Credit and Capital Markets, 47. Jahrgang, Heft 2, Seiten 265–296
Abhandlungen

Austerity Programs under Liquidity Constraints: Stylized Facts of Recession in the Euro Area

Nicos Christodoulakis*

Abstract

Employing a stochastic Brownian pattern for labor income under liquidity constraints, the paper derives closed-form solutions for households' consumption and shows how both the multiplier and the variability of consumption increased after the global crisis and were further enhanced by the austerity programs. The properties of the theoretical model are found to be in line with a number of stylized facts observed in the Southern Euro Area countries, where recession is ravaging for three years after the implementation of austerity programs. A critical factor of such a prolonged recession was that austerity programs were at the same time too harsh and intensive, grossly underestimating the adverse effects on economic activity. The paper argues that the same fiscal adjustment could have been achieved more gradually and with milder effects on households' demand.

Austeritätsprogramme bei Liquiditätsengpässen: Stilisierte Rezessionsfakten in der Eurozone

Zusammenfassung

Unter Anwendung eines Modells für Arbeitseinkommen bei Liquiditätsengpässen der Brown-University leitet diese Arbeit geschlossene Rechenmodelle für die Konsumausgaben des Staates ab und zeigt, wie sowohl der Multiplikator als auch die Variabilität der Konsumausgaben des Staates nach der globalen Krise zugenommen haben und weiter durch Austeritätsprogramme verstärkt worden sind. Die Eigenschaften des theoretischen Modells befinden sich in Übereinstimmung mit einer Reihe von stilisierten Fakten, welche in den südlichen Ländern der Eu-

---

* Acknowledgement: The author is thankful to an anonymous referee and to S. Hajjouz from the University of Warwick for valuable suggestions on an earlier draft. Also participants in the Symposium for the Advancement of Economic Theory in Paris and in seminars held at the Universities of Piraeus and Patras offered useful comments. The usual disclaimer applies.

Corresponding author Nicos Christodoulakis, Athens University of Economics & Business (AUEB), and Hellenic Observatory LSE. 76, Patission street, Athens 10434, Email nchris@aueb.gr

rozone zu beobachten sind, die während eines Zeitraums von drei Jahren nach Durchführung von Austeritätsprogrammen von Rezession geplagt werden. Ein kritischer Faktor einer solchen andauernden Rezession ist darin zu sehen, dass Austeritätsprogramme gleichzeitig zu rigide und zu intensiv waren und dass die schädlichen Auswirkungen auf die konjunkturelle Entwicklung grob unterschätzt worden sind. In dieser Arbeit wird dargestellt, dass dieselbe fiskalische Anpassung durch eine stärker gestufte Konsumnachfrage des Staates bei milderen Auswirkungen hätte erzielt werden können.

Keywords: Debt, Fiscal Policy, liquidity constraints

JEL Classification: H60, H61

I. Introduction

Among many economic tenets that were being questioned after the global crisis of 2008 and are now reconsidered is the long held view of consumption smoothing. According to it, rational households make their consumption and savings decisions by relying on the notion of permanent income rather than responding to temporary fluctuations of their current remuneration. Even in turbulent times when fluctuations of income are strong and persisting, the effect on consumption is – by the same argument – assumed to be dampened away without permanently shifting the long run pattern. Hence it is perhaps surprising to see large differentiations in the way that economies with more or less similar institutional characteristics and a common monetary policy, such as those in the Euro Area, responded after the 2008 crisis against the predictions of the smoothing hypothesis.

Let's start with the facts. The variability of income and consumption in the Euro Area economies is measured as the average standard deviation of GDP and consumption growth respectively over two equal timespans, namely the period 2009–2012 to account for the post-crisis affects and the period 2004–2007 as the pre-crisis benchmark. To avoid idiosyncratic aspects related with the adjustment period of entering the Economic and Monetary Union (EMU), the analysis leaves out the later participants and proceeds with the early members. The remaining eleven countries[1] are classified into two groups: the 'Northern' group includes the economies of Austria, Belgium, Germany, France, Finland and the Netherlands that in the wake of the crisis have suffered less, and the

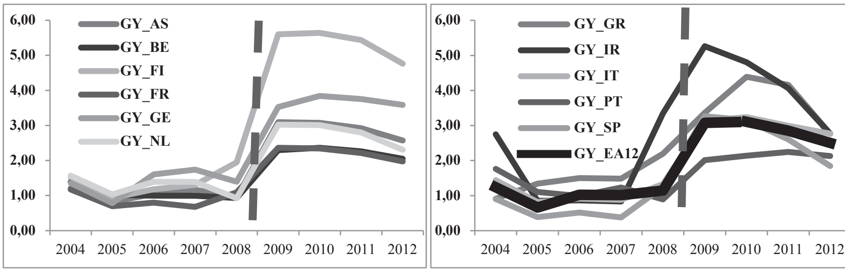
[1] Luxemburg is also left out as an outlier. The main reason is the excessive size of its banking sector relative to population and GDP and that could turn several of the arguments used in the present analysis to look irrelevant. In any case, the average of the initial twelve countries of the Euro Area that also includes Luxemburg is shown in the panels of Fig. 1 as EA12.

'Southern' group consisted of Greece, Ireland, Italy, Portugal and Spain. The latter group – for brevity called GIIPS[2] – was entangled more severely in the debt crisis and underwent substantial fiscal adjustment from 2010 to the present.

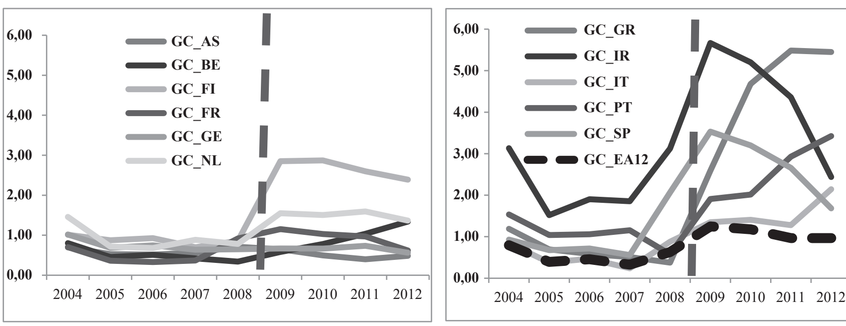
As shown in the top panel of Fig.1, the two groups seem to have experienced almost the same variability of GDP growth before the crisis and were hit by the same degree of output fluctuations afterwards. But the pattern of consumption fluctuations shown in the central panel appears to be vastly different. Before the crisis, both groups seemed to behave in accordance with the smoothing hypothesis and the variability of consumption growth was lower than that of income, though marginally so for the 'southern' group. In the aftermath of the crisis, the hypothesis still seems to hold in the 'northern' part with consumption variability remaining at about a third of that of income, while in the 'southern' part it matched the variability of income. This finding contradicts the prevailing view, originally described by *Muth* (1960), that an income process with a large transitory component would imply a small propensity to consume out of current income and, therefore, consumption variations should have been small. The only case of departing from consumption smoothing could occur if households were either departing from forward-looking rationality or confusing the nature of shocks as permanent; see for example *Romer* (1996, p 315).

A plausible explanation of the vast differences in consumption fluctuations may be found in the way fiscal policy was conducted in order to face credit shortages and the surge of indebtedness in some countries. As shown in the bottom panel of Fig. 1, public debt to GDP ratios were a lot more disturbed in the 'southern' group and, consequently, fiscal corrections were imposed to an extent much larger than in the Euro Area core. The degree of fiscal activism can be measured by the variability of primary balances, and in Table 1 this appears to have risen four times in the 'southern' group as opposed to 0.71 times in the north, relative to the pre-crisis level. Income reductions were effected either by tax hikes in order to raise revenues or by economy-wide wage cuts to improve competitiveness. However, it is noticeable that the variability of tax revenues is substantially higher in the 'southern' group, suggesting that fiscal correction in these economies relied on tax hikes relatively more so than in the 'northern' one.

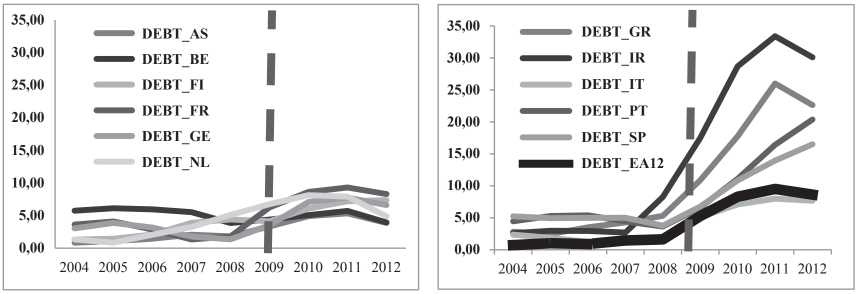
[2] Sometimes they are collectively referred to by other acronyms with a negative connotation. The terms 'northern' and 'southern' initially were meant to divide the Euro Area by geography, but the crisis in Ireland broadened the definition.



(a) Variability of GDP Growth Rate



(b) Variability of Consumption Growth Rate



(c) Variability of Debt to GDP Ratio

Source: Ameco.

