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# Why Simple, When it Can Be Difficult?

## Some Remarks on the Basel IRB Approach

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## I. Introduction

In the focus of the current discussion about the New Basel Capital Accord is the calibration of the risk weight function for the credit risk measurement. From the Quantitative Impact Study it turned out that the initial proposed risk weight function will result in an average capital requirement well above the current 8%, this caused a kind of a storm of protest among market participants and especially among politicians. The committee reacted by proposing potential modifications to the risk weight function, intended to mitigate the average capital burden, which are now part of the recently published third Consultative Document.

But, while altering the parameters, the approach in general has been left unchanged.

The IRB approach and in particular the IRB risk weight function can be considered as one of the major innovations in Basel II.

The calculation of risk weights depends directly on the probability of default of a given obligor, and the connection between *PD* and corresponding risk weights is given by a continuous function. It is obvious, that this risk weight function is one of the crucial building blocks in the IRB approach and in the overall Basel II proposal.[1] The shape, i.e. the graph of the risk weight function is thereby the decisive factor that determines the economy-wide level of regulatory capital that will be required in future. Not at least for this reason, the weighting factors have been vividly disputed in recent discussions.[2] The parties involved not

[1] Recall that it is intended, to induce the majority of banks to use the IRB approach.
[2] See for example the compilation of comments on www.bis.org/bcbs/cacomments.htm.

only included academics and bankers, but even politicians felt that they had to comment on the proposals.

In our opinion, it is a necessary prerequisite for a sound discussion to get a clear picture of the basic underlying interrelations, and the aim of the following section is to provide a detailed treatment of the IRB risk weight function and to disclose its basic assumptions. Further on, we will propose a much easier approach to model the risk weight function.

The paper is organized as follows: In the next section the methodical shift in capital rules will be described. Section III. will provide a detailed derivation of the IRB risk weight function. Section IV. is going to discuss the basic assumptions, and section V. will describe the process as being basically determined by political goals. Section VI. is going to propose a radically simpler approach. We will finish with a conclusion that sums up the main points.

## II. Risk-bucketing Approach and Portfolio Models

As has been already mentioned, the IRB approach can be considered as a *conceptually* new approach to regulatory capital rules. The current rules (Basel I, 1988), as well as the proposed standard approach, follow the so-called *risk-bucketing approach,* whereby the assets of a bank with similar risk characteristics are bundled together in risk-buckets, and are charged with a uniform capital requirement ratio. It is assumed that the single assets in a bucket have the same risk, that is risk-buckets are considered as being homogeneous.

The decisive characteristic of the risk-bucketing approach is the so-called portfolio-invariance. The level of the marginal requirement for single assets does only depend on its specific characteristics and not on the characteristics of the portfolio in which it is embedded. This implies in particular that no diversification effects are taken into account. It need not be stressed that this is a major drawback for modelling credit risk.

Nevertheless, the approach has some important advantages: The administrative burden is relatively low, since the capital requirement of the overall portfolio is simply the weighted sum of the marginal requirements of the single assets. Related to this, risk-bucketing systems do not impose complex reporting requirements. In particular for small and medium sized banks, the simplicity of a risk-bucketing approach is the major argument.

On the other hand, sophisticated credit risk models are available[3], which are not only able to measure risk more accurately, but can account explicitly for portfolio effects.

The committee did recognize the need to apply portfolio models, but also realized that full-blown portfolio models are currently too complex, given the limitations on the part of the majority of banks. A solution for this dilemma is provided by the results due to Gordy.[4]

Gordy showed that under certain assumptions, portfolio models behave like risk-bucketing models. That is, the marginal requirements for single assets do only depend on their specific characteristics.

Therefore, it is possible to retain on the one hand the practical simplicity of the risk-bucketing approach, while on the other hand diversification effects can be captured. From a regulatory point of view, Gordy seems to have cut the Gordian knot.

But there is a price you have to pay for it: The result is subject to mainly two assumptions. These are:

1. There is only a single systematic factor driving correlations across obligors.
2. Portfolios are asymptotically fine-grained, i.e. no single exposure accounts for a substantial share of total exposure.

We will give an assessment of the assumptions in a later section, but first, we will turn to an analysis of this simplified portfolio-model approach.

