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Inter-Variety Equilibrium of Chinese Treasury Futures

Jinzhong Wang*, Hong Zhong** and Zhenjie Yu***

Abstract

Treasury futures, important tools in interest risk management, need to maintain price equilibrium between different varieties. In this paper, we conduct research on ten-, five-, and two-year Treasury futures in China's futures market. The auto-regression model is used to fit and predict the spot yield, the CTD (cheapest to deliver) price is used in valuing Treasury futures, and the transaction cost and market friction are considered in building the arbitrage-free spread interval. By comparing the amount of deviation and the equilibrium reversion speed, we analyse the inter-variety price equilibrium between Treasury futures. We find that there are many arbitrage opportunities among the three varieties, and the market is not fully efficient. Through further analysis of the pairwise spread relationship of the futures, we conclude that longer operation of the Treasury futures market will lead to higher market efficiency, shorter duration of arbitrage opportunities, and a faster return to equilibrium. The existing literature mainly focuses on the equilibrium relationship between two Treasury futures in statistical terms, but this paper examines the equilibrium relationship between all existing varieties of Treasury futures in China's market based on pricing, which expands the subject and methods of research on inter-variety equilibrium in Treasury futures.

Keywords: Arbitrage-free Equilibrium, Arbitrage-free Interval, Inter-variety Arbitrage, Deviation

JEL Classification: G13, G14

I. Introduction

Based on Treasury bonds as underlying assets, Treasury futures are important interest rate derivatives and play an important role in managing interest rate risk (*Kolb* et al., 1982; *Figlewski*, 1984). Futures trading has a price discovery function, which can provide liquidity for the spot market, so as to improve its infor-

* Prof. Jinzhong Wang, Southwestern University of Finance and Economics, 555 Liutai Avenue, Wenjiang District, Chengdu, Sichuan, E-Mail: jzwang@swufe.edu.cn.

** Hong Zhong, Southwestern University of Finance and Economics, 555 Liutai Avenue, Wenjiang District, Chengdu, Sichuan, E-Mail: zhonghong@smail.swufe.edu.cn.

*** Zhenjie Yu, Southwestern University of Finance and Economics, 555 Liutai Avenue, Wenjiang District, Chengdu, Sichuan, E-Mail: jeffreyyzj@163.com.

mational efficiency (*Kim*, 2015; *Tse*, 1995, 1999). In 2018, the China Financial Futures Exchange (CFFE) launched two-year Treasury futures, further enriching the interest rate derivative tools based on the existing five- and ten-year Treasury futures. Along with expanding the variety of Treasury futures, inter-variety arbitrage becomes possible, and more investors are attracted to participation in Treasury futures trading, improving liquidity in the spot market. Trading in five-year Treasury futures has operated smoothly for more than seven years, which provides valuable experience for launching and trading ten- and two-year Treasury futures. We must admit that improving the market is a gradual process.

To better judge the efficiency of the Treasury futures market, many scholars have studied price correlation and equilibrium between Treasury futures and bonds statistically. They believe that the Treasury futures market has a price discovery function on the spot market with significant unidirectional Granger causality (*Wang* et al., 2015; *Zhang* et al., 2019). In their research, *Wang* and *Yao* (2015) find a high correlation between the price of Chinese Treasury futures and the CTD (cheapest to deliver) price, whose relationship is long term and stable. However, it is insufficient to focus merely on the futures-spot equilibrium relationship. To understand China's financial markets, it is necessary to study the inter-variety equilibrium relationship between Treasury futures since the launching of two-year Treasury futures. By studying the equilibrium relationship between the varieties of futures since two-year Treasury futures were launched, we can reveal the characteristics of China's financial futures market. Because the price equilibrium relationship is somewhat vague, unstable, and unreliable in statistical terms, and compared with that of commodity futures, the theoretical pricing of financial futures can be carried out more accurately, we study the inter-variety equilibrium between Treasury futures on an arbitrage-free pricing basis.

In this paper, we study and compare the equilibrium relationship of all existing Treasury futures varieties in China's market based on prices, which expands the scope of research and ideas about inter-variety equilibrium in Treasury futures. On the basis of Treasury bond price data in the interbank market, we use auto-regression models to fit and predict the spot yield, price the three Treasury futures with arbitrage-free pricing models, and then establish the arbitrage-free spread interval with consideration of transaction cost, so as to study the equilibrium between their prices. Our study makes several contributions to current literature as follows.

First, we expand the research sample to all varieties of Treasury futures in China's market, including the newly launched two-year Treasury futures, to obtain a more comprehensive understanding of the characteristics of China's Treasury bond futures market. The existing research focuses mainly on inter-variety equilibrium between five- and ten-year Treasury bond futures (*Hou*, 2018; *Wu*, 2017; *Liu*, 2019) or futures-spot equilibrium of a single variety (*Li* et al., 2019; *Wang* et al., 2015; *Chen*, 2020). Many arbitrage opportunities among the three varieties are available, and the market is not fully efficient. Through further analysis of the pairwise spread relationship in futures, we conclude that longer operation of the Treasury futures market results in higher market efficiency, the shorter existence of arbitrage opportunities, and the more rapid reversion to equilibrium.

Second, we determine the equilibrium relationship among Treasury bond futures from the perspective of no-arbitrage pricing, instead of using the cointegration method commonly used in existing research (*Wu*, 2017; *Liu*, 2019; *Sun*, 2021), which is a novel research perspective on equilibrium among varieties of Treasury bonds futures. The equilibrium relationship based on pricing is more reliable than that based on statistics. The former is a theoretical connection, while the latter is a statistical phenomenon. The Covid-19 pandemic greatly reduced or even reversed the market spread. The auto-regressive model and arbitrage-free pricing model that we adopted quickly captured the impact of the pandemic on the futures price of Treasury bonds and predicted the arbitrage-free equilibrium spread with the same trend.

The remainder of this paper is organised as follows. In section 2, we present a literature review. In section 3, we outline our data. In section 4, we adopt the auto-regressive model to fit and predict the spot yield, which is used in the arbitrage-free equilibrium spread model for pricing Treasury futures. In section 5, we explain how to price Treasury futures and obtain an arbitrage-free equilibrium spread. In section 6, we analyse the inter-variety equilibrium relationship by comparing the degree of deviation and speed of reversion to equilibrium between the market spread and the arbitrage-free equilibrium spread. In section 7, we conclude.

II. Literature Review

Inter-variety arbitrage opportunities are created whenever the price deviates from the equilibrium. When studying inter-variety equilibrium and inter-variety arbitrage, most Chinese and international scholars focus on commodity futures and stock index futures. Little research has been conducted on Treasury futures. This prior research on inter-variety price equilibrium between commodity futures is valuable as a reference for studying the inter-variety price equilibrium in Treasury futures.

Because of the different underlying assets, the prices of commodity futures may be correlated to but lack a direct theoretical relationship with pricing. Price equilibrium between futures is usually examined through the cointegration method, which can show whether the equilibrium is stable using statistical tests. *Emery* et al. (2002) conducted a inter-variety arbitrage study on the futures contracts of correlated varieties of electricity and natural gas. They first analysed the correlation between the two futures prices using the cointegration method and then built arbitrage models to simulate trading. The empirical results showed that the arbitrage strategy had good market performance and could obtain excess returns. *Peng* (2010) adopted a statistical arbitrage method to research inter-variety arbitrage in soybean, soybean meal, and soybean oil futures. They concluded that the China futures market was inefficient. *Yin* (2008) performed Granger-causality tests and a cointegration analysis on the price of palm oil and soybean oil futures contracts and then used an error correction model (ECM) to confirm the long-term equilibrium between their prices. *Wang* (2011) conducted a correlation analysis on the main contract price of copper and zinc futures, designed inter-variety arbitrage schemes, checked the existing frequency and specific form of the inter-variety arbitrage opportunities between copper and zinc futures, and concluded that inter-variety arbitrage opportunities existed. Similar research methods were also employed by *Wahab* et al. (1994), *Mitchell* (2007), *Jiang* and *Wang* (2020), and *Zhang* (2020). Because the equilibrium relationship is statistically somewhat vague, unstable, and unreliable, the results from studying inter-variety futures price equilibrium have only historical statistical significance but cannot accurately predict the future.