Figure 1: Standard Deviations Calculated on a Moving 5-Year Period

At the same time consumption plans were also severed by the enhancement of liquidity constraints either as a result of bank failures or due to the increased stringency in credit provision by the European Central Bank to the financial institutions in the 'southern' countries. As demand was sinking and credit curtailed, unemployment rose and income losses were further multiplying. The possibility that the increased consumption

Table 1

Five-Year Moving Averages of Standard Deviations Across Countries of the 'Northern' and 'Southern' Parts of the Euro Area

	Before the crisis 2004–2007		*After the crisis 2009–2012*		*South vs. North*
Variable	*'North'*	*'South'*	*'North'*	*'South'*	
GDP growth	1.16	1.09	3.20	3.16	similar
Forecasted GDP growth rate for 2013–2018			1.316	1.291	similar
Consumption Growth	0.69	1.05	1.20	3.17	*higher*
Public Debt to GDP	2.86	3.54	6.10	15.86	*higher*
Primary Balance to GDP	1.41	1.16	2.42	4.65	*higher*
Public Spending to GDP	0.82	0.82	2.19	2.60	*higher*
Tax Revenues to GDP	0.62	0.90	0.50	1.15	*higher*

Note: No country weighting. The expected growth rate is the simple average of the IMF forecasts for the period 2013–2018.

Source: AMECO 2000–2012. Growth forecasts from IMF WEO Database, 2013.

variability in the GIIPS does reflect some permanent shifts in underlying incomes was systematically disregarded by the main policy predictions; for example, the IMF forecasts for GDP growth in the period 2013–2018 are virtually the same for the two groups; see Table 1. Somehow, the expectation was that recessionary effects would be both brief and small without leaving any permanent trace. Such unwarranted optimism was relying on two key hypotheses: First that debt reduction would always exert a powerful positive impact on activity, no matter the type of shock impinging upon the economy. Various studies by – among others – *Reinhart/Rogoff* (2010), *Kumar/Woo* (2010), *Cecchetti* et al (2011) suggested that if debt is beyond a range of around 90 % to GDP, Governments should be "… in favour of swiftly implementing ambitious strategies for debt reduction" as explicitly advised by *Checherita/Rother* (2010).

Any fear that such a rapid consolidation in economies already engulfed in the global crisis might entrap them further into deflation was mitigated by a second convenient hypothesis according to which fiscal multipliers are too low for recession to reach threatening levels. An official report by IMF predicted that a fiscal correction by 1 % of GDP would reduce output by only 0.50 % and raise unemployment by a mere 0.30 %; see *WEO* (2010, p 94). As the growth potential would be unleashed by market reforms, such a small recession was expected to bottom-out in late 2010 and rebound afterwards; (*ibid*, p 165).

The above assumptions acted as a soothing device to convince both the public opinion in the stressed countries in accepting the austerity measures as well as the decision-making bodies in the lending institutions in approving the bail-out packages. But real life households quickly discovered that liquidity constraints are truly binding, thus disregarding income fluctuations in planning their consumption is not an option available anymore. As a matter of fact, all prognostications[3] of small contractionary effects faltered, shaking not only the social fabric but the consumption smoothing literature as well. It was only then that the link between the intensity of fiscal consolidation and the recessionary impact started to be noticed.

However, the issue was not a novelty and early warnings on the severity of consumption volatility are plenty in the literature. For example, *Hall/Mishkin* (1982) had already noted the *"excess sensitivity"* of consumption to changes in transitory income, implying that even if some households perceive losses due to austerity cuts as temporary their impact on consumption may be vigorous. A likely explanation is that in the aftermath of the crisis, several households were liquidity-constrained as the value of real-estate was collapsing and banks were curtailing consumer loans. These changes may be responsible for large variations in consumption even by forward-looking households faced with predictable changes in income; for a discussion see *Chah* et al (1995).

The combination of income cuts and liquidity constraints is well-known to trigger wild fluctuations in consumption and output. *Zeldes* (1989) had pointed that the existence of liquidity constraints make "… the marginal propensity to consume out of transitory changes in income to be much larger for households holding few assets relative to future income than for the rest of the population". Similarly, *Deaton* (1991) had argued that under uncertainty the propensity to consume, and consequently the multiplier, rises since "… the combination of the persistence of the random walk and the binding liquidity constraints precludes the accumulation of assets".

[3] For an account of forecasting errors by the IMF and the European Union see *Christodoulakis* (2013).

A suitable framework to examine the effects of austerity programs under uncertainty would be to model households' income in a stochastic pattern affected by liquidity constraints. By doing so, the present paper provides closed-form solutions for the income multiplier under uncertainty and then its properties are examined. The multiplier is found to increase considerably after the crisis and even more so when austerity measures are imposed. The findings seem to be in line with a number of stylized facts regarding the severity of fluctuations and the extent of recession prevailing in the GIIPS economies.

The rest of the paper is organized as follows: Section 2 briefly reviews the literature on optimal debt and argues why predictions may fail if liquidity constraints are ignored. Section 3 describes a model of optimizing households with stochastic income affected by austerity measures and subject to liquidity constraints. A number of stylized facts concerning the impact of austerity in the GIIPS are established in Section 4. A higher level of consumption is shown to be achieved by measures extended to the medium term rather than a front loaded program and this is taken to imply that a more gradual adjustment could have had exerted a milder effect on recession. Conclusions are summarized in Section 5.

II. Public Debt and Multipliers in the Time of Crisis

The literature of growth-inducing fiscal consolidation thrived in the early 1990s as European economies were cutting public debts and deficits in their way toward the Economic and Monetary Union (EMU).[4] If successful, fiscal consolidation would raise the prospects of joining the EMU and – as a result – usher in a period of stability, increased capital flows and low interest rates. The assumption was that international markets would appreciate the determination of Governments to swiftly redress their deficits and award them with renewed confidence and creditworthiness. Thus, redressing public finances would have had both beneficial supply-side effects and demand-augmenting consequences that could spur growth and, by doing so, further facilitate adjustment.

Reinhart/Rogoff (2010) set to prove that the virtues of rapid fiscal consolidation survive just as well in a period of falling demand and capacity

[4] Among many others, see *Giavazzi/Pagano* (1990 and 1996), *Alesina/Perotti* (1997), *Alesina/Ardagna* (1998). For the case of Greece in particular, see *Christodoulakis* (1994).

slackness by looking for simple correlations between growth rates and indebtedness over the period 1946–2009. Although such tests are at best only indicative and in any case meaningful for the long run, the authors reach the conclusion that debt/GDP levels above *90%* are associated with lower growth outcomes. The implication is that in such a case a fiscal correction is due whatever the economic cycle. *Kumar/Woo* (2010) established that a 10 percentage point increase in the debt-to-GDP ratio is associated with a growth slowdown of around 0.2 percentage points per year, and the impact gets stronger if a debt threshold of around 90% is exceeded. In the same line, *Cecchetti* et al (2011) investigated 18 OECD countries from 1980 to 2010 and concluded that beyond a level of around 85% of GDP, public debt is a drag on growth.[5]

Assuming an *ad hoc* non-linear framework, *Checherita/Rother* (2010) set to examine the fiscal impact on growth in the 12-member states of the Euro area over the period 1971–2008 by estimating the following quadratic debt equation for the annual growth rate of per capita GDP:

$$\text{growth}(t) = \psi_0 + \psi_1 \cdot y_{t-L} + \psi_2 \cdot h_{t-1} - \frac{1}{2}\psi_3 \cdot h_{t-1}^2 \qquad (1)$$

where $y(t-L)$ is lagged per capita output in logs to account for catching-up effects, and $h(t)$ the public debt to GDP ratio. With positive parameter values, equation (1) implies that growth is maximized when the debt-to-GDP ratio reaches a level of $h^* = \psi 2/\psi 3$. Using data prior to 2007, *Baum* et al (2012) estimated this level to be 66.4% of GDP, while by including data for up to 2010 the threshold is raised to 95.6% of GDP.

Since all the above approaches were empirical, *Checherita* et al (2012) used an endogenous growth model and determine the growth-maximising debt to GDP ratio as a function of the output elasticity of public capital in the production function. For the Euro Area, the optimal level is found to be around 50% of GDP, implying that growth-enhancing fiscal adjustment should have had been stronger by a multiple.[6]

[5] Such views did not remain entirely unchallenged. *Égert* (2012) found that the negative relationship between debt and growth is rendered insignificant for several countries, while others disputed the calculations by *Reinhart/Rogoff* (2010) causing a prolonged academic uproar; for a discussion see, for example, The Economist website: http://www.economist.com/blogs/freeexchange/2013/04/debt-and-growth.

[6] The optimal level of debt 50% of GDP is consistent with an output elasticity of public capital in the Euro Area equal to $\alpha = 0.236$. However, if the output elas-

But reality did not reward such hypotheses. With the economies of GIIPS trapped in a deep recession and the Euro Area as a whole consistently revising its growth prospects downwards, the above claims were hard to maintain. Attention shifted on how adjustment programs affect multipliers, thus further fuelling fluctuations in the economy rather than dampening them. *Cottarelli/Jaramillio* (2012) argued that prescriptions of swift consolidation "… ignore that fiscal multipliers should be larger when output is below potential", thus excessive fiscal zeal might have serious contractionary effects.

