

## **Long Term Effects of Fiscal and Monetary Policy in a Stock-Flow-Consistent Macro-Framework**

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### **Abstract**

Some fundamental issues of fiscal and monetary policy are discussed within a stock-flow-consistent macroeconomic framework for an open economy. The core of the model is the financial sector with both a capital market and a money market and correspondingly differing interest rates. Further key elements are households which maximize an inter-temporal utility function with respect to both consumption and wealth, profit maximizing firms subject to a production function with decreasing returns to scale, and government expenses funded by both taxes and public debt. Concerning monetary policy, different regimes are discussed, including monetary abstinence, quantitative easing, and monetizing public debt. Typical Keynesian policy measures such as rising wages, expansionary monetary policy, and public deficit spending are also discussed. The main focus is on long term effects of these measures, with the dynamics being only briefly sketched. Because the model is entirely solvable analytically, it may also be useful as a means of didactics. (E10, E40, E50)

### **Zusammenfassung**

#### **Langfristeffekte von Fiskal- und Geldpolitik in einem stock-flow-konsistenten Makromodell**

Einige fundamentale Wirkungen von Geld- und Fiskalpolitik werden innerhalb eines Modells für eine offene Volkswirtschaft diskutiert, in dem alle Strom- und Bestandsgrößen konsistent miteinander und mit dem System der VGR sind. Kern des Modells ist der Finanzsektor, auf dem Kapitalmarkt und Geldmarkt zusammen kommen und auf dem es dementsprechend zwei Zinssätze gibt. Das Modell ist vollständig mikrofundiert mit privaten Haushalten, die ihre laufenden Ausgaben sowie ihr Vermögen optimieren, gewinnmaximierenden Unternehmen mit einer unterlinear homogenen Produktionsfunktion und einer Regierung, die Steuern erhebt und sich verschulden kann. Für die Zentralbank werden unterschiedliche Verhaltensannahmen modelliert, monetäre Abstinenz, quantitative Lockerung und Monetisierung von Staatsschulden. Es werden darüber hinaus typische keynesianische Instrumente wie expansive Lohnpolitik und Deficitspending diskutiert. Das Schwergewicht liegt auf den langfristigen Wirkungen, während die

Anpassungsdynamik nur am Rande behandelt wird. Da das Modell algebraisch lösbar ist, eignet es sich auch für didaktische Zwecke. (E10, E40, E50)

## I. Introduction

Macroeconomics has developed remarkably in recent years, but the appropriate economic policy in the ongoing financial crisis is nevertheless highly controversial. Even among professional economists, there is no consensus on some of the core questions: To what extent can monetary policy change the capital market interest rate in the long run? Does the purchase of government bonds by the central bank necessarily cause inflation, or – if sterilized – rising interest rates in the private sector? Does public deficit spending result in more or less private production in the end? Even within the ECB board, there is considerable disagreement on the impact on inflation and growth of direct government bond purchases by the central bank. There is also substantial disagreement between European and US economists on both the appropriate monetary policy in the crisis and the long-term implications of substantial public debt.

While these issues play an immense role in contemporary economic policy, they cannot adequately be tackled using the relatively simple models of ISLM type which still dominate macroeconomic textbooks, up to the graduate level at least. More sophisticated models of DSGE type (e.g. *Smets/Wouters* (2003)) are more realistic, including rigidities and price contracts. In addition, the empirical validation and/or micro foundation of behavioral assumptions are now standard in macroeconomic modeling. However, this progress comes at the price of increasing complexity and, therefore, decreasing accessibility to those who are responsible for practical policy decisions.

More general models which are simple enough to be understood by graduate students and non-specialized economists, like in *Woodford* (2003), are relatively rare. Moreover, even complex models of DSGE-type are mostly unsatisfactory in some respects. In particular, they normally abstract from a private bank sector, nor do they distinguish between the base rate and long-term interest rate, although these are crucial issues in the prevailing financial environment. Some models do not even include money, and if so, they rarely make explicit how it is introduced into the economy and how this affects the capital market interest rate in the long run. Above all, many models are not stock-flow-consistent, neglecting e.g.

the capacity effect of investment or the long term impact of monetary policy on individual wealth (*Tobin* (1982), 187; *Godley/Lavoie* (2007), 12).

This paper offers a macroeconomic framework which could help to close the gap between standard textbook models and the much more complex models which are used in modern macroeconomic analysis. It is fully stock-flow consistent, and the behavior of all private economic agents is micro-funded. Households maximize an inter-temporal utility function with respect to both consumption and wealth, and firms including commercial banks maximize profits. With these assumptions, different monetary and fiscal strategies are discussed, laying particular emphasis on their long run consequences when the economy eventually reaches a new steady state.<sup>1</sup>

By and large, the framework follows the methodology of *Tobin* (1969), *Taylor* (2004), *Dos Santos* (2005), *Dos Santos/Zezza* (2008), *van Treek* (2009) and *Godley/Lavoie* (2007). A key feature of these approaches is stock-flow-consistency, i.e. all flows which do not immediately perish (like consumption) result in respective stocks in the following period, and the resulting stocks in turn are both subject to economic agent's optimization and affect their future optimal flows. For example, real net investment results in a rising capital stock, which does not only increase labor productivity in the next period, but also contributes to private wealth and thereby has an impact on future saving. Moreover, changes in the quantity of circulating money, caused by either private hoarding or monetary policy, disturb the optimal size and composition of private wealth and thus result in economic responses which are not recognized in pure flow models. Another advantage of stock-flow-consistent models is that they allow for a complete description within the System of National Account (SNA). Thus any inconsistency or incompleteness among flow variables could be detected by just looking over these accounts.<sup>2</sup>

On the other hand, some important limitations of SNA considerations are highlighted by stock flow consistent models as well. In particular, these models throw some light on the relations between saving and hoarding on the one hand and between real investment and money creation on the other hand, thereby taking into account fundamental Austrian criticism of conventional Keynesian views (*Huerta de Soto* (2006);

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<sup>1</sup> An Excel sheet which allows the reader to carry out his own experiments is provided on the author's website [www.cawm.de](http://www.cawm.de).

<sup>2</sup> For an overview on this class of models see e.g. *Papadimitriou, D./Zezza, G.* (2011) and *Taylor* (2008).

*Block* (1999)).<sup>3</sup> For example, from the viewpoint of private households, an increase in their bank deposits is saving, but it may be accompanied by a pure increase in paper money rather than by additional real investment in the economy as a whole. Conversely, when the central bank injects more money into the economy, it thereby creates some additional (involuntarily) saving as well. The respective impacts on interest, income and employment, which have already been analyzed by *von Mises/Hayek*<sup>4</sup>, can only be comprehensively analyzed with a strictly micro-founded, stock-flow-consistent model as it is developed in the following sections.

The present paper is mainly devoted to steady state considerations, i.e. we ask what the long run outcome of various policy measures would be. In particular, we discuss both the effectiveness of a lower base rate in increasing equilibrium total output and employment, and the effect of a higher share of public debt. Here it is where stock-flow-consistency has its main advantages because, in long run equilibrium, not only current income but also total wealth of all economic agents must be optimal in both size and composition.

The dynamics of the model are only briefly sketched, using an exemplary specification which is outlined in the appendix. Generally, the steady state results are fully confirmed by the dynamic behavior of the models. This does not come as a surprise because, with given parameters, the steady state solution of the model is unambiguous and does not show path dependency. Therefore, while other – and maybe more appropriate – assumptions concerning the short term behavior of agents were certainly possible, this would not change the long run equilibrium results of the model at all. For example, while a larger share of public debt may indeed spur total output in the short term, according to the model, it inevitably results in a decline of total output in the long run.

There is of course previous work on the issue with SFC-models, which is mostly based on the standard book by *Lavoie/Godley* (2001/2002) and *Godley/Lavoie* (2007, I). *Zeza/Dos Santos* (2004) have extended the original model of *Lavoie/Godley* by both a government and an independent central bank. However, a foreign sector is still lacking, and behavioral assumptions are not fully micro-founded. For example, the authors introduce liquidity demand of private households, but it is not explicitly

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<sup>3</sup> For an overview on the traditional Austrian theory of the business cycle see e.g. *Ekelund/Hebert* (2011), 516 pp.

