Inflation, Factor Substitution and Growth*

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I. Introduction

Why should central banks pursue price stability? This question posed by Fischer (1996) and Issing (2001) among others seems to have clear answers. From a legal perspective price stability is fixed as one or even the main goal of monetary policy in most central banks' constitutions. From a political perspective the dislike of inflation by the general public strongly supports the case for price stability. Finally, from an economic perspective price stability owes its high importance to the fact that deviations in either direction (inflation or deflation) could severely damage the real sector of the economy leading to higher unemployment or lower growth. It is evident, however, that without convincing support from the economic perspective it would be very hard for the two other perspectives to defend their views over a longer period of time.

In a medium and long-term perspective this makes it interesting to focus on the inflation-growth-relationship. For the three reasons mentioned above central bankers seem to believe that price stability and growth are not opposing or neutral goals of monetary policy but that they are highly complementary. One should expect that the economic perspective on that relationship is unanimously supported in theoretical studies. But over the last decades this support has not been too convincing. Theoretical models relating inflation and growth in the context of the mainstream growth-theoretic approaches have not (yet) produced very clear conclusions. Their benchmark results defend the idea of neu-

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trality or superneutrality of money with respect to real economic variables, whereas slight modifications of the models' assumptions can create either positive or negative growth effects of inflation.

In the empirical evidence from time series as well as cross country studies, however, the support for a negative inflation-growth-relationship has become more and more convincing since the 1990s. Gosh and Philips (1998) made it very clear that inflation ranges among the major determinants of growth strongly interacting with other major determinants such as real capital or human capital formation. Fischer, Sahay and Vegh (2002) underlined that the empirical literature is unanimous in finding that high inflation countries have a bad growth record in the medium and long run. But there is still a controversy about the nature of the relationship in low inflation countries. Bruno and Easterly (1998) had pointed out 40 percent inflation as a danger point, beyond which increases in inflation are very likely to lead to a growth crisis. For transition countries, Fischer, Sahay and Vegh (1996) found that this cut-off point occurs at inflation rates of about 50 percent. Khan and Senhadji (2000) analysed the inflation-growth-relationship separately for industrial and developing countries. They discovered that the threshold level of inflation above which inflation significantly slows growth is estimated at 1-3 percent for industrial countries and 7-11 percent for developing countries. Above this rate, inflation and growth are negatively related; below it, the relationship is not statistically significant.

Against this background this paper presents a new view on the old debate. It aims at a closer synthesis of the results in empirical and theoretical research and tries to identify new areas for advances in monetary growth theory. The paper highlights the negative influence of inflation on the allocative decisions in the real economy and shows how this relationship can be introduced into a simple monetary growth model by endogenising the aggregate elasticity of substitution. Traditionally, the elasticity of substitution is treated as a purely technical parameter of the aggregate production function. Recent contributions to the literature, however, call for a broader understanding of this concept that also comprises important institutional determinants of production and growth. The contribution of the paper to this literature is the following: It defends the idea that inflation is among the most important institutional determinants of the aggregate elasticity of substitution and that changes in the elasticity of substitution due to inflation have significant - and so far neglected - growth effects. In a more general perspective the paper

also contributes to recent efforts of producing more robust estimates of the aggregate elasticity of substitution and its possible determinants.

The paper starts with a short review of the monetary growth literature and tries to identify the reasons that are behind the dominance of the superneutrality benchmark result. It turns out that modern mainstream growth theory has conceptual difficulties in dealing with institutions and institutional changes. This can be very misleading in the case of money. Since money is one of the most important institutions in modern market economies, changes in monetary conditions always have repercussions on the institutional functioning of markets. Policy experiments in monetary growth models which do not sufficiently take these repercussions into consideration are thus subject to some kind of a "Lucas critique". They assume constancy of structural parameters although this cannot be expected in an inflationary environment.

In the following section one possible way to overcome this conceptual problem is described. The essential function of money in the allocation of real resources can be measured by looking at the aggregate elasticity of substitution between factors of production. I give three complementary explanations for this view building on earlier work by Tommasi (1994), Ventura (1997) and Huygens and Smith (1999). Having established a plausible link between deviations from monetary stability and changes in the aggregate elasticity of substitution it is shown in the subsequent section how the allocative effects of inflation is made visible in a very simple monetary growth model. Small changes in the elasticity of substitution are enough to compensate for potential positive effects of inflation and to question the superneutrality benchmark result. The last section deals with possible extensions and modifications of the basic model and investigates possible implications for future empirical research.

II. Monetary Growth Models: Benchmark Results and Modifications

The standard approach of how to include monetary aspects in an intertemporal optimising model of economic growth was introduced by Sidrauski (1967). Including real money balances as an additional argument in the utility function of the representative agent, Sidrauski could derive superneutrality of money with respect to the steady-state growth rates and levels of capital intensity and per-capita income. The only real variable which is negatively influenced by higher rates of inflation is the

real value of money balances. Since the latter enters the utility function, inflation induces a welfare loss.

