 6.27 % and thus 2.85 % higher than the market, while the annualized standard deviation is lower with 12.55 %. The Sharpe ratio of the MV portfolio is 0.350 and higher than the market Sharpe ratio of 0.102. The annualized return is 5.48 % for the LV quintile and 2.06 % higher than the market return. Overall, the standard deviation of the LV quintile is the lowest for the European market with 10.62 %. The resulting Sharpe ratio of the LV quintile is 0.340. On the European market, we do not observe a statistically significant mean of the market return. In contrast, the average annualized return of the MV portfolio and the LV quintile is positive and statistically significant at least at the 10 %-level. Statistical significance gets even stronger considering Fama-French risk-adjusted returns.

Panel C of Table 1 reports the results regarding the German equity market. Compared to the U.S. and European market, the German market portfolio exhibits a considerably lower standard deviation along with higher expected excess returns relative to the German equity market. The latter results in a higher Sharpe ratio of 0.333 for the U.S. market versus –0.067 for the German market. Considering these risk-return trade-off statistics, it becomes apparent that the low-volatility effect is not only existing in specific stock markets but also across international equity markets and stock market indices, respectively. Turning to the MV portfolio, we observe a mean return of 4.70 %, i.e. 4.27 % higher than that of the market portfolio. The standard deviation of the MV portfolio is 15.31 %, which results in a Sharpe ratio of 0.185. Regarding the risk-return trade-off, the LV quintile portfolio also outperforms the market portfolio with an average return of 3.33 % and a corresponding standard deviation of 15.23 %. Consistent to this, the Sharpe ratio of the LV quintile is higher than the market Sharpe ratio with 0.098 compared to –0.066. Considering the risk-adjusted returns, the Fama-French alphas of the MV and LV portfolio are 0.26 %. Although not statistically significant, abnormal returns are positive and high relative to the quintile 5 portfolio (i.e. the high volatility portfolio).

Overall, the results provide evidence that the low-volatility effect still is present in the considered equity markets. Both strategies to construct low-volatility portfolios, MV and LV, outperform the market portfolio. Further, the low-volatility effect is also present across markets, i. e. market portfolios with lower annualized volatility earn higher expected returns.

2. *Robustness of the Anomaly across Bear- and Bull-Markets*

In this section, we perform an additional analysis in order to determine whether our results on the low-volatility effect vary across different states of the economy. Therefore, the sample is divided into bull- and bear-market phases.[5] Results on the U.S. market are reported in Panel A of Table 2. While the U.S. market portfolio earns a lower average return of 12.96 % than the MV portfolio and the LV portfolio are 11.00 % and 11.85 %, we obtain a positive and statistically significant FF-α for the MV portfolio (0.37 %) and the LV portfolio (0.28 %), respectively. Similar results are obtained for the German and the European market in Panel B and C of Table 2. The FF-α is positive for the MV portfolio (0.71 %) and the LV portfolio (0.49 %) and statistically significant at the 1 %-level.

Table 2

Return Statistics for Bull- and Bear-Markets

	Bull-market				Bear-market			
	Return	FF-α	Std dev	Sharpe ratio	Return	FF-α	Std dev	Sharpe ratio
Panel A: U.S. market								
Market portfolio	12.96 %*** [4.31]		11.91 %	0.974	−17.84 %* [−2.00]		18.01 %	−1.130
MV portfolio	11.00 %*** [5.06]	0.37 %** [2.44]	8.41 %	1.148	0.22 % [0.17]	0.49 % [1.21]	13.42 %	−0.208
Quintile 1 (LV)	11.85 %*** [5.63]	0.28 %** [2.35]	8.67 %	1.270	−7.23 % [−0.77]	0.25 % [0.73]	15.15 %	−0.662
Quintile 2	14.35 %*** [5.20]	0.13 % [1.49]	10.98 %	1.214	−15.51 % [−1.35]	0.39 % [0.93]	19.78 %	−0.914
Quintile 3	16.32 %*** [4.51]	0.01 % [0.21]	13.89 %	1.061	−20.62 % [−1.48]	0.54 %* [1.92]	24.38 %	−0.946
Quintile 4	16.53 %*** [4.11]	−0.09 % [−1.01]	15.62 %	0.954	−22.45 % [−1.39]	1.02 %*** [3.28]	27.05 %	−0.918
Quintile 5 (HV)	22.51 %*** [4.03]	0.02 % [0.14]	21.34 %	0.935	−34.21 %* [−1.78]	0.68 % [1.39]	36.79 %	−0.986

(continue next page)

[5] The bear-market period covers the down-markets during the dot-com crisis (January 2000 to September 2002) and the global financial crisis (November 2007 to February 2009).

