

## Interest Rates and Macroeconomic Investment under Uncertainty

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### Abstract

The interest rate is generally considered as an important driver of macroeconomic investment characterised by a particular form of path dependency, “hysteresis”. At the same time, the interest rate channel is a central ingredient of monetary policy transmission. In this context, we shed light on the issue (which currently is a matter of concern for many central banks) whether uncertainty over future interest rates at the zero lower bound hampers monetary policy transmission. As an innovation we derive the exact shape of the “hysteretic” impact of rate changes on macroeconomic investment under different sorts of uncertainty. Starting with hysteresis effects on the micro level, we apply an adequate aggregation procedure to derive the interest rate effects on a macro level. Our results may serve as a guideline for future central banks’ policies on how to stimulate investment in times of low or even zero interest rates and uncertainty.

*Keywords:* Forward guidance, interest rate, investment, monetary policy, path-dependence, hysteresis, uncertainty, zero lower bound.

*JEL Classifications:* C61, E22, E44.

*Acknowledgments:* We are grateful for valuable comments received from participants in the ICMAIF 2019 Conference, Rethymno/Crete, and for the beneficial comments by two anonymous referees.

My dear friend and esteemed colleague Ansgar died while we were completing this paper in July 2020. I later revised and finished the paper. Any remaining mistakes are mine (Matthias Göcke).

### I. Introduction

Leading central banks around the world have been reducing their key interest rates since the financial crisis in 2008, even reaching the zero lower bound. Since the key interest rate works as a main driver of the market interest rate, interest rates on the credit market, for private as well as for corporate lending,

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have been falling as well. Assuming a negative relationship between investments costs and investment, the central banks' intention behind this unconventional monetary policy has been to stimulate consumption and investment as important components of total demand, and, by this, preserving price stability. In this paper, we check where exactly the interest rate thresholds for additional investment (as well as for disinvestment) are located. The interest rate is of great importance for investment decisions, since interest rates as the main determinant of the cost of capital are of crucial relevance for profitability. In other words, our results matter because we shed light on an important element of the monetary policy transmission process.

For this purpose we develop the well-known standard concept of hysteresis, once popularized by *Baldwin/Krugman* (1989) and *Blanchard/Summers* (1986) and only recently revived by *Summers* (2014, 2015), significantly further to derive a nonlinear path-dependent macroeconomic model in which strong reactions of investment occur when the interest rate passes a kind of “band of inaction”.<sup>1</sup> Our model may thus serve as a guideline for future central banks' interest rate policies in times of low or even zero interest rates and high uncertainty on how to be successful in stimulating macroeconomic investment. This is of special importance in the current Corona crisis, where uncertainty about the future is emanating. However, our results also matter because the mechanics of our macroeconomic model is applicable to a variety of other economic issues related to investment-type decisions which involve irreversible fixed costs and uncertainty.

Relations between economic variables are often characterized by a scenario where initial conditions and the past realizations of economic variables matter. I. e. past (transient) exogenous disturbances and past states of the economic system do have an influence on the current economic relations. Typical examples are the dynamics of (un-)employment in business cycles and the dynamics of the nexus of exchange rates and exports. However, the focus is to an increasing extent, from the perspective of monetary policy, also on the dynamics of the relation between the interest rate (which is to a certain extent influenced by the central bank itself) and investment. Since the standard characteristics of hysteresis apply – i. e. permanent effects of a temporary stimulus, resulting in path-dependent multiple equilibria – these economic phenomena are termed “hysteresis” (*Göcke* 2002, *Belke et al.* 2014).

At the same time, uncertainty has gained much attention as one of the key drivers of the depth and duration of the Great Recession (*Bloom et al.* 2018, *Caggiano et al.* 2017). In this context, there is a growing interest in the impacts

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<sup>1</sup> For applications of the standard hysteresis model which often consist of (deviating from our approach) just checking for unit roots in macroeconomic time series see, for instance, *Chang* (2011) and *García-Cintado/Romero-Ávila/Usabiaga* (2015).

of (policy) uncertainty on monetary policy and on hysteretic systems such as capital and labor markets, especially in the context of sustained lower growth (“secular stagnation”). Lawrence Summers, to cite a prominent example, has stated repeatedly that the phenomenon of “secular stagnation” cannot be fully explained without taking recourse to hysteresis (Belke 2018, Buiter et al. 2015, Summers 2014, 2015, Nishi/Stockhammer 2019). Hence, it appears to be highly rewarding to investigate the mechanics of interest rate hysteresis in macroeconomic investment under uncertainty and, on a more cursory level, the *implications for monetary policy* more deeply.

Analogous to magnetism, the pattern of hysteresis depends on the scope: based on sunk-adjustment costs (e.g. entry costs of starting investment activity) *microeconomic* behavior (e.g. of *single firms* as investors) may show a discontinuous switching-pattern (being active as an investor or not) as described by a non-ideal relay, analogous to the magnetism of a single iron crystal. Correspondingly, one may conjecture that the *macroeconomic* dynamics of *aggregate* economic variables (e.g. the investment activity of a *whole country*, based on an aggregation over firms with heterogeneous cost structures) shows a pattern similar to the well-known hysteresis-loop of an entire piece of iron. The aggregate macroeconomic loop would then be characterized by a smooth/continuous transition between different “branches” of the loop, occurring with changes in the direction of, for instance, the interest rate movement.

However, the technical problem we are faced with is that hysteresis in economics is up to now usually based on a representation of a system with only a single input variable, which has an enduring effect on an economic outcome (i.e. the output variable). This input variable typically stands for a price or earnings variable (e.g. the exchange rate, affecting unit earnings in foreign trade, or wages as the price or cost of labour in the context of employment hysteresis). However, in general there is more than one factor influencing economic decision problems. We call this constellation the “multiple input variable scenario”.

Since hysteresis problems are about investment decisions in the broadest sense, the interest rate is particularly important, since the profitability depends on interest rates as the main determinant of the cost of capital. As a stylized fact, the central bank is able to influence investment activity via the asset price channel and the impact of its interest rate setting on Tobin’s  $q$  (Hayashi 1982). In other words, the central bank’s policy rate can be considered as a driver of the market interest rate  $i$  which is in the focus of our modelling efforts in this paper.

The “multiple input variable scenario” (i.e. “vector hysteresis”) in economics has been addressed by Göcke (2019), where it is outlined how the simultaneous influence of several original input variables (e.g. the revenue level and interest rate) is captured by the resulting variations of the present value of an investment as an input signal function. However, following Belke/Göcke (2009) and in con-

trast to Göcke (2019), in the present paper we want to focus on the hysteretic nature of the relation between investments and the interest rate as the relevant driving variable, including explicit stochastic changes of this determinant of investments.

Starting with a microeconomic model, we show the path-dependent nature of investment decisions, resulting in a difference between the (low) interest rate that triggers an investment and the (high) interest rates that triggers a disinvestment of a single firm. The divergence between these both interest triggers may be due to sunk cost, e.g. if the investment is firm specific and cannot be sold at the full purchasing price.