Superneutrality as a benchmark result for steady-state effects of inflation is complemented by an important result for the real effects of inflation along the transition path. As it was first discovered by Fischer (1979), and later explained by Cohen (1985), this convergence effect of inflation is strictly positive as long as the intertemporal elasticity of substitution of the joint utility index is not equal to one. In the case of the logarithmic utility function with an intertemporal elasticity of substitution equal to one, inflation does not even have an effect on the speed of convergence. The positive convergence effect of inflation is due to changes in the structure of nominal interest rates that favour a more rapid accumulation of real capital. The portfolio shifts along the convergence path are thus in line with the old ideas of Tobin (1965), that higher inflation could have positive growth effects by speeding up real capital accumulation.

The two benchmark results - superneutrality of money with respect to the steady state and either positive or no growth effects along the convergence path - are neither in line with most of the empirical literature nor with the typical justifications for price stability as an important goal of monetary policy. Therefore many attempts to improve the standard model have been made. Before these modifications are reviewed it should be reconsidered how the standard model actually treats money. Money creates utility for the representative consumer because it serves as a medium of transaction. As Feenstra (1986) demonstrated, the direct utility of real balances can be regarded as a short-hand description of money helping to overcome a binding cash-in-advance-constraint for the purchase of consumption goods. It is no wonder then that Abel (1985) confirmed the superneutrality result in a growth model with a cash-inadvance-constraint for consumption goods. At the same time, the Sidrauski-model treats real money balances as a part of total real wealth thus serving as a store of value. There is, however, no treatment at all of the monetary functions with respect to the production side of the economy, nor is there any reference to the function of money as a unit of account.

The standard model has been modified in different ways during the last three decades. Various channels have been identified that can theoretically contribute to a positive effect of inflation on the steady-state values of per-capita consumption and capital intensity. These channels are:

- Recursive time preferences which are positively related to the level of total wealth (including real money balances): Epstein and Hynes (1989), Hayakawa (1992).
- A positive status effect of total wealth (including real money balances) in individual utility: Zou (1998).
- A finite time horizon of individual agents (as in OLG models or in perpetual youth models) without an operative bequest motive leading to a redistribution of wealth among generations: Marini and van der Ploeg (1988), Drazen (1981), Petrucci (1999), van der Ploeg and Alogoskoufis (1994), Mino and Shibata (1995), Ho (1996).
- An endogenous labour-leisure choice, if (as in the non-Walrasian setting of a monetary search model) the real balance effect on leisure dominates the consumption effect: Shi (1999).
 - Likewise, a bundle of negative steady-state effects of inflation have been discovered. They rely on:
- A cash-in-advance-constraint for investment goods or the (equivalent) treatment of real money balances as an additional factor of production: Levhari and Patinkin (1968), Stockman (1981).
- Real money balances as a factor of production in human capital formation if human capital then contributes to higher production or higher labour productivity: Marquis and Reffett (1991), Pecorino (1995), Chang (2002).
- Pecuniary transaction costs of inflation that are measured in output units: Zhang (2000), Jha, Wang and Chop (2002).
- Taxes levied on nominal interest rates and nominal capital gains leading to a fall in the real after-tax return on savings when inflation increases: Feldstein (1999).
- An endogenous labour-leisure choice with either money in the utility function, a cash-in-advance-constraint for consumption or a particular shopping-time technology, if the consumption effect of money growth on leisure dominates the real balance effect: Brock (1974), Cooley and Hansen (1989), Wang and Yip (1992), Wu and Zhang (1998). This seems to be the normal case.

Abstracting from the particular models with endogenous labour-leisure choice this overview reveals that positive growth effects of inflation depend on factors in the utility function that stimulate a higher capital

formation, whereas negative effects of inflation originate from changes in net production or in the aggregate productivity of factor inputs.

Judgements on the practical relevance of the different channels should again consider existing empirical evidence. On the one side, Holman (1996) finds some support for the assumption that money enters the utility function. Remember that this is a necessary, but not a sufficient condition for the emergence of a Tobin effect along the transition path. On the other side, there seems to be considerable empirical evidence that money influences the efficiency of aggregate production. This was confirmed in studies by Delorme, Thompson and Warren (1995) as well as by Nourzad (2002). In the latter paper separate stochastic production frontiers are estimated for annual panels of 10 developed and 10 developing countries over the period from 1981-1990. It turned out that the influence of real money balances on the efficiency in the production sector is particularly pronounced in higher-developed countries. A substantial negative effect of inflation on some general measure of aggregate productivity could also be derived in the cross-country growth regressions by Alexander (1997) that are based on an explicit growth-accounting framework. This study covered a sample of OECD countries for the period 1966-1988. So far, there is no direct empirical evidence, however, for the shopping-time cost or the pecuniary transaction costs of inflation.

III. The Missing Link: Inflation and the Elasticity of Substitution

As Khan and Senhadji (2000) note, the estimated relationship between inflation and growth does not provide the precise channels through which inflation affects grow, beyond the fact that is primarily through productivity since investment and employment are controlled for. The need to find a sensible way of how to integrate the complications and annoyances to day-to-day living caused by inflation was also stressed by Ragan (1998). He even proclaimed that a better understanding of these negative growth effects of inflation could be the most important justification for a long-term policy of disinflation.