(Table 2 continued)

	Bull-market				Bear-market			
	Return	FF-α	Std dev	Sharpe ratio	Return	FF-α	Std dev	Sharpe ratio
Panel B: European market								
Market portfolio	10.54 %*** [3.23]		12.62 %	0.736	–25.12 %** [–2.58]		20.57 %	–1.374
MV portfolio	14.03 %*** [6.69]	0.71 %*** [4.77]	7.67 %	1.661	–20.19 % [–1.68]	–0.83 % [–1.04]	20.70 %	–1.136
Quintile 1 (LV)	11.67 %*** [5.14]	0.49 %*** [3.60]	8.65 %	0.826	–12.38 % [–1.44]	–0.05 % [–0.15]	14.61 %	–1.095
Quintile 2	13.44 %*** [4.44]	0.40 %** [1.49]	11.00 %	0.805	–22.06 %* [–2.00]	–0.23 % [–0.54]	19.40 %	–1.305
Quintile 3	12.35 %*** [3.64]	0.25 % [1.27]	13.50 %	0.649	–27.10 %** [–2.40]	–0.42 % [–0.92]	21.79 %	–1.384
Quintile 4	15.35 %*** [3.41]	0.19 % [0.81]	16.60 %	0.644	–30.99 %** [–2.20]	0.06 % [0.09]	27.00 %	–1.256
Quintile 5 (HV)	18.96 %*** [3.06]	0.24 % [0.93]	21.45 %	0.660	–46.38*** [–2.85]	–0.34 % [–0.45]	33.62 %	–1.449
Panel C: German market								
Market portfolio	9.92 %** [2.42]		16.98 %	0.511	–31.68 %** [–2.61]		26.22 %	–1.319
MV portfolio	12.77 %*** [4.19]	0.58 %*** [2.77]	10.71 %	1.074	–25.12 %** [–2.04]	–0.71 % [–1.06]	23.75 %	–1.190
Quintile 1 (LV)	12.47 %*** [4.16]	0.54 %** [2.53]	11.58 %	0.684	–22.39 %* [–1.98]	–0.12 % [–0.22]	22.67 %	–1.131
Quintile 2	15.03 %*** [3.62]	0.42 %** [1.59]	15.65 %	0.663	–36.73 %*** [–2.73]	–1.17 % [–1.38]	27.78 %	–1.419
Quintile 3	18.78 %*** [3.65]	0.50 %* [1.75]	18.42 %	0.760	–35.40 %** [–2.05]	0.26 % [0.34]	33.74 %	–1.131
Quintile 4	19.57 %*** [3.93]	0.51 %* [1.70]	18.66 %	0.791	–20.29 %* [–1.72]	0.92 % [1.05]	32.90 %	–1.010
Quintile 5 (HV)	22.14 %*** [2.83]	0.41 % [1.09]	24.53 %	0.702	–52.04 %*** [–3.09]	–1.39 % [–1.26]	40.25 %	–1.345

Notes: Table 2 summarizes the annualized return statistics of the U.S., the European and the German equity market of bull- and bear-market periods. The observed bull periods range from October 2002 to October 2007 and from March 2009 to December 2018. The bear periods range from January 2000 until September 2002 and from November 2007 until February 2009. Return calculation is based on Eq. (5), volatility is calculated according to Eq. (6) and the Sharpe ratio is calculated according to Eq. (7). Heteroscedasticity and autocorrelation (HAC) robust t-statistics are reported in brackets. Superscripts *, ** and *** denote statistical significance (at least) at the 10 %, 5 % and 1 % level.

During bear-market periods, we obtain higher average returns of the MV and LV portfolio relative to the market portfolio and the HV portfolio. In all three considered markets, however, statistical significance of the risk-adjusted returns completely vanishes for the low-volatility portfolios. Overall, these results provide evidence of a significant and robust low-volatility effect during bull-market periods while the effect gets weaker during market downturns that are characterized by financial distress and turmoil.

3. *Robustness Against Variations in the Rebalancing Frequency*

So far, our results provide evidence of a low-volatility effect on different major stock markets. However, in practice, transaction costs determine whether the effect can effectively be exploited by arbitrageurs. Therefore, we test in the following whether our results are robust to a lower portfolio rebalancing frequency which will imply a remarkable reduction of overall transactions costs. For this, we reduce the rebalancing frequency of the portfolios from one to six months. Table 3 reports the annualized statistics of monthly returns in the U.S., the European and the German market implied by a semi-annually portfolio rebalancing frequency.