In a situation with uncertainty, e.g. related to stochastic future revenues and/or to future interest rate changes, the hysteresis property is even amplified due to option value effects. While Belke/Göcke (2009) and Göcke (2019) only calculate simple ‘symmetric’ stochastic revenue changes (with a binary stochastic impact, either positive or negative, on revenues of the same size and the same probability), we apply (as a microeconomic base for aggregation) the micro-hysteresis model presented by Belke/Frenzel Baudisch/Göcke (2020), which augments the established model in three aspects. First, in order to be able to directly model consequences of monetary policy on the option value of sunk investments stochastic interest rate changes are included instead of only relying to stochastic revenue changes. Second, to make the model able to illustrate the present monetary policy stance in, for instance, the Euro area at the zero lower bound (ZLB), ‘asymmetric’ future changes of the interest rate are modelled that allow a representation of a situation with very low interest rates, where a future increase is possible (and more or less probable) but no further decrease. While Belke/Frenzel Baudisch/Göcke (2020) is focused on empirical aspects by directly applying a linearized approximation to estimate empirical macroeconomic hysteresis dynamics (called “play-hysteresis”, similar to mechanical backlash), the current paper is concentrating on other more theoretical aspects. First, as an innovation an explicit application of the adequate (Preisach-Mayergoyz) aggregation procedure is presented, which allows the derivation of macroeconomic hysteresis dynamics even in a situation where the (dis-)investment triggers are different for heterogeneous (microeconomic) firms. This application case is new, since it is based on a relation between aggregate investments as the dependent and interest rate changes as the relevant input variable. This leads to a negative slope of the macro hysteresis loop, which is different to the standard application of the aggregation procedure that is related to revenue changes. The difference in the hysteresis loop’s slope is due to the fact that interest rate decreases foster investment – while in the standard application increasing revenues have a positive impact on investments. Second, the specific investment dynamics implications in a situation characterised by a low interest rate (ZLB) and uncertainty in fu-

ture revenues and interest rates are discussed. This discussion especially focuses on the consequences of stochastic *future* interest rates on the investment incentives. This relates not only to the current interest rate policy but as well to the credibility of central bank announcements addressing future monetary policy.

In order to derive the exact shape of the impact of changes in the interest rate (to a certain extent driven by the central bank) on macroeconomic investment under the scenarios of both certainty and uncertainty, we proceed as follows. In section 2, we deal with the representation of sunk cost hysteresis by means of non-ideal relays. For this purpose, we differentiate between scenarios of (1) sunk cost hysteresis and interest changes in a situation with no uncertainty, (2) non-ideal relay in a situation with stochastic revenue changes, (3) non-ideal relay in a situation with stochastic interest rate changes and (4) non-ideal relay especially related to interest rates as an input variable in a stochastic situation. In Section 3, we aggregate our micro results to the macroeconomic level and formally derive the main pattern of macroeconomic, interest rate-driven investment hysteresis. Section 4 finally concludes and draws some implications for monetary policy and its transmission to the real sector (investment), for instance in the Euro area.

## II. Sunk Cost Hysteresis and Non-ideal Relays

### 1. Sunk Cost Hysteresis and Interest Changes in a Scenario with no Uncertainty

In order start with the illustration of hysteresis effects on a microeconomic level, we will apply a simplistic microeconomic model that shows a couple of similarities to the model presented by *Belke/Göcke* (2009). This microeconomic hysteresis model presented in the following subsections, where a situation based on certainty and different uncertainty scenarios are included (related to revenues and interest rates) is similar to the micro level model in *Belke/Frenzel Baudisch/Göcke* (2020). A price-taking firm  $j$  decides in period  $t$  whether or not to invest into one unit of capital  $K_j$  ( $= 1$ ). Additionally, the firm has to pay the sunk investment costs  $H_j$  ( $\geq 0$ ) if it invests and starts production. The value created by  $H_j$  is completely firm specific and can not be regained if the firm is disinvesting. If production is shut down, selling the unspecific capital stock  $K_j$  at price 1 is possible. The specific part  $H_j$  decays immediately as soon as the firm does not produce and sell. Thus,  $H_j$  represents sunk adjustment costs. Using capital, the firm produces and sells the production immediately resulting in a revenue  $e_{j,t}$ . Since one additional unit of capital is applied,  $e_{j,t}$  is the (marginal gross) rate of return. Two different components of costs have to be paid. Based on using unspecific capital as an input factor, the interest rate  $i_t$  (influenced by the central bank) has to be paid on the firm's capital stock  $K_j$  as an opportunity cost. If the

firm has not produced in the preceding period, it has to pay the starting costs  $H_j (\geq 0)$  in addition. On the other hand, if it has been active in the preceding period and is just continuing production, only the interest costs on unspecific  $K_j$  are relevant.

Disregarding the adjustment costs, the rate of profit in period  $t$  is:

$$(1) \quad R_{j,t} = e_{j,t} - i_t \cdot K_j = e_{j,t} - i_t$$

As a simple example we assume the firm is for the next period ( $t+1$ ) with certainty expecting a single “once and forever” change  $\rho$  in the interest rate (imposed, for instance, by announcements of the central bank), which remains constant for the whole *infinite future*:  $i_{t+\tau} = i_t + \rho$  (for all  $\tau > 0$ ). The future gross rate of return is (with certainty) expected constant as well:  $e_{j,t+\tau} = e_{j,t}$ . Corrected for interest costs, the future rate of return is:

$$(2) \quad R_{j,t+\tau} = e_{j,t} - i_{t+\tau} = e_{j,t} - (i_t + \rho) \quad (\text{with } \tau > 0)$$

Under the assumed/expected interest rate dynamics, the present value of future profits as an annuity (with payments at the end of the periods) is in the case of an ongoing activity:

$$(3) \quad V_{j,t} = \frac{e_{j,t} + \frac{e_{j,t}}{i_t + \rho}}{1 + i_t} - K_j = \frac{(1 + i_t + \rho) \cdot e_{j,t}}{(1 + i_t) \cdot (i_t + \rho)} - 1$$

Under certainty – to make an entry a profitable investment – the present value of revenues has (at least) to cover the value of the capital stock  $K_j = 1$  plus the sunk entry costs  $H_j$ . Solving ( $V_{j,t} - H_j = 0$ ) results in the firm’s investment/entry trigger rate of return  $\alpha_j$  under certainty:

$$(4) \quad \alpha_j = \frac{(1 + i_t) \cdot (i_t + \rho) \cdot (1 + H_j)}{1 + i_t + \rho} \quad \text{entry if } e_{j,t} > \alpha_j$$

A firm that was active in the preceding period will leave the market and sell the unspecific capital  $K_j$ , if the revenue is too low. Thus, an exit of the firm is optimal if ( $V_t < 0$ ), and the exit/disinvestment trigger rate of return  $\beta_j$  is:

$$(5) \quad \beta_j = \frac{(1 + i_t) \cdot (i_t + \rho)}{1 + i_t + \rho} \quad \text{exit if } e_{j,t} < \beta_j$$

In the even simpler case of an expectation of unchanged and constant future interest rates ( $\rho = 0$ , i. e. the central bank credibly commits itself to a constant interest rate path in the future), the entry and exit triggers of the gross rate of return are:

$$(6) \quad \text{for } \rho = 0: \quad \begin{aligned} \alpha_{j,\rho=0} &= i_t \cdot (1 + H_j) && \text{entry if } e_{j,t} > \alpha_j \\ \beta_{j,\rho=0} &= i_t && \text{exit if } e_{j,t} < \beta_j \end{aligned}$$

The results show that an entry requires interest costs on both, unspecific capital (normalized to  $K_j=1$ ) and sunk investment costs ( $H_j \geq 0$ ), based on the future interest rate ( $i_t + \rho$ ) to be covered by the revenue, i.e. by the gross rate of return  $e_{j,t}$ , to make an investment profitable. On the other hand an exit will only occur, if the revenue  $e_{j,t}$  no longer covers the interest opportunity costs just on the unspecific part of the capital  $K_j$ . Thus, a disinvestment occurs if the rate of return falls below the future interest rate.