<sup>4</sup> See *Nentjes* (2007) and *Thalenhorst/Wenig* (1984) for a modern interpretation of *Hayek's* contribution.

derived from utility-maximization but simply assumed to be proportional to consumption expenses (Zezza/Dos Santos (2004), 5). As will be shown below, this is not at all implied by a neoclassical utility maximization approach, where liquidity demand can rigorously be derived from wealth maximization along with the assumption of a given liquidity preference parameter. Similar criticism can be raised against their heuristic assumptions on investment and (mark-up) pricing of firms. The latter follow indeed the conventional approach of SFC-models, but they are not necessarily consistent with a rigorous neoclassical profit maximization approach as it is used in the present paper.

In a later paper, *Godley/Lavoie* have also examined the sustainability of public debt policy within a quite simple SFC-model for a closed economy (*Godley/Lavoie* (2007), II). In that model, however, there is neither a productive sector, nor is the behavior of private households micro-founded. Moreover, there is no private bank sector, and the model has only a single interest rate. On the other hand, it is analytically solvable, as it is the case in the present model.

The SFC-model by *Le Heron/Mouakil* (2008) is much more similar to our model, having the same sectors (including a private bank sector) and being focused on the financial and monetary markets. It pursues roughly the same approach as it is used in *Le Heron* (2009), where it is also extended to a two-country-open economy. An advantage of their approach over our model is its design as a growth model. On the other hand, their model is neither micro-founded nor analytically solvable. Moreover, it explicitly focuses on short term results. The authors even doubt that long run equilibriums are relevant at all, as in reality parameters constantly change anyway (*Heron/Mouakil* (2008), 430). Our answer to this statement would be that, even with changing parameters, the steady state equilibrium at least tells us into which direction the results tend and, even more important, which states of the economy are definitely not sustainable in the long run. In terms of a physical analogy: Although Newton's law of gravity does never work perfectly in reality, without the knowledge of it one would hardly be able to make airplanes fly and spaceships reach the moon.

The remainder of the paper is organized as follows: Section II. outlines the theoretical framework and introduces the basic equations in the context of the SNA. Section III. defines economic agents' behavior. Section IV. gives the steady state solution of the model and discusses the long run implications of selected non-monetary and monetary policy

measures. Section V. summarizes and comments on both the limitations and possible extensions of the model. Appendix I contains a proof of the consequences of monetizing public debt. In appendix II, the assumptions concerning short term behavior of agents are outlined.

## II. The General Framework: Overview

Consider an open economy which produces nothing but corn by means of labor, real capital (seed), and an absolutely limited factor which may be land or entrepreneurship. Corn lives only for one period and can either be consumed or sown in order to produce more corn in the following period. The corn-producing firms are owned by private households (H) which also provide labor as the second factor of production and consume corn ( $C_H$ ) at the corn price  $p$ . Firms (F) pay a nominal wage rate  $w$  to workers as well as interest  $i$  on their nominal debt capital  $pK_F$ .

Land is unlimited and only used by homogenous, private firms for the purpose of corn production. However, because fertility decreases in the amount of used land by each firm, the degree of homogeneity of the production function is less than unity. As a result, there are pure profits (resp. land rents)  $\Pi_F$  which accrue to private households in equal shares.

It is assumed that firms cannot be traded, because otherwise the assumption of homogenous households would no longer hold. Therefore, the value of firms (the present value of future land rents) is neglected by private households when they optimize their wealth, as is the implicit value of future wage earnings and future tax payments. Note that, while these assumptions are both standard and empirically plausible, they are not entirely innocuous concerning the general results. In particular, with equity being part of the private wealth function, there would be another channel by which policy measures could affect the equilibrium interest rate.

Real capital (in the form of seed) exists for only one year and must be bought by firms at the corn market, where they compete with the consumption demand of both private households and the government. Although corn is the only good in the economy, it is assumed that it is traded at nominal terms by the use of money.<sup>5</sup> Moreover, money is the only

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<sup>5</sup> The one good-assumption is only made in order to avoid index problems with the definition of both capital and real income. In particular, with only one homogenous good, the so called Cambridge Controversy on the proper definition of capital is not relevant.

asset which is available to private households for the sake of both saving and providing for economic shocks. It is partly held in cash ( $L_H$ ) and partly lent to the commercial banks in the form of private deposits  $D_H$ . Firms in turn borrow nominal capital ( $pK_F$ ) from commercial banks, which is part of the latter's supply at the capital market ( $pK_B$ ). Banks possibly lend capital to both the government and the foreign sector, and they borrow money in the form of deposits not only from private households but possibly also from the central bank ( $D_M$ ). Moreover, commercial banks also hold some liquidity  $L_B$  as a reserve. Accordingly, in order to avoid losses, interest on deposits  $i_d$  must be lower than the capital market interest rate  $i$ . Banks are owned by private households, so bank profits  $\Pi_B$  accrue to the latter as well. In equilibrium, bank profits are assumed to be zero.

The central bank (monetary authority M) can issue paper money in the following ways: (i) Purchasing corn in exchange for cash money, (ii) lending money to private banks in the form of deposits, or (iii) purchasing either government bonds or corporate bonds at the capital market and thereby offering direct central bank credit. While the first option (buying corn) is mainly relevant for the initial supply of cash money,<sup>6</sup> options (ii) and (iii) are what central banks normally do when they want to change the quantity of money. Note that both  $D_M$  and  $pK_M$  may well be negative, then reflecting a contractive monetary course. Central bank profits accrue to the government (G). Paper money is only held by commercial banks and private households as a liquidity reserve  $L_B$  and  $L_H$  respectively. Note that  $\Delta L_j > 0$  constitutes saving in the view of the respective sector  $j$  but, for the whole economy, it is just hoarding.

In addition to central bank profits, the government receives income from a proportional wage tax  $\tau$  on labor income. It can also go into debt by issuing government bonds  $pK_G$ . The government does not hold any liquidity, but spends its total net receipts on corn consumption ( $pC_G$ ).

When the economy's capital market interest rate is below the respective interest rate in the rest of the world (X), demand for foreign capital is positive. Part of the home economy's wealth is then held in the form of foreign assets  $pK_x$ . Respective interest payments from abroad are mir-

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<sup>6</sup> The purchase of corn by the central bank would actually mean consumption by the monetary authority, e.g. in order to pay salaries or to buy a new office. Of course, the initial amount of cash money could also simply be donated to private households, as it was e.g. the case in the German monetary reform after World War II.

rored by real corn imports of the economy. Thus, with differing interest rates, there is trade of real corn although the latter is a homogenous good. Analogously, if the domestic interest rate is above the world level, there are net corn exports along with net interest payments to the rest of the world. Hence, a steady-state equilibrium implies a balanced current account but does not necessarily require equilibrium in the balance of trade. Corn is then just used as a means of temporal exchange.

Because individuals live for only two periods, deposits are lent for just one period respectively and must then be renewed. Thus, the terms  $L_j$  represent flows and stocks at the same time. The same is true for real capital  $K$  (seed), which lives for only one period and thus represents both real investment and the capital stock. Note that, in contrast to definition-al balancing in national accounts, the monetary circle is open at three sides: While the central bank can create additional money  $\Delta M$  and channel it into the economy, both private households and banks may hoard some money and thus withdraw it from circulation ( $\Delta L_H$  and  $\Delta L_B$  respectively). As will be shown below, these options are of key importance for the effects of monetary policy.

The complete system of budgetary equations is shown in Table 1. For the sake of brevity, the time subscript of all variables which refer to the current period  $t$  is omitted in the table (and also throughout this paper).<sup>7</sup> Because seed is not storable, and individuals live only for two periods, lent real capital is fully refunded and, as appropriate, reinvested in each period. While this is less important in the steady state, it must be kept in mind concerning transition periods, where investment and repayment do not cancel out necessarily.

In a stationary state equilibrium, no net savings exist, although there is some gross saving in the form of the annual seed (note that  $Y$  denotes GDP rather than net income). With a constant population and with both given preferences and technology, there exists a definite stationary state, which is not path-dependent and, thus, also independent of the design for the short run dynamics of the model. However, the long run equilibrium crucially depends on the respective economic policy chosen. In particular, different levels of the base rate and/or of government debt result in different levels of gross domestic income  $Y$  in the steady state, as will

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<sup>7</sup> Debt capital demand and supply  $K_j$  of all sectors  $j$  add up to zero, see equation (5) below. Analogously, in steady state equilibrium, additional money supply  $\Delta M$  must equal additional liquidity demand  $\Delta L_H + \Delta L_B$ , see equation (3) below.