This was confirmed by *Blanchard/Leigh* (2013) who found that fiscal multipliers are more likely to be in the range of 0.90 to 1.70 rather than around 0.50 as formerly assumed by the IMF. Thus austerity programs applied in the wake of the debt crisis more likely have had hindered growth rather than boosting it. Wiser after the event, an official study for the Euro area underlined that "… fiscal multipliers are higher now *(i.e. during the crisis)* than they would be in normal circumstances"; see *European Economy* (2012, Box I.5, pp 42–43, emphasis added). Finally, an IMF assessment on 28 economies found that the stronger the correction in the fiscal stance the wider the error in predicting GDP growth; see *WEO* (2012, Box 1.1). The reasons why multipliers were so strongly influenced by the uncertainty that prevailed after the global crisis and then further enhanced by the fiscal consolidation programs are elaborated in the next section.

III. Consumption under Fiscal Austerity and Credit Constraints

In this section the effect of austerity policies on consumption is examined in a framework more suitable to reflect the post-crisis realities. To capture the unusual uncertainty and losses that incurred on incomes, households are assumed to optimize an intertemporal utility function with an income subject to stochastic shocks and impaired by credit constraints.

Due to the complexity of stochastic calculus, most solutions to this problem are obtained numerically and this hinders an analytic examination of how some key properties of the consumption function are affected. For example, *Seater* (1996) showed that optimal consumption under liquidity constraints still satisfies the standard Euler equation as

ticity is set equal to $\alpha = 0.39$ as estimated by *Aschauer* (1989) for the US economy, the optimal debt becomes 102 % of GDP.

in the unconstrained case, but the solution was described only in non-closed-form. *Park* (2006) showed that consumption is increasing and concave in wealth but a closed-form solution was derived only for the inverse function. More recently, *Travaglini* (2008) obtained a solution by assuming a quadratic utility function and income following a Brownian pattern.

The present model extends the above framework by including income cuts as an additional random process and considering the effect of public debt stabilization on consumption. After closed-form solutions are obtained, the impact of austerity policies upon the Marginal Propensity to Consume (MPC) and the size of the multiplier are thoroughly analyzed and found to be in accordance with actual developments in the Euro Area.

1. *Households*

On the onset, households are at each period *(t)* assumed to receive income (y_t) exclusive of interest payments, consume (c_t) and have a discount rate equal to the real rate of interest *(r)*. With an over-dot denoting the time derivative and current period subscripts omitted for simplicity, savings (a_t) accumulate as

(2) $$\dot{a} = ra + y - c$$

The stream of consumption $\{c_{t+s}, 0 \leq s < \infty\}$ is chosen so as to maximize the intertemporal utility, i.e.

(3a) $$\max_{\{c\}} \int_0^\infty e^{-rs} u(c_{t+s}) ds$$

subject to the usual transversality condition for wealth

(3b) $$\lim_{s \to \infty} e^{-rs} a_{t+s} = 0$$

A quadratic utility function is defined as in *Zeldes* (1989):

(4) $$u(c) = c - \frac{\gamma}{2} c^2$$

where $\gamma > 0$ and $u'(c) = 1 - \gamma c$, $u''(c) = -\gamma$, $u'''(c) = 0$. The specific assumption is made for ensuring a closed-form solution, though it has the drawback that its third derivative is zero and, thus, measures of prudence cannot be obtained; for a discussion of the issue see *Carroll/Kimball*

(1996). The problem can be mitigated if one thinks of the above formulation as an approximation around long-term trends of consumption and income.[7]

Optimality implies that expected consumption is constant over time, that is $c_t = \mathrm{E}_t\{c_{t+s}\}$ with operator $E_t\{\cdot\}$ describing the expectation based on information available at time t. Taking (4) into account and solving (3a, 3b), optimal consumption is obtained as the well-known function of savings and human capital:

(5) $$c_t = r \cdot (a_t + H_t)$$

Human capital is defined as the present value of expected future incomes:

(6) $$H_t = \int_0^{\infty} e^{-rs} E_t\left\{ y_{t+s} \right\} ds$$

As in *Dixit/Pindyck* (1994, p. 87) one can treat (*H*) as an asset and equate the return on it to the sum of the dividend (i.e. the current income) and the expected capital gain:

(7a) $$rH_t \cdot dt = y_t \cdot dt + E_t(dH)$$

or after rearranging and omitting subscripts for simplicity:

(7b) $$rH = y + \frac{1}{dt} E_t(dH)$$

This is a dynamic stochastic equation that will be solved in the presence of austerity measures. To model them, income is assumed to have three components: a systematic trend that affects "permanent income", a random part that represents "transitory" changes and finally a component that is determined by Government policies. Income changes according to the augmented Brownian pattern:

(8) $$dy = \mu \cdot dt + \sigma \cdot dz + g \cdot dw$$

where *(μ)* is the drift, *(σ)* the volatility, and *{dz}* a random process normally distributed with zero mean and variance dt. Component *(g)*, if pos-

[7] Another limitation of the model is that in order to be as simple as possible and obtain closed-form solutions, it lacks a production sector thus the effect of tax policies on investment decisions cannot be captured. To lessen the problem, tax effects are assumed to be lump-sums as described later.

itive, takes the form of transfers or of tax hikes if negative, and is applied through another process *{dw}*.

The policy process *{dw}* is also assumed to be stochastic, since in several cases Government intervention follows irregular patterns outside the pre-planned budgeting framework. In this case, contingent or implicit Government liabilities arise. As described by *Lojsch* et al (2011), contingent liabilities are cashed in on the occurrence of a random event, while the implicit ones may lack a legal basis but arise as a consequence of expectations created by past practices or pressures from interest groups. In other cases, Governments may choose to unexpectedly boost their appeal before elections and transfers may be interpreted as a signal of prospective performance as suggested by the literature on political business cycles following *Rogoff* (1990).[8] Policy fuzziness may also be the result of institutional slackness that prevails around the election period and allows pressure groups to appropriate privileges as described by *Skouras/Christodoulakis* (2013).

In the case of tax hikes, implementation may also be stochastic, though for different reasons: First, because tax hikes as well as other forms of income cuts are mostly effective by surprising households so that they cannot relocate their incomes to offshore fiscal jurisdictions and minimize the burden. Second, because it reflects the realities of adjustment programs in the Euro Area, where a precondition for the approval of bail-out financing was the swift implementation of austerity measures by avoiding lengthy parliamentary procedures. Moreover, in several instances austerity measures were unexpectedly revised to meet conditionality targets just before deadlines expired.

In the present context, Government policies are modeled to follow a Poisson process with implementation rate equal to (λ), i.e.

$$dw = \begin{cases} 1 \; with \; probability \; (\lambda dt) \\ 0 \; with \; probability \; (1 - \lambda dt) \end{cases} \tag{9}$$

For example, if a month is assumed to be the module of time *(dt)*, a rate $\lambda = 1$ implies Government interventions occurring at a monthly frequency, while $\lambda = 1/12$ implies once a year in average.

[8] A most recent example of such signaling is the decision by the Greek Government to distribute transfers of up to € 800 to crisis-stricken families just one week before local and European elections take place in May 2014.

2. *Crisis Effects*

The effects of the global crisis are captured in a number of ways: first, liquidity constraints due to credit shortage imply that savings should remain non-negative[9] $\{a_t \geq 0\}$. Second, earnings are restrained by imposing an upper bound on income $\{0 \leq y \leq \Omega\}$, while the drift (μ) of the income streams is reduced and variability (σ) increases. Third, private sector savings (a_t) are deposited in the banking sector and subsequently invested either in domestic Government bonds (b_t) or in foreign assets (z_t) abroad. The balance sheet condition implies:

$$a_t = b_t + z_t \tag{10}$$

If the off-shore assets turn to be toxic for some extraneous reason (i.e. they suffer an exogenous shock $dz < 0$), the Government has to issue new bonds (i.e. $\Delta bt > 0$) in order to cover the losses in the banking sector. This augments indebtedness and triggers a series of austerity measures to restore sustainability of public finances as examined next.

3. *Government Debt*

In every period, the Government is engaged in net fiscal transfers $g_t > 0$ to the households and collects revenues (τ_t) which for simplicity are assumed to be non-distortionary lump-sums, such as rents on public ownership, privatization proceeds or various forms of levies. The following definitions express future debt liabilities and revenue-generating capacity respectively in present value terms:

$$d_t = \int_0^\infty e^{-rs} E_t\{g_{t+s}\}ds, \text{ and } \quad f_t = \int_0^\infty e^{-rs} E_t\{\tau_{t+s}\}ds \tag{11}$$

Public debt $\left(b_t\right)$ accumulates according to the budget constraint:[10]

$$\dot{b} = rb_t + g_t - \tau_1 \tag{12a}$$

Debt sustainability requires that the transversality condition holds i.e.

$$\lim_{s\to\infty} e^{-rs} b_{t+s} \leq 0 \tag{12b}$$

[9] In general, the constraint could be set at any negative constant.

[10] In the absence of income growth and population change, variable *(b)* has similar dynamics with variable *(h)* used to denote the debt to GDP ratio in equation (1).