Summarising, if interest rate expectation is fixed, an (unexpected) change in the (current and future) rate of return on capital  $e_{j,t}$  results in an investment/disinvestment pattern of the firm  $j$  which is described by a so called ‘non-ideal relay’:<sup>2</sup>

$$(7) \quad K_j = \begin{cases} 1 & \text{if } (K_{j,t-1} = 0 \wedge e_{j,t} > \alpha_j) \\ 1 & \text{if } (K_{j,t-1} = 1 \wedge e_{j,t} \geq \beta_j) \\ 0 & \text{if } (K_{j,t-1} = 0 \wedge e_{j,t} \leq \alpha_j) \\ 0 & \text{if } (K_{j,t-1} = 1 \wedge e_{j,t} < \beta_j) \end{cases} \quad \text{with } \alpha_j \geq \beta_j$$

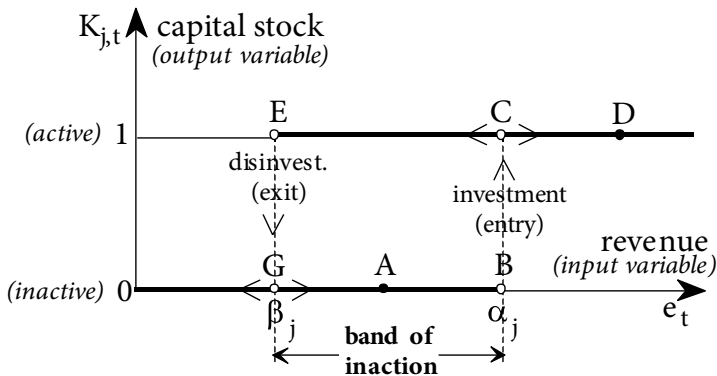


Figure 1: Investments According to a ‘Non-ideal Relay’ Related to the Rate of Return as the Input Variable

<sup>2</sup> For a general description of relay-hysteresis see *Krasnosel'skii/Pokrovskii* (1989, p. 263 and 271) and *Brokate/Sprekels* (1996, pp. 23 f.).

A non-ideal relay is characterized by path-dependent multiple-equilibria. Starting in an inactivity situation at point A (Figure 1) a revenue increase exceeding the trigger  $\alpha_j$  induces an investment (“entry”), i. e. a “jump” from the ( $K_j=0$ )-inactivity-line to the ( $K_j=1$ )-activity-line (point C). Disinvestment (“exit”, point E) only results if the rate of return falls below the exit trigger  $\beta_j$ . Passing a trigger is necessary to induce a switch between the two equilibrium-branches – otherwise the activity status remains the same. Therefore, the area GB (or CE) can be described as a ‘*band of inaction*’ or ‘*hysteresis-band*’ (Baldwin 1989, 7f.; Baldwin/Lyons 1989, 11). Dependent on the past, two different equilibria of the output/dependent variable (firm’s activity) are possible for the same level of the current level of the input variable (revenue). If a *temporary* change of the input variable (by passing a trigger) results in a switch between equilibria, a *permanent* output effect remains (called ‘*remanence*’). This after-effect is the constituting feature of hysteresis.

In the above representation, the level of the revenue  $e_{j,t}$  was implicitly assumed as the single input variable, and the entry trigger condition ( $V_{j,t} = H_j$ ) was solved for  $e_{j,t}$  in order to derive revenue triggers. However, if alternatively the rate of return is *ceteris paribus* expected as constant ( $e_{j,t} = e_j$ ), and the interest rate is assumed to be the single input variable of the system, the entry condition can be solved for the interest rate, resulting in an entry trigger  $a_j$  for the interest rate.<sup>3</sup> We assume the simplest case of  $\rho = 0$  for illustration, so the entry/investment interest rate trigger is:

$$(8) \quad \text{for } \rho = 0: \quad a_{j,\rho=0} = \frac{e_j}{1 + H_j} \quad \text{entry if } i_t < a_j$$

If the interest is low, capital costs for  $K_j$  and  $H_j$  are low, and a high present value of future revenues implies a profitable investment. A similar calculation determines the disinvestment interest trigger rate  $b_j$  for a scenario with a high interest rate:

$$(9) \quad \text{for } \rho = 0: \quad b_j = e_j \quad \text{exit if } i_t > b_j$$

For a constant level of the rate of return  $e_{j,t} = e_j$  an (unexpected) change of the interest rate  $i_t$  again shows a ‘non-ideal relay’ pattern, this time with respect to

<sup>3</sup> See Göcke (2019) for a discussion of this interest rate related microeconomic hysteresis loop. In that paper a multiple inputs situation with simultaneous changes in the revenues as well as in the interest rates, is addressed. However, this “vector-hysteresis” with multiple input changes is shown to be simplified in economic problems, if multi-dimensional simultaneous changes of the interest rate and revenues are reduced to changes of a one-dimensional “signal” given by the present value of the investment.



the interest rate as the input variable as depicted in Figure 2 (see Göcke 2019, p. 103, for a similar representation):

$$(10) \quad K_{j,t} = \begin{cases} 1 & \text{if } (K_{j,t-1} = 0 \wedge i_t < a_j) \\ 1 & \text{if } (K_{j,t-1} = 1 \wedge i_t \leq b_j) \\ 0 & \text{if } (K_{j,t-1} = 0 \wedge i_t \geq a_j) \\ 0 & \text{if } (K_{j,t-1} = 1 \wedge i_t > b_j) \end{cases} \quad \text{with } a_j \leq b_j$$

Again, a range of path-dependent multiple equilibria (i.e. an interest rate related “band-of-inaction” between  $a_j$  and  $b_j$ ) emerges due to a difference between an investment and disinvestment trigger level of the input variable. Starting at point L, a decreasing interest rate, makes an investment profitable (at point M) by reducing interest costs on both types of capital ( $K_j$  and  $H_j$ ). A subsequent interest rate increase will result in disinvestment (point N with exit trigger rate  $b_j$ ), if the opportunity costs only on the unspecific  $K_j$  are no longer covered by revenues.

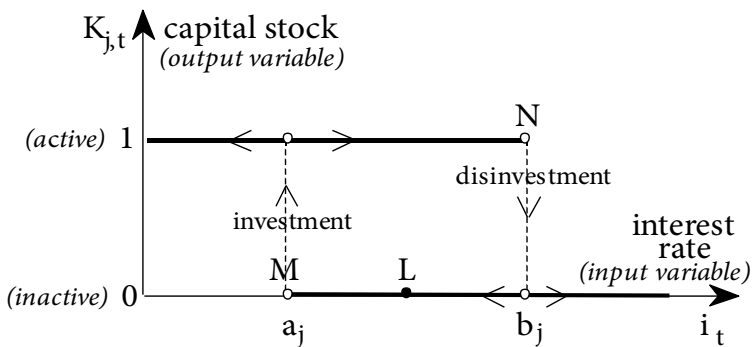


Figure 2: ‘Non-ideal Relay’ Related to the Interest Rate as the Input Variable

## 2. Non-ideal Relay in a Scenario with Stochastic Revenue Changes

In a situation with uncertainty, e.g. due to stochastic changes in future revenues, a real option approach applies (*Majd/Pindyck* 1987; *Pindyck* 1988, 1991; *Dixit* 1989; *Bentolila/Bertola* 1990; *Dixit/Pindyck* 1994; *Belke/Göcke* 1999, 2009).<sup>4</sup> We again apply the simple case of no expected change in the interest rate ( $\rho = 0$ ) for an illustration of the option effects related to revenue uncertainty. A firm which is currently inactive has to decide whether to invest now or not, including the option to invest later. The option to decide on the investment in the future limits the risk by a “wait-and-see” strategy. By staying passive, a firm can avoid future losses if the stochastic future rate of return will be unfavourable. An instantaneous investment eliminates this option to enter later if the future revenue will be favourable. Thus, the option value of waiting has to be covered in addition to the sunk costs in order to trigger an immediate investment.<sup>5</sup>