*Table 1*  
**Matrix of Flows**

	H	F	B	G	M	X
H	$\Delta L_H = L_H - L_{Ht-1}$	$pC_H$	$D_H$	$[\tau wN]_{t-1}$		
F	$[wN + p\Pi_F]_{t-1}$		$[(1+i)pK_F]_{t-1}$			$[ipK_X]_{t-1}$
B	$[(1+i_d)D_H + p\Pi_B]_{t-1}$	$pK_F$	$\Delta L_B = L_B - L_{Bt-1}$	$pK_G$	$[(1+i)pK_M]_{t-1} + [(1+i_d)D_M]_{t-1}$	$pK_X$
G		$pC_G$	$[(1+i)pK_G]_{t-1}$			
M			$pK_M + D_M$	$[\Pi_M]_{t-1}$	$\Delta M = M - M_{t-1}$	
X			$[(1+i)pK_X]_{t-1}$			

*Table 2*  
**Flows in System of National Account**

				+ Private Household Interest Income	$+i_d \frac{D_H}{P}$
+ Wages	$+\frac{w}{p}N = \gamma Y$	+ Private Household Consumption	$+C_1 + C_2$	+ Wages	$+\frac{w}{p}N = \gamma Y$
+ Firm Interest	$+iK_F = \beta Y$	+ Public Consumption	$+C_G$	+ Profits	$+\Pi_F + \Pi_B + \Pi_M$
+ Firm Profits	$+\Pi_F = (1-\gamma-\beta)Y$	./.. Commodity Import	$Y_X = iK_X$	./.. Interest on foreign and public Debt	$i(K_X + K_G)$
+ Re-Investment	$+K_F$	+ Depreciation	$+K_X$	+ Depreciation	$+K_F$
= GDP	$= Y$	= GDP	$= Y$	= GDP	$= Y$

be shown below. The main interest of the present paper is analyzing such long run effects of economic policy.

In Table 2, the main flows are shown within the System of National Account. Note that neither interest on government debt nor interest receipts from abroad are part of the GDP, although they contribute to private disposable income. The double-accounting of SNA shown in Table 2

must not be confused with the actual flows of receipts and expenses shown in Table 1. In particular, there are differences when money is either hoarded by the private sector or freshly printed by the central bank.

Note that, while the dynamics of the model depend on economic adjustments in nominal terms, the steady state equilibrium is exclusively determined in real terms, because so are the objective functions of all economic agents. Thus, in order to investigate the steady state solution, all variables including liquidity must be defined in corn units, although actual trades are made in nominal terms.

To begin with, real wealth of private households consists of real bank deposits and real liquidity:

$$(1) \quad V = \frac{D_H}{p} + \frac{L_H}{p}$$

The balance sheet of the commercial bank sector in real terms is given by

$$(2) \quad K_B + \frac{L_B}{p} = \frac{D_H}{p} + \frac{D_M}{p}$$

where central bank deposits  $D_M$  can well become negative (the deposits then convert into reserves of the commercial banks). We assume that neither the government nor firms nor foreign agents hold domestic currency. Then, because all paper money in circulation must be held by someone as liquidity, it follows that

$$(3) \quad M = \bar{M} + D_M + pK_M = L_H + L_B$$

where  $\bar{M}$  is cash money, and  $K_M$  is central bank credit supply at the capital market. Because  $D_M$  depends on commercial banks' demand of deposits, the total amount of money  $M$  can only indirectly be controlled by the central bank, e.g. by varying the base rate (see below).

After some manipulation of terms, from (3) it follows that the corn price is given by

$$(4) \quad p = \frac{Y_{nom}}{Y} = \frac{\bar{M}}{\frac{L_H}{p} + \frac{L_B}{p} - \frac{D_M}{p} - K_M}$$

Finally, equilibrium in the capital market requires that

$$(5) \quad K_H + K_M = K_F + K_G + K_X$$

where we have the supply of capital on the left hand side and the demand for capital on the right hand side of the equation. From (1), (4) and (5) it follows that a positive price level requires

$$(6) \quad K_X + K_G + K_F \leq V$$

i.e. real capital demand in the form of corn cannot exceed the amount of real wealth which is voluntarily held by private households in a steady state.

### III. Economic Agent Behavior

#### 1. Productive Firms

Let the corn production function be

$$(7) \quad Y = N^\gamma K_{F,t-1}^\beta \quad \gamma + \beta < 1$$

where  $Y$  is gross domestic production, which includes depreciation but not foreign interest payments. Both labor  $N$  and capital  $K_F$  are paid their marginal (gross) product, so their shares equal the respective exponents and the share of pure firm profits is  $1 - \gamma - \beta$ . Note that the factor price of capital is  $1 + i$ , because the seed completely depreciates in one period and, hence, gross capital income must cover both capital regeneration and pure interest.

In a stationary state we have  $K_{F,t-1} = K_F$ . Thus, with the marginal productivity condition, equilibrium labor input is given by

$$(8) \quad N = \left( \frac{\beta}{1+i} \right)^{\frac{-\beta}{\gamma+\beta-1}} \left( \frac{w/p}{\gamma} \right)^{\frac{1-\beta}{\gamma+\beta-1}}$$

and total corn output is

$$(9) \quad Y = \left( \frac{w/p}{\gamma} \right) N = \left( \frac{1+i}{\beta} \right) K_{F,t-1}$$

Note that, unlike in the case of linear homogeneity, both factor demand and output are uniquely determined by the factor prices.

## 2. Private Households

Following Peter Diamonds well known OLG-model (Diamond 1965), private households live for two periods (1; 2) and maximize the following utility function:

$$(10) \quad U_H = C_1^\alpha C_2^{1-\alpha} C_G^\omega (Z - N)^\phi + \left( \frac{D_H}{p} \right)^{id} \left( \frac{L_H}{p} \right)^{il}$$

The first summand in (10) denotes utility from private consumption. We assume a constant population, so for private consumption we have a constant steady state volume  $C_H = C_1 + C_2$ . The remaining variables in the first summand are public goods (government consumption  $C_G$ )<sup>8</sup>, and leisure, which is total time budget  $Z$  minus labor time  $N$ .

The second summand represents utility resulting from wealth as an optimal composition of liquidity and deposits, with  $i_d$  being the money market interest rate on deposits and  $i_l$  denoting the (non-pecuniary) advantage of holding liquidity (Keynes' liquidity preference).

For simplicity it is assumed that individuals work only in the first period of their life, earning total real wage income  $wN/p$ , but no capital income. Conversely, in the second period, they are retired and earn all profits as well as the returns from their former savings. Thus, concerning the first part of (10), the relevant restriction is given by

$$(11) \quad \left( \frac{w}{p} \right) N(1 - \tau) - C_1 - \frac{C_2}{1 + i_v} = 0$$

As equation (11) reveals, only labor is assumed to be taxed by the proportional tax rate  $\tau$ , while all capital income is tax-free. Note that the average interest on former savings  $i_v$  in (11) is lower than the interest on deposits  $i_d$ , because liquidity yields only a "virtual" interest  $i_l$ . In particular, when  $V$  is total wealth of households, we have

$$(12) \quad i_v = \frac{\left( \frac{D_H}{p} \right) i_d + \left( \frac{L_H}{p} \right)}{V}$$

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<sup>8</sup> Because the volume of public goods is not directly chosen by private households,  $C_G$  is treated as a constant in maximizing (10). However, if taxes were the only source for government expenses, it would also be possible to maximize (10) with respect to the tax rate  $\tau$  in order to find a welfare-optimal mixture of private and public consumption. The respective solution is easily derived as  $\tau^* = (1 + 1/\omega)^{-1}$ , where  $\omega$  is the exponent of  $C_G$  in (10), i.e. the relative weight of the public good in the individual's utility function.

Maximizing (10) with respect to (11) yields solutions

$$(13.1) \quad N = \frac{Z}{1 + \phi}$$

$$(13.2) \quad C_2 = (1 - \alpha)(1 - \tau)(1 + i_v)N \frac{w}{p}$$

$$(13.3) \quad C_1 = \alpha(1 - \tau)N \frac{w}{p}$$

$$(13.4) \quad S = V = (1 - \alpha)(1 - \tau)N \frac{w}{p}$$

An advantage of this approach is that both optimal labor supply  $N$  and optimal savings  $S$  are independent from the interest rate. Moreover, savings are identical to total wealth  $V$ , because individuals live only for two periods. These features greatly facilitate the algebraic solution of the model. In particular, the two parts of the utility function (10) can be optimized separately. Note that (13.4) confirms Keynes' view that savings are independent from interest but depend on disposable income, although we have applied a pure neoclassical utility-maximization approach in this model.