Other scholars have established price equilibrium between different futures through the construction of common volatility models after determining the correlation between different futures prices. *Wahab* et al. (1994) used ECM, an auto-regressive and moving average (ARMA) model, and an auto-regressive conditional heteroscedasticity (ARCH) model on the basis of cointegration analysis to analyse the main contract price of gold and silver futures and obtained inter-variety arbitrage with the moving average method. In studying the inter-variety arbitrage between soybean, soybean meal, and soybean oil futures, *Simon* (1999) used a generalised auto-regressive conditional heteroscedasticity (GARCH) model on the basis of cointegration analysis to illustrate the long-term equilibrium between their prices and proved that arbitrage opportunities existed due to insufficient efficiency in the futures market. *Haigh* and *Holt* (2002) used M-GARCH, B-GARCH, and other methods to conduct arbitrage comparison research on crude oil and its derivative futures, finding inter-variety arbitrage opportunities. The M-GARCH method yielded the highest profit in trading. Using a hypothesis of a neutral market with an arbitrage proportion of 1:1, *Yang* and *Chen* (2018) studied the inter-variety arbitrage relationship between China's five- and ten-year Treasury futures, establishing a statistical arbitrage strategy based on the GARCH method. In a study on arbitrage strategy, *Cui* et al. (2015) found that the price may be influenced by asymmetric information. The introduction of a T-GARCH model in an arbitrage model rather than a GARCH model might lead to higher and stabler profit. Similar research methods were also adopted by *Zhou* (2017), *Zhu* et al. (2015), *Miao* and *Zhu* (2019), and *Zou* et al. (2019). It is worth noting that the transaction cost is also a very important factor in inter-variety futures price equilibrium, which has a direct influence on whether the arbitrage strategy will be profitable. In analysing the price equilibrium between the FTSE 100 index and FTSE 250 index futures, *Butterworth* et al. (1999) found that the transaction cost might turn a profitable arbitrage opportunity into a deficit. Based on the volatility model, the analysis of inter-variety futures price equilibrium relies on a hypothesis of common volatility changes, whose rationality is hard to judge, so the applicability to different futures could vary.

Price correlation is much higher between Treasury futures and spot Treasury bonds than between other commodity futures, so theoretical pricing could be more accurate. *William* (1978) proved a noteworthy correlation between Treasury futures and spot Treasury bonds, even taking transaction costs into account. *Comell* et al. (1988) deduced a cost of carry model of stock index futures. *Chen* (2015) pointed out that the model also applies to Treasury futures pricing when conversion options are excluded. With reference to the cost of carry model which is widely used in futures pricing, *Cornell* (1983) made some corrections based on the characteristics of Treasury futures, proposing a Treasury futures pricing model. Following the research of global scholars, *Luo* (2003) deduced a cost of carry model for Chinas Treasury futures. By establishing an equilibrium model between futures and the spot market, *Rendleman* et al. (1984) determined the futures equilibrium price. All the underlying assets of different Treasury futures are Treasury bonds, with the difference coming from residual maturity. Because they can employ the same pricing model, we can start with pricing theory to establish the price relationship between different futures and then conduct empirical research based on trading data.

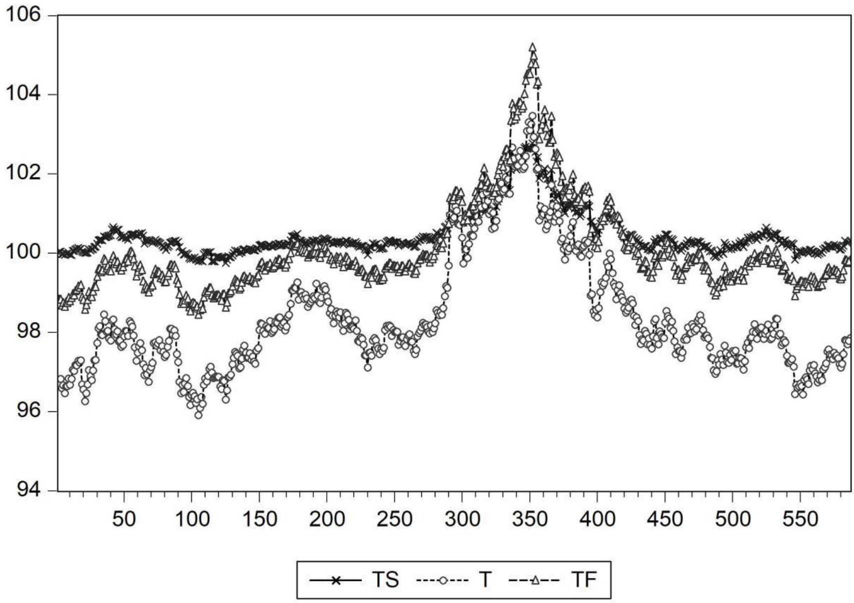
The existing literature on Treasury futures arbitrage *focuses mainly on intertemporal arbitrage and futures-spot arbitrage (Chen*, 2020; *Li*, 2019; *Liu*, 2015; *Fang*, 2014; *Chen*, 2014), and few papers examine inter-variety arbitrage, all of which concentrate on the statistical arbitrage method (*Wu*, 2017; *Liu*, 2019; *Sun*, 2021). Through our literature review, we determined the feasibility of studying the price relationship between different Treasury futures based on pricing. As a result, we fit and predict the term structure of interest rates using a simple and flexible auto-regression model, price Treasury futures based on the market price of Treasury bonds and the cost of carry model, and compare and analyse the equilibrium and arbitrage between two-, five-, and ten-year Treasury futures.

III. Data

Our research requires data on the Treasury futures market price, the CTD (cheapest to deliver) market price, and the spot yield (alternative to the risk-free rate), all obtained from the Wind financial terminal. The data-processing methods are as follows.

1. Selection of Treasury Futures Contract

Treasury futures can have various contracts with different maturity dates trading at the same time, and each contract lasts for several months, so we need to build a continuous price sequence to represent the price changes across those various contracts. In contrast, only one trading contract serves every day as the main contract, based on its highest trading activity and largest trading volume. To some extent, building a continuous price sequence by choosing the main Treasury futures contract overcomes the weakness of having a small trading volume and large price volatility in the delivery month. In general, the trading vol-



Note(s): This figure shows the closed price curve for two-year (TS), five-year (TF), and ten-year (T) Treasury futures.

Source(s): Wind financial terminal

Figure 1: Treasury Bond Futures Closed Price Curve

ume of futures contracts takes an inverse U-shape and is time varying. We build a continuous price sequence of the daily closing quotation of the main contract by replacing the main futures contract in the month before maturity with the next contract. The two-year Treasury futures were launched on 17 August 2018, and many values are missing in the first three months, perhaps because of initially inactive and unstable trading. As a result, we select market data from 19 November 2018 to 19 April 2021, a total of 587 sets of samples. We obtain three continuous closing quotation sequences as shown in Figure 1.

2. *Selection of CTD*

The underlying assets of Treasury futures are uncertain. For each main contract, there is a basket of deliverable bonds. The short side has a selection of the most beneficial bonds, i.e. CTD, to settle the contract on the delivery day. To price Treasury futures on the basis of Treasury bonds, we need to determine the relevant CTD for the main contracts. We obtain statistics for all the deliverable bonds for the main contracts on two-, five-, and ten-year Treasury futures, gather and compare their CTD time, second CTD time and third CTD time. The deliverable bond with the most CTD time in contract duration or most time for the total CTD, second CTD, and third CTD is determined to be CTD for the relevant contract. In practice, the deliverable bond with the most CTD time takes priority; if the CTD time accounts for a relevant small proportion, then we choose the deliverable bond with most total CTD, second CTD, and third CTD time. The futures contracts and their corresponding CTD selected to build the continuous closing quotation sequence are shown in Table 1.

Table 1

Statistics of Futures Contracts and Corresponding CTD (Cheapest to Deliver)

Futures contract	*CTD*	*Time of CTD*	*Time of second CTD*	*Time of third CTD*	*Proportion of CTD*	*Conversion factors*
TS1903	180007.IB	28	36	26	21.54 %	1.0083
TS1906	160015.IB	80	22	5	40.61 %	0.993
TS1909	160015.IB	67	26	7	36.81 %	0.9938
TS1912	160015.IB	82	33	6	43.62 %	0.9946
TS2003	180021.IB	82	46	32	45.56 %	1.0025
TS2006	190003.IB	79	51	30	44.63 %	0.9947
TS2009	190003.IB	124	47	9	68.51 %	0.9954

(continue next page)

(Table 1 continued)