Panel A reports the statistics for the U.S. market. The market portfolio's annualized expected return yields 6.35 % and thus, it is almost equal to the one-month rebalancing period with 6.39 %. The standard deviation of 13.98 % is also almost equal to the portfolio standard deviation implied by one-month rebalancing frequency. Similarly, the returns and standard deviations of the MV portfolio and the LV quintile do not change much either. The MV portfolio yields an annualized expected return of 8.62 % which is only 0.11 % lower than the average return generated by one-month rebalancing, while the volatility is marginally higher with 10.31 % compared to 10.13 %. The expected return of the LV quintile is 6.52 % and therefore 0.02 pp. higher than with the one-month rebalancing frequency. The volatility of the LV quintile also increases only marginally from 10.88 % to 10.97 %. The returns of the MV portfolio and LV quintile are still highly significant at the 1 %-level, and the market return is significant at the 5 %-level. The risk-adjusted returns in Column 2 are identical for the LV portfolio and similar for the MV portfolio. Further, the alphas are significant at the 1 %-level.

Similar results are reported for the European market in Panel B. The return of the MV portfolio changes to 6.63 % and the standard deviation slightly rises to 12.67 %. The LV portfolio return decreases to 5.37 % and the volatility rises to 10.83 %. The average returns of the MV portfolio and the LV quintile portfolio are significant at the 5 %-level, while the expected return on the market portfolio remains insignificant. Considering the risk-adjusted returns, the alphas do not change remarkable and are statistically significant at least at the 5 %-level.

Table 3

Return Statistics, Semi-Annually Rebalancing Frequency

	Return	FF-α	Std Dev	Sharpe Ratio
Panel A: U.S. market				
Market portfolio	6.35 %** [2.15]		13.98 %	0.333
MV portfolio	8.62 %*** [3.63]	0.50 %*** [3.48]	10.31 %	0.668
Quintile 1 (LV)	6.52 %*** [2.67]	0.33 %*** [2.65]	10.97 %	0.439
Quintile 2	5.95 %** [1.99]	0.19 %* [1.50]	14.59 %	0.292
Quintile 3	5.93 %* [1.75]	0.10 % [0.84]	17.02 %	0.249
Quintile 4	5.94 % [1.64]	0.07 % [0.52]	18.88 %	0.225
Quintile 5 (HV)	6.02 % [1.48]	0.05 % [0.29]	25.87 %	0.167
Panel B: European market				
Market portfolio	3.43 % [1.20]		15.51 %	0.102
MV portfolio	6.63 %** [1.20]	0.47 %** [2.46]	12.67 %	0.374
Quintile 1 (LV)	5.37 %** [2.04]	0.34 %** [2.21]	10.83 %	0.323
Quintile 2	3.55 % [1.14]	0.11 % [0.61]	14.21 %	0.121
Quintile 3	2.29 % [0.84]	0.01 % [0.04]	16.66 %	0.028
Quintile 4	2.66 % [0.89]	–0.01 % [–0.06]	19.89 %	0.042
Quintile 5 (HV)	0.38 % [0.53]	–0.14 % [–0.61]	25.30 %	–0.056

	Return	FF-α	Std Dev	Sharpe Ratio
Panel C: German market				
Market portfolio	0.40 % [0.53]		20.73 %	–0.067
MV portfolio	5.42 % [1.63]	0.33 % [1.32]	15.46 %	0.230
Quintile 1 (LV)	3.51 % [1.25]	0.29 % [1.24]	15.89 %	0.105
Quintile 2	1.54 % [0.73]	0.02 % [0.09]	20.21 %	–0.013
Quintile 3	0.93 % [0.63]	0.00 % [0.00]	22.52 %	–0.039
Quintile 4	6.40 % [1.52]	0.39 % [1.36]	23.62 %	0.191
Quintile 5 (HV)	1.11 % [0.71]	0.02 % [0.06]	29.31 %	–0.024

Notes: The table summarizes the annualized return statistics of the U.S., the European and the German market (semi-annually portfolios rebalancing). The geometrical mean returns (Eq. (5)), the annualized volatility of the monthly returns (Eq. (6)) and the Sharpe ratio (Eq. (7)) are calculated over the sample period from January 2000 to December 2018. Heteroscedasticity and autocorrelation (HAC) robust t-statistics are reported in brackets. Superscripts *, ** and *** denote statistical significance (at least) at the 10 %, 5 % and 1 % level.

Finally, Panel C reports the results for the German market. The MV portfolio reports a substantially higher return of 5.42 % for the one-month rebalancing frequency, while the volatility increases to 15.46 %. The return of the LV portfolio is also higher at 3.51 % compared to the one-month rebalancing frequency (3.33 %). The volatility increases from 15.23 % to 15.89 %. However, the average and risk-adjusted returns implied by the low-volatility strategies on the German market are not statistically significant at the 1 %-, 5 %- or 10 %-level.