From (13.4) it follows that the second summand in (10) is subject to the restriction

$$(14) \quad \left( \frac{D_H}{p} \right) + \left( \frac{L_H}{p} \right) = V = (1 - \alpha)(1 - \tau)N \frac{w}{p}$$

The respective optimality conditions are

$$(15.1) \quad \left( \frac{D_H}{p} \right) = \frac{i_d}{i_d + i_l} (1 - \alpha)(1 - \tau)N \frac{w}{p}$$

$$(15.2) \quad \left( \frac{L_H}{p} \right) = \frac{i_l}{i_d + i_l} (1 - \alpha)(1 - \tau)N \frac{w}{p}$$

Thus the equilibrium share of liquidity declines in the money market interest rate  $i_d$ , which is again in accordance with Keynesian theory. Note that, in transition periods, involuntarily held liquidity balances can emerge such that conditions (15) are not satisfied. Respective real-balance effects are a well-known driver of business cycles and in fact play a central role in the dynamics of the model (see below).

### 3. Labor Market

From the firm's labor demand (8) and the household's labor supply (13.1) the equilibrium real wage rate is easily calculated as

$$(16.1) \quad \left(\frac{w}{p}\right)^* = \gamma \left(\frac{\beta}{1+i}\right)^{\frac{\beta}{1-\beta}} \left(\frac{Z}{1+\phi}\right)^{\frac{\gamma+\beta-1}{1-\beta}}$$

Alternatively, it can also be assumed that the real wage rate  $w/p$  is above its equilibrium level and held constant either by law or by a wage contract with price level indexation:

$$(16.2) \quad \left(\frac{w}{p}\right) = \text{const.}$$

In this case, labor input  $N$  is directly given by labor demand (8).

In the dynamic version of the model, also a fixed nominal wage rate can be assumed:

$$(16.3) \quad \left(\frac{w}{p}\right) = \left(\frac{\bar{w}}{p}\right)$$

The latter assumption is not appropriate for the calculation of the steady state solution, because this would require numerical methods.

### 4. Commercial Banks

Commercial banks provide real capital  $K_B$  by lending their deposits to other agents and earning the capital market interest rate  $i$ . It is assumed that banks hold a certain fraction  $l$  of total deposits in the form of liquidity, either by law or voluntarily as a precaution. Hence we have

$$(17) \quad \frac{L_B}{p} = l \left( \frac{D_H}{p} + \frac{D_M}{p} \right)$$

and

$$(18) \quad K_B = \frac{D_H}{p} + \frac{D_M}{p} - \frac{L_B}{p}$$

In equilibrium, bank profits  $\Pi_B$  must be zero because, unlike in the case of farms, there is no absolutely limited factor. Hence, in equilibrium we have

$$(19) \quad iK_B = i_d \left( \frac{D_H}{p} + \frac{D_M}{p} \right)$$

From (17) to (19) it follows that the relation between the money market interest rate and the capital market interest rate in equilibrium is

$$(20) \quad i_d = (1 - l)i$$

### 5. Current Account

Because corn is the only good, there are no gains from foreign commodity exchange within one period. However, gains from international intertemporal commodity exchange in the form of lending and borrowing are well possible. It is reasonable to assume that lending from abroad is only an imperfect substitute to domestic lending, because it involves additional risks in terms of exchange rate fluctuations and lower trust in foreign debtors. Then, even with differing interest rates, the share of capital held abroad (or borrowed from abroad), in relation to total income, is limited. In particular, it is assumed that

$$(21) \quad K_X = \frac{a_X(i_X - i)}{1 + i} Y = \frac{Y_X}{i}$$

In (21),  $a_X$  is a measure of capital elasticity concerning international interest rate divergence,  $K_X$  is the real value of capital (in corn units) which has been invested abroad, and  $Y_X$  is the real corn import which, in equilibrium, is equal to interest payments from abroad on that capital.<sup>9</sup>

The denominator in (21) is only motivated as a means to facilitate the calculation of the equilibrium interest rate.<sup>10</sup> Note that the parameter  $a_X$  must not exceed certain limits because of the fundamental restriction (6). In economic terms, with a too high value of  $a_X$ , more capital would be required for foreign investment than is available from the domestic capital market.

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<sup>9</sup> Note that, in steady state equilibrium, there are no net capital exports or imports, but only interest payments, while foreign capital itself is refunded and instantly renewed in each period (as is all other capital). In terms of SNA, we then have for Gross National Income  $GNI = Y + Y_X$  and for Gross Domestic Product  $GDP = Y = C_H + C_G + K_X - Y_X$ .

<sup>10</sup> See equation (27) below. With reasonable values for  $i$ , the denominator is close to unity and thus does not make a substantial difference.

## 6. Government

As was already mentioned, the government raises a proportional tax  $\tau$  on wage income, but there is no tax on profits and interest. The latter assumption is again a mere simplification and could well be relaxed.

The government can also issue debt  $K_G$  at the capital market interest rate  $i$ , and in addition earns the central bank's profits  $\Pi_M$ . All government revenues are consumed, while there is no public investment. This assumption is mainly motivated by the fact that otherwise a second (public) good would have to be modeled. Concerning the general results of the model, the assumption is innocuous as long as public debt is at least consumed to a greater part than private debt.

In order to derive the steady state equilibrium, it is useful to define public debt as a fraction of GDP, i.e.

$$(22) \quad K_G \equiv gY = \frac{\tau N(w/p) + \Pi_M - C_G}{i}$$

The right hand side of (22) simply reflects the government's budget equation in a stationary state, where the sum of revenues from taxes and central bank profits must equal government consumption plus interest on public debt (all being here defined in real terms).

Like parameter  $a_X$  in (21), the parameter  $g$  cannot be chosen arbitrarily high because of the fundamental restriction (6). In the dynamic version of the model, it is also possible – and more reasonable – to have any nominal value  $pK_G$  (instead of  $g$ ) as the exogenous variable.

Note that, in the steady state, government consumption decreases in  $g$ , because of the interest payments on public debt. This is not necessarily the case in the short term.

## 7. Central Bank

The central bank controls the base rate  $i_b$ , but can also lend directly to the capital market at the capital market interest rate  $i$ . Analogously to government debt, for the steady state solution of the model it is useful to define the direct capital market supply of the monetary authority as a fraction of GDP:

$$(23) \quad K_M \equiv mY$$



Again, in transition periods, it is more reasonable to assume a given nominal value  $pK_M$ .

Concerning central bank deposits at the commercial bank sector, we allow for two different policy regimes:

#### a) Monetary Abstinence (Taylor Rule Regime)

With a Taylor rule, when both the output gap and the inflation gap are zero, the base rate is set equal to the long run equilibrium money market interest rate  $i_d^*$ . Thus, in a steady state, we have by definition

$$(24) \quad \frac{D_M}{p} = 0$$

$$(25) \quad i_b = i_d^* = (1 - l)i^*$$

where  $i^*$  denotes the equilibrium capital market interest rate with  $D_M / p = 0$ .

This regime means that the central bank does not try to manipulate steady state output by means of interest policy, but confines itself to the mitigation of business cycle fluctuations and external shocks. As long as only steady states are considered, this strategy can therefore be labeled as monetary abstinence.

#### b) Quantitative Easing (Fixed Base Rate)

By quantitative easing we mean an unlimited money supply by the monetary authority at a given base rate, with the latter being arbitrarily chosen by the central bank. In a way, this regime is just opposite to monetary abstinence. Because the base rate is also paid to commercial banks in case of negative central bank deposits, it definitely determines the money market interest rate:

$$(26) \quad i_d = \bar{i}_b$$

From (20) it follows that, under this regime, in equilibrium we have  $i = \bar{i}_b / (1 - l)$ , i.e. the equilibrium capital market interest rate is nearly perfectly controlled by the central bank as well. The only external variable is the commercial bank's liquidity parameter  $l$ , which can well rise above its legal minimum, e.g. in a recession.