## III. Derivation of the Risk Weight Function

In the following, we will restrain ourselves to the risk weight function for corporate exposures. The risk weight function ($RW_{CD3}$) according to the third Consultative Document[5] has the following structure:

$$RW_{CD3} = LGD \cdot N\left(\frac{N^{-1}(PD)}{\sqrt{1-\rho}} + \sqrt{\frac{\rho}{1-\rho}} \cdot N^{-1}(0.999)\right) \cdot MA \cdot 12.5 \tag{1}$$

where $N(\cdot)$ denotes the cumulative distribution function of the standard normal distribution and $N^{-1}(\cdot)$ denotes its inverse. $LGD$ denotes the loss

[3] E.g. CreditMetrics, CreditRisk$^+$, CreditPortfolioView, KMV.
[4] *Gordy* (2000), *Gordy* (2001).
[5] *Basel* (2003).

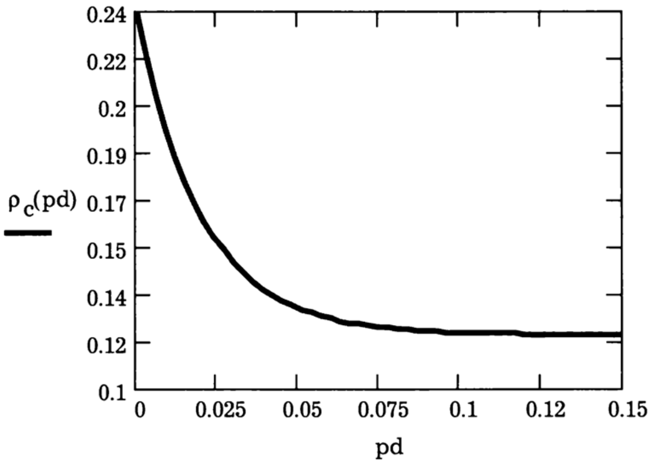


*Fig. 1: Correlation for corporate exposure*

given default, $MA$ stands for a maturity adjustment factor to account for different effective maturities ($m$) and is determined by the equation:

(2) $$MA(m, PD) = \frac{1 + (m - 2.5) \cdot (0.08451 - 0.05898 \cdot \log(PD))^2}{1 - 1.5 \cdot (0.08451 - 0.05898 \cdot \log(PD))^2}$$

and $\rho$ is a correlation parameter, determined by the equation:

(3) $$\rho(PD) = 0.12 \cdot \left(\frac{1 - e^{-50 \cdot PD}}{1 - e^{-50}}\right) + 0.24 \cdot \left(1 - \frac{(1 - e^{-50 \cdot PD})}{1 - e^{-50}}\right)$$

$\rho$ is specified as a decreasing function of $PD$ and ranges from a minimum of 0.12 to a maximum of 0.24.[6] Note by the way, that $e^{-50} = 1.9 \cdot 10^{-22}$, so that, for all practical purposes, the denominator in (3) equals 1, and you can simply write: $\rho(PD) = 0.12 + 0.12 \cdot e^{-50PD}$.

Figure 1 shows the graph of the correlation as a function of $PD$.

The formula ($RW_{CD3}$) differs from the formula put forward in the second Consultative Document in some aspects, but from a conceptual point of view, both have the same structure. The $CD2$-formula reads as follows:

(4) $$RW_{CD2} = 0.02 \cdot LGD \cdot N\Big(1.118 \cdot N^{-1}(PD) + 1.288\Big) \cdot M \cdot 976.5$$

[6] Or more precisely: $0.12 \leq \rho \leq 0.2382$, since $\min(PD) = 0.0003$ is given.

In comparison to the $CD2$-formula, the fixed coefficients of 1.118 and 1.288 have been replaced by a term, involving the correlation parameter $\rho$.

The second important modification concerns the scaling factor of 976.5, which does not appear in the $CD3$-formula. Instead, the implicit level of confidence has increased from 99.5% to 99.9%. (The confidence level is reflected in the term $N^{-1}(0.999)$ in the $CD3$-formula.)

The $LGD$ factor remains unchanged and is still assumed to be 0.45 in the IRB basic approach.

The factor 12.5 in the $CD3$-formula only serves as to transform the capital requirements into risk weights. If $RW_{CD3}$ is multiplied by the solvency ratio of 8% this factor cancels out.

So far, those are the main formulas, as they have been presented in the Basel II proposals. But besides some comments made in the supporting document[7], most of the theoretical background, from which the formulas have been derived, remains unexplained.

One may argue that the majority of readers might have a more user-orientated approach and are not very much interested in the theoretical foundations of the risk weight function. But, we think that more transparency would increase the acceptance of the approach and would enable a well-founded discussion.