Howitt (1993) had already underlined that the "transaction-impeding aspects of inflation" are much more difficult to incorporate into a simple general equilibrium framework than the "money-taxing aspect", even if the former aspect seems to be more important in the real world. These "transaction-impeding aspects" rely on the use of money as a unit of

account and a standard of deferred payment. They can only be incorporated indirectly in a simple monetary growth model by making some other key variable of the model depend upon inflation. Howitt favoured an explicit transaction cost function to include inflation and showed that all significant negative effects of inflation depend on the existence of some active transaction-impeding mechanism. The main problem with such a transaction cost function is that it affects actual production or the actual supply of one of the factors of production. It does not, however, catch the idea that a malfunctioning of markets reduces potential output. Therefore Klump and De La Grandville (2000) and Klump and Preissler (2000) proposed to look at the aggregate elasticity of substitution as a much more reasonable link between the flexibility of markets and a country's growth results. If the elasticity of substitution can really be used in this context it should also be possible to link it to monetary factors and disturbances.

In order to get a first idea of what might determine the elasticity of substitution it is convenient to remember what Hicks (1963) regarded as the three different ways in which the substitution between factors of production can take place: as inter-sectoral substitution of production, as intra-sectoral substitution of the known methods of production or as substitution by innovations. Klump and Preissler (2000) investigate different possible determinants of the elasticity of substitution. All relevant determinants, like the level of trade integration, the strength of trade unions or the contestability of markets by competitors influence the reallocation of real inputs following a change of relative prices and have an impact on the general efficiency of the production system. Thus the aggregate elasticity of substitution should be regarded as an aggregate index of flexibility that characterises a given market economy.

There are different ways of how to establish a link between inflation and the aggregate elasticity of substitution. One way stresses the information content of prices which is lowered by higher inflation (Tommasi 1994). Another way highlights the interaction between price stability and the integration of single actors in one common market (Ventura 1997). The focus of the last way (Huybens and Smith 1999) is the significant role of the financial system for the long-term functioning of developed market economies. The authors point out the potential risks for an efficient allocation of real resources that deviations from price stability may cause by undermining the financial system. In general, all three ways study very much in detail the role of money and markets as social insti-

tutions that are responsible for the exploitation of the wealth effects of an efficient division of labour.

In Tommasi's (1994) approach anticipated higher inflation is the cause of higher relative price variability and thus of increased uncertainty about future prices. Tommasi studies the market for a homogeneous good that can be either a consumption good or a factor of production. The good is being sold and purchased by a continuum of agents every period. Each visit on the market entails a search cost which is different for different agents. This heterogeneity maps into downward-sloping demand curves for individual sellers who set prices to maximise their expected profits. Sellers face downward-sloping demand curves because buyers are not fully informed: if they were, all output would be produced by the lowest-cost firms. In such a world, inflation exacerbates the informational problem by depreciating the information that current relative prices convey about future relative prices. Buyers react by holding smaller information stocks. This translates into higher reservation prices and enables sellers to charge higher prices. Additionally, production gets allocated toward high-cost producers, increasing overall production cost. To summarize, price instability moves the economy away from perfect competition, generating the loss of many of its efficiency properties.

Ventura (1997), building on earlier work by Azariadis (1996), analyses the structural transformation in a small country that opens up to the world market. He assumes that final good production uses a CES technology with two intermediate goods as input factors. One intermediate good is produced using only capital whereas the other is using only labor. It turns out from this model that via exporting intermediate capital goods on perfectly elastic world markets a small country can overcome the problem of decreasing returns to further capital accumulation and can thus grow without limits. In terms of the aggregate production function this means that market integration leads to an increase of the economy-wide elasticity of production. Recent papers by Ferreira and Trejos (2002) and Miyagiwa and Trejos (2003) support the view that the removal of barierrs to international trade leads to changes in the aggregate production technology that can be measured by an increase in the flexibility of factor substitution. Therefore the economy-wide elasticity of substitution should not be regarded as a purely technical parameter but as determined by the whole institutional framework that regulates the allocation of an economy's resources. Within this broad concept of determinants of factor substitution the important role of

price stability can be added quite easily. If inflation increases the transaction costs of trade market integration with domestic and foreign trading partners will shrink and the aggregate elasticity of substitution will decrease.

The last channel that becomes relevant in this context is a country's financial system. As Huybens and Smith (1999) demonstrate in a model with an explicit role for banks and secondary capital markets, higher inflation leads to lower capital accumulation and lower real activity. Inflation reduces not only the volume of bank lending activity, but also the trade in equity markets relative to GDP. Credit rationing may occur which prevents that the most efficient technique is used for production as it cannot be implemented. Khan, Senhadji and Smith (2001) generalise the idea that predictable changes in the rate of inflation should have significant real effects because this price instability increases the adverse selection and moral hazard problems in credit markets. The flexibility or inflexibility of the financial system to contribute to the most efficient reallocation of resources could be measured with the help of the aggregate elasticity of substitution. It should be noted that Huybens and Smith (1999) assume an aggregate CES production function with $\sigma > 1$. This can be taken as an indicator that countries with a more sophisticated financial system will also be those with a more intensive division of labour, more trade and higher wealth. Schreft and Smith (1997) show in a monetary growth model with explicit banking activities that an aggregate CES production function with $\sigma < 1$ can be responsible for endogenous volatility around the steady state.