#### IV. Steady State Solution and Selected Results

In this section, we examine the steady state results of the model under varying assumptions concerning both individual behavior parameters and policy strategies. In particular, the long run impacts of different levels of liquidity preference, government debt, and monetary measures are discussed. The short run dynamics of these variations are also shown, using an exemplary specification of the respective adjustment behavior which is described in the appendix. These considerations are, however, of minor importance in the present paper and serve only as an illustration. While numerous different specifications for the short term reactions are conceivable, with given fundamental parameters, they all result in the same long run equilibrium in the present model. Thus, although the model is not immediately ready for empirical calibration, it can improve our understanding of complex long run linkages between equilibrium stocks and flows which cannot be detected in purely short run oriented, partial models.

The key variable of the model is the capital market interest rate. It must generally be calculated from the capital market equilibrium condition (5), with the only exception of the quantitative-easing regime where  $i$  is indirectly set by the central bank.

In the more general case, we must transform (5) by inserting (17) and (18):

$$(27) \quad K_F + K_G + K_X = \frac{D_H}{p}(1-l) + \frac{D_M}{p}(1-l) + K_M$$

Equation (27) is the link between the capital market and the money market. By inserting the right hand side of (9) as well as (15.1), (21), (22), and (23), and using that  $N(w/p) = \gamma Y$ , (27) can be rewritten as follows:

$$(28) \quad \frac{a_X(i_X - i)}{1+i}Y + gY + \frac{\beta}{1+i}Y = mY + \frac{i_d}{i_d + i_l}(1-\alpha)(1-\tau)\gamma Y(1-l) + \frac{D_M}{p}(1-l)$$

The resulting equilibrium interest rate depends on the monetary regime chosen. In the monetary abstinence regime, we have  $D_M = 0$  and, hence,  $Y$  cancels out in (28). The equilibrium interest rate can then be calculated from the quadratic equation

$$(29) \quad i^* = -\frac{H_2}{2H_1} \pm \sqrt{\left(\frac{H}{2H_1}\right)^2 - \frac{H_3}{H_1}}$$

where

$$(29.1) \quad H_1 \equiv (m - g)(1 - l) + (1 - l)^2(1 - \alpha)(1 - \tau)\gamma + a_X(1 - l)$$

$$(29.2) \quad H_2 \equiv (m - g)(1 - l) + \phi i_l + (1 - l)^2(1 - \alpha)(1 - \tau)\gamma \\ - \beta(1 - l) - a_X i_X(1 - l) + a_X i_l$$

$$(29.3) \quad H_3 \equiv (m - g)i_l - \beta i_l - a_X i_X i_l$$

In contrast, in the quantitative easing monetary regime, the capital market interest rate is immediately determined by the base rate according to (20). In that regime, its totally elastic deposit supply enables the central bank to generate whatever interest rate it prefers. The consequences of such a policy for the price level are discussed below.

Once the capital market interest rate is known, all other variables in the model can be calculated by simple insertion into the equations given above.

The long term responses to policy measures predicted by the model can be examined by just varying the respective parameters such as e.g.  $g$  or  $m$ , or by changing the respective monetary or wage policy regime. We consider first some non-monetary measures, in particular expansionary wage policy and fiscal deficit spending. In order to separate effects, this is done under the monetary abstinence regime, i.e. there is no additional monetary impulse, neither positive nor negative. We then turn to monetary policy, which is the main issue in the present paper.

### 1. *Rising Wages*

From (8) it is immediately clear that the firm's labor demand c.p. declines in the real wage rate, because the relevant exponent is negative. As Figure 1a shows, the effect of a pure increase in the real wage rate  $w/p$  above its equilibrium level indeed reduces total output, while the price level increases.

However, the so-called purchasing power theory of wages claims that there might be a compensating effect: Because workers save less than capitalists, consumption demand would rise with an income shift from firm-

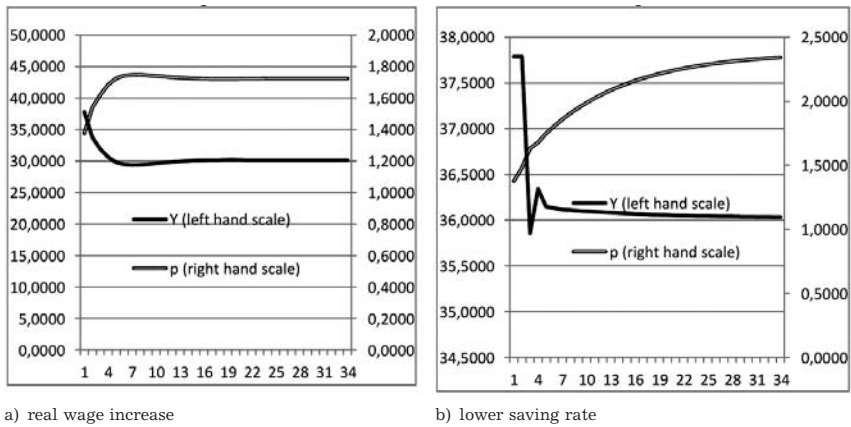


Figure 1: Effects of Expansionary Wage Policy

owners to workers, and this could lead to a shift of the labor demand curve (8) which outweighs or even dominates the effect of rising labor costs.

Because our model does not explicitly distinguish between worker households and capitalist households, we cannot directly incorporate this effect. However, as a substitute, we can assess the effect of a rising propensity to consume ( $\alpha$ ) in the private saving function (14.4). Obviously, the only way how  $\alpha$  could affect labor demand is a decreasing interest rate, because there is no other endogenous variable in the labor demand equation (8). However, according to (13.4), (14) and (27), a higher  $\alpha$  reduces private saving  $S = V = (L_H + D_H) / p$  and thereby increases the steady state capital market interest rate. Hence, with a higher propensity to consume, the equilibrium wage rate declines according to (16.1), and so does total output, as is revealed by Figure 1b.

A recent variant of the purchasing power argument admits that rising wages come at the expense of declining capital income and thus do not necessarily increase total demand including investment. However, it is claimed that a substantial part of profits and interest income does not really flow into real investment but is only held in the form of pure financial assets or liquidity. Hence, shifting income from capitalists to workers would increase total demand for real goods and, at the same time, reduce unproductive and dangerous financial speculation.

In terms of our model, this variant of the argument can be translated into a decrease in liquidity preference of either households ( $i_l$ ) or com-

mercial banks ( $l$ ). According to (28), in both cases the capital market interest rate decreases, because less wealth is held in the form of liquidity and more real assets are demanded at the capital market. Hence, when rising wages lower average liquidity holdings, there is indeed a positive impact on total output, although the price level rises as well (see also Section IV. 3. below).

Hence we are left with three effects of a rising wage rate, two negative and one positive concerning total income. However, both the negative cost effect and the negative saving effect appear much more direct and probable than the indirect liquidity effect, if the latter is relevant at all. Moreover, the natural way to fight a rising liquidity preference would be the provision of additional money rather than taking the somehow strange detour of a rising wage bill. As will be shown below, this would be a much better cure which does not have the disadvantage of the two negative effects and, thus, is clearly the dominant strategy in case of doubt.

## 2. Public Debt

In order to analyze the pure effect of additional government debt, we first assume monetary abstinence again. Then, in the long run, any increase in public debt clearly reduces both steady state output and the equilibrium wage rate, while the price level increases (see Figure 2a). In formal terms, according to (29), the equilibrium capital market interest rate increases in  $g$ , while the equilibrium wage rate  $w/p$  unambiguously decreases in  $i$  according to (16). The rising price level results from the decrease in real income going along with unchanged liquidity in the market. The economic intuition of the negative effect on total output is straightforward: Because all revenues from public debt are consumed by assumption, a crowding out effect occurs which can be neither avoided nor compensated in the long run, although there may occur opposite effects in the short term.<sup>11</sup>

When public deficit spending is accompanied by an increase in money supply, the negative effect on total income can be avoided in the long run,

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<sup>11</sup> Because both total income and government consumption decrease in  $g$  in the steady state, so does aggregate consumption. Hence, from the steady state point of view, social welfare tends to decrease in the amount of government debt. On the other hand, there might be compensating effects in transition periods, if individuals should prefer more public goods today over (a higher amount of) public goods tomorrow.