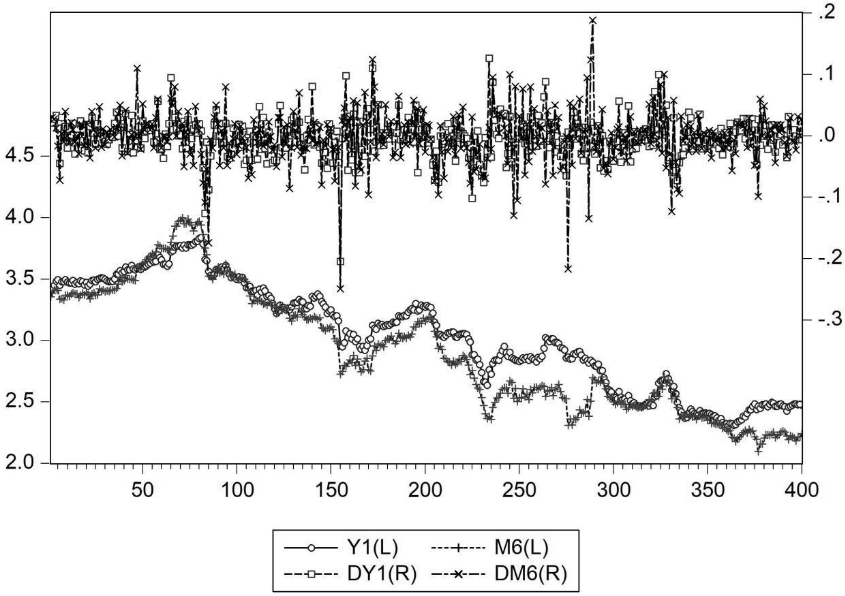
Futures contract	*CTD*	*Time of CTD*	*Time of second CTD*	*Time of third CTD*	*Proportion of CTD*	*Conversion factors*
TS2012	190011.IB	97	74	4	53.30 %	0.9959
TS2103	180009.IB	86	17	2	58.11 %	1.0033
TS2106	200003.IB	82	34	25	58.16 %	0.9872
TF1903	160014.IB	113	23	1	62.78 %	0.998
TF1906	170006.IB	101	15	2	57.39 %	1.0086
TF1909	170013.IB	88	34	9	48.35 %	1.0248
TF1912	170020.IB	94	14	14	50.00 %	1.03
TF2003	190004.IB	63	66	44	34.81 %	1.0072
TF2006	190013.IB	101	50	7	57.71 %	0.9975
TF2009	190013.IB	85	62	28	46.96 %	0.9977
TF2012	200005.IB	84	46	30	46.15 %	0.9595
TF2103	200005.IB	75	38	26	48.70 %	0.9617
TF2106	200013.IB	85	35	--	70.83 %	1.0007
T1903	160023.IB	54	27	9	60.00 %	0.9796
T1906	160023.IB	52	32	9	55.91 %	0.9802
T1909	160023.IB	47	30	7	55.95 %	0.9808
T1912	180027.IB	52	32	24	48.15 %	1.0194
T2003	180027.IB	60	51	31	42.25 %	1.0189
T2006	180027.IB	62	45	31	44.93 %	1.0185
T2009	180027.IB	32	42	38	28.57 %	1.018
T2012	2000004.IB	87	6	4	89.69 %	0.9884
T2103	2000004.IB	86	56	6	58.11 %	0.9887
T2106	2000004.IB	157	8	2	94.01 %	0.9889

Note(s): This table shows the futures contracts and the corresponding CTD selected to construct the continuous closing quotation sequence of Treasury futures.

Source(s): Wind financial terminal.

3. Spot Yield Data

The Treasury futures contract has a duration limit of no more than one year, so we can fit the spot yields of six-month (M6) and one-year (Y1) Treasury bonds, instead of the complete yield term structure. Therefore, we select the spot yield data for 902 market days, from 4 September 2017 to 19 April 2021. We use the data for the first 400 market days as samples to estimate an AR model, using the model and historical market data for a day and the previous day to make a one-step-ahead prediction, obtaining the predicted value of the next day's spot yield. Then we take the second 502 market days as out-of-sample data to perform a comparative evaluation of the static prediction effect of the AR model. Figure 2 shows the curve for the spot yield of six-month and one-year Treasury bonds in the interbank market as well as their first-order difference. Table 2 shows the descriptive statistics for the spot yield data and the adjusted Dickey-Fuller (ADF) test results of the series. We find that the original time-series data are not stable but turn stable after the first-order difference, so we use the first-order difference data for M6 and Y1 to construct an auto-regressive model.



Note(s): This figure shows the curve of the spot yield of six-month and one-year Treasury bonds in the interbank market as well as their first-order difference.

Source(s): The original data comes from wind financial terminal, and the chart data is processed.

Figure 2: Spot Yield Curve

Table 2

Descriptive Statistics of Spot Yield Data

	Mean	*Median*	*Maximum*	*Minimum*	*Std*	*p-value of ADF*
M6	2.5457	2.4999	4.0000	0.9785	0.5723	0.8741
Y1	2.6821	2.6347	3.8384	1.1057	0.53289	0.8345
DM6	–0.0013	–0.0006	0.2047	–0.2492	0.04248	0.0000
DY1	–0.0010	0.0000	0.2179	–0.2752	0.03289	0.0000

Note(s): This table shows the descriptive statistics for spot yield data, including M6, Y1, DM6, and DY1. M6 represents the spot yields of six-month Treasury bond, and Y1 represents the spot yields of one-year Treasury bond. DM6 and DY1 represent their corresponding first-order difference. We tested the stationarity of time-series data with the adjusted Dickey-Fuller (ADF) method. We find that the original time-series data are not stable but become stable after the first-order difference. Therefore, it is feasible to establish the auto-regressive model using the first-order-difference data for M6 and Y1.

Source(s): Wind financial terminal.

IV. Spot Yield Prediction Model

In order to price Treasury futures using the spot bond price and the cost of carry model, we need to fit and predict the changes in the risk-free interest rate. In this section, we fit and predict the spot rate (alternative to the risk-free rate) with simple and flexible auto-regression models because of the significant auto-correlation in the time-series data for the spot rate. Since Treasury bonds have nearly no credit risk, and the interbank market has active transactions with a large volume, the spot yield of Treasury bonds in the interbank market is a good substitute for the risk-free interest rate.

To determine the lag order in the auto-regression model, we check the partial auto-correlation function in the first-order difference sequence for M6 and Y1. We found significant correlation in the second-order lag, so it is appropriate to build an AR(2) model. Because the first-order and partial auto-correlation are not significant, it is necessary to consider whether we should eliminate the first-order lag. So, we construct four auto-regressive models, in which DM6 is the first-order difference of M6, and DY1 is the first-order difference of Y1; the estimated results of the model are shown in Table 3.

$$\mathrm{DM6}_t = c + \beta_1 \times \mathrm{DM6}_{t-1} + \beta_2 \times \mathrm{DM6}_{t-2} + \upsilon_t \tag{1}$$

$$\mathrm{DM6}_t = c + \beta_1 \times \mathrm{DM6}_{t-2} + \upsilon_t \tag{2}$$

$$\mathrm{DY1}_t = c + \beta_1 \times \mathrm{DY1}_{t-1} + \beta_2 \times \mathrm{DY1}_{t-1} + \upsilon_t \tag{3}$$

$$\mathrm{DY1}_t = c + \beta_1 \times \mathrm{DY1}_{t-2} + \upsilon_t \tag{4}$$

Table 3

Estimation Result of the Spot Yield Model

Variables	*DM6*	*DM6*	*DY1*	DY1
Constant	–0.002777	–0.002767	–0.002291	–0.002295
	(0.002409)	(0.002536)	(0.002086)	(0.001965)
AR(1)	–0.049489		0.049044	
	(0.042781)		(0.051086)	
AR(2)	0.085093**	0.087853**	0.176481***	0.179313***
	(0.039174)	(0.037621)	(0.039794)	(0.039902)
N	399	399	399	399
R2	0.010213	0.007766	0.034765	0.032359
Log likelihood	681.603	681.111	809.652	809.156
AIC	–3.396506	–3.399054	–4.038354	–4.04088

Note(s): This table shows the estimation results for the spot yield model (Equations 1–4). DM6 is the first-order difference of M6, and DY1 is the first-order difference of Y1. AR(1) represents the first-order lagged term of the dependent variable, and AR(2) represents the second-order lagged term of the dependent variable.We find that the influence of the first-order lagged term is not significant, so it can be eliminated. Based on the Akaike information criterion(AIC), an estimation model with smaller AIC should be selected. The AIC of the models for DM6 and Y1 with only a second-order lagged term is smaller than that of the models with first-order and second-order lagged terms. Therefore, we choose the auto-regressive model with only a second-order lag, that is, the models represented by Equations (2) and (4). ***, **, and * denote significance at the 1 %, 5 %, and 10 % levels, respectively.

Source(s): The original data comes from wind financial terminal, and the table data is processed.

As shown in Table 3, the effect of the first-order lag is not significant, so it should be eliminated. According to the Akaike information criterion (AIC), the estimation model with smaller AIC should be selected. The AIC of the DM6 and DY1 models with only a second-order lag is smaller than that of the models with a first-order and second-order lag. Therefore, we choose the auto-regressive model with only a second-order lag, i. e. the models represented by Equations (2) and (4).