Empirical studies of the aggregate elasticity of substitution have so far come to mixed results. Rowthorn (1999) found evidence that the elasticity of substitution is well below unity for a group of industrial countries. A similar result is presented by Ripatti and Vilmunen (2001) in a study of the Finnish economy. Duffy and Papageorgiou (2000) find in a large cross country study, supplementing nicely the study by Khan and Senhadji (2000), that elasticities of substitution significantly differ in developed and non-developed countries. Typically, the latter group has a σ well below unity, whereas in the former group σ is measured to range above unity. This proves in a certain sense the conjecture by Klump and De La Grandville (2000) that the process of economic development itself, usually regarded as a steady increase in an economy's division of labour and market flexibility, should be positively correlated with the aggregate level of σ .

Summarizing the different arguments there are plausible reasons to treat the rate of inflation Π as a factor that strongly influences the aggregate elasticity of substitution. Thus in the following section the implications of such a hypothesis:

$$\sigma = \sigma(\Pi)$$
 with $\sigma'(\Pi) < 0$

are analysed. It will be shown that inflation via marginal changes of the elasticity of substitution can cause rather significant effects for the real economy.

IV. Factor Substitution and Growth in a Monetary Optimising Model

In order to study potential indirect growth effects of inflation on real growth via changes of the elasticity of substitution the standard money growth model has to be adapted to incorporate a CES production function. Although the CES function was already introduced by Solow (1956) as a natural complement of neoclassical growth theory it took a long time before the elasticity of substitution was established as a possible determinant of growth in the modern growth literature. As De La Grandville (1989) and Klump and De La Grandville (2000) have shown consistent results concerning the growth effects of the elasticity of substitution can only be expected if they are derived within a particular "family" of CES functions.

Given the definition of the elasticity of substitution $\sigma = \frac{f'(k) \left[f(k) - kf'(k) \right]}{-kf''(k)f(k)}$ as a point elasticity, one particular "family" of CES function consists of all those functions that share the same baseline values for capital intensity (k_0) , per-capita production (y_0) and the marginal rate of substitution (s_0) . Members of one family are distinguished by different values of σ only, whereas different families are distinguished by different baseline values. One particular family of CES production functions is thus generated by "normalising" the parameters of the standard specification of the CES production function:

(1)
$$y = f(k) = C[\alpha k^{\psi} + (1 - \alpha)]^{\frac{1}{\psi}}, \quad \psi = \frac{\sigma - 1}{\sigma}$$

$$\alpha = \alpha(\sigma) = \frac{k_0^{1-\psi}}{k_0^{1-\psi} + s_0}, \quad C = C(\sigma) = y_0 \left[\frac{k_0^{1-\psi} + s_0}{k_0 + s_0} \right]^{\frac{1}{\psi}}.$$

y stands for per-capita production and k for the capital intensity. C, α are usually known as the CES function's efficiency and distribution parameter, respectively. They are now normalised with respect to the three baseline values. ψ is the constant distribution parameter, that can take values between $-\infty$ and 1.

(1) and (2) describe a normalised family of CES functions. Each member of this family shares one common point of reference in which only the elasticity of substitution may differ. It can be shown that each family includes three special members when σ takes the value of 1, 0 or $-\infty$. These are the linear, the Cobb-Douglas and the Leontief production functions, respectively. It should be noted, that only with a normalised CES production function the Leontief case is not restricted to equal values of the production coefficients for capital and labour. Instead, a general specification of the Leontief case can be derived, where the production coefficients are determined by the given baseline values of the capital intensity.

Klump and De La Grandville (2000) as well as Klump and Preissler (2000) could derive that an increase of the elasticity of substitution has (for $k \neq k_0$) an unambiguously positive influence on per-capita production:

$$(3) \qquad \frac{\partial y}{\partial \sigma} = -\frac{1}{\sigma^2} \frac{1}{\psi^2} y \left\{ \pi \ln \left(\frac{\pi_0}{\pi} \right) + (1 - \pi) \ln \left(\frac{1 - \pi_0}{1 - \pi} \right) \right\} > 0.$$

The influence of the elasticity of substitution on the profit share:

(4)
$$\pi = \frac{f'(k) \cdot k}{f}$$

depends, however, on the actual level of the capital intensity relative to its baseline level:

(5)
$$\frac{\partial \pi}{\partial \sigma} = \frac{1}{\sigma^2} (1 - \pi) \pi \ln \left[\frac{k}{k_0} \right] \le 0 \quad \text{if and only if } k \le k_0$$

From the definition of the profit share π (4) as well as the derivations (3) and (5) it follows that $k > k_0$ is sufficient to ensure that the influence of the elasticity of substitution on the marginal product of capital is unambiguously positive:

(6)
$$\frac{\partial f'(k)}{\partial \sigma} = \frac{y}{k} \cdot \frac{\partial \pi}{\partial \sigma} + \frac{\pi}{k} \cdot \frac{\partial y}{\partial \sigma} > 0, \quad \text{if } k > k_0.$$

In Klump (2001) the result (6) is of central importance for the proof that in an intertemporal optimising growth model the comparative static effect of a change in the elasticity of substitution on capital intensity and per capita consumption is strictly positive (for $k = k^* > k_0$):

$$\frac{dk^{*}}{d\sigma} = \frac{1}{-f''(k^{*})} \cdot \frac{\partial f'(k^{*})}{\partial \sigma} > 0$$
(7)
$$\frac{dc^{*}}{\partial \sigma} = \frac{1}{-f''(k^{*})} \left\{ -f''(k^{*}) \frac{\partial y^{*}}{\partial \sigma} + \left[f'(k^{*}) - (\delta + n) \right] \frac{\partial f'(k^{*})}{\partial \sigma} \right\} > 0.$$

Here δ is the constant depreciation rate and n is the constant growth rate of the labour force. It should be noted that $k^* > k_0$ can regarded as the usual case for economies where capital plays a major role in the production process and that standard CES production function implicitly assume a normalisation $k_0 = 1$.