Solving (12a), condition (12b) is ensured as long as:[11]

$$b_t + d_t \leq f_t \tag{13}$$

Thus, debt is sustainable as long as future revenue capacity covers current outstanding debt and future Government liabilities; for a discussion on fiscal soundness and debt sustainability see *Giammaroli* (2007). Sustainability breaks down whenever future debt liabilities become uncontrollable ($\Delta dt > 0$), fiscal capacity collapses ($\Delta ft < 0$) or there has been a sudden issuance of Government bonds to cover an imbalance in (10). Such types of fiscal shocks have actually impinged upon the GIIPS and destabilized their economies. For example, in Greece public revenues collapsed from an average of around 35 % of GDP during the period of 2000–2008 bottomed down at 32 % of GDP in 2009 signaling a major loss of control in public finances. Italy and Portugal were faced with a long recession that weakened their revenue capacity and, at the same time, augmented their future liabilities such as pensions and unemployment allowances. Finally, Ireland and Spain suffered major losses in commercial banks which prompted rescue operations by the Government.

In all the above cases, harsh stabilization policies had to be subsequently implemented in various forms, such as salary and wage cuts, higher taxation and outright dismissals. For simplicity, we assume here that austerity measures take the form of random non-distortionary tax hikes (i.e. negative net transfers, $g = -k,\ k > 0$) for a period *(T)* long enough so as to correct excessive indebtedness. This is ensured as long as:

$$\Delta b_t = \int_0^T e^{-rs} E_t \{k_{t+s} dw_{t+s}\} ds \tag{14}$$

To achieve a certain fiscal correction the Government may choose a combination of implementation frequency *(λ)* and application period *(T)* to determine a tax hike *(k)* that is levied randomly. Tax hikes are modeled as impulse functions, thus we have that $E_t \left\{ k_{t+s} dw_{t+s} \right\} = \lambda k$, and expression (14) is then easily calculated as:[12]

---

[11] Note that the possibility of the banks investing abroad makes the households' transversality condition (3b) not to be sufficient for automatically ensuring the respective condition (12b) for the public sector.

[12] This expression differs from the well-known formula derived by Merton (1971) where random increments in income are assumed to be step functions and, therefore, the compounding factor is squared $(1/r^2)$.

(15a) $$\Delta b_t = \frac{1}{r}\lambda k\left[1 - e^{-rT}\right]$$

Rearranging, the tax intensity that stabilizes a certain fiscal correction (Δb) is obtained as:

(15b) $$k = \frac{r \cdot \Delta b}{\lambda\left[1 - e^{-rT}\right]}$$

The tax hike increases with the debt burden that has to be corrected and is inversely affected by the austerity period *(T)* and the frequency *(λ)* at which policy measures are implemented. The effects on consumption are investigated next.

4. Optimal Consumption

Applying Ito's lemma for human capital as in *Dixit/Pindyck* (1994, p 86) gives:

(16) $$\frac{1}{dt}E_t\left(dH\right) = \frac{\partial H}{\partial t} + \mu\frac{\partial H}{\partial y} + \frac{1}{2}\sigma^2\frac{\partial^2 H}{\partial y^2} + E_w\left\{\lambda \cdot \left[H\left(y-k\right) - H\left(y\right)\right]\right\}$$

with $E_w\left\{\cdot\right\}$ denoting the expected value of process *{dw}*. By Taylor's rule the second-order approximation of the term due to the Poisson process is:

(17) $$H\left(y-k\right) - H\left(y\right) \approx -k\frac{\partial H}{\partial y} + \frac{1}{2}k^2\frac{\partial^2 H}{\partial y^2}$$

Substituting (17) into (16) and (7b), the differential equation for human capital is obtained:

(18) $$\frac{1}{2}(\sigma^2 + \lambda k^2)H'' + \left(\mu - \lambda k\right)H' - rH + y = 0$$

In the above formulation, the variance and the drift of income in the standard Brownian process have been adjusted by *(λk^2)* and *(λk)* respectively[13] due to the Government policies. Expected future income is given as

(19) $$E_t\left(y_{t+s}\right) = y_t + \left(\mu - \lambda k\right)s$$

[13] Note that for a Poisson process the mean and the variance are equal to *(λ)*.

Thus a partial solution of *(H)* is evaluated from (6) as

(20) $$H_t^P = \frac{y_t}{r} + \frac{\mu - \lambda k}{r^2}$$

Finally, the general solution for human capital is given by

(21) $$H_t = \frac{y_t}{r} + \frac{\mu - \lambda k}{r^2} + \frac{1}{r}[Ae^{\alpha y_t} + \mathrm{B}e^{\beta y_t}]$$

where *A, B* are constants and $\alpha < 0 < \beta$ denote the negative and positive roots of the quadratic characteristic equation:

(22) $$\phi(x;k) \equiv \frac{1}{2}(\sigma^2 + \lambda k^2)x^2 + (\mu - \lambda k)x - r = 0$$

Constants A and B are determined so as the consumption function is well-behaved and the 'smooth pasting' condition $c'(y)|_{y=\Omega} = 0$ holds, i.e. the Euler equation is satisfied even when liquidity constraints become binding; for details and properties of the solution see the Appendix. Consumption is finally obtained from (5) as:

(23) $$c_t = ra_t + \frac{\mu - \lambda k}{r} + y_t - \frac{1}{\beta}e^{\beta(y_t - \Omega)}$$

The first term in the r.h.s. denotes consumption out of wealth, while the second is the present value of future income increments net of tax hikes. The last two terms express consumption out of current income, and drive the stochastic properties of the multiplier as examined below.

The multiplier

The marginal propensity to consume (MPC) is derived by differentiating (23):

(24) $$MPC = \frac{\partial c}{\partial y} = 1 - e^{\beta(y - \Omega)}$$

Then the income multiplier (m) is neatly obtained by:

(25) $$m = \left[1 - \left(\frac{\partial c}{\partial y}\right)\right]^{-1} = e^{\beta(\Omega - y)}$$

Given the income constraint $y \leq \Omega$, the above expression implies that the multiplier does not fall below unity and may be well above. The multiplier is itself a stochastic process and its properties can be derived by applying Ito's lemma as in *Dixit/Pindyck* (1994, p 80):

$$dm = \left\{ \frac{\partial m}{\partial t} + \mu \frac{\partial m}{\partial y} + \frac{1}{2}\sigma^2 \frac{\partial^2 m}{\partial y^2} \right\} dt \tag{26}$$
$$+ \sigma \frac{\partial m}{\partial y} dz + E_w \left\{ \lambda \cdot \left[m(y-k) - m(y) \right] \right\} dt$$

Approximating the Poisson process as in (17), taking into account that $m' = -\beta m$, $m'' = \beta^2 m$, and the fact that *(β)* is a root of (22), after some cumbersome but straightforward manipulations in (26) the following process is obtained:

$$\frac{dm}{m} = \left[(\sigma^2 + \lambda k^2)\beta^2 - r \right] dt + (\beta\sigma)(-dz) \tag{27}$$

Thus the multiplier follows a *geometric* Brownian pattern with a drift growth rate equal to $\left[\left(\sigma^2 + \lambda k^2 \right) \beta^2 - r \right]$ and variability *(σβ)*. It is noticeable that random changes in the multiplier take the opposite direction of those affecting households' income.

To examine the effects of the multiplier on consumption, one can consider (23) as a stochastic function of income and apply Ito's lemma exactly as before to get:

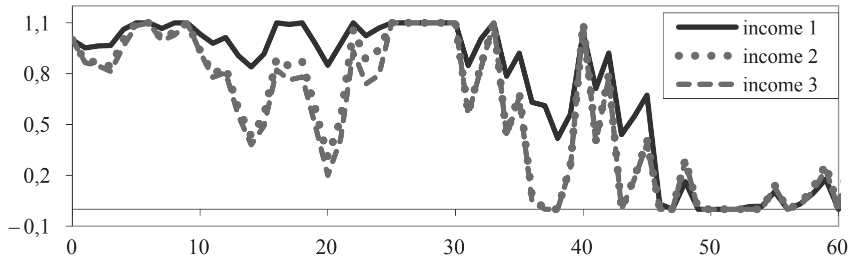
$$dc = \left\{ \frac{\partial c}{\partial t} + \mu \frac{\partial c}{\partial y} + \frac{1}{2}\sigma^2 \frac{\partial^2 c}{\partial y^2} \right\} dt + \sigma \frac{\partial c}{\partial y} dz + E_w \left\{ \lambda \cdot \left[c(y-k) - c(y) \right] \right\} dt \tag{28}$$

Taking into account that $c' = 1 - (1/m)$ and $c'' = -\beta/m$, the consumption process is obtained after similar manipulations as a standard Brownian pattern

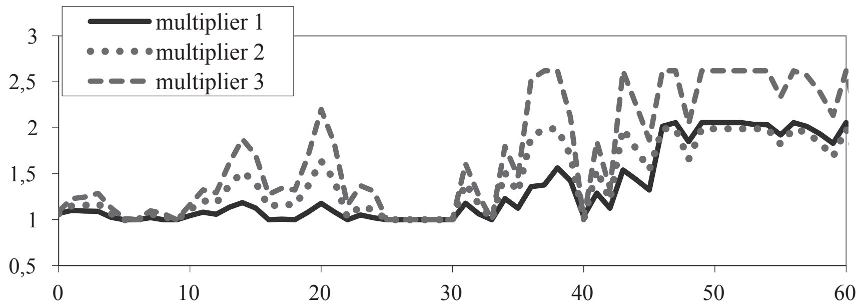
$$dc = \left[\mu - \frac{r}{m\beta} - \frac{r\Delta b}{1 - e^{-rT}} \right] \cdot dt + \left[\left(1 - \frac{1}{m} \right) \sigma \right] \cdot dz \tag{29}$$

As an illustration of how consumption and the multiplier are affected by uncertainty and austerity, a random realization of the stochastic processes expressed by (8), (26) and (28) is depicted in Fig. 2 under three alternative patterns of parameter values. Common parameter values are set as $r = 2\,\%$, $\mu = 0.03$, $\Omega = 1.10$. Variability is $\sigma = 0.04$ for pattern 1, and 0.08 for 2 and 3. Pattern 3 additionally includes a tax hike $k = 0.03$ at a frequency $\lambda = 1/3$. Means and standard deviations are shown in Table 2.