To test the prediction effect of the model, we perform one-step-ahead prediction by adding the latest market spot yield into the auto-regression model. The effect of in-sample fitting and out-of-sample prediction is shown in Figure 3. The fitted value and predicted value are close to the real value in the market. The results show that the second-order lag can explain and predict the spot yield two days later well. In the process of building the model, we checked its stability and reliability. In the following section, based on the spot yield predicted by the model, we conduct an empirical study on pricing Treasury futures, calculating the theoretical spread interval and analysing the inter-variety equilibrium between Treasury futures.

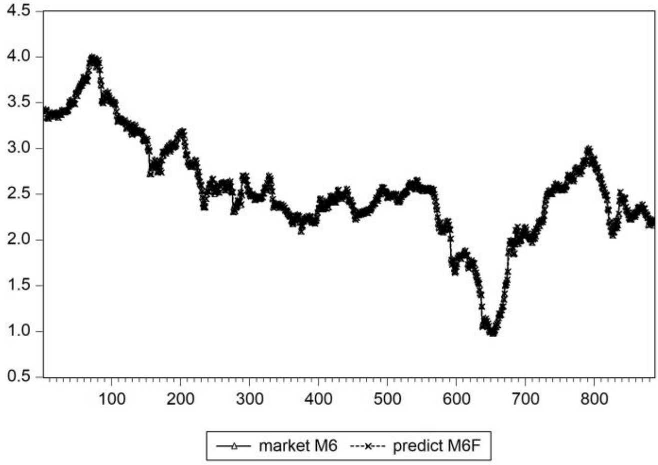


Figure 3a. Fitting and Prediction Results for M6

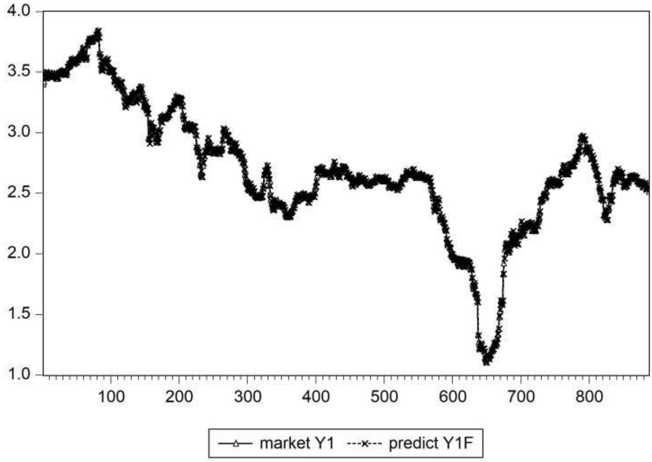


Figure 3b. Fitting and Prediction Results for Y1

Note(s): This figure shows the fitting and prediction results for M6 and Y1. The second-order lagged term explains and predicts the spot yield two days later well.

Source(s): The original data comes from wind financial terminal, and the chart data is processed.

Figure 3: Fitting and Prediction Effect of M6 and Y1 Return Sequence

V. Arbitrage-Free Equilibrium Spread Models

In this section, we construct an arbitrage-free equilibrium spread model. We begin with the theoretical pricing of Treasury futures, determine the equation for the equilibrium spread (basic arbitrage-free equilibrium spread models) for inter-variety Treasury futures, and analyse how the market price returns to equilibrium after deviation. Then, we revise the basic arbitrage-free equilibrium spread model by considering the arbitrage transaction cost and conversion option value to construct an arbitrage-free spread interval, so as to make it as close as possible to actual market conditions. If the market spread is in the arbitrage-free spread interval, we think that the market is balanced.

1. *Basic Arbitrage-Free Equilibrium Spread Models*

First, we consider price equilibrium between Treasury bonds and futures. Using the theoretical futures pricing equation by the CFFE and the futures-spot market equilibrium arbitrage-free analysis model by *Wang* (2015) for reference, we posit the following hypotheses. First, the capital market is perfect, with no taxes, no transaction costs, and no short limit. The assets are perfectly divisible, and the market has perfect competition and many traders. Both buyers and sellers are price takers. Resources can flow freely with no information asymmetry. Second, the prices of Treasury futures and forwards with identical underlying interest rates are equal. Third, the CTD is known, which means there is no conversion option value.

Meanwhile, we define T as the delivery date of a Treasury futures contract, S_t as the spot price of a Treasury bond at time t, CF as its corresponding conversion factor, I_t as the accrued interest from the last interest date on which interest was paid to time t, C as the interest payment during the holding period, whose payoff times is n with corresponding time nodes $t < t_1 < t_2 < \ldots < t_n < T$, and r_{t_i} as the spot yield at period t_i. The theoretical pricing equation for Treasury futures is as follows:

$$F_t = \frac{\left(S_t + I_t - \sum_{i=1}^{n} \frac{C_i}{1 + r_{t_i}}\right) \times \left(1 + r_T\right) - I_T}{CF} \tag{5}$$

Second, considering the market spread relationship between different Treasury futures, for example, two- and five-year Treasury futures, they both have the same theoretical pricing equation, shown as Equation (5). The price of two-year Treasury futures is F_{2t}, and the price of five-year Treasury futures is F_{5t}. The theoretical spread of both futures is as follows:

$$\Delta F = F_{2t} - F_{5t} \tag{6}$$

When the market spread differs from the theoretical spread, keen arbitrageurs can identify arbitrage opportunities and build an arbitrage strategy. An analysis of the arbitrage strategy is shown in Table 4.

Table 4

Analysis of Arbitrage Strategy

Arbitrage opportunities	*Arbitrage strategy*	*Arbitrage results*	*Offset gain or loss in arbitrage position*
When the actual spread is larger than the theoretical spread: $\Delta F = F_2 - F_5 > \Delta F^*$	Short 1 lot TS + Long 2 lots TF	Short positions drive TS prices down: $F_2' = F_2 - \Delta_2,\ \Delta_2 >= 0$ Long positions drive TF prices up: $F_5' = F_5 + \Delta_5,\ \Delta_5 >= 0$ Spread narrows, Δ_2 and Δ_5 do not equal 0 at the same time: $\Delta F' = F_2' - F_5' = \Delta F - (\Delta_2 + \Delta_5) < \Delta F$	$(F_2 - F_2')*20000 + (F_5' - F_5)*2*10000 = (\Delta_2 + \Delta_5)*20000$
When the actual spread is smaller than the theoretical spread: $\Delta F = F_2 - F_5 < \Delta F^*$	Long 1 lot TS + Short 2 lots TF	Long positions drive TS prices up: $F_2' = F_2 + \Delta_2,\ \Delta_2 >= 0$ Short positions drive TF prices down: $F_5' = F_5 - \Delta_5,\ \Delta_5 >= 0$ Spread widens, Δ_2 and Δ_5 do not equal 0 at the same time: $\Delta F' = F_2' - F_5' = \Delta F + (\Delta_2 + \Delta_5) > \Delta F$	$(F_2' - F_2)*20000 + (F_5 - F_5')*2*10000 = (\Delta_2 + \Delta_5)*20000$

Note(s): This table shows how an arbitrage strategy is built when arbitrage opportunities appear. As long as the market spread deviates from the theoretical spread, arbitrageurs will be attracted to relevant arbitrage strategies in trades. If the price of two-year Treasury futures (TS) is F_2, the price change driven by arbitrage is Δ_2, the price after change is F_2', the price of five-year Treasury futures (TF) is F_5, the price change driven by arbitrage is Δ_5, the price after change is F_5', the theoretical spread is ΔF^*, and the actual spread is ΔF.

Table 4 indicates that as long as the market spread deviates from the theoretical spread, arbitrageurs will be able to employ relevant arbitrage strategies in trading. The greater the deviation in the market price spread from the theoretical price spread, the more arbitrageurs that will be attracted to arbitrage trading, expanding the arbitrage position in the market and the power to push price changes. When the market price spread reverts to the theoretical price spread, arbitrage opportunities disappear, and the market returns to equilibrium. Therefore, if the market is close to efficiency, our belief that the market spread can revert to the theoretical spread is reasonable, which means that the market equilibrium spread equals the theoretical spread calculated. We assume that the market is in equilibrium if the market spread equals the theoretical spread.

2. Modification of Arbitrage-Free Equilibrium Spread Models in Reality

The models and assumptions above are based on an ideal market, but the real capital market has transaction costs and market friction, with different forward and futures prices and uncertain CTD for Treasury futures. In order to make the study closer to reality, we modify the models and assumptions and loosen some conditions and then try to build an arbitrage-free spread interval on this basis. If the market spread is in the arbitrage-free spread interval, we conclude that the inter-variety market is in equilibrium.