Overall, the results show that lowering the rebalancing frequency does not negatively affect the low-volatility portfolio returns while a lower rebalancing implies higher net returns (after transaction costs) for active low-volatility investors.

4. *Drivers of the Anomaly*

Stock market anomalies can often be explained by other effects. In the following section we control for different characteristics by performing double portfolio sortings as suggested by *Ang* et al. 2006, for example. Each month, we first sort stocks based on individual characteristics such as volume, size, operating profitability and the dividend yield.[6] Then, within each quintile, we sort stocks based on past three-year volatility. The five volatility portfolios are then averaged over each of the five characteristic portfolios. This process aims to eliminate potential effects due to stock characteristics and enables to investigate the risk-return statistics for each of the characteristics-sorted portfolio.

a) Trading Volume

First, we measure the trading volume of the portfolios for the single- and for the double-sorting. The trading volume is based on the daily traded number of shares for each stock averaged to a monthly basis. In general, the standard deviation of a stock is supposed to be positively related to trading frequency, and hence, the LV quintile is expected to exhibit a lower average monthly trading volume.

As the left-hand side of Table 4 shows, the trading volume of the LV quintile in the U.S. market is the lowest with 3535.60k, while the HV quintile has a trading volume of 8037.39k. Similar results are observed for the European market. However, in the German market, the LV quintile has the highest average trading volume with 2149.55k compared to 1405.43k of the HV quintile. Thus, the hypothesis that stocks with lower volatility have lower trading volume holds only for the U.S. and European market. On the right-hand side of Table 4, we find that the difference in trading volume between the LV and HV quintile gets smaller after the double-sorting, but it is not eliminated completely. The returns of the quintile portfolios have now changed. In the U.S. market, the LV quintile now has the lowest average return with 5.63 % while the quintile with the highest volatility has a return of 7.32 %. Furthermore, the FF-α of the LV portfolio is 0.32 % and statistically significant at the 1 %-level after the single-sorting while the double-sorting yields a lower FF-α of 0.27 % that is significant at least at the 5 %-level. In contrast, we obtain different results for the European and German market. In both markets, the LV quintiles have higher returns than the HV quintiles after controlling for trading volume. Further, in both markets the risk-adjusted returns of the LV portfolios does not show significant differences performing a single-sorting or double-sorting. Thus, one can conclude that the low-volatility anomaly may be explained by trading volume in the U.S. market, but not in the European and German market.

[6] All data on individual stock characteristics is obtained from Thomson Datastream.

Table 4

Double-Sorting on Trading Volume

	First portfolio sorting				Second portfolio sorting			
	Return	FF-α	Std dev	Volume	Return	FF-α	Std dev	Volume
Panel A: U.S. market								
Quintile 1 (LV)	6.43 %*** [2.70]	0.32 %*** [2.65]	10.67 %	3535.60	5.63 %** [2.32]	0.27 %** [2.14]	10.97 %	4656.98
Quintile 2	6.21 %** [2.14]	0.22 %* [1.94]	13.66 %	4288.09	6.08 %** [2.11]	0.20 %** [2.00]	13.51 %	5040.86
Quintile 3	6.06 %* [1.80]	0.12 % [0.94]	17.02 %	4782.68	5.83 %* [1.77]	0.10 % [0.89]	16.99 %	5111.79
Quintile 4	5.61 % [1.55]	0.04 % [0.31]	18.94 %	5159.65	5.57 % [1.52]	0.03 % [0.23]	19.29 %	5150.04
Quintile 5 (HV)	5.67 % [1.43]	0.01 % [0.09]	26.10 %	8037.39	7.32 % [1.68]	0.13 % [0.74]	25.53 %	6140.86
Panel B: European market								
Quintile 1 (LV)	5.43 %** [2.12]	0.35 %** [2.32]	10.50 %	3951.01	5.52 %** [2.18]	0.35 %** [2.38]	10.68 %	3990.57
Quintile 2	4.00 % [1.27]	0.15 % [0.82]	13.75 %	4143.28	3.34 % [1.12]	0.09 % [0.49]	13.81 %	4392.03
Quintile 3	2.44 % [0.89]	0.01 % [0.03]	16.22 %	4320.22	2.26 % [0.81]	–0.02 % [–0.12]	16.81 %	4913.46
Quintile 4	2.49 % [0.88]	–0.01 % [–0.06]	19.86 %	5082.50	2.50 % [0.97]	–0.01 % [–0.07]	19.88 %	4964.41
Quintile 5 (HV)	–0.40 % [0.42]	–0.20 % [–0.83]	25.73 %	6431.61	0.35 % [0.53]	–0.11 % [–0.48]	25.65 %	5921.49
Panel C: German market								
Quintile 1 (LV)	3.35 % [1.25]	0.25 % [1.16]	13.92 %	2149.55	3.28 % [1.18]	0.24 % [1.04]	15.06 %	1789.93
Quintile 2	0.69 % [0.52]	–0.04 % [–0.15]	18.35 %	1578.23	2.57 % [0.91]	0.13 % [0.49]	19.13 %	1500.37
Quintile 3	3.58 % [1.11]	0.17 % [0.65]	21.61 %	1205.74	2.77 % [0.97]	0.06 % [0.21]	21.94 %	1359.06
Quintile 4	5.38 % [1.44]	0.32 % [1.30]	21.25 %	1409.20	6.83 % [1.57]	0.47 % [1.56]	26.15 %	1491.99
Quintile 5 (HV)	–0.02 % [0.53]	–0.10 % [–0.32]	27.77 %	1405.43	–1.20 % [0.38]	–0.18 % [–0.55]	28.08 %	2116.43