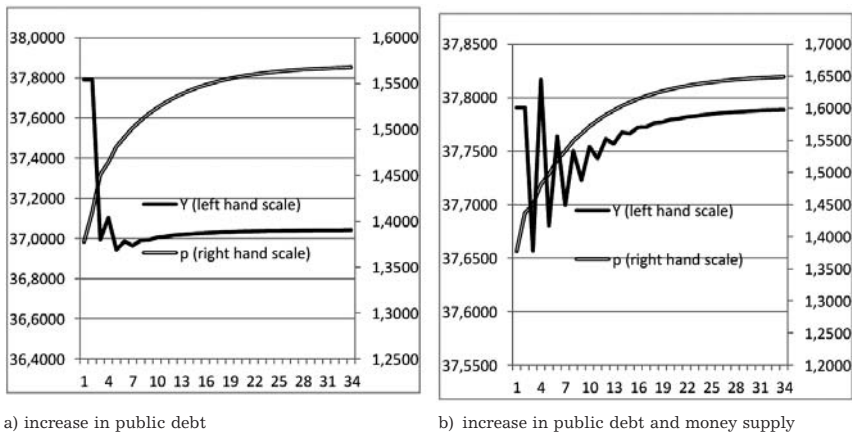


Figure 2: Effects of Fiscal Debt

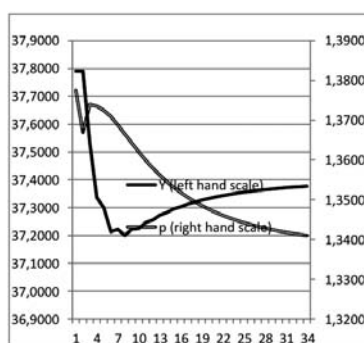
but this comes at the cost of an even steeper increase in the price level. In Figure 2b it is assumed that the central bank, while generally sticking to the monetary abstinence strategy, provides more direct credit which is just sufficient to meet additional government debt (i.e.  $\Delta K_M = \Delta K_G$ ). In this case, total output eventually returns to its initial level despite the additional public debt, which is now fully funded by (temporary) inflation. Now there is only a temporary sacrifice of the private sector in terms of corn, which mirrors the (also temporary) increase in real public consumption during the transition periods.<sup>12</sup> We will return to this issue in Section IV.5. on monetizing public debt below.

### 3. Rising Liquidity Preference

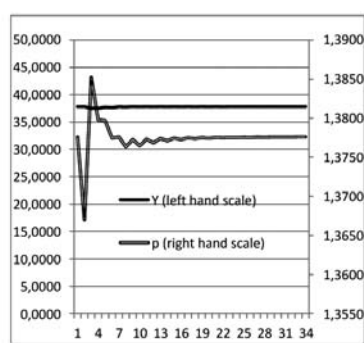
Both hoarding and the creation of additional money disturb, in a way, the monetary circle. While monetarist and Austrian economists usually believe that such disturbances are only temporary, although harmful, we will demonstrate that they have an impact on the long run equilibrium as well.

As was already argued in Section IV.1., a rise in private household's liquidity preference  $i_l$  reduces capital supply and, hence, tends to increase

<sup>12</sup> Note that, in the steady state, government debt cannot exceed the sum of government income, divided by the interest rate, because otherwise government consumption would become negative.



a) increased liquidity preference



b) increased liquidity preference compensated

Figure 3: Effects of Rising Liquidity Preference

the capital market interest rate. As a consequence, both total output and the price level decrease, unless the monetary authorities react (see Figure 3.a).

Note that the private savings rate  $(1 - \alpha)$  may well remain unchanged at the same time. Thus, with rising liquidity preference, output does not decrease because people save too much, but because they prefer to hold more liquidity and less real investments in their portfolio. A similar effect arises when commercial banks' liquidity share  $l$  increases. As can be seen from (28), the supply of real capital decreases and the capital market interest rate goes up again. The deflationary results are similar to those of a rising liquidity preference of households.

As was already stated before, the natural answer to such a development would be an increase in money supply. In case of the quantitative easing strategy, this is automatically achieved because, under this strategy, every additional demand for money is met by the central bank at a fixed base rate. Hence, there is no effect of a changing liquidity preference at all. The same is true, in the long run at least, when the central bank follows the monetary abstinence strategy, but compensates for the increased liquidity preference by providing more capital (i.e.  $\Delta K_M > 0$ ). Because this kind of monetary support requires some time and must be fine tuned as well, there are some short term effects, but in the end, both the initial output and price level can be retained (see Figure 3.b).

#### 4. Increase in Money Supply

The simplest expansionary monetary policy is printing some additional fiat money  $\bar{M}$ , which can be brought into circulation by either donations (Milton Friedman's helicopter case) or by commodity purchases of the central bank. It is immediately clear from (4) that, with given real demand for both liquidity and deposits, the result is simply a price increase which is proportional to the increase in  $\bar{M}$  (see Figure 4a).

Because  $\bar{M}$  is not an element of (28), there is also no direct impact on the interest rate. Only in the short term there may (and presumably will) be some indirect effects, because people feel temporarily richer than before due to the additional money. In the long run, however, all elements of capital demand and supply in (28) are exclusively determined by real parameters, which are independent of  $\bar{M}$ . Hence, so far, the conventional Monetarist's view is supported by the model.

The latter result does not hold, however, when the additional money is brought into circulation by either lowering the base rate  $i_b$  or by a (permanent) increase of central bank loans  $K_M$  (Figure 4.b). In both of these cases, not only the price level, but also total real income increases in the long run. The latter result is in sharp contrast to the Monetarist's view, but was already derived by *Metzler* (1951, 97).<sup>13</sup> In formal terms, now there is not only an increase in money supply, but also a rising supply of capital by the central bank, as can be seen from (28). Thus, the interest rate  $i$  decreases which in turn increases equilibrium real income.<sup>14</sup>

An intuitive explanation could point to a kind of illusion by the private agents, who do not realize that they effectively own the central bank. Otherwise they would know that, along with the additional credit supply, a seignorage accrues which tends to reduce their future tax burden. In other words: Private agents would then consider the central bank's capital supply as a perfect substitute for their own capital supply and, hence, reduce the latter as the former increases. In case of such a Ricardian counter-effect, no decrease in the capital market interest rate would occur and thus any kind of expansive policy would be ineffective concerning total output and employment in the long term (*Barro* (1974)).<sup>15</sup> Nor-

<sup>13</sup> See also *Niehans* ((1978), 87).

<sup>14</sup> Note that government consumption is increased at the expense of private consumption. The reason is the seignorage from increased central bank profits.

<sup>15</sup> See appendix II for a formal proof.



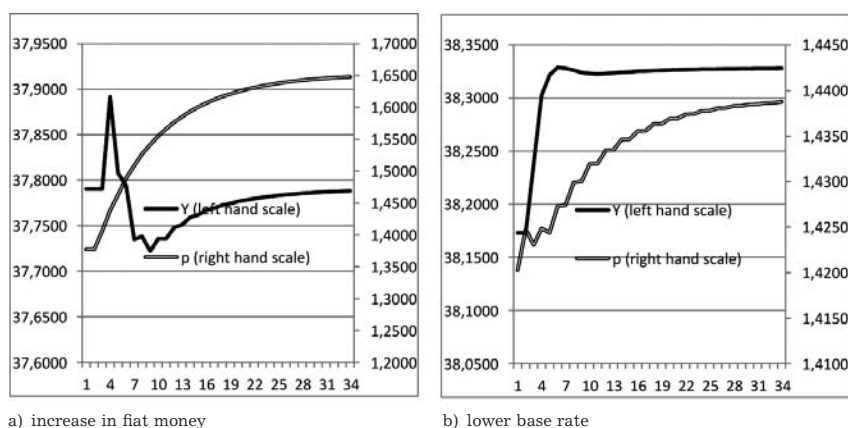


Figure 4: Effects of Expansionary Monetary Measures

mally, however, strict Ricardian equivalence does not hold, so the expansionary effect on real output remains.

On the other hand, the latter comes at the price of temporary inflation because, according to (4), a rise in either  $K_M$  or  $D_M / p$  tends to increase  $p$  as well. Hence, there is a kind of static Phillips curve (i.e. a permanent increase in total output does not imply permanent, but only temporary inflation). Moreover, even with a zero interest rate, the conceivable rise in real income that could be generated thereby is limited, because the marginal productivity of productive capital would then eventually tend to zero. Remember that  $\delta Y / \delta K_F$  denotes the marginal increase in gross domestic product. Accordingly, while there is still some additional gross production, it falls short of gross investment as the marginal product of capital approaches zero. Therefore, at this point at latest, expansionary monetary policy would reduce social welfare.