First, arbitrage has a transaction cost, so the strategy will be implemented only when the gain from arbitrage is greater than the transaction cost, which means that the transaction fee and margin occupancy cost affect implementation of the arbitrage strategy. If the arbitrage cost is C, then the theoretical spread in the inter-variety market equilibrium model turns into an arbitrage-free spread interval $[\Delta F - C, \Delta F + C]$. In this paper, we mainly consider the impact of the transaction cost. The margin occupancy cost depends on the size of the arbitrage position, holding period, and interest rate. Normally, the opportunity for arbitrage exists for only a few days, which means the margin occupancy cost is negligible if it is calculated by the risk-free interest rate. The settings and calculation of transaction fees are as follows.

According to the contract specifications published by the CFFE, commission charges for Treasury futures are RMB 3/lot. For ten-year Treasury futures (T), its face value is RMB 1 million, and the minimum margin is 2.0% of the contract value; for five-year Treasury futures (TF), its face value is RMB 1 million, and the minimum margin is 1.0% of the contract value; and for two-year Treasury futures (TS), its face value is RMB 2 million, and the minimum margin is 0.5% of the contract value. The commission charge per RMB 100 quoted of Treasury futures is as follows:

For ten-year Treasury futures:

$$\frac{3 \times 100}{1000000 \times 2\%} = 0.015$$

For five-year Treasury futures:

$$\frac{3 \times 100}{1000000 \times 1\%} = 0.03$$

For two-year Treasury futures:

$$\frac{3 \times 100}{2000000 \times 0.5\%} = 0.03$$

Each arbitrage trade involves building and settling a position once, respectively, so the commission charge per RMB 100 quoted of arbitrage strategy is as follows:

For two- and five-year Treasury futures:

$$\frac{0.03 + 0.03}{2} \times 2 = 0.06$$

For two- and ten-year Treasury futures:

$$\frac{0.03 + 0.015}{2} \times 2 = 0.045$$

For five- and ten-year Treasury futures:

$$\frac{0.03 + 0.015}{2} \times 2 = 0.045$$

Arbitrage has an impact on Treasury futures price, so, in an inactive market, the cost of this impact greatly affects profits. *Qin* (2014) used the mean value of the bid-ask spread to measure the impact cost in studying calendar spread arbitrage between Treasury futures and found that a high impact cost is about RMB 0.006 per RMB 100 quoted, a normal impact cost is about RMB 0.003 per RMB 100 quoted, and a low impact cost is about RMB 0.002 per RMB 100 quoted. Considering the high volume and frequent trading in the interbank Treasury futures market, we adopt the mean value of these three as the impact cost: RMB 0.0037 per RMB 100 quoted. The transaction cost of arbitrage strategy per RMB 100 is as follows:

For two- and five-year Treasury futures:

$$C_{2\&5} = 0.06 + 0.0037 = 0.0637$$

For two- and ten-year Treasury futures:

$$C_{2\&10} = 0.045 + 0.0037 = 0.0487$$

For five- and ten-year Treasury futures:

$$C_{5\&10} = 0.045 + 0.0037 = 0.0487$$

Second, the assumption that futures and forward prices are constant does not affect the accuracy of the model. *Cox* et al. (1981) believed that Treasury futures should be less expensive than the relevant forward, which means that our estimated futures price is on the high side. But in other empirical studies, such as *Rendleman* et al. (1984), the price difference between futures and forwards is not significant. Because market interest rates in China do not fluctuate very much over a short period, we do not consider the difference in the model.

Last, we consider the conversion option value for deliverable bonds. Deliverable bonds for Treasury futures are not single underlying asset but a basket of conforming Treasury bonds, and it is stipulated that short sellers of Treasury futures contracts can choose the delivery bond. Although most Treasury futures contracts are settled through hedging transactions before maturity, some are still held to maturity and be delivered. The possibility of delivery and uncertainty in delivery bonds influences investor expectations of Treasury futures prices. Using a cost of carry model, *Hemler* (1990) calculated Treasury futures prices. In combination with historical data on market prices, the conversion option value of Treasury futures accounts for 0.3 % of the price. The CITICS Futures Co., using simulated transaction data and Hemler's three methods, calculated the conversion option value in China's market, believing that under current conditions, the conversion option value of Treasury futures accounts for 0.1 % – 0.2 % of the price. Drawing on research findings in China and elsewhere, we use 0.2 % of the futures price as the conversion option value. For example, after modification, the arbitrage-free spread for two- and five-year Treasury futures is as follows.

For two-year Treasury futures (TS), we assume that the option-free price is F_2, the option-embedded price is OF_2, and the option price is P_2, whereas for five-year Treasury futures (TF), the option-free price is F_5, the option-embedded price is OF_5, and the option price is P_5. The conversion option value is set at 0.2 % of the Treasury futures price, i. e.:

$$P_2 = F_2 \times 0.2\%, P_5 = F_5 \times 0.2\% \tag{7}$$

The option-embedded price of TS and TF is as follows:

$$OF_2 = F_2 - P_2\, , OF_5 = F_5 - P_5 \tag{8}$$

Considering the option value, the theoretical spread between TS and TF is as follows:

$$\Delta F_{2-5} = OF_2 - OF_5 = (F_2 - F_5) - (P_2 - P_5) \tag{9}$$

Because the option value constantly fluctuates, 0.2 % should be understood as the upper limit of the option value, rather than a fixed value. The lower limit of the option value is 0. So, considering the option value, the theoretical spread bounds are as follows:

$$\max\Delta F_{2-5} = (F_2 - F_5) - \min(P_2 - P_5) = (F_2 - F_5) + P_5 \tag{10}$$

$$\min\Delta F_{2-5} = (F_2 - F_5) - \max(P_2 - P_5) = (F_2 - F_5) - P_2 \tag{11}$$

Considering the option value and the transaction cost, the arbitrage-free spread interval between two- and five-year Treasury futures is as follows:

$$[(F_2 - F_5) - F_2 \times 0.2\% - 0.0637, (F_2 - F_5) + F_5 \times 0.2\% + 0.0637] \tag{12}$$

We can obtain the arbitrage-free spread interval of the other two groups in a similar way.

The arbitrage-free spread interval between two- and ten-year Treasury futures:

$$[(F_2 - F_{10}) - F_2 \times 0.2\% - 0.0487, (F_2 - F_{10}) + F_{10} \times 0.2\% + 0.0487] \tag{13}$$

The arbitrage-free spread interval between five- and ten-year Treasury futures

$$[(F_5 - F_{10}) - F_5 \times 0.2\% - 0.0487, (F_5 - F_{10}) + F_{10} \times 0.2\% + 0.0487] \tag{14}$$

VI. Empirical Results

We use the CTD spot bond price and Equation (5) to calculate the theoretical price of Treasury futures (with no conversion option value). Using Equation (6), we calculate the theoretical spread between the three varieties of futures. And in combination with arbitrage-free spread interval Equations (12) – (14), we calculate the bounds of the theoretical spread. In order to learn the extent of the market spread deviation from the arbitrage-free spread interval more intuitively, using the indicators proposed by *Wang* (2015) for reference, we define the deviation PL_t as the portion of the arbitrage-free spread interval in which the market spread exceeds the upper or lower limit. We assume that the deviation is PL_t (Equation 15), the market spread is ΔF, and the upper and lower limits of the arbitrage-free spread interval are, respectively, U_u and U_d. In order to further understand the duration of deviation and to determine the existence of arbitrage opportunities, we define the equilibrium reversion speed as the time from the breakthrough spread bounds to reversion to the interval, measured in days.

$$PL_t = \text{Max}\,\{\Delta F - U_u\,, 0\} + \text{Min}\,\{\Delta F - U_d\,, 0\} \tag{15}$$

In this section, we analyse the inter-variety equilibrium by examining the spread curve, the deviation, and the equilibrium reversion speed. When the market spread is within the arbitrage-free spread interval, no arbitrage opportunity among Treasury futures exists. The inter-variety market price is in equilibrium, and the futures market is efficient. When futures prices exceed the interval, then the market deviates from equilibrium, and arbitrage opportunities exist. The efficiency of the futures market can be measured by the amount of deviation and the speed of reversion to the equilibrium. Greater market efficiency is indicated by having less deviation and more rapid reversion to equilibrium.

1. Analysis of the Market Spread Curve

We draw a trend chart of the market spreads and their corresponding arbitrage-free bounds for two-, five- and ten-year Treasury futures, shown in Figure 4.