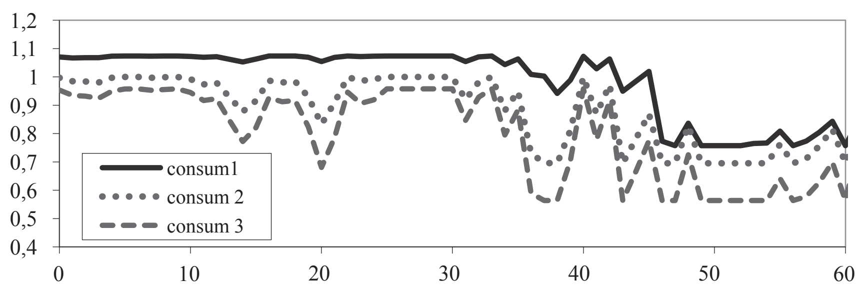
Variability is increasing with uncertainty for all three variables and is further enhanced by austerity for consumption and the multiplier. Mean values are lower for income and consumption, but the multiplier is substantially increasing with austerity. More formally, a number of proper-



(a) The Income Process



(b) The Multiplier Process



(c) The Consumption Process

Note: Common parameter values $r = 2\,\%$ $\mu = 0.03$ $\Omega = 1.10$. Variability is set at $\sigma = 0.04$ for pattern 1, and 0.08 for 2 and 3 respectively. Pattern 3 additionally includes a tax hike $k = 0.03$ at a frequency $\lambda = 1/3$

Figure 2: A Random Realization of Three Stochastic Process, as in Table 2

ties are discussed for the multiplier and consumption function after the following Lemma for the positive root of (22) is established:

Lemma 1: The positive root $\beta = \beta(k)$ is a concave function of fiscal cuts (k), thus it is increasing for a reasonable range of (k). The proof is given in the Appendix.

Table 2

A Random Realization of the Stochastic Process Under Alternative Assumptions on Variability and Austerity

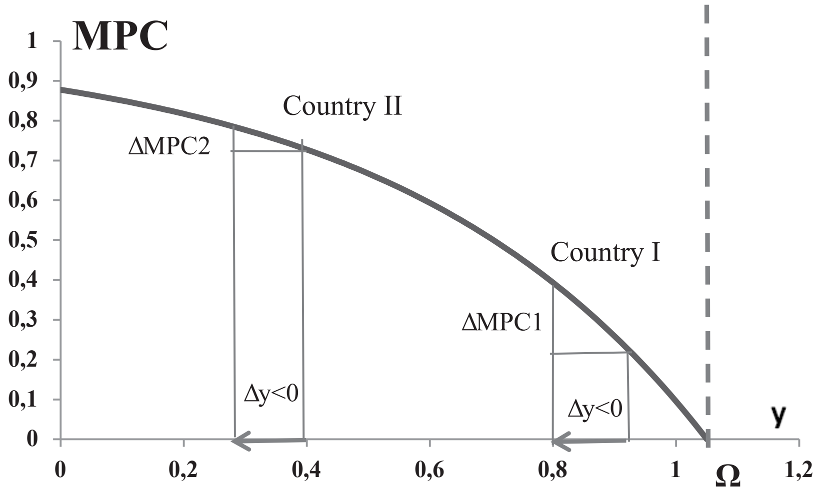
	income	*consumption*	*multiplier*
Standard deviations #1	0.39	0.11	0.37
pattern #2	0.46	0.13	0.41
pattern #3	0.46	0.17	0.67
Mean #1	0.62	0.97	1.42
pattern #2	0.52	0.86	1.49
pattern #3	0.51	0.77	1.80

Note: Common parameter values $r = 2\,\%$ $\mu = 0.03$ $\Omega = 1.10$. Variability is $\sigma = 0.04$ for pattern 1, and 0.08 for 2 and 3. Pattern 3 additionally includes a tax hike $k = 0.03$ at a frequency $\lambda = 1/3$.

Proposition 1: The consumption rule and the MPC are concave everywhere with respect to (w.r.t.) income (y). The proof is given in the Appendix. This is in agreement with previous findings in the literature describing the concavity of the consumption rule in the presence of uncertainties in households' income; see for example *Zeldes* (1989) for an empirical investigation, and *Carroll/Kimball* (1996) for a formal proof by using the intertemporal optimization framework.

The concavity of MPC has two important implications: at the onset of a slump that reduces incomes, both the MPC and the multiplier rise but they would rise *more* for an economy where households are in the vicinity of the upper income threshold (say of type I); in contrast, in an economy where households are farther from the upper income threshold (say of type II), MPC and the multiplier would rise to a *lesser* extent; see Fig. 3 for an illustration. This assertion implies that if credit limits are common – as one may roughly assume to be the case for the Euro Area economies – the recessionary impact per unit of austerity would be stronger in a richer country (such as Italy) than in a poorer one (such as Portugal, for example). This counterintuitive result is due to the fact that households in type II economies contemplate that their income may eventually rise, while in type I it is more likely to fall farther away from the upper limit. The issue is empirically discussed in the next Section.

A second implication of the MPC concavity is that, by analogy, the marginal propensity to save is convex. At the onset of a boom households



Note: If in a slump income is reduced by Δy, the closer is the representative household to the upper limit (Ω), the more the MPC is rising. The same holds for the MPS in a boom. Parameters are set at $\beta = 2$ and $\Omega = 1.05$.

Figure 3: The MPC as a Concave Function of Income

would increase both their savings and the marginal propensity to save, but the latter will rise *by less* for lower-income economies (Country of type II) as compared to higher-income ones. This confirms *Deaton* (1991) who asserts that the savings ratio may not be as strongly procyclical as expected by the permanent income hypothesis.

Proposition 2: The multiplier drift rate rises with the variance (σ) of the transitory income component and/or a fall in the interest rate. These are obvious by inspecting (27). The first property implies that multipliers should have been expected to rise in the aftermath of the crisis since uncertainty about future incomes widened. This finding is in sharp contrast with the permanent income hypothesis, according to which changes in transitory income reduces the marginal propensity to consume and, therefore, the multiplier falls; see, for example, *Romer* (1996, p 313). The second implication is that multipliers increased as monetary policy was easing in the aftermath of the crisis; this is in line with *Christiano* et al (2011) who find that fiscal multipliers get considerably larger when the zero lower bound on nominal interest rates state becomes binding.

Proposition 3: The multiplier drift rate rises when fiscal cuts get stronger. This follows from (25) and Lemma 1. The implication is that as austerity gets more intense, income multiplier rises and its impact on recession and fluctuations becomes stronger.

Proposition 4: Consumption drift is falling as the fiscal correction gets more demanding (i.e. higher Δb), and is concave w.r.t. the adjustment period (T). The first part follows immediately from (29). The concavity of the drift is analytically proven in the Appendix.

Proposition 5: Variability of consumption is less than that of income, and is rising with (m). This follows immediately from the second term in square brackets in (29). The connection between the severity of consumption fluctuations and the size of the multiplier was completely neglected in devising the austerity programs, though it seemed to be a long tradition in economics. No less than *Keynes* (1936, p 125) had warned that an economy "… in which saving is a very small proportion of income [i.e. implying that the multiplier is larger] will be more subject to violent fluctuations than a wealthy where saving is a larger proportion of income and the multiplier consequently smaller."

In the aftermath of the global crisis, consumption fell and fluctuations widened for a variety of reasons. Within the stochastic context of (8), some effects can be expressed by a fall in (μ) and/or a rise in volatility (σ) to indicate adverse prospects and higher uncertainty respectively. But the main adversity for households came from the austerity programs that were implemented in the GIIPS countries especially after 2010 in order to address the unsustainability of public finances. These policies profoundly affected consumption dynamics and recession as examined below.

IV. Stylised Facts of Austerity

Using the above Propositions, some crucial developments that took place in the Euro Area economies – and in GIIPS more specifically – may be explained. Five stylized facts are described, namely the rise in multipliers, the increased variability of consumption and its relation to the intensity of austerity, the impact of the latter on recession, and how it is related to the concentration of households near the upper income bound.