Notes: The table shows the effect of trading volume on the low-volatility anomaly. Trading volume is measured in 1,000. Panel A shows the results for the single-sorting of the data. Panel B shows the results for the double-sorting. Heteroscedasticity and autocorrelation (HAC) robust t-statistics are reported in brackets. Superscripts *, ** and *** denote statistical significance (at least) at the 10 %, 5 % and 1 % level.

b) Size

Second, we control for size measured by market capitalization. Small stocks are usually younger firms with less-diversified business models resulting in a higher risk that is reflected by a higher return volatility. Therefore, the LV quintile is supposed to contain stocks with higher market capitalization while the HV quintile should be composed of stocks with lower market capitalization.

The left side of Table 5 shows that low-volatility stocks are mostly large-cap stocks. In the U.S. market, the mean market capitalization of the LV quintile is 38337.74m, while the HV quintile has an average market capitalization of 15291.34m. The LV quintile has also the highest return with 6.50 % compared to 5.69 % of the quintile with the highest volatility. Similar results are reported for

Table 5

Double-Sorting on Size

First portfolio sorting					Second portfolio sorting			
	Return	FF-α	Std dev	Size	Return	FF-α	Std dev	Size
Panel A: U.S. market								
Quintile 1 (LV)	6.50 %*** [2.69]	0.33 %*** [2.65]	10.88 %	38337.74	6.74 %*** [2.73]	0.35 %*** [2.68]	11.00 %	30297.41
Quintile 2	6.27 %** [2.14]	0.23 %* [1.94]	13.82 %	32146.54	7.36 %** [2.43]	0.30 %*** [2.73]	13.91 %	28166.65
Quintile 3	6.11 %* [1.80]	0.12 % [0.94]	17.22 %	28462.11	6.73 %** [1.97]	0.16 % [1.35]	16.88 %	28301.83
Quintile 4	5.65 % [1.55]	0.04 % [0.31]	19.16 %	21986.84	6.56 %* [1.72]	0.11 % [0.73]	19.01 %	25704.24
Quintile 5 (HV)	5.69 % [1.43]	0.01 % [0.09]	26.42 %	15291.34	3.01 % [1.03]	–0.18 % [–1.10]	26.35 %	23771.59
Panel B: European market								
Quintile 1 (LV)	5.48 %** [2.12]	0.35 %** [2.32]	10.62 %	25078.55	5.71 %** [2.17]	0.37 %** [2.35]	10.81 %	22635.40
Quintile 2	4.03 % [1.27]	0.15 % [0.82]	13.90 %	24377.67	4.79 % [1.51]	0.23 % [1.23]	13.54 %	22338.93
Quintile 3	2.45 % [0.89]	0.01 % [0.03]	16.40 %	21986.57	2.43 % [0.87]	–0.01 % [–0.03]	16.45 %	21554.09
Quintile 4	2.48 % [0.88]	–0.01 % [–0.05]	20.08 %	19419.90	2.35 % [0.83]	–0.02 % [–0.12]	20.21 %	20019.88
Quintile 5 (HV)	–0.46 % [0.42]	–0.20 % [–0.83]	26.02 %	15573.01	–1.19 % [0.32]	–0.27 % [–1.11]	25.84 %	19871.82