## *1. Portfolio Model Underlying the IRB Approach*

The portfolio model, which underlies the committees proposal, is mainly based on results due to Gordy[8] and has the following basic assumption:

A risk index for a given obligor $i$ is assumed, which is composed of two components. On the one hand, it is driven by the idiosyncratic risk of the corresponding obligor and on the other hand, it is driven by a systematic risk factor, which underlies every obligor. Let $Z_i$ denote the risk index for obligor $i$, $X$ the systematic risk, and $\epsilon_i$ the idiosyncratic risk, then $Z_i$ is specified according to:

$$Z_i = w_i \cdot X + \sqrt{1 - w_i^2} \cdot \epsilon_i \qquad i = 1, \cdots, n \tag{5}$$

7 Basel 2001b, section 172.

8 See e.g. *Gordy* (2000), (2001), see also *Finger* (1999, 2001) and *Vasicek* (1991) for related work.

where $w_i$ can be thought of as the weight by which the risk index of obligor $i$ is driven by the systematic risk factor $X$. It can be assumed that every obligor has the same weight, i.e. $w_i = w_j$, so that we can drop the subscript in what follows.

Further, it is assumed that both $X$ as well as $\epsilon_i$ are standard normally distributed random variables and that they are mutually independent, i.e.:

$$E(X, \epsilon_i) = 0, \quad E(\epsilon_i, \epsilon_j) = 0, \quad i \neq j, \forall i, j \tag{6}$$

It is also important to understand why $X$ and $\epsilon_i$ are weighted with $w_i$ and $\sqrt{1 - w_i^2}$ respectively, since these terms eventually show up in the risk weight function.

The expectation in the present context is zero, while $Var(wX) = w^2 Var(X) = w^2$ and $Var(\sqrt{1 - w^2} \cdot \epsilon_i) = 1 - w^2$ and thus, we have:

$$E(Z_i) = 0 \qquad Var(Z_i) = 1 \tag{7}$$

which means that the weights are chosen such that $Z_i$ is *again* standard normally distributed.

If we further assume that the portfolio consists of $N$ loans, each having an exposure of $1/N$, a loss given default of $LGD = 1$ and each having a probability of default of $p$, then obligor $i$ defaults, if

$$Z_i < \alpha \tag{8}$$

where $\alpha = N^{-1}(p)$.

For a given realization of $X = x$ (i.e. for the conditional probability $p(\cdot|X)$), the condition of default is given by the inequality:

$$\epsilon_i < \frac{\alpha - w \cdot x}{\sqrt{1 - w^2}} \tag{9}$$

Since $\epsilon_i$ is by definition standard normally distributed, the conditional probability that obligor $i$ will default is given by:

$$\begin{aligned} p(i|X) &= P\left(\epsilon_i < \frac{\alpha - w \cdot X}{\sqrt{1 - w^2}} \, |X = x\right) \\ &= N\left(\frac{\alpha - w \cdot X}{\sqrt{1 - w^2}}\right) \end{aligned} \tag{10}$$

The crucial point is that once we have conditioned on the common factor $X$, the individual obligor defaults are driven only by the idiosyncratic terms and are therefore *independent*.

This independence in turn is eventually the justification to apply risk bucketing rules.

Next, we can ask who accounts for the variance in the portfolio? Is it largely driven by the variance stemming from the market, or is it driven by idiosyncratic variance. In the appendix it is shown, that for an asymptotically fine-grained portfolio the variance of the portfolio is completely determined by the variance of the market. Or speaking more technically, we have[9]

$$\lim_{n \to \infty} Var(PF_n) - Var(E(PF_n|X)) = 0 \tag{11}$$

An important implication of this asymptotical behavior of the portfolio is, that it enables us to compute the $q^{th}$ quantile of the portfolio distribution. Given the fact, that the portfolio value of this asymptotic portfolio depends solely on the value of $X$, it is possible to map the quantile of the $X$-distribution to the portfolio quantile.

And if we impose the additional restriction, that the individual conditional expected loss functions are non-decreasing, we can find the relationship:

$$\alpha_q(E(PF_n|X)) = E(PF_n|\alpha_q(X)) \tag{12}$$

In words: The $q^{th}$ quantile of the conditional expected loss function equals the expected loss function conditional on the $q^{th}$ quantile of the distribution of the systematic factor $X$.