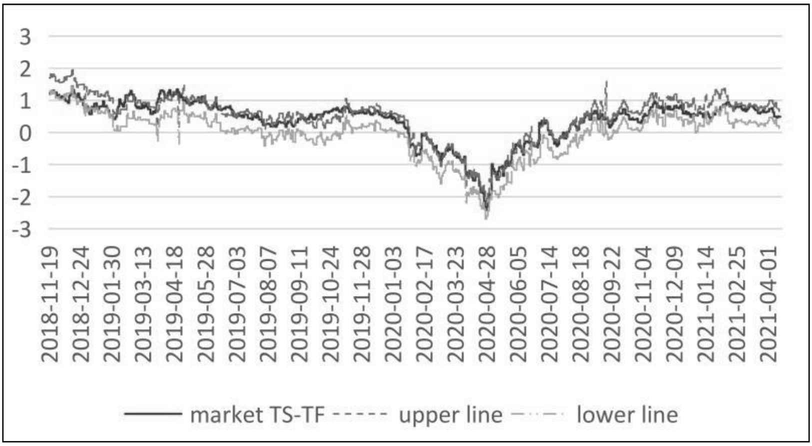


Figure 4a. Spread Curve for Two-Year and Five-Year Treasury Futures

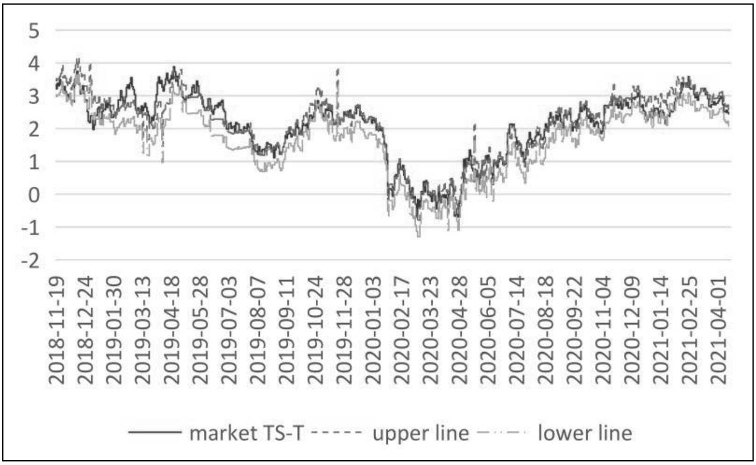


Figure 4b. Spread Curve for Two-Year and Ten-Year Treasury Futures

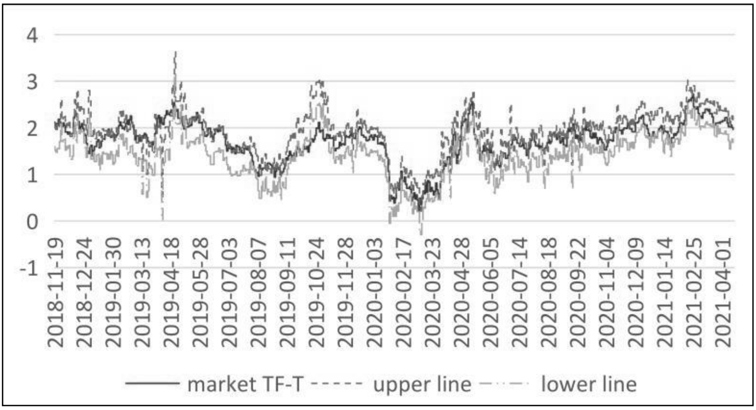


Figure 4c. Spread Curve for Five-Year and Ten-Year Treasury Futures

Note(s): Figures 4a, 4b, and 4c separately show the trends for market spread and relevant arbitrage-free spread bounds for two- and five-year Treasury futures, for two- and ten-year Treasury futures, and for five- and ten-year Treasury futures.

Source(s): The original data comes from wind financial terminal, and the chart data is processed.

Figure 4: Market Spread Curve and Relevant Arbitrage-Free Spread Bounds

On the whole, the trend in the arbitrage-free spread bounds is in line with that of the market spread, and the spread is generally positive. In some periods, the spread is greatly narrowed, even leading to a reversed price relationship, mainly because of the vast impact of the pandemic on the macroeconomy and the financial market, which caused big changes in issuance, Treasury bond transactions, and market expectations of interest rates. This effect began to emerge in February 2020 and diminished around June 2020, which generally coincided with the outbreak of the pandemic in China and the time when it was generally seen as under control there. Compared to the other two curves, the spread curve of five- and ten-year Treasury futures is smoother and steadier and less affected by the pandemic. Figure 4c has a smaller spread, narrower range, and shorter duration of effects. The main reason is that the pandemic affected short-term interest rates and Treasury bond supply but had a smaller impact on interest rates for medium- and long-term Treasury bonds. Although the spread narrowed greatly, the arbitrage-free spread interval set by the model incorporated the relevant information and predicted the same trend, and the arbitrage strategy remained effective as long as arbitrage opportunities existed. The trend chart illustrates that the market spread was not always in the arbitrage-free spread interval, and it usually approaches the upper limit and then exceeds it.

2. *Analysis of the Extent of Deviation*

We calculate the amount of deviation and depict it in a bar chart (see Figure 5). Descriptive statistics of the deviation are in Table 5.

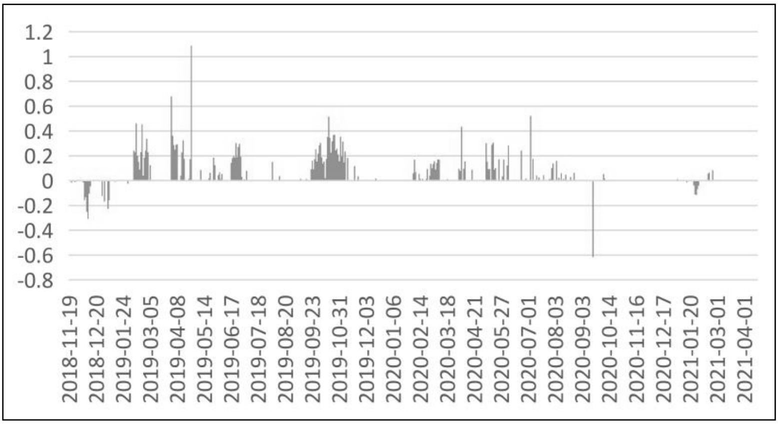


Figure 5a. Spread Deviation between Two-Year and Five-Year Treasury Futures

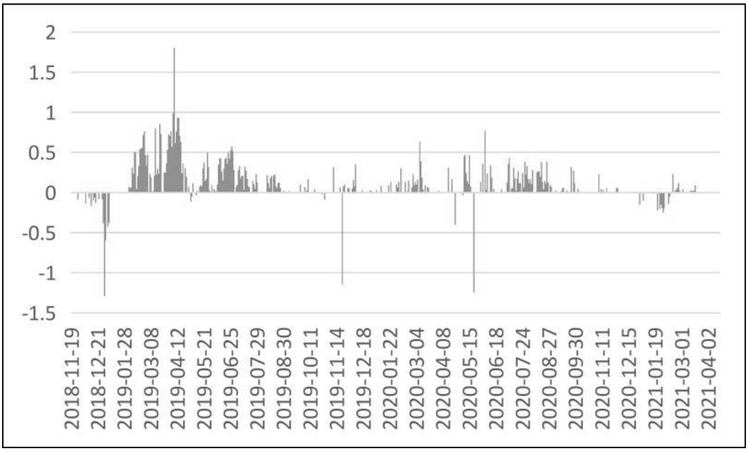


Figure 5b. Spread Deviation between Two-Year and Ten-Year Treasury Futures

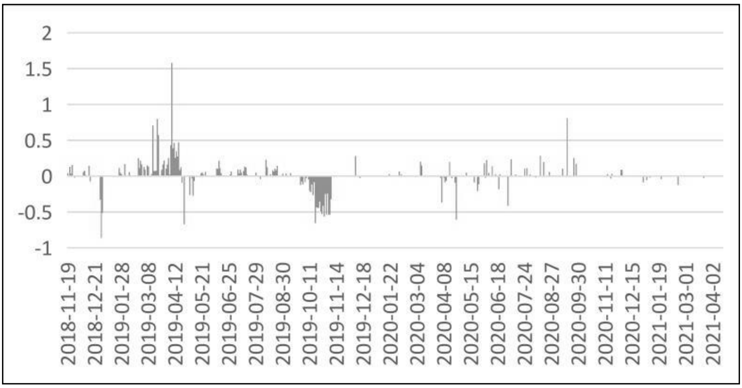


Figure 5c. Spread Deviation between Five-Year and Ten-Year Treasury Futures

Note(s): Figures 5a, 5b and 5c separately show the extent of market spread deviation from arbitrage-free spread intervals for two- and five-year Treasury futures, of two- and ten-year Treasury futures, and of five- and ten-year Treasury futures.

Source(s): The original data comes from wind financial terminal, and the chart data is processed.