The option value effects are demonstrated based on a very simplistic example: assume a single non-recurring stochastic change of the rate of return, which can be either positive ( $+\varepsilon_u$ ) or negative ( $-\varepsilon_d$ ) (and  $\varepsilon_{ud} \geq 0$ ). The probability of a negative change ( $-\varepsilon_d$ ) is  $P$ , and  $(1 - P)$  for a positive revenue change ( $+\varepsilon_u$ ). From period  $t+1$  on, the firm will decide under certainty again. Instead of deciding to invest now or never, in a scenario with uncertainty there is an option to wait and to decide on the investment later. The firm can still invest in the next period if the future revenue level turns out to be favourable ( $+\varepsilon_u$ ). However, by staying passive, potential future losses can be avoided if the revenue change will be negative ( $-\varepsilon_d$ ). To wait and stay passive implies zero profits in the current period  $t$ . Conditional on a ( $+\varepsilon_u$ )-realisation, the firm will use its option to invest in  $t+1$ . This causes discounted sunk investment costs and an annuity of ( $e_{j,t} + \varepsilon_u - i_t$ ). In case of a ( $-\varepsilon_d$ )-realisation the firm will remain passive. The expected present value of the wait-and-see strategy is  $E(W_{j,t}^{\text{entry}})$  and has to be compared with the expected present value [ $E(V_{j,t}) - H_j$ ] of an immediate entry (without a re-exit):

$$(11) \quad E(W_{j,t}^{\text{entry}}) = \frac{1-P}{1+i_t} \cdot \left( \frac{e_{j,t} + \varepsilon_u - i_t}{i_t} - H_j \right)$$

$$(12) \quad E(V_{j,t}) - H_j = P \cdot \left( \frac{e_{j,t} + \frac{e_{j,t} - \varepsilon_d}{i_t}}{1+i_t} \right) + (1-P) \cdot \left( \frac{e_{j,t} + \frac{e_{j,t} + \varepsilon_u}{i_t}}{1+i_t} \right) - 1 - H_j$$

<sup>4</sup> Based on a microeconomic (real options) hysteresis model *Dosi/Moretto/Tamborini* (2019) analyse the effects of subsidies on triggering investments in a situation with increasing uncertainty in future revenues.

<sup>5</sup> The option value effects described below may even be amplified in a situation with an investment project which needs “time to build” (*Majd/Pindyck* 1987).

In the case of uncertainty, the investment revenue trigger  $\alpha_j$  can be calculated for a situation of indifference between immediate entry and wait-and-see, i. e. if:  $E(W_{j,t}^{entry}) = E(V_{j,t}) - H_j$ :

$$(13) \quad \alpha_j = i_t \cdot (1 + H_j) + \frac{P \cdot \epsilon_d}{i_t + P} \quad \text{in period } t: \text{ entry if } e_{j,t} > \alpha_j$$

A currently active firm, deciding to leave the market now or to stay active, with an option to exit later if an unfavourable ( $-\epsilon_d$ ) price change will occur, has an analogous decision problem. Currently remaining active and waiting for a period results in a current profit of  $(e_{j,t} - i_t)$ . Conditional on a ( $-\epsilon_d$ )-realisation, the firm will use its option to exit in  $t+1$ . For a ( $+\epsilon_u$ )-realisation the firm will continue activity with a future annuity of  $(e_{j,t} + \epsilon_u - i_t)$ . The expected present value of the wait-and-see strategy  $E(W_t)$  is:

$$(14) \quad E(W_{j,t}^{exit}) = P \cdot \left( \frac{e_{j,t} - i_t}{1 + i_t} \right) + (1 - P) \cdot \left( \frac{e_{j,t} + \frac{e_{j,t} + \epsilon_u}{i_t}}{1 + i_t} - 1 \right)$$

The present value of waiting  $E(W_{j,t}^{exit})$  has to be compared with the expected present value of an immediate exit (without a re-entry), which is zero. The exit/disinvestment-trigger revenue level  $\beta_j$  in the case of revenue uncertainty can be calculated for a situation of indifference between wait-and-see and immediate exit in  $t$ , i. e.  $E(W_{j,t}^{exit}) = 0$ :

$$(15) \quad \beta_j = i_t - \frac{(1 - P) \cdot \epsilon_u}{i_t + (1 - P)} \quad \text{in period } t: \text{ exit if } e_{j,t} < \beta_j$$

Thus, the investment trigger rate of return  $\alpha_j$  is under uncertainty augmented by the positive term  $[(P \cdot \epsilon_d)/(i_t + P)]$ , and the option value effect on the disinvestment trigger revenue  $\beta_j$  is negative:  $-(1 - P) \cdot \epsilon_u/[i_t + (1 - P)]$ . Thus, the option value effects due to (revenue) uncertainty result in a widening of the ('band-of-inaction')-range between both triggers  $\beta_j$  and  $\alpha_j$ . However, considering the typical pattern of the path-dependence, the non-ideal-relay type dynamics of microeconomic hysteresis do not have changed.

### 3. Non-ideal Relay in a Scenario with Stochastic Interest Rate Changes

A qualitatively similar widening effect on the 'band of inaction' will result, if uncertainty is not based on revenue changes, but on stochastic future changes of

the interest rate (for instance, induced by – from a firm’s perspective stochastic – future central bank policy changes).<sup>6</sup>

The option value effects are again demonstrated by a simplistic example: now a single non-recurring stochastic change of the interest rate, which can be either positive ( $+\rho_u$ ) or negative ( $-\rho_d$ ) (and  $\rho_{u,d} \geq 0$ ). The probability of a positive interest change ( $+\rho_u$ ) is  $B$ , and  $(1-B)$  for a negative interest rate change ( $-\rho_d$ ). From period  $t+1$  on, the situation is assumed to be unchanged/constant again. The option to wait and to decide on the investment later has again to be taken into account in this scenario. If the future interest change is negative ( $-\rho_d$ ), and thus favourable for investors, the firm can still invest in the next period, and by staying passive, potential future losses can be avoided if the positive interests change ( $+\rho_u$ ) leads to higher interest cost. Regarding eq. (1) the current profit rate is  $R_{j,t} = e_{j,t} - i_t$ . The future interest rate, relevant for the interest costs and for future discounting, is (with probability  $B$ )  $i_{t+\tau} = i_t + \rho_u$  and with probability  $(1-B)$ :  $i_{t+\tau} = i_t - \rho_d$ . The future profit follows from eq. (2), if  $i_t + \rho$  is substituted by either  $i_t + \rho_u$  or  $i_t - \rho_d$ . Using eq. (3), the resulting present values of future revenues for both possible cases can be calculated by the same substitution:

$$(16) \quad V_{t,u} = \frac{(1+i_t+\rho_u) \cdot e_{j,t}}{(1+i_t) \cdot (i_t+\rho_u)} - 1 \quad \text{and} \quad V_{t,d} = \frac{(1+i_t-\rho_d) \cdot e_{j,t}}{(1+i_t) \cdot (i_t-\rho_d)} - 1$$

A currently inactive firm decides to invest now or not, including the option to invest in the next period, if the interest costs are decreasing – i.e. conditional on a ( $-\rho_d$ )-realisation of the stochastic interest rate, which has a probability of  $(1-B)$ . The expected present value of the wait-and-see strategy is  $E(W_{j,t}^{\text{entry}})$  and is compared with the expected present value of an immediate entry [ $E(V_{j,t}) - H_j$ ]:

$$(17) \quad E(W_{j,t}^{\text{entry}}) = \frac{1-B}{1+i_t} \cdot \left( \frac{e_{j,t}}{i_t-\rho_d} - H_j \right)$$

$$(18) \quad E(V_{j,t}) - H_j = B \cdot V_{t,u} + (1-B) \cdot V_{t,d}$$

The investment revenue trigger  $\alpha_j$  results for indifference between both alternatives as:

$$(19) \quad \alpha_j = i_t \cdot (1+H_j) + \frac{\rho_u \cdot B \cdot (1+H_j)}{i_t + \rho_u + B} \quad \text{in period } t: \text{ entry if } e_{j,t} > \alpha_j$$

<sup>6</sup> See *Dias/Shackleton* (2011) for another model of investment hysteresis with stochastic interest rates. However, their mathematically more complex approach is based on stochastic differential equations, and thus only numerical solutions are computable, while our more simplistic model will allow algebraic closed form expressions that are directly interpretable.