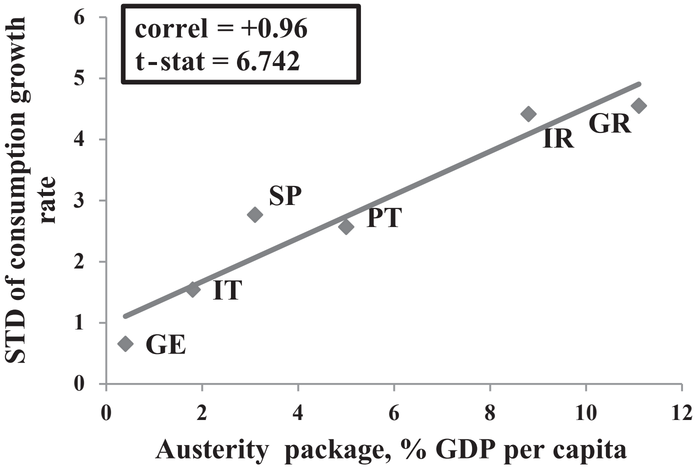
Fact 1: Post-crisis multipliers are higher

Proposition 2 is in agreement with the *ex post* calculations of the fiscal multipliers by IMF and the European Union discussed in Section 2. More formally, fiscal elasticities of growth can be obtained by estimating a first-difference version of (1) so as to avoid the presence of unit roots. If post-crisis data are included in the estimation, multipliers are found to range[14] between 1.146 and 1.440; for econometric details see *Christodoulakis* (2013).

A consequence of the higher multipliers was that post-crisis savings as a proportion to income were reduced. As households were unable to cover contingencies with increased borrowing, consumption fluctuations became more pronounced as shown in Table 1. While variation of consumption growth nearly doubled in the 'northern' part of the Euro Area, it got trebled in the 'southern' part where income-restricting measures were implemented by austerity programs at a much higher intensity and frequency.

Fact 2: The variance of consumption increased in the aftermath of the crisis.

This is depicted in Fig. 1(a) and Table 1 as discussed in the introduction. It follows from the previous fact and Proposition 4.



Source: Ameco and Financial Times (see footnote 16).

Figure 4: Consumption Variability and Austerity Intensity

Fact 3: The variance of consumption is increasing with the level of austerity.

[14] This is with regards to primary fiscal balances. Elasticities of gross balances are found to range somewhere lower at between 1.032 and 1.294 as changes may be due to interest rate variations, not necessarily affecting taxation or other current expenditure.

This follows from Propositions 3 and 4. To examine the effects of austerity, its intensity in each particular country (j) is defined as the ratio (A_j) of the size of the adjustment programs being applied during the recent years to per capita GDP. This is calculated for the GIIPS group and also for Germany as a benchmark by the Financial Times wherein further details can be found.[15] In Fig. 4, a strong positive correlation is found between fluctuations in the growth rate of consumption and the intensity of austerity.

Fact 4: The impact on recession is increasing with the level of austerity.

This follows from Proposition 5. Though seems self-evident, it is nevertheless in sharp contrast with the early optimism adopted by IMF according to which, as discussed in Section 2, for any given fiscal correction a front-loaded adjustment (i.e. by adopting a higher k) would have only small and transient effects. The recessionary impact (R_j) per country (j = Germany and each of the GIIPS) is defined as the ratio of cumulative recession impacting upon a specific economy, namely:

$$R_j = \sum_{t=2009}^{T=2012} \frac{\hat{y}_t - y_t}{y_{2008}} \tag{30}$$

In the above expression y_t denotes per capita real GDP at period t, $\hat{y}_t$ is a simple time-trend projection[16] evaluated over the period 2000–2007, and their difference is expressed as a ratio to the level at 2008. Fig. 5 reveals a strong negative correlation between the impact of recession and the intensity of austerity (R_j, A_j).[17]

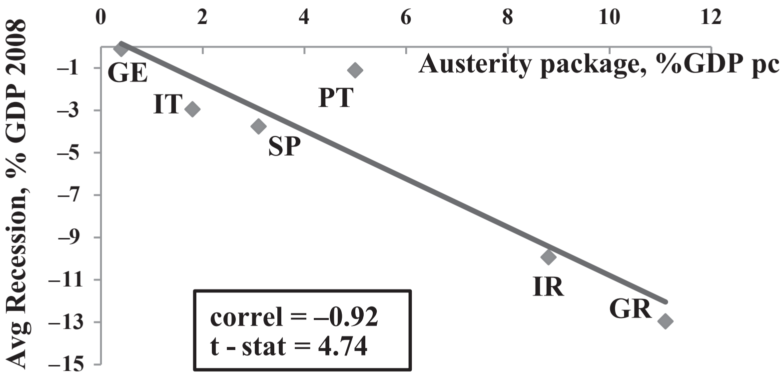
Recessionary effects notwithstanding, the size of austerity reached such unprecedented levels that risked political stability in most countries.[18] This undermined the effectiveness of the programs but surpris-

[15] For Greece, Germany, Spain and Italy data are for the 2011 austerity program. For Portugal it is for 2012, while for Ireland for 2009 and 2011 but only the latest is used here to conform to the other economies. Source: http://www.ft.com/cms/s/0/feb598a8-f8e8-11e0-a5f7-00144feab49a.html#axzz2JSOwncys.

[16] The simple time-trend is more suitable for short periods. Other measures of recessions, such as Hodrick-Prescott filters or output gaps, were also used with similar results.

[17] Using the same set of data, *De Grauwe/Ji* (2013) find a striking correlation between the pressures exercised by international markets to the particular economies and the size of austerity programs applied on them.

[18] Since 2009, no Government in the 'southern' part of the Euro Area survived a full term and all were outvoted in the ensuing elections, for reasons mostly at-



Source: Ameco and Financial Times (see footnote 16).

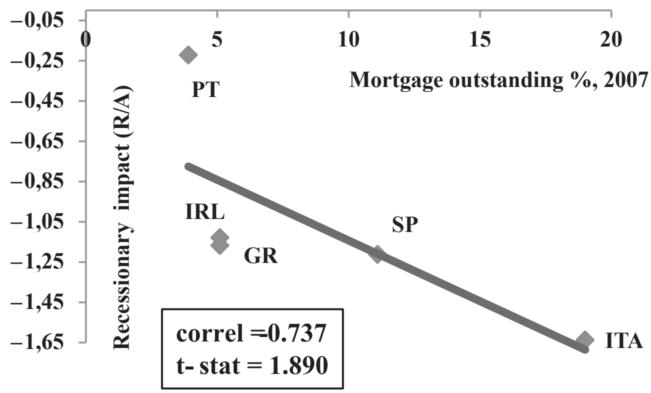
Figure 5: Average Recession and Austerity Intensity

ingly, since fiscal correction targets were not revised, it precipitated even harsher austerity measures afterwards.

Fact 5: The austerity impact is increasing with the proximity to upper income bound.

This follows from Proposition 1. To examine it in the context of GIIPS, the recessionary impact of austerity is measured by the ratio (R_j/A_j) as defined before. A rough proxy for the degree of concentration of households' incomes near their upper bound, is taken by the number of mortgages outstanding as a percent of total house loans in each country in 2007; for a discussion see *Gomez-Salvador* et al (2011), where the relevant data have been taken from. The correlation is shown in Fig. 6, confirming the intuition of Proposition 1. One can see that mortgage indebtedness mostly afflicts countries with higher-income households (such as Italy and Spain), since they get closer to the binding upper bound and are, therefore, less likely to see their income rising and repay their loans.

tributed to the enactment of austerity measures. In Greece, Italy and Portugal, Governments were serially collapsing during 2011–2012. In the 'northern' part of the Euro Area, incumbents were outvoted in Belgium, France and Finland, while the Dutch Government collapsed but finally survived the elections. Only in Germany the Government went through the elections unscathed and in Austria it suffered only marginally.



Source: Data for (R, A) as in Figures 4 and 5. Mortgage data for 2007 from Gomez-Salvador et al (2011, Annex 2, Table 2).

Note: A higher ratio implies that the economy is more liquidity constrained and this makes the impact harsher.

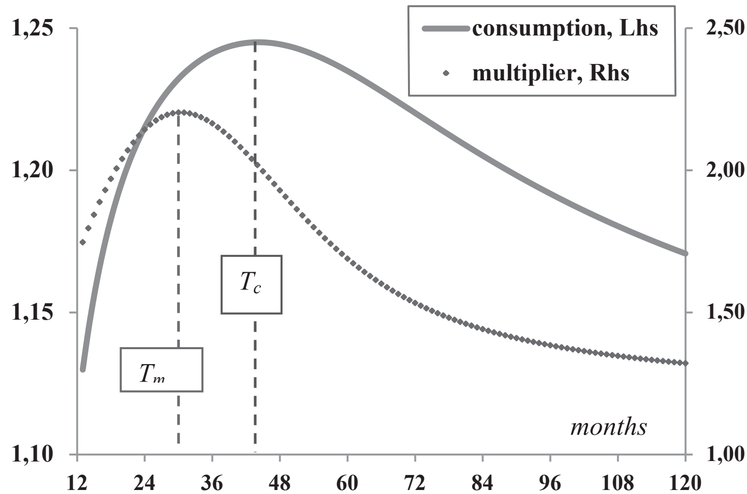
Figure 6: The Recessionary Impact of Austerity versus Mortgage Outstanding as Percent of Total House Loans in Each Country

Optimal austerity

A byproduct of the above analysis is that a more gradual adjustment might have had milder recessionary effects. Given the concavity of the consumption process as shown in Proposition 4, an optimal level of implementation period (T_c) can be computed so as to maximize consumption as expressed in (23) or, equivalently, its drift as in (29).