Figure 5: Spread Deviation

Table 5

Descriptive Statistics for Deviation

	TF-T	TS-TF	TS-T
Num	168	164	268
Mean	0.016055	0.126465	0.165274
Std	0.075485	0.030629	0.086112
Median	0.038053	0.118778	0.122498
Maximum	1.576785	1.084795	1.802819
Minimum	−0.860063	−0.614224	−1.289057

Note(s): This table shows the descriptive statistics for deviation, which represents the extent of market spread deviation from an arbitrage-free spread interval. We define the deviation as the portion of the arbitrage-free spread interval in which the market spread exceeds the upper or lower limit. (Equation 15)

Source(s): The original data comes from wind financial terminal, and the table data is processed.

Figure 5 illustrates that arbitrage opportunities mainly appear in the first half of the time window with a clustered distribution, perhaps because the period since the launching of the Treasury futures is short, and a relatively stable equilibrium has not been established. Fewer arbitrage opportunities emerge in the second half of the time window, and they are more scattered. The market spread deviation for two- and five-year Treasury futures is mostly between [−0.5, 0.5], with only three samples exceeding the interval and only one exceeding ±1. The market spread deviation for two-year and ten-year Treasury futures is mostly between [−0.7, 0.7], larger than the main interval between two- and five-year Treasury futures, and four samples have a deviation exceeding ±1, which means that the price equilibrium between two- and ten-year Treasury futures is weaker. Ten-year Treasury futures were launched later than five-year Treasury futures, which could explain the greater deviation. In the latter portion of the time window, the deviation is smaller and more scattered, which means that the market effectiveness increased, the inter-variety market equilibrium increased, and the arbitrage opportunities gradually diminished. The market spread deviation is smaller and the arbitrage opportunities are more scattered for five- and ten-year Treasury futures than for the other two spread relationships because of they were launched earlier. The deviation is mostly between [−0.5, 0.5], and only one sample has deviation that exceeds ±1. In sum, the market spread is not always within the arbitrage-free interval, short-term arbitrage opportunities exist and then gradually decrease with an increase in the duration of market operation, which makes the market more efficient.

3. *Analysis of the Speed of Reversion to Equilibrium*

The statistics on speed and frequency of reversion to equilibrium are in Table 6. Out of a total of 587 samples, there are 268 days (45.7 %) on which the market spread of two- and ten-year Treasury futures exceeds the arbitrage-free interval calculated by spot Treasury bonds, a larger proportion than for two- and five-year Treasury futures. There are a total of 88 arbitrage opportunities that samples exceed the arbitrage-free interval bounds and then revert to equilibrium. The proportion of arbitrage opportunities disappearing within five days is close to 86 %. There are 164 days (27.9 %) on which the market spread of two- and five-year Treasury futures exceeds the arbitrage-free interval. There are 65 arbitrage opportunities that the samples exceed the arbitrage-free interval bounds and then revert to equilibrium, nearly 90 % of them last no more than five days. There are 167 days (28.4 %) on which the market spread of five- and ten-year Treasury futures exceeds the arbitrage-free interval. There are 77 arbitrage opportunities and more than 95 % of them disappear within five days. All three spreads indicate that arbitrage opportunities exist for a short time, so dis-

Table 6

Speed of Reversion to Equilibrium and Frequency Statistics for the Market Spread

Days to return equilibrium (Speed of reversion to equilibrium)	*TS-TF*		*TS-T*		*TF-T*	
	Frq.	*Proportion (%)*	*Frq.*	*Proportion (%)*	*Frq.*	*Proportion (%)*
1	40	61.5	47	53.4	45	58.4
2	10	15.4	15	17	13	16.9
3	3	4.6	6	6.8	11	14.3
4	5	7.7	3	3.4	3	3.9
5	1	1.54	5	5.7	2	2.6
6	1	1.54	1	1.1	0	0
7	1	1.54	1	1.1	1	1.3
8	0	0	2	2.3	0	0
9	1	1.54	0	0	0	0
10 or more days	3	4.64	8	9.2	2	2.6
Total	65	100	88	100	77	100

Note(s): This table shows the speed of reversion to equilibrium and frequency statistics for the market spread. When the market spread reverts quickly to the interval from breakthrough spread bounds, that indicates a more efficient market. Frequency indicates how many times the arbitrage opportunity has occurred. We define the speed of reversion to equilibrium as the time from the breakthrough spread bounds to reversion to the interval, measured in days.

Source(s): The original data comes from wind financial terminal, and the table data is processed.

equilibrium in the market for Treasury bond futures does not last very long. The proportion of arbitrage opportunity that exists for no more than 5 days increased in turn(85 % – 90 % – 95 %), which is related to the launch of five-, ten- and two-year Treasury bond futures in turn, which proves again that as the time that the market is operational grows longer, its efficiency rises, and arbitrage opportunities disappear more quickly.

4. *Comparison of the Three Spreads*

The statistics of the relevant data on deviation in the three spreads is shown in Table 7. The spread between five- and ten-year Treasury futures is Δ_{FT}, the spread between two- and five-year Treasury futures is Δ_{SF}, and the spread between two- and ten-year Treasury futures is Δ_{ST}.

Table 7

Statistical Comparison on Deviation

	Δ_{FT}	Δ_{SF}	Δ_{ST}
Number of deviation sample	168	164	268
Percentage of deviation sample	28.4 %	27.9 %	45.7 %
Number of sample deviating from upper limit	107	143	234
Mean deviation	0.016055	0.126465	0.165274
Variance in deviation	0.075485	0.030629	0.086112
Number of arbitrage opportunities	77	65	88
Mean speed of reversion to equilibrium	2.18	2.52	3.07
Proportion of arbitrage opportunities lasting more than 10 days	2.60 %	4.64 %	9.20 %

Note(s): This table shows the statistics for relevant data on the deviation between the three spreads. The spread between five- and ten-year Treasury futures is Δ_{FT}, the spread between two- and five-year Treasury futures is Δ_{SF}, and the spread between two- and ten-year Treasury futures is Δ_{ST}. We define the speed of reversion to equilibrium as the time from the breakthrough spread bounds to reversion to the interval, measured in days.

Source(s): The original data comes from wind financial terminal, and the table data is processed.

A comparison of the number of deviation sample, $\Delta_{SF} < \Delta_{FT} < \Delta_{ST}$, indicates that Δ_{SF} has the fewest days in which it deviates from arbitrage-free spread interval, close to Δ_{FT} but more days on which it deviates from the upper limit. A comparison of the mean deviation, $\Delta_{FT} < \Delta_{SF} < \Delta_{ST}$, shows that Δ_{FT} has the least deviation from the arbitrage-free spread interval. A comparison of the variance in deviation, $\Delta_{SF} < \Delta_{FT} < \Delta_{ST}$, demonstrates that Δ_{SF} has the least fluctuation in deviation. And a comparison of the proportion of arbitrage opportunities lasting more than 10 days, $\Delta_{FT} < \Delta_{SF} < \Delta_{ST}$, shows that Δ_{FT} usually deviates from the arbitrage-free spread interval for a shorter duration before returning to equilibrium, and the arbitrage opportunities disappear sooner. These comparisons demonstrate that Δ_{FT} has a stabler inter-variety price equilibrium, with higher market efficiency, Δ_{SF} is in second place, and Δ_{ST} is last. This order is related to the duration of operations in the three markets: longer time in operation leads to greater development of the market mechanism, higher market efficiency, and a stabler price equilibrium.

VII. Conclusion

In this paper, we build models for predicting spot yields and the arbitrage-free equilibrium spread. By testing the pairwise spread relationship between three varieties of Treasury futures, we find that many arbitrage opportunities arise in trading, the inter-variety market of Treasury futures is not in equilibrium, and market efficiency needs to increase, which means that China's Treasury bond futures market offers opportunities for obtaining risk-free returns – an attractive discovery for investors. In addition, we calculate and analyse the extent of deviation and the speed of reversion to equilibrium between the market spread and the arbitrage-free equilibrium spread interval, by comparing the spread relationships of three groups. This comparison demonstrates that the longer the Treasury futures market remains in operation, the more efficient the market will be. The reason might be that when a market is more developed, traders in it are more familiar with the products and trading rules, which increases the number and stability of traders, creating more stable market liquidity. China's financial market should be more open, allowing foreign arbitrageurs to participate in the Treasury bond futures market more easily, which will push price changes that make market spreads return to theoretical spreads more quickly and enable the market to return to equilibrium. This could help to raise efficiency in China's Treasury bond futures market.

Our study makes both academic and practical contributions. Academically, we offer a new perspective for research on equilibrium among different varieties of Treasury bond futures, based on no-arbitrage pricing, rather than the cointegration method, as commonly used in existing research. Using the equilibrium relationship based on the pricing method is more reliable than that based on statistical methods. Practically, we expand research to encompass all the varieties of Treasury futures available in the Chinese market, including the newly launched two-year Treasury futures, so that we can gain a more comprehensive understanding of the characteristics of this market. We find that China's Treasury bond futures market has a lot of arbitrage opportunities, which makes it very attractive to investors. At the same time, we make suggestions on how to increase the efficiency of the market.