The result for the investment trigger rate of return shows that for the decision whether or not to invest now, instead of waiting for a while, only the potential increase of the interest rate is relevant. In this case, an immediate investment will later turn out to be “wrong” due to increasing costs of the capital. This potential risk can be avoided just by waiting with an option to decide on the investment later, when the future interest rate (i.e. the monetary policy stance) is known. On the other hand, with regard to the investment decision, the chance of a decreasing interest rate is not relevant for an immediate entry, since for this case there is no risk of having conducted the “wrong” investment. Thus, for the investment decision, the option value of waiting is only based on avoiding the stochastic risk of higher future interest rate (i.e. the risk of a more contractionary monetary policy *in the future*).

A currently active firm can disinvest now or stay active, with the option to leave later, if an unfavourable interest rate increase ( $+\rho_u$ ) will happen. Currently remaining active and to wait up to the next period results in a current profit of  $(e_{j,t} - i_t)$ . Conditional on a  $(+\rho_u)$ -realisation, the firm will use its disinvestment option in  $t + 1$ . For a  $(-\rho_d)$ -realisation the firm will continue activity with a future annuity of  $(e_{j,t} - i_t + \rho_d)$  and the corresponding present value  $V_{t,d}$ . The expected present value of the wait-and-see strategy  $E(W_{j,t}^{\text{exit}})$  is:

$$(20) \quad E(W_{j,t}^{\text{exit}}) = B \cdot \left( \frac{e_{j,t} - i_t}{1 + i_t} \right) + (1 - B) \cdot V_{t,d}$$

An immediate disinvestment results in zero profits. Hence, indifference is given with  $E(W_{j,t}^{\text{exit}})$ , determining the exit trigger:

$$(21) \quad \beta_j = i_t - \frac{\rho_d \cdot (1 - B)}{i_t - \rho_d + 1 - B} \quad \text{in period } t: \text{ exit if } e_{j,t} < \beta_j$$

The results for the immediate disinvestment versus continuing activity for a while with an option to exit later, is only related to the potential decrease of the interest rate (i.e. induced by a lower monetary policy rate). An immediate disinvestment would turn out as the “wrong” decision, if the future interest cost decreases, while an immediate exit would turn out to be right in the case of an increasing interest rate. Waiting prevents the risk of a “wrong” decision in the case of reduced interest costs. Hence, only the risk of decreasing future interest rates (i.e. a more expansionary future monetary policy stance) is relevant for an immediate disinvestment. In this case, the option value of waiting is merely based on avoiding the stochastic risk of a lower future interest rate.

In a situation with no stochastic change ( $+\rho_u = -\rho_d = 0$ ), the results resemble the triggers stated in eq. (6) where the special case with no expected future interest rate change ( $\rho = 0$ ) is assumed. Again, as in the case of stochastic revenue changes, option value effects widen the hysteretic “band of inaction”. The entry

trigger  $\alpha_j$  is increased by a  $[(+\rho_u) \cdot B]$ -based term, describing the danger of an unfavourable interest rate increase which could be avoided by waiting. The exit trigger  $\beta_j$  is reduced related to  $[(-\rho_d) \cdot (1 - B)]$ , since there may be a favourable potential future interest rate decrease.

4. *Non-ideal Relay Related to Interest Rates in a Stochastic Scenario*

In the previous two subsections, we solved the entry/exit triggers of the non-ideal relay for the rate of return  $e_{j,t}$  as the relevant input variable of the hysteresis-system, triggering investments and disinvestments, respectively. Of course, the underlying triggering conditions could be solved for the interest rates as the alternative input variable of the system as well, resulting in a kind of “mirrored” non-ideal relay pattern, as depicted in Figure 2.

Indifference between immediate entry and wait-and-see,  $E(V_{j,t}) - H_j = E(W_{j,t}^{entry})$ , results in an entry trigger condition for the case of revenue uncertainty:

$$(13') \quad e_{j,t} = i_t \cdot (1 + H_j) + \frac{P \cdot \epsilon_d}{i_t + P} \Leftrightarrow i_t = \frac{e_{j,t}}{1 + H_j} - \frac{P \cdot \epsilon_d}{(1 + H_j) \cdot (i_t + P)}$$

In the case of interest rate uncertainty the analogous entry trigger condition is:

$$(19') \quad e_{j,t} = i_t \cdot (1 + H_j) + \frac{\rho_u \cdot B \cdot (1 + H_j)}{i_t + \rho_u + B} \Leftrightarrow i_t = \frac{e_{j,t}}{1 + H_j} - \frac{\rho_u \cdot B}{i_t + \rho_u + B}$$

Explicitly solving these conditions for the current interest rate  $i_t$  to exactly calculate the interest rate entry trigger  $a_j$  will lead to a confusing result including some root expressions. However, the direction of the option value effects on the level of the interest rate triggering an investment is obvious for both different cases of stochastic effects. Eqs. (13') and (19') make clear that there is a negative effect of the option value related to the risk of potentially decreasing future revenues and of a potentially increasing future interest rate on the current interest rate that triggers an immediate investment. In Figure 2 this would be depicted as a shift of the investment trigger  $a_j$  to the left. Analogously, solving the indifference condition between an immediate exit and wait-and-see,  $E(W_{j,t}^{exit}) = 0$ , leads to the following conditions, inducing an exit in period t:

$$(15') \quad i_t > e_{j,t} + \frac{(1 - P) \cdot \epsilon_u}{i_t + (1 - P)} \quad (\text{if revenues are stochastic})$$

$$(21') \quad i_t > e_{j,t} + \frac{\rho_d \cdot (1 - B)}{i_t - \rho_d + 1 - B} \quad (\text{if interest rate is stochastic})$$

For the disinvestment trigger we see a positive option value effect on the level of the exit trigger interest rate in both cases of different uncertainty sources, the “risk” (i. e. chance) of rising future revenues and the “risk” of future interest rate decreases, creating an option value of waiting with the disinvestment decision. The interest rate  $b_j$  that triggers an immediate exit is shifted to the right in Figure 2 in both cases. Hereby, the option value effects lead to a widening of the “band of inaction” related to the interest rate triggers, as it is similarly the case for the band of inaction related to the revenue based triggers.

### 5. Implications for Monetary Policy in an Uncertain Crisis Situation

The results have clear implications for monetary policy if we consider the *consequences of an interest rate close to zero* (the so-called “zero lower bound”), as experienced for years in the US as well as in the Euro area in the wake of the global financial crisis and later on as a reaction on the Corona virus related recession, *as an investment incentive*. At first glance, an interest rate level near the “zero lower bound” should completely avoid the interest costs on an investment, and thus, result in massive investment activities, if the firms’ rates of return on investments are only slightly positive. However, even if the current interest rate is  $i_t=0$ , there is still a risk of future revenue decreases and/or future interest rate increases (i. e. policy rate hikes by the central bank), which in combination with the sunk investment costs  $H_j$  results in an investment trigger of the rate of return that is still pretty far in the positive range, since the investment still not only has to cover the interest costs (which are zero in this specific situation), but additionally the option value of waiting. This option value is driven by the potential size of uncertainty, which is included in the model by an uncertain revenue decrease  $(-\varepsilon_d) < 0$  and/or an uncertain future increase  $\rho_u > 0$  of the interest rate, and by the probabilities  $P$  or  $B$  of these unfavourable potential changes. The revenue related entry trigger results from eq. (13) for a stochastic revenue case and from eq. (19) for stochastic interest rates are – for a situation with zero interest rates ( $i_t=0$ ) – are reducing to:

$$(13'') \quad \alpha_j = i_t \cdot (1 + H_j) + \frac{P \cdot \varepsilon_d}{i_t + P} \Rightarrow$$

$$\text{for } (-\varepsilon_d) < 0, P > 0, \text{ and } i_t = 0: \alpha_{j,\varepsilon(i=0)} = \varepsilon_d > 0$$

$$(19'') \quad \text{for } \rho_u > 0, B > 0, \text{ and } i_t = 0: \alpha_{j,\rho(i=0)} = \frac{\rho_u \cdot B \cdot (1 + H_j)}{\rho_u + B} > 0$$