### 5. Monetizing Public Debt

A currently intensively disputed issue is monetizing public debt, i.e. the purchase of public bonds by the central bank. Because in our model public and private bonds are perfect substitutes, we have implicitly considered this case in the previous section already: With given capital demand, the result of additional capital supply by the central bank is an increase in both  $Y$  and  $p$ , i.e. a static Phillips curve relation. However, this does not necessarily mean that monetizing public debt is inflation-

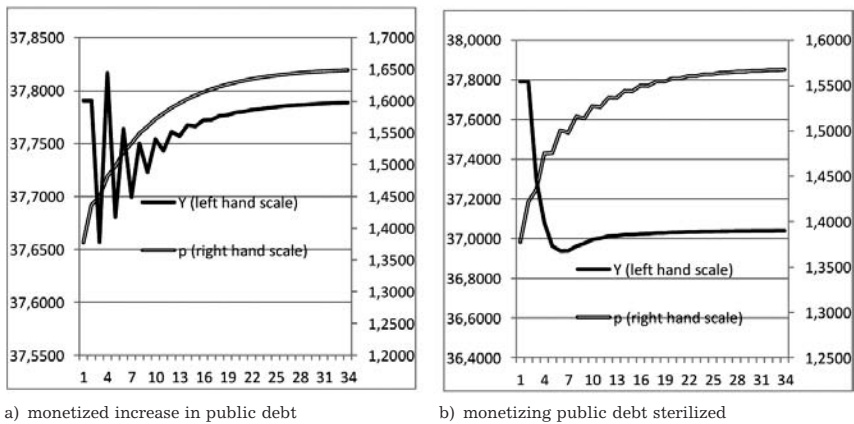


Figure 5: Effects of Monetizing Public Debt

ary, because the additional money supply could well be sterilized, e.g. by a compensating increase in the base rate. On the other hand, the respective sterilization measures raise the question what this would imply for the capital market interest rate and, thus, for the provision of capital for the private sector.

Figure 5a (which is quite similar to Figure 4b above) shows the effect of an increase in public debt which is accompanied by a respective increase in central bank credit, i.e. with  $\Delta K_M = \Delta K_G > 0$ . In contrast, in Figure 5b it is assumed that the inflationary effect of monetizing public debt is sterilized by an appropriate increase in the base rate. As a comparison of Figure 5a with Figure 2a (in Section IV.2.) reveals, this is by all means possible, with both total output and the price level being just the same as in case without any monetary action (i.e. as in Figure 2a above).

Indeed, as is proved in the appendix, with  $\Delta K_M > 0$  and  $\Delta p = 0$ , the required increase in the base rate (or any other contractive monetary measure) implies an unchanged capital market interest rate. Hence, our model supports the view that monetizing a given amount of public debt can be sterilized such that neither the price level is higher nor total output is lower than it would be anyway with a pure increase in public debt. Lastly, by such a strategy, former private lending to the government is merely substituted by respective lending by the central bank. Thus, more private lending is left for private capital needs, so eventually nothing changes at all on the capital market.

On the other hand, with the central bank always meeting the borrowing needs of the government, it is quite likely that public borrowing increases. This, however, would indeed decrease aggregate income, as we have shown in Section IV.2. Thus monetizing public debt does little harm when the level of public debt is not affected thereby, while otherwise it indeed threatens both total income and price stability.

## V. Concluding Remarks

For the sake of brevity, we did not fully exploit the potential of the framework which was developed above. For example, both foreign trade issues and welfare considerations have only been addressed very briefly, although the model offers a wide range of possibilities to do this. The same is true for the incorporation of technical progress and population growth.

We have also only touched upon the dynamics of the model. Generally, the dynamics show cyclical approaches towards the respective new equilibrium and thus confirm the results of the steady state considerations. Note that a new steady state general equilibrium is attained only after several periods, although individual life was restricted to only two periods. This might be seen as a quite strange feature of the model. On the other hand, it does not at all affect the steady state results which are the main focus of the present paper. Moreover, with more than one period of individual life, compound interest would creep into the formulas and the model would lose its tractability, which is, however, one of its main advantages.

Anyway, even in the dynamic version, the present model is not immediately suitable for an empirical application. The main problem is again, that we have restricted the span of both the individual's lifetime and the production process to only two periods. This keeps the model mathematically tractable, but it also leads to an unrealistically low capital/income-relation. Moreover, phenomena like long-run accumulation of capital and the emergence of bubbles cannot be analyzed within this limited framework. This does not rule out, however, that a more realistic version might be developed from it later on.

Concerning the dynamics, we have also saved some details which would have to be taken into account in an empirically relevant model. For example, there is no distinction between nominal and real interest rates, the latter being adjusted for the (expected) inflation rate. Concerning the

steady state results, this does not make a difference, because there is no inflation in the steady state in this model. Nonetheless, for a more realistic and interesting modeling of the short term dynamics, it would be desirable to take account of those effects.

### Appendix I: Proof on Monetizing Public Debt

Proposition:  $\Delta K_M > 0$  and  $\Delta p \stackrel{!}{=} 0$  implies  $\Delta i = 0$

Proof: According to (4), an unchanged price level requires

$$(A1) \quad \Delta K_M = \frac{\Delta L_H}{p} + \frac{\Delta L_B}{p} - \frac{\Delta D_M}{p}$$

With given capital demand, a constant capital market interest rate  $i$  would also require an unchanged capital supply. Thus, according to (5) and (17), we have to show that

$$(A2) \quad \Delta K_B + \Delta K_M = (1-l)\Delta \left( \frac{D_H}{p} + \frac{D_M}{p} \right) + \Delta \left( \frac{L_H}{p} \right) + \Delta \left( \frac{L_B}{p} \right) - \Delta \left( \frac{D_M}{p} \right) \stackrel{?}{=} 0$$

By inserting (17) into (A2) and dividing the result by  $Y$ , (A2) reduces to

$$(A3) \quad \Delta \left( \frac{D_H}{Yp} \right) + \Delta \left( \frac{L_H}{Yp} \right) = \Delta \left( \frac{V}{Y} \right) \stackrel{?}{=} 0$$

According to (13.1) and (13.4), private wealth  $V$  is a constant fraction of the wage bill, which is in turn a constant fraction  $\gamma$  of aggregate income  $Y$ . Thus we have  $\Delta(V/Y) = 0$  q.e.d.

### Appendix II: Dynamics of the Model

Neither the price level nor real income is known at the beginning of a period yet, so economic agents must make their (tentative) decisions in nominal terms. The general succession of economic actions assumed is as follows: At the beginning of any Period  $t$ , both private households and the government start with the money they have earned in the previous period. In addition, private households hold some liquidity  $L_H$ . In transition periods, this is generally not the optimal level  $L_H^*$ . It is assumed that the respective excess liquidity (which may also be negative) is allocated in the same way concerning consumption and deposits as nominal income is.<sup>16</sup> In particular, the household's budget set of restrictions (11) is modified as follows:

<sup>16</sup> For an overview on the diverse real balance effects see e.g. *Hynes* (1974) and *Piergallini* (2006).

$$(A4) \quad \begin{aligned} Y_{H1}^{nom} &= w_{t-1}N_{t-1}(1-\tau) + (L_{H,t-1} - L_H^*) = C_1^{nom} + V^{nom} \\ &= \alpha Y_{H1}^{nom} + (1-\alpha)Y_{H1}^{nom} \end{aligned}$$

$$(A5) \quad V^{nom} = D_H^* + L_H^* = \frac{i_d}{i_d + i_l} V^{nom} + \frac{i_l}{i_d + i_l} V^{nom}$$

$$(A6) \quad \begin{aligned} Y_{H2}^{nom} &= D_{H,t-1}(1 + i_{d,t-1}) + L_{H,t-1} + \Pi_{F,t-1}^{nom} + \Pi_{B,t-1}^{nom} \\ &= C_1^{nom} \end{aligned}$$

where  $V^{nom}$  is nominal wealth of private households,  $Y_{H1}^{nom}$  is nominal disposable income in their active period of live, and  $Y_{H2}^{nom}$  is nominal disposable income after retirement.

Nominal government consumption is derived from government receipts minus debt service plus a voluntarily chosen amount of new public debt in the respective period:

$$(A7) \quad C_G^{nom} = \tau N_{t-1}w_{t-1} + \Pi_{M,t-1}^{nom} - K_G^{nom} - K_{G,t-1}^{nom}(1 + i_{t-1})$$

Firm's nominal capital demand depends on expected future commodity demand  $Y_{t+1; \exp.F}$  according to (9), on the expected price level  $p_{\exp.F}$ , and on the current capital market interest rate  $i$ . The latter is derived from nominal demand and supply of capital analogously to (27). Both nominal capital supply by the central bank ( $pK_M$ ) and nominal capital demand from abroad ( $pK_X$ ) are exogenous variables.