First portfolio sorting					Second portfolio sorting			
	Return	FF-α	Std dev	Size	Return	FF-α	Std dev	Size
Panel C: German market								
Quintile 1 (LV)	3.33 % [1.20]	0.26 % [1.09]	15.23 %	18378.52	3.00 % [1.08]	0.21 % [0.93]	15.70 %	12705.66
Quintile 2	0.33 % [0.48]	−0.05 % [−0.20]	19.99 %	16829.50	6.24 % [1.58]	0.38 % [1.40]	19.79 %	13184.56
Quintile 3	3.51 % [1.10]	0.18 % [0.65]	23.43 %	13212.96	1.25 % [0.66]	0.04 % [0.14]	21.74 %	12657.44
Quintile 4	5.46 % [1.42]	0.35 % [1.29]	23.15 %	9911.90	1.12 % [0.74]	−0.05 % [−0.17]	25.06 %	13222.46
Quintile 5 (HV)	−0.82 % [0.49]	−0.12 % [−0.36]	30.18 %	5610.57	0.99 % [0.68]	0.02 % [0.05]	28.48 %	11819.00

Notes: The table shows the effect of stock size on the low-volatility anomaly. Size is measured as the market capitalization of a stock (in million dollar). Panel A shows the results for the single-sorting on volatility, while Panel B shows the results for the double-sorting. Heteroscedasticity and autocorrelation (HAC) robust t-statistics are reported in brackets. Superscripts *, ** and *** denote statistical significance (at least) at the 10 %, 5 % and 1 % level.

the European and German market in Panels B and C. On the right side, we observe that the returns of the LV and HV quintile further spread apart. The LV quintile has a return of 6.74 % while the HV quintile has a return of 3.01 %. The difference in size gets smaller due to the double-sorting, but it is not eliminated completely. Similar results are reported for the European market. However, the difference in returns gets closer in the German market, but the low-volatility anomaly is still evident. Therefore, we can confirm that low-volatility stocks have a higher market capitalization, but the anomaly stays alive after controlling for stock size. This is supported considering the risk-adjusted returns in the U.S. and the European stock market. The values of FF-α only slightly differ between the first and second portfolio-sorting. Still, the low-volatility anomaly persists.

c) Operating Profitability

Third, we examine the operating profitability of the quintile portfolios, which is measured return-on-equity (ROE).[7] Firms with high operating profitability mostly offer a well-diversified business model and are rarely associated with bad news which, in general, should result in a lower level of average stock return volatility.

[7] Data on ROE is obtained from Thomson Datastream and defined as follows: (Net Income before Preferred Dividends – Preferred Dividend Requirements)/Average of Last Year's and Current Year's Common Equity * 100.

Table 6

Double-Sorting on Operating Profitability

First portfolio sorting					Second portfolio sorting			
	Return	FF-α	Std dev	ROE	Return	FF-α	Std dev	ROE
Panel A: U.S. market								
Quintile 1 (LV)	6.61 %*** [2.71]	0.34 %*** [2.66]	10.71 %	35.65 %	5.58 %** [2.26]	0.25 %* [1.96]	11.23 %	21.73 %
Quintile 2	6.21 %** [2.13]	0.22 %** [1.99]	14.04 %	26.34 %	6.04 %** [2.05]	0.19 %* [1.70]	13.96 %	18.60 %
Quintile 3	5.92 %* [1.74]	0.10 % [0.78]	16.74 %	25.23 %	6.66 %* [1.88]	0.16 % [1.25]	16.99 %	18.85 %
Quintile 4	5.60 % [1.54]	0.04 % [0.26]	19.03 %	16.79 %	5.12 % [1.43]	–0.02 % [–0.15]	19.14 %	14.73 %
Quintile 5 (HV)	5.29 % [1.38]	–0.01 % [–0.07]	26.09 %	–5.07 %	5.86 % [1.50]	0.04 % [0.32]	24.66 %	11.94 %
Panel B: European market								
Quintile 1 (LV)	5.52 %** [2.15]	0.35 %** [2.37]	10.57 %	20.51 %	4.52 %* [1.69]	0.24 % [1.60]	11.20 %	16.73 %
Quintile 2	4.12 % [1.28]	0.16 % [0.86]	13.94 %	21.03 %	3.65 % [1.20]	0.11 % [0.63]	14.26 %	16.91 %
Quintile 3	2.36 % [0.88]	0.01 % [0.04]	16.50 %	20.13 %	2.50 % [0.86]	0.01 % [0.04]	16.78 %	17.92 %
Quintile 4	2.39 % [0.85]	–0.02 % [–0.08]	20.06 %	16.57 %	2.31 % [0.85]	–0.03 % [–0.14]	19.78 %	18.80 %
Quintile 5 (HV)	–0.22 % [0.45]	–0.18 % [–0.74]	25.97 %	8.84 %	1.31 % [0.66]	–0.01 % [–0.04]	24.54 %	10.95 %
Panel C: German market								
Quintile 1 (LV)	3.49 % [1.24]	0.27 % [1.15]	15.24 %	14.31 %	3.56 % [1.25]	0.21 % [0.92]	16.02 %	12.39 %
Quintile 2	–0.08 % [0.40]	–0.10 % [–0.36]	19.98 %	13.62 %	3.64 % [1.11]	0.20 % [0.77]	18.38 %	12.62 %
Quintile 3	3.56 % [1.12]	0.21 % [0.76]	23.52 %	13.39 %	1.10 % [0.65]	–0.01 % [–0.04]	23.72 %	12.03 %
Quintile 4	5.75 % [1.46]	0.36 % [1.30]	23.00 %	14.50 %	1.95 % [0.76]	0.03 % [0.10]	25.10 %	12.87 %
Quintile 5 (HV)	–0.28 % [0.54]	–0.09 % [–0.27]	29.79 %	6.00 %	3.45 % [1.08]	0.24 % [0.80]	27.11 %	11.89 %