In this paper, we study the equilibrium relationship between the three varieties of Treasury futures by their daily closing quotation. Intra-day trading of Treasury bonds and futures has become very active, therefore, a direction for further research to build on our study would be the use of intra-day high-frequency data.

References

Butterworth, D./*Holmes,* P. (2005): The Hedging Effectiveness of UK Stock Index Futures Contracts Using an Extended Mean Gini Approach: Evidence for the FTSE 100 and FTSE Mid250 Contracts, Multinational Finance Journal, Vol. 9(3/4), 131 – 160.

Chen, J. M./*Yang*, J. F. (2014): Empirical Study on Futures-Spot Arbitrage of China's Treasury Futures at Present Stage, Zhejiang Finance, Vol. 3, 48 – 52.

Chen, J. Y. (2020): Empirical Study on Futures-Spot Arbitrage and Hedging Strategies of Treasury Futures, Modern Business, Vol. 25, 138 – 139.

Chen, R./*Ge*, J. (2015): On the Pricing of Treasury Bond Futures: Principles and a Literature Review, Journal of Xiamen University (Arts & Social Sciences),Vol. 1, 33 – 40.

Cornell, B./ *French*, K. R. (1983): The pricing of stock index futures, Journal of Futures Markets (pre-1986),Vol. 3(1), 1 – 14.

Cox, J. C./*Ingersoll*, J. E., Jr./*Ross*, S. A. (1981): A re-examination of traditional hypotheses about the term structure of interest rates, Journal of Finance, Vol. 36(4), 769 – 799.

Cui, L./*Huang*, K./*Cai*, H. J. (2015): Application of a TGARCH-wavelet neural network to arbitrage trading in the metal futures market in China, Quantitative Finance, Vol. 15(2), 371 – 384.

Elton, E. J./*Gruber*, M. J./*Rentzler*, J. (1984): Intra-day tests of the efficiency of the Treasury bill futures market, Review of Economics and Statistics, 129 – 137.

Emery, G. W./*Liu*, Q. (2002): An analysis of the relationship between electricity and natural-gas futures prices, Journal of Futures Markets: Futures, Options, and Other Derivative Products, Vol. 22(2), 95 – 122.

Fang, Y. X./*Zheng*, X. (2014): Study on arbitrage strategies based on China's Treasury futures restart data, Price: Theory & Practice, Vol. 11, 83 – 85.

Figlewski, S. (1984): Hedging performance and basis risk in stock index futures, Journal of Finance, Vol. 39(3), 657 – 669.

Haigh, M. S./*Holt*, M T. (2002): Crack spread hedging: accounting for time-varying volatility spillovers in the energy futures markets, Journal of Applied Econometrics, Vol. 17(3), 269 – 289.

Hemler, M. L. (1990): The quality delivery option in Treasury bond futures contracts, Journal of Finance, Vol. 45(5), 1565 – 1586.

Hou, Y. H. (2018): Equilibrium relationship and arbitrage opportunity between five-year and ten-year treasury bond futures in China. Southwestern University of Finance and Economics.

Jiang, R. J./*Wang*, L. M. (2020): Empirical analysis on statistical arbitrage based on China's different futures markets, Statistics & Decision, Vol. 36(3), 131 – 135.

Kim, A. (2015): Does futures speculation destabilize commodity markets?, Journal of Futures Markets, Vol. 35(8), 696 – 714.

Kolb, R. W./*Chiang*, R. (1982): Duration, immunization, and hedging with interest rate futures, Journal of Financial Research, Vol. 5(2), 161 – 170.

Li, J. (2019): Empirical study on futures-spot arbitrage opportunities of China's Treasury futures, Fujian Quality Management, Vol. 15, 11 – 111.

Li, S./*Bao*, Y./*Peng*, C./*Zhao*, Y. L. (2019): Pricing of government bond futures embedded quality option and timing option, Journal of Systems Science and Mathematical Sciences, Vol. 3, 341 – 352.

Liu, C./*Wang*, F./*Liu*, X. D./*Liu*, L. (2015): Study on arbitrage, hedging and investment strategies of Treasury futures, Communication of Finance and Accounting, Vol. 27, 11 – 113.

Liu, W. W. (2019): Research on the cross-breed arbitrage strategy of Treasury bonds, Shanxi University of Finance and Economics.

Luo, X. L./*Li*, Y. Z./*Rao*, H. H. (2003): The application of carry-cost theory to the pricing of financial futures, Journal of Central South University (Social Science Edition), Vol. 9(6), 784 – 787.

Miao, Y./*Zhu*, J. M. (2019): Empirical study of arbitrage strategy on Hushen 300 Index futures base on GARCH, Journal of Beijing Institute of Graphic Communication, Vol. 27(5), 77 – 80.

Mitchell, J. B. (2007): Soybean crush spread arbitrage: Trading strategies and market efficiency. Available at SSRN 987507.

Peng, J. F. (2010): Research about statistical arbitrage on futures: Based on soybean, soybean meal and soybean oil futures, Shandong University.

Poole, W. (1978): Using T-bill futures to gauge interest rate expectations, Economic Review, (Spr), 7 – 19.

Qin, J. J. (2014): Empirical study of pricing and arbitrage in Treasury bond future market, Shanghai Jiao Tong University.

Rendleman, R. J., Jr./*Carabini*, C. E. (1979): The efficiency of the Treasury bill futures market, Journal of Finance, Vol. 34(4), 895 – 914.

Simon, D. P. (1999): The soybean crush spread: Empirical evidence and trading strategies, Journal of Futures Markets: Futures, Options, and Other Derivative Products, Vol. 19(3), 271 – 289.

Sun, Z. Y. (2021): Research on the strategy of cross variety arbitrage of Treasury bond futures based on AR-GARCH model, Northwest Normal University.

Tse, Y. K. (1995): Lead-lag relationship between spot index and futures price of the Nikkei stock average, Journal of Forecasting,Vol. 14(7), 553 – 563.

Tse, Y. (1999): Price discovery and volatility spillovers in the DJIA index and futures markets, Journal of Futures Markets. Vol. 19(8), 911 – 930.

Wahab, M./*Cohn*, R./*Lashgari*, M. (1994): The gold-silver spread: Integration, cointegration, predictability, and ex-ante arbitrage, Journal of Futures Markets, Vol. 14(6), 709 – 756.

Wang, J. Z./*Hu*, X. F. (2015): Research on the effectiveness of Chinese Treasury bond futures market, Economic Review, Vol. 6, 55 – 68.

Wang, L./*Feng*, Q. N. (2015): The interest rate liberalization, the price-discovery and risk-aversion functions of Treasury bond future, Finance Forum, Vol. 20(4), 36 – 45.

Wang, W./*Yao*, Y. (2015): The application of Treasury futures in commercial bank duration gap management., Shanghai Finance, Vol. 4, 91 – 95.

Wang, Z. (2011): Chinese metal futures arbitrage and risk management research: A case study of SHFE copper and zinc, Zhejiang University.

Wu, J. T. (2017): Research on arbitrage strategy of Treasury bonds based on high frequency data, Shanxi University of Finance and Economics.

Yang, Y. J./*Chen*, S. C. (2018): Study on China's Treasury bond futures spread based on high-frequency data, Accounting and Finance, Vol. 2, 1 – 6.

Yin, X. M./*Sun*, T./*Xu*, Z. D. (2008): Feasibility study on cross-commodity arbitrage between palm oil and soybean oil futures on the basis of high frequency data, Rural Economy and Science-Technology, Vol. 8, 84 – 86.

Zhang, J. F./*Tang*, Y. W./*Gang*, J. H./*Fan*, L. L. (2019): Price discovery in China's interest rate markets: Evidence from the Treasury spot, futures, and interest rate swaps markets, Journal of Financial Research, Vol. 1, 19 – 34.

Zhang, J. W. (2020): Empirical study on cross-variety arbitrage strategy of commodity futures: Based on iron ore and coke, Modern Business, Vol. 28, 90 – 92.

Zhou, L. (2017): Study on commodity futures-spot arbitrage based on GARCH model, Journal of Jilin Business and Technology College, Vol. 33(6), 78 – 84.

Zhu, L. R./*Su*, X./*Zhou*, Y. (2015): Empirical study of statistical arbitrage in Chinese future market, Journal of Applied Statistics and Management, Vol. 34(4), 730 – 740.

Zou, Z. Y./*Chen*, H. C./*Li*, X. (2019): An empirical study on the asymmetry of soybean futures price fluctuation, Journal of Shaoguan University, Vol. 40(1), 88 – 93.