Now, equation (7) can be used for studying the effects of inflation in a monetary growth model. Following Sidrauski (1967) real money balances per head $(m=\frac{M}{PN})$ are considered as an additional argument of the individual utility index. We specify instantaneous utility as a CIES function, where $\theta(0 \le \theta \le 1)$ denotes the weight of real money balances in the utility index:

(8)
$$u(c,m) = \frac{1}{1-n} \left[(c^{1-\theta} m^{\theta})^{1-\eta} - 1 \right]$$

For $\eta=1$ one has the special case of a logarithmic utility function $u(c,m)=(1-\theta)\ln c + \theta \ln m$.

Money is supplied by the government via transfers of z to each household. Since these transfers are assumed to be proportional to real money balances per head the government's budget constraint is given by:

$$(9) z = \mu m$$

 μ denotes the growth rate of nominal money balances. Since we assume perfect foresight, expected and actual inflation rates coincide, $\Pi^e = \Pi$. The dynamics of real money are given by:

$$\dot{m} = \left[\mu - \Pi - n\right]m$$

The individual's real wealth v consists of real capital k and real money balances m. The dynamics of real wealth are described by:

(11)
$$\dot{v} = f(k) - c - (n+\delta)k + z - (\Pi + n)m$$

Finally, we define by $x = \frac{k}{v}$ and $(1-x) = \frac{m}{v}$ the share of real capital and real balances in total real wealth, respectively. This enables us to consider total real wealth v as a second state variable and the share of real capital in total real wealth x as an additional control variable.

The intertemporal optimising problem now becomes:

(12)
$$\max_{c,x} \int_{0}^{\infty} u(c,m)e^{-\rho t}dt = \max_{c,x} \int_{0}^{\infty} u[c,(1-x)v])e^{-\rho t}dt$$

subject to the intertemporal budget restriction (11) and making use of the specifications (1), (2) and (8). The Hamiltonian of the optimisation problem can be written as:

(13)
$$H = u[c, (1-x)v] + \phi[f(xv) - c - (n+\delta)xv + z - (\Pi+n)(1-x)v]$$

From (13) we can derive a modified Keynes-Ramsey equation for the dynamics of per capita consumption together with two equations which describe the dynamics of real money balances and real capital per head:

$$g_{c} = \frac{\dot{c}}{c} = \frac{1}{\theta + \eta(1 - \theta)} \left[f'(k) - n - \delta - \rho \right]$$

$$+ \frac{\theta(1 - \eta)}{\theta + \eta(1 - \theta)} \left[f'(k) + \mu - \delta - n - \frac{\theta}{1 - \theta} \frac{c}{m} \right]$$

$$g_{k} = \frac{\dot{k}}{k} = \frac{f(k)}{k} - (n + \delta) - \frac{c}{k}$$

$$g_{m} = \frac{\dot{m}}{m} = f'(k) + \mu - \delta - n - \frac{\theta}{1 - \theta} \frac{c}{m}$$

The steady state of the system is given by the following conditions:

(15)
$$f'(k) = \delta + n + \rho$$

$$c^* = f(k^*) - (n + \delta)k^*$$

$$\frac{c^*}{m^*} = \frac{1 - \theta}{\theta} (\rho + \mu)$$

In addition we know that in the steady state the rate of inflation is:

$$\Pi^* = \mu - n$$

The steady state represented by (15) and (16) reflects superneutrality of money in the sense that changes in the rate of money expansion only influence the rate of inflation Π^* and the level of real money balances m^* . They do not have any impact on the steady values c^* and k^* , as long as σ is treated as exogenous. If σ is negatively correlated with the rate of inflation, the influence of changes in σ on these steady-state values is the same as in the non-monetary model. From the equations (15), (16) and (7) it can be concluded that:

(17)
$$\frac{dk^*}{d\mu} = \frac{d\Pi}{d\mu} \sigma'(\Pi) \frac{dk^*}{d\sigma} < 0$$

$$\frac{dc^*}{d\mu} = \frac{d\Pi}{d\mu} \sigma'(\Pi) \frac{dc^*}{d\sigma} < 0.$$

A higher monetary growth rate is thus responsible not only for higher inflation but also for a lower capital intensity and lower per-capita consumption in the steady state.