We only have to take care of the fact that $X$ enters the formula with a negative sign, so that the $q^{th}$ quantile of the loss distribution equals the $(1-q)^{th}$ quantile of the distribution of $X$. Substituting into (10) gives:

$$p(PF_n|X_q) = N\left(\frac{\alpha - w \cdot N^{-1}(1-q)}{\sqrt{1-w^2}}\right) \tag{13}$$

Finally, substituting $N^{-1}(PD)$ for $\alpha$, we get:

$$p(PF_n|X_q) = N\left(\frac{N^{-1}(PD) - w \cdot N^{-1}(1-q)}{\sqrt{1-w^2}}\right) \tag{14}$$

9 See *Gordy* (2001), p. 7.

Now, we are close to the formula, we wanted to arrive at. The last step is merely to replace $w^2$ by $\rho$.

This is justified, since in this special setup, we can replace the weighting parameter $w$[10] by $\sqrt{\rho}$, which stands for the asset-correlation between pairwise assets. This follows from the fact that we have assumed the idiosyncratic factors to be independent and identically distributed random variables. So evaluating the correlation of the risk index of two obligers $i$ and $j$, we get

$$\rho = corr(Z_i, Z_j) = E(Z_i \cdot Z_j) = E\left((wX)^2\right) = w^2 \tag{15}$$

where we have repeatedly used the fact that $X$ and $\epsilon_i$ are standard normally distributed.

So, finally we arrive at the *conditional portfolio loss function* for the $q^{th}$ quantile as a function of the probability of default ($PD$) and the asset-correlation parameter ($\rho$):

$$p(PF_n|X_q) = N\left(\frac{N^{-1}(PD)}{\sqrt{1-\rho}} - \frac{\sqrt{\rho}}{\sqrt{1-\rho}} \cdot N^{-1}(1-q)\right) \tag{16}$$

We can think of this formula as being the marginal contribution of a given asset to the portfolio capital requirements. And since in the context of this model, marginal requirements are portfolio-invariant, i.e. they do not depend on how they affect diversification, they can simply be summed up to arrive at the capital requirement for the overall portfolio.

Going back to (1), we see that the above conditional portfolio loss function (choosing $1-q=0.999$) is the key ingredient of the IRB-risk weight function.

What is added in (1) are merely scaling parameters, the loss given default ($LGD$), and the maturity adjustment factor ($MA$).

With the above function it gets clear, where the coefficients 1.118 and 1.288 in the initial (CD2) formula stem from. They have been calculated by assuming a fixed correlation of $\rho = 0.2$ (or a weight of $w = 0.447$) and a confidence level of $1-q = 0.995$.[11]

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10 Recall that $w$ indicates the weight, by which the risk index of an individual obligor is driven by the common systematic factor.

11 See Basel 2001b, section 172.

## IV. Discussion of Assumptions

In general, the appropriateness of a model depends on its assumptions. So, it seems to be reasonable to recall briefly the crucial assumptions, the presented model is based upon. We may note the following four:

$A1$. The portfolio is asymptotically fine-grained

$A2$. There is a single systematic risk factor

$A3$. Random variables are assumed to be normally iid

$A4$. The conditional expected loss functions are non-decreasing

To summarize, the following equation shows, where the assumptions enter the model.

(17) $$\alpha_q(PF_n) \xrightarrow{A1} \alpha_q(E(PF_n|X)) \overset{A2,\,A4}{\Longleftrightarrow} E(PF_n|\alpha_q(X))$$

Under these assumptions, the marginal capital requirements are portfolio-invariant and the risk-bucketing approach can be applied.

The last assumption may be considered as the least innocent one. However, although an obligor exhibiting contra-cyclical behavior may not be the rule, it is nonetheless conceivable, thus challenging the assumption of non-decreasing conditional expected loss functions.

The normal distribution assumption is notoriously known to be flawed. Its use is mainly due to its simple computational implementation, but there is extensive literature showing the problems and pitfalls in its application. We may mention here especially the fact that VaR methods fail to be coherent risk measures as defined by Artzner et al.[12] under non-elliptical distributions.[13] It is almost sure, that the distribution of credit events may be poorly described by a normal distribution.[14]

Assuming only a single systematic risk factor may be grossly acceptable for a homogenous market, like national markets. But what about internationally diversified portfolios? Is it reasonable to assume one single world-wide systematic factor? This is especially questionable because Basel is explicitly designed for internationally operating banks.

Finally, it need not be stressed, that an asymptotically fine-grained portfolio will rarely be encountered in reality.

12 See *Artzner* et al. (1999).

13 The normal distribution being a member of the family of elliptical distributions.

14 See e.g. *Danielsson* (2001) and *Danielsson* et al. (2002).

This last point was also recognized by the committee to be a major drawback of the model. Real portfolios are never asymptotically fine-grained and always have some degree of lump risk. To cope with this, the so-called granularity adjustment was introduced. The aim is to correct the calculated capital requirements for real portfolios by an *add-on,* which recognizes the real degree of granularity or diversification of the given portfolio. This is done in principle by mapping the characteristics of the given (lumpy) portfolio to those of a fine-grained portfolio, and by calculating a so-called efficient number of exposures in that comparable portfolio.