These results show that in a stochastic environment a monetary policy stance based on low current interest rates (as currently in the Euro area) may be not

very effective in stimulating investments, if the future includes the risk of subsequent revenue decreases and/or future interest rate increases (i.e. policy rate hikes), since this uncertainty is creating an option value of waiting with investments. Eq. (13'') shows, that in a zero interest situation even a small positive probability  $P$  of a future revenue decrease ( $-\varepsilon_d$ ) is sufficient, to demand a "full compensation capacity"  $\varepsilon_d$  against the potential negative shock in the revenue investment trigger  $\alpha_j$ , as the level of  $P$  is cancelled out from the expression. Moreover, eq. (13'') points out the importance of the size  $\varepsilon_d$  of a potential future revenue decrease on the investment trigger. This may be relevant for the credibility of announcements related to future fiscal policies. If e.g. due to a credible announcement of future Coronavirus aid payments for companies and workers and/or of countercyclical macroeconomic demand policies the expectation of a potential downturn in revenues is reduced, this would be represented in a small size of  $\varepsilon_d$  (and, in a more general case, of the downturn probability  $P$  as well), and consequently in a relatively moderate size of the trigger interest rate stimulating investments. These arguments are pointing on the interrelation between monetary and fiscal policies, especially in a situation with uncertainty.

The exit trigger in an uncertain situation is related to an option value based on a potential revenue increase ( $\varepsilon_u > 0$ , see eq. (15)) and/or an interest rate decrease ( $-\rho_d < 0$ , see eq. (21)), occurring with probability  $(1-P)$  or  $(1-B) > 0$ , mathematically resulting in even negative rates of return necessary to trigger disinvestments. For a potential interest rate decrease, the entry trigger is:

$$(21'') \quad \text{for } \rho_u > 0, (1-B) > 0, \text{ and } i_t = 0: \beta_{j,\rho(i=0)} = \frac{-\rho_d \cdot (1-B)}{-\rho_d + 1 - B}$$

However, if in a situation with zero interest rates, a further interest rate decrease is not feasible [i.e.  $\rho_d \rightarrow 0$ ], or not probable [ $(1-B) \rightarrow 0$ ], the disinvestment trigger of the rate of return on investment will then converge to zero as well. The exit trigger revenue including the "risk" of a future revenue increase (if the interest rate is expected to be zero) reduces to:

$$(15'') \quad \beta_1 = i_t - \frac{(1-P) \cdot \varepsilon_u}{i_t + (1-P)} \Rightarrow$$

$$\text{for } \varepsilon_u > 0, (1-P) > 0, \text{ and } i_t = 0: \beta_{j,\varepsilon(1=0)} = (-\varepsilon_u) < 0$$

As a mirror image of the entry trigger in a zero interest rate situation, the corresponding exit trigger is again characterized by a "full compensation" property: The current rate of return can be negative up to the absolute size of the potential future revenue increase  $\varepsilon_u$ , before disinvestment is executed. In the entry/investment trigger case, this property makes monetary policy ineffective due to un-



certainty effects, however, in the exit/disinvestment case the analogous property is protecting the ‘activity status’ against disinvestments.

As demonstrated, the inclusion of option values is in an uncertain volatile situation widening the “band of inaction”, in both model versions – i. e. calculated for revenue triggers as well as related to interest rate triggers. Vice versa, a reduction of uncertainty – if during a crisis the fluctuations become smaller and, thus, the future development (related to revenues as well as for future interest rate changes) is seen as less volatile by the investors – will reduce the option value of waiting and as a consequence result in a smaller width of the “band of inaction”. This narrowing of the hysteresis-band if the economic situation is stabilizing can lead to different reactions, depending on a firm waiting with an entry or, alternatively, with an exit decisions. A firm that was not entering during the crisis, but instead was waiting with an investment, will in a more stable situation start to invest (due to a down-shift of the entry revenue trigger and/or an up-shift of the interest rate entry trigger). As a consequence of less volatile expectations investments will be triggered, though current revenues and/or current interest rates may not have changed, but only because of the decreasing option value of waiting. In this situation a shift of the triggers levels (of revenues and/or interest rates) leads to the passing (i. e. the entry) even if the input variables remain constant. On the other hand, a firm that has waited with an exit/disinvestment, may stop activity due to an analogous reduction of the option value of waiting (with the exit), if a situation becomes more stable. Therefore, these option values do have complex and counteracting effects on activity and employment in an uncertain crisis situation (especially with low/zero interest rates), depending on the initial activity/inactivity status of the firms in the economy.

Of course our simple model has some limitations related to the interpretation of the interest rate. In practice even negative interest rates are possible. In principle an adequate calculation in the spirit of our simple model could represent even this situation, however, only based on calculating finite amortisation periods. But this would imply not to use the very simple formulas for present values of a perpetuity (i. e. an indefinitely continued annuity) as applied in this paper – since present values of a perpetuity formulas are only meaningful for positive discount rates. As the difference between the profit rate (revenues) and the interest rate (costs) is relevant for the profitability of an investment, the general results of our model related to the hysteresis band should analogously apply to situations with negative interest rates. Moreover, our calculations are based on a single interest rate, not differentiating between deposit and borrowing rates. If the interest rate is interpreted as a borrowing rate for financing investments, limiting the explicit analysis to positive interest rates seems not too unrealistic. Furthermore, though the investor is assumed to be risk neutral, the option to wait limits the negative effects of stochastic disturbances. If instead a risk-averse

investor would be assumed, limiting risks would become even more relevant. Thus, the effects of uncertainty on the trigger levels would even be amplified due to risk aversion (*McDonald/Siegel 1986; Hugonnier/Morellec 2005*).

In the last subsections we have addressed stochastic revenue changes and interest rate changes independently. Actually, in real world business cycles, an increase of the revenues is typically positively correlated with an increase in the interest rate. Both effects on the expected future profitability of the investment are partly compensating each other. Thus, a positive correlation between these both determinants would dampen the option value effects on the band of inaction. Moreover, in real world business cycles the profitability of an investment could be positively correlated with the investment decisions of other investors. Nevertheless, for reasons of simplicity, these macroeconomic stochastic correlation effects are not explicitly modelled in this paper. In the next section an aggregation procedure for independent investment decisions of a multitude of investors related to only interest rate changes as the relevant determinant is presented.

### III. Aggregation and Macroeconomic Investment Hysteresis

So far, we depicted explicitly only the microeconomic effects of hysteresis, resulting in a non-ideal relay dynamics in both perspectives, related to the revenues as well as to interest rates triggering an immediate investment or disinvestment. For macroeconomic dynamics the *Mayergoyz (1986)-Preisach (1935)*-procedure is typically applied – describing an explicit aggregation procedure for a multitude of heterogeneous non-ideal relay agents ( $j = 1, \dots, n; n \gg 0$ ), having different entry/exit triggers due to differences in the firms' cost or revenue structures.<sup>7</sup> The standard procedure is based on the microeconomic relay-type as depicted in Figure 1 for revenue variations, where the entry trigger has a higher level compared to the exit trigger ( $\alpha_j \geq \beta_j$ ) for all heterogeneous firms. Since this relation is “mirrored” if the triggers are stated in terms of interest rates as shown in Figure 2, with an investment/entry trigger below the exit/disinvestment trigger ( $a_j \leq b_j$ ), the standard Preisach aggregation procedure needs some modifications. We present this modified version (where the axes of coordinates of the exit and the entry trigger were changed in comparison to the standard case based on revenue triggers) below.

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<sup>7</sup> This procedure was introduced to economics by *Amable et al. (1994)* and *Cross (1993)*. See e.g. *Göcke (1994)*, *Piscitelli et al. (2000)*, *Belke/Göcke (2005)*, and *Motal/Vasconcelos (2012)* for applications of the Preisach-Mayergoyz procedure in foreign trade and in labour market economics. See *Bassi/Lang (2016)* for an agent-based model of investment-hysteresis related to capital utilisation as an input variable with a simulation for heterogeneous agents.