Notes: The table shows the effect of return on equity (ROE) on the low-volatility anomaly. Panel A shows the results for the single-sorting on volatility, Panel B shows the results for the double-sorting. Heteroscedasticity and autocorrelation (HAC) robust t-statistics are reported in brackets. Superscripts *, ** and *** denote statistical significance (at least) at the 10 %, 5 % and 1 % level.

The left-hand side of Table 6 indeed shows that in the U.S. market, the LV quintile exhibits the highest operating profitability with 35.65 % and averages a return of 6.61 %. In contrast, he HV quintile has a negative operating profitability of –5.07 % but still generates an average return of 5.29 %. After the double-sorting, the low-volatility anomaly is not alive anymore in the U.S. market. The HV quintile has a higher return of 5.86 % compared to the 5.58 % of the LV quintile. The LV quintile has still a higher operating profitability than the HV quintile with 21.73 % compared to 11.94 %, but the difference is not as remarkable as for the single-sorting. Similar results are observed considering risk-adjusted returns. While the FF-α of the LV quintile is positive and highly significant (0.34 %) after the first portfolio sorting, the risk-adjusted return decreases to 0.25 % performing the double sorting. In contrast, the low-volatility anomaly still exists in the European market after controlling for operating profitability. However, the FF-α decreases from 0.35 % to 0.24 % and does not show any statistical significance after the double-sorting. For the German market, the returns are very similar in magnitude across the quintiles. This indicates that the low-volatility anomaly cannot be supported after controlling for operating profitability. Again, the FF-α analysis does not show significant values for the German market before or after the double-sorting procedure. Overall, considering the risk-adjusted returns, operating profitability might provide an explanation for the low-volatility anomaly in the U.S. market.

d) Dividend Yield

Finally, we run a double-sorting on the dividend yield of the stocks. Dividend yield is calculated on gross dividends (including tax credits). Furthermore, the dividends are based on anticipated annual dividends and exclude special or once-off dividends. Generally, mature firms, that are not growing very quickly pay higher dividend yields. The reason is that they do not rely as heavily on retained earnings as younger companies. As mentioned before, mature firms often have lower volatilities, than younger firms. Therefore, the stocks in the LV quintile should have higher dividend yield, as the stocks in the HV quintile.

On the left of Table 7, we report the average dividend yield of the portfolio quintiles for the single-sorting on volatility. In the U.S. market, the LV quintile has an average dividend yield of 3.12, which is much higher than the average dividend yield of the HV quintile of 0.78. Similar results are found in the European and German market. Thus, our theory is confirmed, that stocks with higher dividend yield correlate with lower volatility. On the right side of Table 7, the results of the double-sorting on dividend yield and volatility are reported. In the U.S. market, the average dividend yield of the LV quintile diminishes to 1.86, while the average dividend yield of the HV quintile rises to 1.65. Thus, the difference in average dividend yield between the LV and the HV quintile almost disap-