The transitional dynamics of our model are much more complex. It was shown by Fischer (1974) and Cohen (1985) for the original Sidrauski model that monetary policy can have positive real effects along the transition path as long as the utility function (8) is non-logarithmic. These positive growth effects of inflation are due to changes in the structure of nominal interest rates along the transition path provoking a reallocation in the wealth portfolio and an acceleration in real capital formation. Quantitatively this effect was found to be not very strong, however. So it can be conjectured that in our extended Sidrauski model the positive growth effect of inflation is already dominated for low rates by a negative effect of inflation on the elasticity of substitution that decreases the

marginal productivity of capital. For the proof of this conjecture we log-linearise the system (14). The Jacobian matrix J becomes:

(18)
$$\begin{pmatrix} -\frac{\theta(1-\eta)(\rho+\mu)}{\theta+\eta(1-\theta)} & \frac{1+\theta(1-\eta)}{\theta+\eta(1-\theta)} k^* f''(k^*) & \frac{\theta(1-\eta)(\rho+\mu)}{\theta+\eta(1-\theta)} \\ -\frac{c^*}{k^*} & \rho & 0 \\ -(\rho+\mu) & k^* f''(k^*) & (\rho+\mu) \end{pmatrix}$$

We find that $trace\ J>0$ and that $det\ J=\frac{\left(\rho+\mu\right)c^*f''(k^*)}{\theta+\eta(1-\theta)}<0$. This implies that the system has two positive and one negative root. The negative root represents the unique equilibrium path.

From (18) we derive the characteristic equation, taking into account the steady-state values (15):

(19)
$$\psi = -\varepsilon^{3} + \varepsilon^{2} \left[\rho + \frac{(\rho + \mu)\eta}{\eta + \theta(1 - \eta)} \right] - \varepsilon \left[\frac{\rho(\rho + \mu)\eta}{\eta + \theta(1 - \eta)} \right] - c^{*}f''(k^{*}) \left[\varepsilon \frac{1 + \theta(1 - \eta)}{\eta + \theta(1 - \eta)} - \frac{\rho + \mu}{\eta + \theta(1 - \eta)} \right] = 0.$$

We denote by $\varepsilon < 0$ the negative root of the dynamic system by $\lambda = -\varepsilon$ the speed of convergence towards the steady state. As a first step we can analyse how the rate of convergence is influenced by increased monetary growth in the context of an endogenous elasticity of substitution. It follows from (19) that:

(20)
$$\frac{d\lambda}{d\mu} = -\frac{\partial\psi}{\partial\mu} \bigg/ \frac{\partial\psi}{\partial\varepsilon} \frac{\partial\varepsilon}{\partial\lambda} = \frac{\partial\psi}{\partial\mu} \bigg/ \frac{\partial\psi}{\partial\varepsilon}.$$

Since for $\varepsilon = 0$ and $\theta > 0$, ψ is negative, for $\varepsilon < 0$ we have $\partial \psi / \partial \varepsilon < 0$. We can also derive from (19):

$$\frac{\partial \psi}{\partial \mu} \left[\theta + \eta (1 - \theta) - \frac{\eta(\rho + \mu)}{\varepsilon} \right]$$

$$= cf''(k^*) \left[\frac{\theta (1 - \eta)^2}{\theta + \eta (1 - \theta)} \right] + \frac{\partial (cf''(k^*))}{\partial \mu} (\rho + \mu) \left[1 - \frac{\eta(\rho + \mu)}{\varepsilon (\theta + \eta(1 - \theta))} \right]$$

The term in parenthesis on the left-hand side of (21) is unambiguously positive as the root ε is negative. The first term of the left-hand side is

clearly negative, if the utility function is not logarithmic ($\eta \neq 1$). In order to show that also the second term is negative one should recall that:

$$(22) c^*f''(k^*) = -\frac{1}{\sigma}(1-\pi^*)(\rho+n+\delta)\left[\frac{\rho+n+\delta}{\pi^*}-(n+\delta)\right] < 0$$

and therefore:

(23)
$$\frac{\partial (c^*f''(k*))}{\partial \mu} = \sigma'(\Pi) \frac{\partial (c^*f''(k*))}{\partial \sigma}$$

$$= \sigma'(\Pi) \left\{ \frac{1 - \pi^*}{\sigma} \left[\frac{\rho + n + \delta}{\pi^*} - (n + \delta) \right] + \frac{\partial \pi^*}{\partial \sigma} \left[\frac{\rho + n + \delta}{\pi^*} - (n + \delta) + (1 - \pi^*) \left(\frac{\rho + n + \delta}{\pi^{*2}} \right) \right] \right\}$$

From (5) it follows that (23) is strictly negative for $k^*>k_0$. This result implies in (21) that $\frac{\partial \psi}{\partial \mu}<0$ and in (20) that $\frac{\partial \lambda}{\partial \mu}>0$. We can conclude that with an endogenous elasticity of substitution the effect of a higher monetary growth rate on the speed of convergence is positive even if the utility function in non-logarithmic ($\eta\neq 1$). This is not surprising as a lower elasticity of substitution reduces the level of the state-state that has to be approached.

As a second step we look at the linearised evolution of the capital intensity around the steady state:

$$\dot{k} = \lambda (k^* - k_t).$$

In the original Sidrauski model with an exogenous elasticity of substitution the steady-state value k^* in (24) is not changed by monetary growth, and only the speed on convergence λ increases. In our extended model with an endogenous elasticity of substitution we can derive from (24) that:

(25)
$$\frac{d\dot{k}_t}{du} = \frac{d\lambda}{du}(k^* - k_t) + \lambda \frac{\partial k^*}{\partial u}.$$

It is obvious from (25) that three different effects exercise an influence on the evolution of the capital intensity during transition. These are the influence of monetary growth on the speed of convergence, the distance from the steady state and the change of the steady-state capital intensity that is caused by higher monetary growth and inflation, respectively.