Every potentially active firm  $j$  is characterized by a twin-set of entry/exit triggers  $(a_j/b_j)$ . In an  $a_j/b_j$ -diagram (see Figure 3), the firms are represented by points in a triangle area above the 45°-line, if the exit trigger interest rate  $b_j$  is depicted on the ordinate axis (since  $a_j \leq b_j$ ). The aggregation procedure can be performed without any serious restriction of the heterogeneity of the firms' distribution over the triangle area (i.e. of the sunk cost  $H_j$  and revenue  $e_{i,j}$  structure of the firms). Points on the 45°-line describe non-hysteretic firms ( $H_j=0 \Rightarrow a_j=b_j$ ). A non-ideal relay characterizes firms with a position above the 45°-( $a=b$ )-line. For reasons of simplicity we assume the firm specific revenues ceteris paribus during the aggregation procedure as constant ( $e_{j,t} = e_j$ ), thus the points representing a firm's capital stock activity pattern remain in their position.

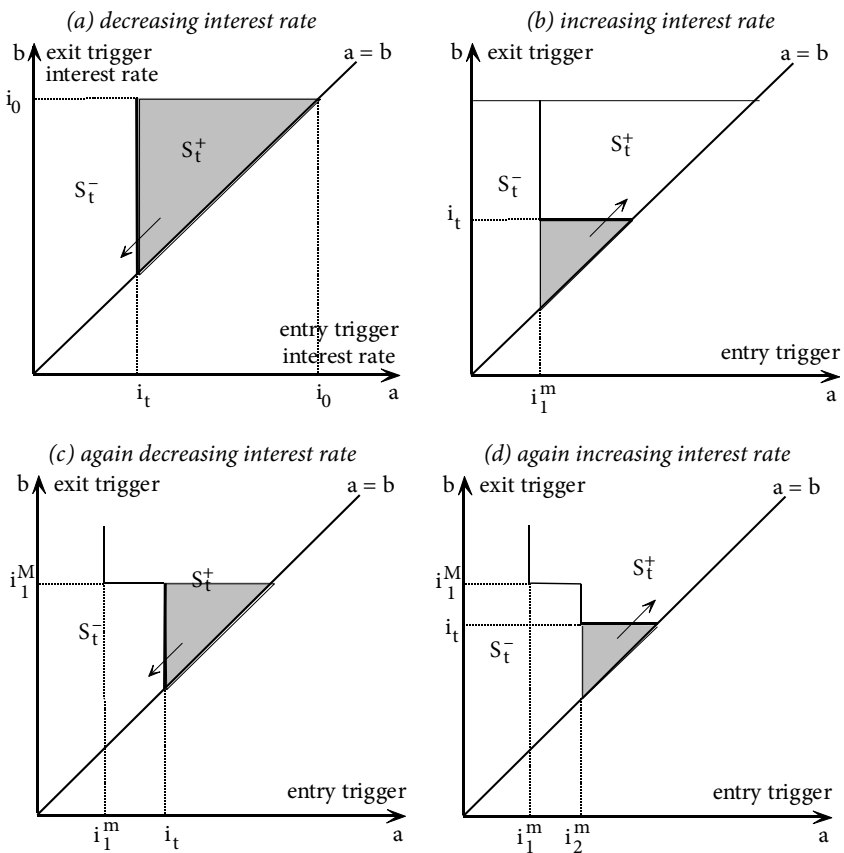


Figure 3: Application of the Modified Mayergoyz/Preisach Procedure – Active Firms under a Volatile Interest Rate

Just to avoid a long description of the past development, we assume an initial situation with a very high interest rate ( $i_0 \gg 0$ ) as a starting point, implying that no firm is initially active. Now, a decreasing interest rate (i.e. as a consequence of an expansionary monetary policy) makes investments affordable and results in investments/entries by firms with the highest rates of return  $e_j$  – i.e. the highest investment interest rate triggers  $a_j$ . The aggregate capital stock increases (and if this capital stock is used for production, aggregate output supply of the entire economy), as traced in Figure 3 (a). This is graphically shown by a growing space of the hatched triangle  $S_t^+$  representing the active firms which have invested and increased their capital stock (while  $S_t^-$  represents the inactive firms). For a decreasing current interest rate (i.e. an expansionary monetary policy), the  $S_t^+$ -expansion is indicated by a shift to the left of the vertical borderline between the area of active and inactive firms. The path AB in Figure 4 depicts the corresponding aggregate macro reaction.

Figure 3 (b) traces a subsequent increase of the interest rate:  $i_t$  rises from the lowest value, the (local) minimum  $i_1^m$ . Therefore, the area  $S_t^+$ , representing active firms, now shrinks, since firms that have recently invested, now disinvest as the interest rate rises above their exit trigger  $b_j$ . For an increasing interest rate, the activity changes (hatched area) are illustrated by a horizontal shift in the lower part of the  $S_t^- - S_t^+$ -borderline. In Figure 4 the corresponding path is BC.

If the interest rate falls again after reaching the local maximum  $i_1^M$ , area  $S_t^+$  again expands, depicted in Figure 3 (c) by a vertical shift to the left of the lower part of the borderline. The corresponding macroeconomic reaction is path CD in Figure 4. The result of the subsequent shifts is a “staircase-shape” of the border between the two areas. If the recently reached (local) minimum is not as low as the lowest minimum  $i_1^m$ , a staircase step in the borderline remains – characterised by the coordinates ( $a = i_1^m / b = i_1^M$ ). If the signal level had continued to decrease and had passed the original minimum, the a-coordinate of the “ $i_1^m$ -step” would have been “wiped out” and replaced (Mayergoyz 1986, p. 605). However, if the new local minimum is higher than the “old”  $i_1^m$  (as traced in Figure 3 (c)), this “old” minimum remains and the new local minimum becomes the second lowest, labelled  $i_2^m$ .

Figure 3 (d) illustrates a subsequent increase in the interest rate. The borderline is changed by an upward shift of the lower horizontal part (path DE in Figure 4). If  $i_t$  does not rise above  $i_1^M$  a new local maximum would become the second highest maximum  $i_2^M$ . If the input were to rise above the “old”  $i_1^M$ , the b-coordinate of the corresponding staircase-step would be eliminated. If subsequent local minima and maxima are not as “extreme” as the preceding extrema, a new corner in the staircase border is created. However, local minima which are lower than preceding minima will erase the a-coordinate of the correspond-

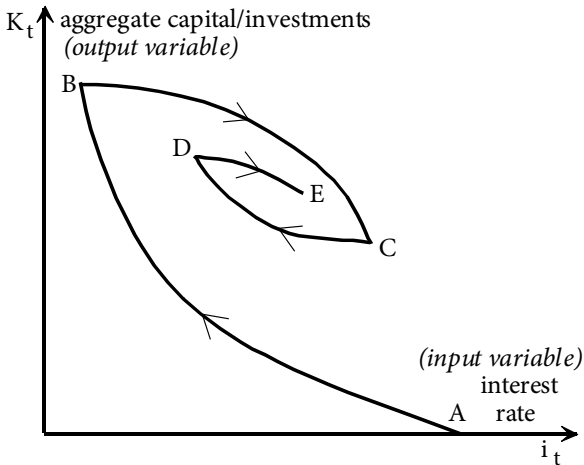


Figure 4: The Continuous “Macroeconomic” Hysteresis Loop for the Aggregate Investment Activity Related to the Interest Rate

ing corners; subsequent local maxima will ‘wipe-out’ the b-coordinate of corners corresponding to lower preceding maxima (Amable et al. 1994).

The aggregate system for the entire economy displays a memory of non-erased (*non-dominated*) past input (i. e. interest rate) extrema – represented by the “staircases” in the borderline of the area S of active firms. Aggregation leads to a change in the pattern of hysteresis. A passing of triggers is necessary at the micro/firm level, in order to induce a permanent effect, whereas *every local extremum* in the time-path of the input variable will have a persisting effect (called “remanence”) in the aggregate loop (see Figure 4). For this reason, this macro hysteresis pattern is called ‘strong’ hysteresis (e. g. Amable et al. 1991, 1994).