The f.o.c. is derived in the Appendix as equation (43) but is complicated enough for an analytic derivation to be obtained, thus a numerical solution is presented. Parameter values are set as the simple averages across GIIPS shown in Table 1 for the period after the crisis at $\mu = 0.013$ and $\sigma = 0.0316$. The upper bound is set at $\Omega = 1.05$ to denote the deterioration in income prospects, while real interest rates are set at $r = 4\,\%$ to reflect both the credit tightening for households as well as a higher degree of discounting the future. Fiscal correction is set $\Delta b = 0.61$, which is calculated as the average[19] deterioration of the debt to GDP ratios in GIIPS during 2007–2013.

[19] The deterioration between 2007–2013 was 98 % for Ireland, 68 % for Greece, 55 % for Spain, 28 % for Italy and 55 % for Portugal; data from AMECO.



Note: T_c denotes the duration of austerity program that maximizes consumption drift; T_m the one that maximizes the multiplier.

Parameter values $r = 4\,\%$, $\mu = 0.013$, $\Omega = 1.05$, $\sigma = 0.0316$, $\lambda = 1/3$, $\Delta b = 0.61$.

Figure 7: Consumption (l.h.s.) and the Multiplier (r.h.s.) as Functions of the Adjustment Period (T) Plotted in Months

Assuming that the adjustment period (T) is expressed in months, the frequency is set equal to $\lambda = 1/3$, implying that austerity measures were implemented roughly every quarter as has actually been the case in GIIPS. The consumption and multiplier functions are depicted in Fig. 7 and an optimal adjustment period is found at $T_c = 44$ months.

Suppose now that the multiplier gets its higher value when program duration reaches a value (T_m). In the Appendix it is shown that $T_m < T_c$. In conjecture with the concavity of the multiplier function in (27) and Lemma 1, this property implies that at the point consumption is maximized the multiplier obtains a lower value, thus making the effect of austerity to be less pronounced. In contrast, if a program has a short duration which is closer to (T_m), the multiplier gets higher leading to a lower consumption drift and making the austerity effect to be harsher.

Though highly schematic, these results indicate that had the austerity programs been extended, consumption could have been above the level actually experienced and extreme recession would have probably been avoided. With the above numerical values, the programs should have been extended over nearly twice as longer than the two-year enforcement period followed in practice. In this case, consumption could have been up by 12 %.

This numerical finding is in line with a growing literature on how the severity of front-loaded austerity programs in the Euro Area has actually accentuated recession and indebtedness rather than dissipating them. For example, *Holland* (2012) finds that excessive fiscal correction was a self-defeating policy and suggests that less austerity can be traded with more growth. In a similar vein, *Schulten/Müller* (2013) argue that recessionary policies based on wage cuts trigger a race to the bottom leading their economies straight into a deflationary trap. Concern is raised even from within the organizations that carried out the austerity plans as now they fear that further measures may deepen the slump; in an IMF direct letter, *Cottarelli* (2012) warns against those who "… may feel inclined to preserve their short-term plans through additional tightening, even if hurts growth more" and advices bluntly: "unless you have to, you shouldn't."

V. Conclusions

Despite optimistic assumptions that a swift fiscal consolidation applied in the GIIPS countries in the aftermath of the global crisis would have small and only transient recessionary effects, the outcome proved to be a lot more painful and lasting. Due to the rise of fiscal multipliers after the crisis, the front-loaded character of the adjustment programs further exacerbated fluctuations and deepened recession.

Using a stochastic framework for labor income under liquidity constraints, the paper derived closed-form solutions for households' consumption under austerity. Results show that the multiplier as well as the variability of consumption not only increased after the global crisis but were further enhanced by the austerity programs. This suggests that a more gradual adjustment could have had milder recessionary effects and would be perhaps politically more acceptable and socially sustainable.

References

Alesina, A./*Ardagna*, S. (1998): "Tales of Fiscal Adjustment", Economic Policy, 13 (27), 489–517.

Alesina, A./*Perotti*, R (1997): "Fiscal Adjustments in OECD Countries: Composition and Macroeconomic Effects", IMF Staff Papers, 44 (2), pp 210–248.

Aschauer, D. A. (1989): "Is Public Expenditure Productive?", Journal of Monetary Economics, 23, 177–200.

Baum, A./*Checherita-Westphal*, Cr./*Rother*, Ph. (2012): "Debt and Growth New Evidence for the Euro Area", ECB Working Paper No. 1450, July.

Blanchard, Ol./*Leigh,* D. (2013): "Growth Forecast Errors and Fiscal Multipliers", IMF Working Papers, WP/13/1, January.

Carroll, C./*Kimball,* M. (1996): "On the Concavity of the Consumption Function", Econometrica, 64 (4), 981–992.

Cecchetti, St./*Mohanty,* M./*Zampolli,* F. (2011): "The Real Effects of Debt", BIS Working Papers, 352, Bank for International Settlements.

Chah, E./*Ramey*, V./*Starr,* R. (1995): "Liquidity Constraints and Intertemporal Consumer Optimization: Theory and Evidence from Durable Goods", Journal of Money, Credit and Banking, 27 (1), 272–287.

Checherita, C./*Hallett,* A. H./*Rother,* Ph. (2012): "Fiscal sustainability using growth-maximising debt targets", ECB Working Paper No. 1472, September.

Checherita, C./*Rother,* Ph. (2010): "The Impact of High and Growing Government Debt on Economic Growth an Empirical Investigation for the Euro Area", ECB Working Paper, No. 1237, August.

Christiano, L./*Eichenbaum,* M./*Rebelo,* S. (2011): "When Is the Government Spending Multiplier Large?", Journal of Political Economy, 119 (1), 78–121.

Christodoulakis, N. (1994): "Fiscal Developments in Greece 1980–93: A Critical Review", European Economy: Towards Greater Fiscal Discipline, No. 3, 97–134, Brussels.

– (2013): "From Grexit to Growth: On Fiscal Multipliers and how to End Recession in Greece", National Institute Economic Review, 224, 66–76, London, April.

Cotarelli, C. (2012): "Fiscal Adjustment: Too Much of a Good Thing?", IMF direct, posted on January 29, http://blog-imfdirect.imf.org/2012/01/29/.

Cottarelli, C./*Jaramillo*, L. (2012): "Walking Hand in Hand: Fiscal Policy and Growth in Advanced Economies", IMF Working Papers No. 137, May.

De Grauwe, P./*Ji*, Y. (2013): How Much Fiscal Policy in a Monetary Union", in The Crisis in the Euro Area", Conference organized by the Bank of Greece, May 23–24.

Deaton, A. (1991): "Saving and Liquidity Constraints", Econometrica, 59 (5), 1221–1248, September.

Dixit, A./*Pindyck*, R. (1994): Investment under Uncertainty, Princeton University Press.

Égert, B. (2012): Public Debt, Economic Growth and Nonlinear Effects: Myth or Reality?, OECD, Economics Department, Working Papers No. 993.

European Economy (2012): The Second Adjustment Programme for Greece: Fifth Review, Occasional Papers No. 94, March.

DOI https://doi.org/10.3790/ccm.47.2.265 | Generated on 2025-08-13 16:06:32

Giammaroli, N./*Nickel*, C./*Rother,* Ph./*Vidal*, J.-P.(2007): "Assessing Fiscal Soundness: Theory and Practice", ECB Occasional Paper No. 56, March.

Giavazzi, F./*Pagano*, M. (1990): "Can Severe Fiscal Contractions be Expansionary? Tales of Two Small European Countries", NBER Macroeconomics Annual, Vol. 5, 75–122, MIT Press.

– (1996): "Non-Keynesian Effects of Fiscal Policy Changes: International Evidence and the Swedish Experience", Swedish Economic Policy Review, 3 (1), 67–103.

Gomez-Salvador, R./*Lojschova*, A./*Westermann*, T. (2011): "Household Sector Borrowing in the Euro Area: A Micro Data Perspective", *ECB Occasional Paper,* No. 125, April.

Hall, R./*Mishkin*, F. (1982): "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households", Econometrica, 50 (2), 461–481.

Holland, D. (2012): "Less Austerity, More Growth?", NIESR Discussion Paper No. 400, London, National Institute of Economic and Social Research. (http://www. niesr.ac.uk/pdf/301012_110656.pdf)

Keynes, J. M. (1936): The General Theory of Employment, Interest and Money, MacMillan Cambridge UP, (reprinted 1978).

Kumar, M./*Woo*, J. (2010): "Public Debt and Growth", IMF Working Papers, No. 10/174.

Lojsch, D. H./*Rodriguez-Vives*, M./*Slavik*, M. (2011): "The Size and Composition of Government Debt in the Euro Area", ECB Occasional Paper, No. 132, October.

Merton, R. (1971): "Optimum Consumption and Portfolio Rules in a Continuous-Time Model", Journal of Economic Theory, 3 (4), 373–413.

Muth, J. (1960): "Optimal Properties of Exponentially Weighted Forecasts," Journal of the American Statistical Association, 55 (290), 299–306.