While the latter effect is cleary negative as it was shown in (17), the two former effects are positive. With increasing monetary expansion one can expect that the negative allocation effect of inflation becomes more and more dominant. The positive effect of monetary expansion on the speed of convergence was found to be extremely small in quantitative experiments with a wide range of possible parameter values (Fahr 2002). Large deviation from the steady state can create the basis for positive effects of monetary growth during transition, but they are finally reversed once the system approaches its long term equilibrium.

Finally we look at the transitional growth of per-capita income. From $\dot{y}=f'(k)\dot{k}$ it follows that:

$$(26) \qquad \frac{\partial \dot{y}}{\partial \mu} = \frac{\partial (f'(k)\dot{k}_t)}{\partial \mu} = \frac{\partial (f'(k)\dot{k}_t)}{\partial \sigma} \sigma'(\Pi) + \frac{d\lambda}{d\mu} f'(k)(k^* - k_t) + \lambda f'(k) \frac{\partial k^*}{\partial \mu}.$$

Beside the three effects from (25) we find a further impact of monetary expansion that relies on a direct influence of inflation on the marginal product of capital. Considering (6) this effect is strictly negative for all $k > k_0$. Summing up the four different effects leads us to a proof of our conjecture: Close to the steady state the total impact of higher monetary expansion on the transitional growth of the per-capita income can already become negative at very low rates of inflation, if the important consequences of inflation for factor substitution are taken into account in an adequate way. Our simple monetary growth model with endogenous elasticity of substitution is therefore capable of generating results that are in accordance with most contributions to the empirical literature.

V. A Simulation of the Model

For a further illustration of the results we present results of a numerical simulation of the model applying the method proposed by Brunner and Strulik (2002). We choose the following basic parameters, partly inspired by comparable assumptions in Turnovsky's (2002) numerical analysis of convergence paths in the neoclassical growth model:

$$\rho = 0.04
n = 0
\delta = 0.01
\eta = 0.4
\theta = 0.1$$

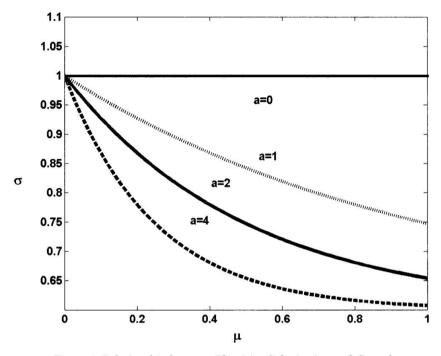


Figure 1: Relationship between Elasticity Substitution and Growth in Nominal Money Supply

We further assume that $k_0 = 1$ as in standard CES functions and then according to (2) the constant CES parameters:

$$C = 2$$
 $\alpha = 0.3$

For the functional relationship between the elasticity of substitution and the inflation rate we choose the following exponential form:

$$\sigma = 0.6 + 0.4 \cdot \exp(-a \cdot \Pi)$$

As it can be seen in Figure 1, the parameter a determines how strongly the factor elasticity reacts to changes in the growth of nominal money supply. As a starting point we take the Cobb-Douglas case with a=0 and therefore $\sigma=1$.

This relationship can now be used to study the effects of monetary expansion and inflation on production growth in the neighbourhood of the steady state at a capital intensity of k=3.8. The results for various level

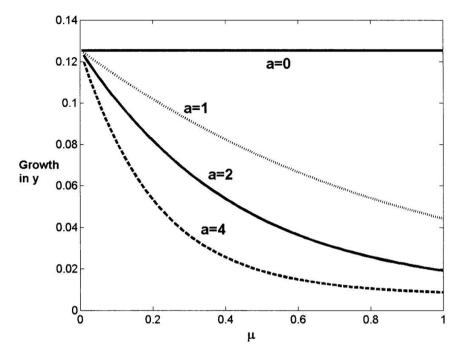


Figure 2: Relationship between Monetary Expansion and Production Growth at k = 3.8

of the parameter a can be seen in Figure 2. Even when the negative relationship between inflation and the elasticity of substitution is not very pronounced (and despite a small positive convergence effect of higher inflation) higher growth in money supply reduces production growth significantly.

VI. Conclusions

In two important survey articles, Issing (2000) and Temple (2000) reviewed the latest contributions to monetary growth theory and their implications for monetary policy. Their judgements were rather sceptical. Issing (2000, 324) underlined that "... the formal theoretical literature regarding the impact of money on the long-run growth path of output yields ambiguous results." And Temple (2000, 419) summarises that "... many of the ideas surrounding inflation and growth are essentially short stories. The ideas are often difficult to formalise, and testable implica-

tions of specific theories are few and far between." Given the high political importance of the inflation-growth-relationship this not very precise knowledge is rather disappointing.