The *distribution* of the heterogeneous firms in the  $(a_j \geq b_j)$ -triangle is of course relevant (for the transmission of monetary policy/interest rate changes to the real economy). A *continuous* (called ‘strong’) macro loop as in Figure 4 requires a continuous distribution of the firms in the  $(a_j \geq b_j)$ -region. The exact density and the  $(a_j, b_j)$ -distribution of firms determines the curvature of branches of the macro loop. The less heterogeneous the firms are, the more these firms are clustered in a specific area in the  $(a_j, b_j)$ -diagram, and as a consequence the more “curved” are the macro branches. In the special case of a multiplicity of homogeneous firms represented by the similar point, the macro loop degenerates to a non-ideal relay.

The sunk investment costs of a firm  $H_j$  and the uncertainty effects determine the difference between the entry and the exit trigger. The higher the level of

sunk costs and of different types of uncertainty, the bigger is the difference between the entry and the exit trigger, i.e. the wider is the “band-of-inaction”. Thus firms with high sunk costs and/or affected by a high level of uncertainty (i.e. high option values) will be located far above the  $45^\circ$ -( $a=b$ )-line. The more firms are located far away from the “non-hysteretic” ( $a=b$ )-line, the more “inflated” is the macro-loop (i.e. the greater is the distance between an increasing upward branch in a situation of a decreasing interest rate compared to a downward branch in a situation with a rising interest rate. In the special/border case of no sunk costs and no uncertainty, all firms are located on the ( $a=b$ )-line, resulting in a standard-type macroeconomic investment function with a negative impact of the interest rate on investments, without a differentiation of “upward” and “downward” directions.

#### IV. Conclusions

The purpose of this paper is to show how microeconomic and, derived from that, also macroeconomic investment depends on (present and future) changes in the interest rate. Since we accept that interest rate changes are driven, at least partly, by central banks, important implications for monetary policy and the monetary policy transmission to the real sector of the macroeconomy (investment) emerge. If interest rate expectations are assumed to be fixed, an unexpected change in the current and future rate of return on capital was shown to result in an investment/disinvestment pattern of a firm which can be described by a so called ‘non-ideal relay’. However, if alternatively the interest rate is assumed to be the single input variable, the entry and exit conditions could be solved for the interest rate.

In a next step we dealt with the effects of uncertainty. We derived that a non-ideal relay emerges in a scenario with stochastic revenue changes. Then we demonstrated that a qualitatively similar widening effect on the ‘band of inaction’ results if uncertainty is not based on stochastic revenue changes, but on stochastic future changes of the interest rate. However, these results necessarily rely on a kind of “mirrored” non-ideal relay pattern. In a stochastic environment, this implies that monetary policy based on *low current interest rates* as currently in the Euro area may *not* be overly *effective in providing an incentive to invest*. Only if the central bank implements monetary policy strategies such as “forward guidance” and is able to credibly commit to low interest rates also for the foreseeable future, our quite strong verdict may be of less relevance (*Bernanke/Reinhart* 2004; *Eggertsson/Woodford* 2003).

In this regard, we see the Euro area long-term interest rate as the result of the expected patterns for short-term rates which, in turn, are driven by the Europe-

an Central Bank's inflation forecasts.<sup>8</sup> Reflecting the increasingly active central bank transparency debate in the literature, the ECB has moved in line with other central banks (*Kedan/Stuart* 2014) and delivers an *outlook on its future path of policy rates* ("forward guidance", *Belke* 2018a). Our model could thus, for instance, contribute to establishing the optimal extent of a central bank's "forward guidance".

However, with respect to forward guidance it can be argued that the ECB does not describe anything else than a *policy rule for its future interest rate path*. Forward guidance then intends to strengthen the credibility of the ECB's monetary policy strategy. Another important caveat is that "forward guidance" cannot correspond with any long-run commitment to a specific interest rate level because this would insinuate an intended change in the medium-term oriented "monetary policy strategy" (*Belke* 2018a). If this is true, however, it is even clearly *not appropriate* to speak of a *change in the reaction function* of the ECB driven by "forward guidance". Moreover, the ECB will not be capable to decouple its monetary policies from those of the Fed during and after the process of "normalisation". As soon as the crisis is over and the world economy will have entered more "normal times", the ECB should and most probably will not abide by this instrument. This is so because inflation "forecasts" underlying forward guidance are vexed by much uncertainty and entail risks also for the reputation of central banks (which have to "invest in their reputation under uncertainty").

What is more, the ECB's announcements on their "forward guidance" may not necessarily be conceived as credible – for instance due to election dates located between the announcement and the dates for which the underlying inflation outlook is published. In that case, deviations of the markets' action and the central bank's ideal projection of market behaviour cannot be excluded. By indicating the need to curb official rates also for the next years could convey the impression that the bank anticipates the crisis lasting for several years to come. But if markets get more pessimistic, consumer and investment spending suffer (*Belke* 2018). Seen on the whole, thus, the instrument of "*forward guidance*" *does not make our considerations regarding the "option value of waiting with investment under uncertainty" less relevant.*

As a final step, we aggregated the micro hysteresis effects to the macroeconomic level. The standard procedure is based on the microeconomic relay-type for revenue variations, where the entry price/revenue trigger has a higher level compared to the exit trigger for all heterogeneous firms. Since this relation is "mirrored" with the triggers being stated in terms of interest rates, we modified

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<sup>8</sup> *Belke* (2018a), for instance, argues that the current low-interest rate policy in the Euro area affects also the long-term yields of euro area savings negatively (in contrast to Mario Draghi's view), exactly because long-run interest rates are driven by the sequence of the inflation forecasts by the ECB staff.

the standard Preisach aggregation procedure, with investment/entry triggers of the interest rate below the exit/disinvestment triggers. We then derived the continuous “macroeconomic” hysteresis loop for the aggregate investment activity related to the interest rate.

In this paper, we developed a tool to model a leverage point for monetary policy to impact on macroeconomic investment under uncertainty. This allowed us to draw some conclusions about the effects of low or even zero interest rate monetary policies on macroeconomic investment. Furthermore, the presented model can be the theoretical base for empirical studies of macroeconomic investment dynamics in periods such as the current one where the economies have reached the zero lower bound and is characterised by global policy uncertainty due to the uncertain (consequences of the Corona crisis on the) future economic development of firms’ revenues and interest costs. Moreover, this tool may now as a next step be integrated into a fully specified general equilibrium model. However, the main challenge will be to model the feedbacks to the hysteretic sub-system derived here. In a situation with hysteresis the capital stock (as the dependent variable) shows a path-dependent equilibrium level related to past interest rate dynamics (as the input variable). However, the size of the capital stock may be a determinant for the equilibrium level of the interest rate as well, since the level of investments is e.g. important for the demand on credit markets. In a real economy the interest rate is an endogenous variable as well, and not a completely exogenous input variable. Consequently, persistence/hysteresis effects on the capital stock may eventually have a hysteretic feedback effect on the equilibrium level of the interest rate itself.<sup>9</sup> And since the level of the interest rate is a determinant for other macroeconomic variables, these hysteresis/path-dependence effects may even spill over to different areas of the whole economy.

Finally, as stated already in the introduction, the attractiveness of the micro-based macro model derived in this paper lies in the fact that the mechanics of our macroeconomic model is applicable to a variety of other economic issues related to investment-type decisions which involve irreversible fixed costs and uncertainty.

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<sup>9</sup> See Göcke (2001) for an example of these macroeconomic feedbacks for the case of hysteresis in foreign trade. In foreign trade hysteresis a temporary change of the exchange rate (as the forcing variable) leaves behind a path-dependent effect on the level of exports and imports of an economy if there are sunk costs of entering a foreign market. These persistent changes in the current account eventually (as a feedback) result in a permanent change of the equilibrium level of the exchange rate itself.



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