Park, M. (2006): "An Analytical Solution to the Inverse Consumption Function with Liquidity Constraints", *Economics Letters*, 92, 389–394.

Reinhart, C./*Rogoff*, K. (2010): "Growth in a Time of Debt", American Economic Review, 100 (2), 573–78.

Rogoff, K. (1990): "Equilibrium Political Budget Cycles", American Economic Review, vol. 80, 21–36, March.

Romer, D. (1996): Advanced Macroeconomics, McGraw-Hill, NY.

Schulten, T./*Müller*, T.(2013): "A new European Interventionism? The Impact of the new European Economic Governance on Wages and Collective Bargaining", in Natali D. and B. Vanhercke (eds.) Social developments in the European Union, ETUI, Brussels, 181–214.

Seater, J. (1996): "An Optimal Control Solution to the Liquidity Constraint Problem", *Economics Letters*, 54 (2), 127–134.

Skouras, S./*Christodoulakis*, N. (2013): “Electoral Misgovernance Cycles: Evidence from Wildfires and Tax Evasion in Greece”, Public Choice, February. http://link.springer.com/article/10.1007/s11127-013-0071-0.

Travaglini, G. (2008): “An Exact Consumption Rule with Liquidity Constraints and Stochastic Income”, Economics Bulletin, 5 (5), 1–9.

WEO (2010): “Recovery, Risk and Rebalancing”, IMF, October.

– (2012): “Coping with High Debt and Sluggish Growth”, IMF, October.

Zeldes, S. (1989): “Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence”, The Quarterly Journal of Economics, 104 (2), 275–98.

Appendix I: Consumption Rule and Proposition 1

Substituting (21) into (5) and differentiating thrice w.r.t. (y) we get:

(31a) $$MPC = c'(y) = 1 + \alpha A e^{\alpha y} + \beta B e^{\beta y}$$

(31b) $$MPC' = c''(y) = \alpha^2 A e^{\alpha y} + \beta^2 B e^{\beta y}$$

(31c) $$MPC'' = c'''(y) = \alpha^3 A e^{\alpha y} + \beta^3 B e^{\beta y}$$

Two cases are examined with respect to the constant A. If $A > 0$, consumption tends to $+\infty$ as $y \to -\infty$, which is absurd; thus A has to be non-positive. Since $\alpha < 0 < \beta$, expression (31a) requires that $B < 0$ in order to have $MPC = 0$ at $y = \Omega$, as implied by the pasting condition. If $A < 0$, it is easily derived from expression (31a) that if income is sufficiently low such that

(32) $$y < \frac{1}{\beta - \alpha} \cdot \ln\left[\frac{\alpha A}{-\beta B}\right]$$

then $MPC > 1$. This however is ruled out in the presence of credit constraints, thus $A = 0$. Solving for the smooth-pasting condition, $c'(\Omega) = 0$, and substituting into (21) and (5), (23) is easily obtained.

To prove Proposition 1, note that for $A = 0$ and $B < 0$, it follows from (31b) and (31c) that $c'' = MPC' < 0$ and $MPC'' < 0$. Thus both the consumption rule and the marginal propensity to consume are concave everywhere. By analogy, the marginal propensity to save $MPS = 1 - MPC$ is convex everywhere.

Appendix II: Proof of Lemma 1 and Proposition 3

Differentiating (25) w.r.t. k we get:

(33) $$\frac{\partial m}{\partial k} = (\Omega - y) \cdot e^{\beta(\Omega - yt)} \cdot \frac{\partial \beta}{\partial k}$$

For $\Omega \geq y$, a necessary and sufficient condition for the above expression to be non-negative is that the positive root (β) is increasing with k. To see under which conditions this holds, suppose that for a tax perturbation dk, the positive root moves to $\beta + d\beta$. By Taylor's approximation:

(34) $$\phi(\beta + d\beta; k + dk) = \phi(\beta; k) + \frac{\partial \phi(\beta; k)}{\partial x} d\beta + \frac{\partial \phi(\beta; k)}{\partial k} dk$$

Since $\phi(\beta + d\beta; k + dk) = \phi(\beta; k) = 0$, the above expression easily gives:

(35) $$\frac{d\beta}{dk} = -\left[\frac{\partial \phi(\beta; k)}{\partial x}\right]_{x=\beta}^{-1} \cdot \frac{\partial \phi(\beta; k)}{\partial k}$$

Partial derivatives are calculated from (22) as:

(36) $$\frac{\partial \phi(\beta; k)}{\partial x} = \left[(\sigma^2 + \lambda k^2)x + (\mu - \lambda k)\right]_{x=\beta} = \frac{\sigma^2 + \lambda k^2}{2}\beta + \frac{r}{\beta} > 0$$

and

(37) $$\frac{\partial \phi(\beta; k)}{\partial k} = \left[\lambda k x^2 - \lambda x\right]_{x=\beta} = -\lambda\beta(1 - \beta k)$$

Substituting (36) and (37) into (35) we get

(38) $$\frac{d\beta}{dk} = \frac{2\lambda\beta^2(1 - \beta k)}{2r + (\sigma^2 + \lambda k^2)\beta^2}$$

This expression is non-negative for $\beta \leq 1/k$. Given that (22) is convex, the slope at $x = \beta$ is positive and thus the restriction on (β) is equivalent to:

(39a) $$\phi\left(\frac{1}{k}; k\right) \geq 0$$

After some straightforward manipulations this leads to:

(39b) $$\frac{\sigma^2}{k^2} + \frac{2\mu}{k} - \lambda - 2r \geq 0$$

If (ρ) is the positive root of the equation

(40) $$\sigma^2 x^2 + 2\mu x - \lambda - 2r = 0$$

then (39b) is satisfied for $1/k \geq \rho$, or $k \leq 1/\rho$, that is for fiscal cuts not very excessive. For parameter values set at $\mu = 0.03$, $\sigma = 0.08$, $r = 0.04$ and $\lambda = 1/3$ this corresponds to $k \leq 0.218$, which is twice the maximum size of austerity cuts currently applied in GIIPS as shown in Fig. 2 and 3.

From (38) it is apparent that $\left(\frac{\partial \beta}{\partial k}\right)$ is decreasing with k, thus $\frac{\partial^2 \beta}{\partial k^2} < 0$ and $\beta(k)$ is concave.

Appendix III: Proof of Proposition 4

The consumption drift in (29) is rewritten as

$$\eta = \mu - \frac{r}{m\beta} - \frac{r\Delta b}{1 - e^{-rT}} = \mu - \frac{r}{\beta} e^{-\beta(\Omega - y)} - \lambda k \tag{41}$$

The second derivative of the consumption drift w.r.t. the implementation period (T) is given by the well-known formula:

$$\frac{\partial^2 \eta}{\partial T^2} = \frac{\partial^2 \eta}{\partial k^2} \cdot \left[\frac{\partial k}{\partial T}\right]^2 + \frac{\partial \eta}{\partial k} \cdot \frac{\partial^2 k}{\partial T^2} \tag{42}$$

Next, we prove that the drift is concave with (k). Differentiating (41) w.r.t. (k) we get:

$$\frac{\partial \eta}{\partial k} = -\lambda + \frac{r}{\beta^2} \cdot [1 + \beta(\Omega - y)] \cdot e^{-\beta(\Omega - y)} \cdot \frac{\partial \beta}{\partial k} \tag{43}$$

The second derivative is derived after some tedious but straightforward manipulations as:

$$\frac{\partial^2 \eta}{\partial k^2} = \frac{r}{\beta^2} \left\{ \left[1 + \beta(\Omega - y)\right] \frac{\partial^2 \beta}{\partial k^2} - \left[1 + \left\{1 + \beta(\Omega - y)\right\}^2\right] \cdot \frac{\partial \beta}{\partial k} \right\} \cdot e^{-\beta(\Omega - y)} \tag{44}$$

Given that $\Omega \geq y$, the term in the first square brackets is positive and so is the second. Since $\beta(k)$ is concave as shown before, it follows that $\frac{\partial^2 \eta}{\partial k^2} < 0$, thus the drift is concave in (k). Expression (15b) is easily checked to be convex, so that $\frac{\partial^2 k}{\partial T^2} > 0$. Naturally the drift is decreasing with (k) and finally (42) implies that $\frac{\partial^2 \eta}{\partial T^2} = (-)(+) + (-)(+) < 0$, thus the drift is concave in (T). The consumption drift is maximized by setting $\frac{\partial \eta}{\partial k} = 0$ at $k = k_c$ and then obtaining the optimal implementation period $T = T_c$ from (15b).

Appendix IV: Proof of $T_m < T_c$

The period $T = T_m$ that maximizes multiplier (m) is determined by setting $\frac{\partial \beta}{\partial k} = 0$ and recalling that $\frac{\partial^2 \beta}{\partial k^2} < 0$. From (43) it is easy to see that at this point

$$\left.\frac{\partial \eta}{\partial k}\right|_{k = k_m} = -\lambda < 0 = \left.\frac{\partial \eta}{\partial k}\right|_{k = k_c} \tag{45}$$

Since the function $\eta = \eta(k)$ is concave w.r.t. (k), the above inequality implies that $k_c < k_m$, and therefore $T_c > T_m$.