Total output produced in Period  $t$  depends on the firm's capital which was generated in the previous Period  $t-1$  and on labor input in the current period  $t$  according to (7). Labor input depends on the (expected) wage rate, which is either fixed (in nominal or in real terms) or the result of labor demand and supply according to (16).<sup>17</sup>

The resulting equilibrium production level is only an upper limit, because firms may be constrained by insufficient expected aggregate demand. Thereby, typical Keynesian short term effects can also occur in the model. Note that in Period  $t$  capital input  $K_F$  is already determined by the investment decision from the previous period. Thus production in Period  $t$  can only be varied by varying labor input in that period.

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<sup>17</sup> In the latter case, not only firms but also workers must estimate the price level at the end of the period. It is assumed that the price expectation by workers is more sluggish than is the respective expectation of firms (see below).

Given these assumptions, labor input can be derived from (8) and (9):

$$(A8) \quad N = \min \left[ \begin{array}{l} Y_{\text{exp.}F} \left( \frac{\gamma}{(w/p)_{\text{exp.}F}} \right) \\ \left( \frac{\gamma K_{F,t-1}^{\beta}}{(w/p)_{\text{exp.}F}} \right)^{1/(1-\gamma)} \end{array} \right]$$

where the upper term would prevail under the (possible) rationing of total demand and the lower term is labor demand without such a rationing. The expected commodity demand in Period  $t$  is assumed to be

$$(A9) \quad Y_{\text{exp.}F} = Y_{t-1} \frac{C^{\text{nom}}}{C_{t-1}^{\text{nom}}} + b_1 (M - L_H^* - L_B^*)_{t-1} + \mu$$

i.e. firms believe that real commodity demand rises by the rate of consumption increase which they see in the current period plus the excess liquidity which was observed in the previous period. Parameter  $b_1$  defines the elasticity concerning the latter determinant and  $\mu$  is an exogenous confidence parameter through which the model can be shocked.

Concerning their investment decision in Period  $t$ , firms must also have an expectation concerning commodity demand in the following period. For simplicity, it is assumed that  $Y_{\text{exp.}F} = Y_{\text{exp.}F,t+1}$ .

Price expectations of firms and workers are assumed to be adaptive, but asymmetric:

$$(A10) \quad p_{\text{exp.}F} = p_{\text{exp.}F,t-1} + b_F (p_{t-1} - p_{\text{exp.}F,t-1})$$

$$(A11) \quad p_{\text{exp.}W} = p_{\text{exp.}W,t-1} + b_W (p_{t-1} - p_{\text{exp.}W,t-1})$$

We assume  $b_W < b_F$ , i.e. price expectations of workers are more sluggish than price expectations of firms. By the latter assumption, a wage gap is created similar to the well-known Goodwin-Pohjola model.

The resulting price level is then given by

$$(A12) \quad p = \frac{Y^{\text{nom}}}{Y} = \frac{C_{H1}^{\text{nom}} + C_{H2}^{\text{nom}} + C_G^{\text{nom}} + K_F^{\text{nom}} - Y_X^{\text{nom}}}{Y} \quad (A10)$$

While (A12) is equivalent to (4) in the steady state, (4) is not generally valid in the dynamic model, because there is normally no monetary equilibrium in transition periods.

For the monetary abstinence regime, we assume the following version of a Taylor rule:

$$(A13) \quad i_b = \max \left[ \begin{array}{l} 0 \\ i_d^* + h_Y \left( \frac{Y_{\text{exp.}F}}{Y_{\text{exp.}F,t-1}} - 1 \right) + h_p \left( \frac{p_{\text{exp.}F}}{p_{\text{exp.}F,t-1}} - 1 \right) \end{array} \right]$$

i.e. the central bank leaves the money market rate at its equilibrium level unless either output expectations or price expectations rise. Note that the interest rate cannot fall below zero.

## References

- Barro, R. J. (1974), Are Government Bonds Net Wealth? *Journal of Political Economy* 82, pp. 1095–1117.
- Block, W. (1999), Austrian theorizing: Recalling the foundations, *Quarterly Journal of Austrian Economics* 2, pp. 21–39.
- Diamond, P. (1999), (1965), National debt in a neoclassical growth model, *American Economic Review* 55 (5), pp. 1126–1150.
- Dos Santos, C. H. (2005), A stock-flow-consistent general framework for formal Minskyan analyses of closed economies, *Journal of Post Keynesian Economics* 27, pp. 711–36.
- Dos Santos, C. H./Zezza, G. (2008), A simplified, benchmark, stock-flow consistent post- keynesian growth model. *Metroeconomica* 59, pp. 441–78.
- Ekelund, R. B./Hebert, R. F., (2007), *A History of Economic Theory and Method*, 5th ed. Illinois.
- Godley, W./Lavoie, M. (2007, I), *Monetary Economics, An Integrated Approach to Credit, Money, Income, Production and Wealth*, New York.
- (2007, II), Fiscal policy in a stock-flow consistent (SFC) model, *Journal of post-Keynesian economics* 30, pp. 79–100.
- Huerta de Soto, J. (2006), *Money, bank credit and economic cycles*, Auburn, Ludwig von Mises Institute.
- Hynes, J. A. (1974), On the Theory of Real Balance Effects, *Journal of Money, Credit and Banking* 6, pp. 65–83.
- Lavoie, M./Godley, W. (2001/2002), Kaleckian Growth Models in a Stock and Flow Monetary Framework. A Kaldorian View, *Journal of Post Keynesian Economics*, Winter 2001–2002.
- Le Heron, E. (2008), Monetary and fiscal policies in a Post Keynesian stock-flow consistent model, *Keynes and macroeconomics after 70 years*, Cheltenham (et al.), Elgar, pp. 279–308.
- (2009), Fiscal and monetary policies in a Keynesian stock-flow consistent model, *Current thinking on fiscal policy*, Basingstoke (et al.), Palgrave Macmillan, pp. 145–175.
- Le Heron, E./Mouakil, T. (2008), A post-Keynesian stock-flow consistent model for dynamic analysis of monetary policy shock on banking behaviour, *Metroeconomica*, 59, 3, pp. 405–440.

- Metzler, L. A.* (1951), Wealth, Saving, and the rate of Interest, *Journal of Political Economy*, 59, pp. 93–116.
- Nentjes, A.* (2007), Hayek and Keynes: A comparative analysis of their monetary views, *Journal of Economic Studies* 15, pp. 136–151.
- Niehans, J.* (1978), *The theory of money*, Baltimore.
- Papadimitriou, D./Zezza, G.* (ed.) (2011), *Contributions in Stock-Flow Consistent Modeling: Essays in Honor of Wynne Godley*, New York.
- Piergallini, A.* (2006), Real Balance Effects and Monetary Policy, *Economic Inquiry* 44, pp. 497–511.
- Smets, F./Wouters, R.* (2003), An estimated dynamic stochastic general equilibrium model of the Euro-area, *Journal of the European Economic Association* Vol 1, pp. 1123–1175.
- Taylor, L.* (2004), *Reconstructing macroeconomics: Structural proposals and critiques of the mainstream*, Cambridge MA.
- (2008), A foxy hedgehog: Wynne Godley and macroeconomic modeling, *Cambridge Journal of Economics*, 32, pp. 639–663.
- Thalenhorst, J./Wenig, A.* (1984), F.A. Hayek’s “Prices and Production” re-analyzed, *Jahrbücher für Nationalökonomie und Statistik* 199, pp. 213–236.
- Tobin, J.* (1969), A general equilibrium approach to monetary theory, *Journal of Money, Credit, and Banking* 1, pp. 15–29.
- (1982), Money and finance in the macroeconomic process, *Journal of Money, Credit, and Banking*, 14, pp. 171–204.
- van Treeck, T.* (2009), A synthetic, stock-flow consistent macroeconomic model of ‘financialisation’. *Cambridge Journal of Economics*, 33, pp. 467–493.
- Woodford, M.* (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton.
- Zezza, G./Dos Santos, C. H.* (2004), The role of monetary policy in post-Keynesian stock-flow consistent macroeconomic growth models, *Central banking in the modern world*, Cheltenham (et al.), Elgar, pp. 183–2018.