Our conclusions, in contrast, are much more optimistic. The results of empirical studies seem to converge to the finding that for developed countries a threshold at a rather low level of inflation (not more than 2 or 3 percent) exists beyond which the inflation-growth relationship becomes significantly negative. And in the field of theoretical modelling the concept of the elasticity of substitution can help to emphasise the powerful (negative) link between inflation, aggregate productivity and growth performance. It is shown in an analysis of the transitional dynamics of a simple monetary growth model with endogenous elasticity of substitution, that this link can already dominate all possible positive effects of inflation at very low rates. In particular such positive effects decline the more the system approaches its long-term equilibrium.

In a more general perspective this paper tries to contribute to the ongoing debate of institutional influences on the process of growth and development. The eminent role of institutions and institutional quality as determinants of growth is strongly defended in a recent paper by Rodrik, Subramanian and Trebbi (2002). Money is certainly one of the most important social institutions. It enables a society to exploit the positive welfare effects of an increasing division of labour. Hence, the most important justification for price stability should be to safeguard this beneficial social role of money. Since modern growth models have difficulties in giving an explicit meaning to problems of institutional change it is understandable that also monetary growth theory did not consider this aspect in an adequate way. This paper tried to shed some light on this difficult relationship by taking the aggregate elasticity of substitution as the main channel on which the institutional role of money enters a simple monetary growth model.

Highly industrialised countries can be assumed to have a relatively high elasticity of substitution reflecting the highly sophisticated structure of institutions that play a role in the increasing division of labour. The more sophisticated the institutional framework of a market economy becomes, however, the more fragile it becomes in the case of monetary disturbances. Thus for a central bank of a highly developed country the goal of price stability within very narrow limits seems to be the most promising way to contribute to high future growth. In countries where other factors (like rigidities on the labour markets or imperfect competi-

tion on other markets) have already reduced the general flexibility of the production system, it is important that monetary policy does not additionally affect the aggregate elasticity of substitution by tolerating increasing price instability.

The agenda for future research in this field includes the following topics. On the theoretical level it seems worth to look not only at inflation as a determinant of the aggregate elasticity of substitution but also at real money balances. This would make it possible to investigate in much more detail interactions between the individual and the average level of real balances that might be responsible for some interesting externalities in the use of money. As in most contributions to monetary growth theory it would also be worth considering an endogenous treatment of the labour supply. Klump (2001) has already studied interactions between the elasticity of substitution, monetary policy and endogenous labour supply in a monetary growth model. However, the endogenous change of the elasticity of substitution following an increase in the inflation rate has not yet been considered.

The agenda for empirical research contains, of course, more detailed studies of the value and of the determinants of the aggregate elasticity of substitution. These studies should take into account the normalisation procedure that was shown to be essential for a consistent treatment of CES functions in growth models. Normalisation puts additional restrictions on the estimated parameters and might help to produce statistically stable estimation results. The supply system approach proposed by Willman (2002) seems to be flexible enough to integrate the idea of normalisation and should therefore be considered a good framework for further empirical studies of CES production functions. First results in this field can be found in Klump, McAdam and Willman (2004).

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Summary

Inflation, Factor Substitution and Growth

Recent empirical studies on the inflation-growth-relationship underline that inflation has negative growth effects under relatively modest rates. Most contributions to monetary growth theory, however, have difficulties in explaining such a pattern. It is shown in this paper that this problem can be overcome by establishing a link between monetary instability and the aggregate elasticity of factor substitution. Several microeconomic justifications can be found for a negative influence of inflation on factor substitution. It turns out that in a simple neoclassical monetary growth model this effect is usually strong enough to question the superneutrality benchmark result in the steady state and to dominate all potential positive effects of inflation along the convergence path. (JEL E52, O11, O41)

Zusammenfassung

Inflation, Faktorsubstitution und Wirtschaftswachstum

Jüngere empirische Studien über den Zusammenhang zwischen Inflation und Wachstum zeigen, dass Inflation auch schon bei relativ geringen Raten negative Wachstumseffekte besitzen kann. In der monetären Wachstumstheorie fehlt es jedoch an Erklärungen dafür. Wie im vorliegenden Beitrag gezeigt wird, können die negativen Wachstumseffekte von Inflation unter Rückgriff auf inflationsbedingte Veränderungen der gesamtwirtschaftlichen Substitutionselastizität erklärt werden. In einem einfachen Modell der monetären neoklassischen Wachstumstheorie lässt sich nachweisen, dass dieser Effekt normalerweise groß genug ist, um mögliche positive Wachstumseffekte der Inflation zu dominieren.

Résumé

Inflation, substitution de facteurs et croissance économique

Des études empiriques récentes sur le rapport entre l'inflation et la croissance montrent que l'inflation, même à des taux peu élevés, peut affecter négativement la croissance économique. La plupart des théories sur la croissance monétaire ont néanmoins des difficultés à expliquer ce phénomène. Dans ce travail, l'auteur montre que ce problème peut être surmonté en établissant un lien entre l'instabilité monétaire et l'élasticité aggrégée de substitution de facteurs. Plusieurs justifications microéconomiques montrent une influence négative de l'inflation sur la substitution de facteurs. Dans un modèle simple de la théorie de la croissance monétaire néoclassique, on peut prouver que cet effet est normalement suffisamment important pour dominer les effets positifs potentiels de l'inflation sur la croissance.