Table 7

Double-Sorting on Dividend Yield

First portfolio sorting					Second portfolio sorting			
	Return	FF-α	Std dev	DY	Return	FF-α	Std dev	DY
Panel A: U.S. market								
Quintile 1 (LV)	6.50 %*** [2.69]	0.33 %*** [2.65]	10.88 %	3.12	7.86 %*** [3.10]	0.42 %*** [4.53]	11.18 %	1.86
Quintile 2	6.27 %** [2.14]	0.23 %* [1.94]	13.82 %	2.05	7.28 %** [2.42]	0.30 %*** [2.79]	13.23 %	1.83
Quintile 3	6.11 %* [1.80]	0.12 % [0.95]	17.22 %	1.61	5.52 % [1.65]	0.06 % [0.49]	16.66 %	1.75
Quintile 4	5.65 % [1.55]	0.04 % [0.31]	19.16 %	1.23	6.14 % [1.70]	0.10 % [0.72]	18.82 %	1.72
Quintile 5 (HV)	5.69 % [1.43]	0.01 % [0.09]	26.42 %	0.78	4.23 % [1.20]	−0.15 % [−1.07]	24.29 %	1.65
Panel B: European market								
Quintile 1 (LV)	5.48 %** [2.12]	0.35 %** [2.32]	10.62 %	3.25	5.49 %** [2.02]	0.35 %** [2.20]	10.83 %	2.90
Quintile 2	4.03 % [1.27]	0.15 % [0.82]	13.90 %	3.14	3.89 % [1.25]	0.18 % [1.03]	14.33 %	2.92
Quintile 3	2.45 % [0.89]	0.01 % [0.03]	16.40 %	3.02	3.22 % [1.03]	0.06 % [0.33]	16.59 %	2.90
Quintile 4	2.48 % [0.88]	−0.01 % [−0.05]	20.08 %	2.79	1.56 % [0.68]	−0.04 % [−0.22]	20.04 %	2.86
Quintile 5 (HV)	−0.46 % [0.42]	−0.20 % [−0.83]	26.02 %	2.15	0.07 % [0.48]	−0.25 % [−1.13]	24.66 %	2.78
Panel C: German market								
Quintile 1 (LV)	3.33 % [1.20]	0.26 % [1.09]	15.23 %	2.96	1.25 % [0.64]	0.15 % [0.61]	16.85 %	2.39
Quintile 2	0.33 % [0.48]	−0.05 % [−0.20]	19.99 %	2.70	4.59 % [1.33]	0.30 % [1.20]	18.86 %	2.28
Quintile 3	3.51 % [1.10]	0.18 % [0.65]	23.43 %	2.30	2.72 % [0.89]	0.11 % [0.36]	23.06 %	2.26
Quintile 4	5.46 % [1.42]	0.35 % [1.29]	23.15 %	1.99	4.93 % [1.33]	0.27 % [1.01]	23.78 %	2.19
Quintile 5 (HV)	−0.82 % [0.49]	−0.12 % [−0.36]	30.18 %	1.40	−0.03 % [0.55]	−0.12 % [−0.43]	27.41 %	2.20

Notes: The table shows the effect of the dividend yield of a stock on the low-volatility anomaly. Panel A shows the results for the first sorting according to volatility, Panel B shows the results for the double-sorting. Heteroscedasticity and autocorrelation (HAC) robust t-statistics are reported in brackets. Superscripts *, ** and *** denote statistical significance (at least) at the 10 %, 5 % and 1 % level.

pears. However, the LV quintile has still the highest return with 7.86 %, while the HV quintile has a much lower return of 4.23 %. This is supported by the FF-α's. It even rises after the second sorting from 0.33 % to 0.42 %, still at the 1 % significance level. Similar results are reported for the European market. However, in the German market we observe contrary results. The average return of the LV quintile diminishes to 1.25 %, and while the HV quintile has a lower return of –0.03 %, the other quintiles have higher returns than the LV quintile. Therefore, we conclude that dividend yield can explain the low-volatility anomaly in the German market, but not in the U.S. and the European market. In the German market, the analysis of the FF-α's shows once again that no significant excess returns can be found after the first and second portfolio sort. So, the α's cannot show whether or not the dividend yield is a driver of the low-volatility anomaly in the German market.

IV. Conclusion

Considering the more recent period from January 2000 to December 2018, this study provides evidence that the low-volatility anomaly still is a robust phenomenon in the U.S., the European and the German equity market. Thereby, the low-volatility effect is particularly strong in bull-markets and low-volatility strategies generate positive and significant abnormal returns during these periods. In bear-markets, low-volatility portfolios still outperform the benchmark market portfolio, however, we cannot observe positive and statistically significant abnormal returns. Further results show that the low-volatility effect is robust against changes in the rebalancing frequency of the portfolios. Even a severe reduction of the rebalancing frequency from one to six months does not diminish the expected returns of the low-volatility portfolios. We additionally control for differences in stock characteristics such as trading volume, size in terms of market capitalization, the dividend yield and operating profitability measured by the return-on-equity. While the low-volatility effect in the European equity market cannot be explained by any of these stock characteristics, the higher expected and risk-adjusted returns of low-volatility stocks in the U.S. market are mainly due to differences in the average trading volume and operating profitability. Differences in operating profitability and the dividend yield play a major role in explaining the average outperformance of low-volatility stocks in the German equity market.

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