A detailed treatment of the granularity adjustment is not our aim here, but it turned out, that the granularity adjustment does not account for the realistic dimension of the granularity effect in real world heterogeneous bank portfolios, and so the adjustment was cancelled in the third consultative paper. But without an appropriate adjustment for real-world "lumpiness", the "fine-grained-assumption" will become critical again.

Note also, that the current risk weight function is *not* calibrated to a homogeneous portfolio, as one might expect, but to a "typical large bank portfolio"[15]. Which means that the basic IRB-function already includes some adjustment for lump risk.

Up to this point, we have provided a rather detailed treatment of the model underlying the IRB-risk weight-function. We have done this, because we consider it necessary to get a clear picture where the formulas come from, and under what conditions they have been derived.

To sum up, we have seen, that the shape of the risk weight-function is the result of a quite complex model, and is depending on the four assumptions specified above. As we have argued, we do not want to question them altogether, but we want to stress that one has to be aware of the fact, that they can only be viewed as rough approximations to real conditions.

Given this limitations, it follows that even if we have accurate input data, the model might provide us only with rough estimates of the appropriate capital charge.

Obviously, to this model-inherent uncertainty adds the vagueness about data quality. Sound default data – i.e. stable estimates of the probability of default – is difficult to obtain, given the low frequency of de-

15 Basel (2001b), section 457.

fault events. Especially for small and medium-sized institutions, the data problem is the major limitation to apply model-based capital rules.

We will come back to this point, but now let us turn to the various modifications in the risk weight-function, that were proposed in Basel (2001c).

## V. Politically Determined Calibration Process

In response to its Quantitative Impact Study as well as other evidence, such as the comments received by market participants, the committee published a paper in November 2001, where it proposed several modifications of the proposals put forward in the second Consultative Document. These modifications then became largely part of the recently issued third Consultative Document. Amongst other changes, notably the IRB-risk weight function for corporate exposures has been altered substantially, as has been already mentioned. In figure 2 we visualize the modifications.

What can be seen, is that the graph of the new corporate risk weight-function $RW^c_{CD3}$, is now substantially flatter, implying for example a reduction of 40% for a *PD* of 20%, or a reduction of 20% for a *PD* of 1%.[16]

Together, these modifications imply a substantial relief to the capital burden. Schwaiger, Lawrenz[17] quantified the relief due to the new risk weight function up to 30%.

This is broadly consistent with the results of the second Quantitative Impact Study[18], where the increase in regulatory requirements under the initial IRB approach was estimated at 22% (for the corporate portfolio).

This increased capital requirement caused somewhat of a storm of protest. Even politicians felt that they had to act in order to "protect the economy" and especially the small and medium-sized enterprises. However, even the committee itself did not have the intention to increase overall capital requirements. It was right from the outset one of the

16 $RW^c_{pm}$ shows the graph for the risk weight-function put forward in the Potential Modifications (Basel (2001d)), and which grossly coincides with $RW^c_{CD3}$.

17 See *Schwaiger, Lawrenz* (2002), *Lawrenz, Schwaiger* (2002a), *Lawrenz, Schwaiger* (2002b).

18 Basel (2001d).

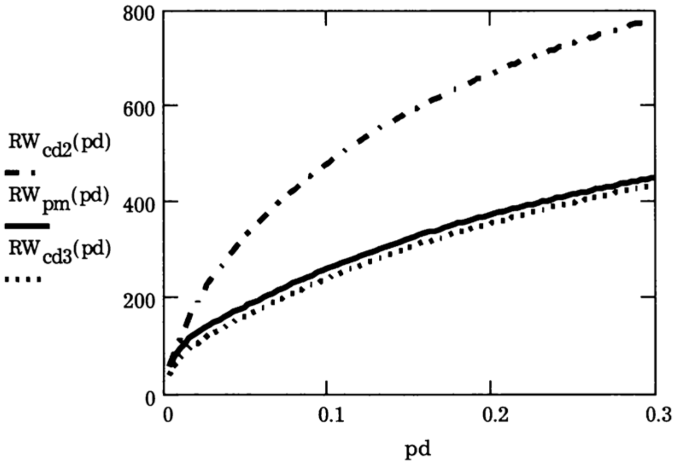


*Fig. 2: Risk weight function for corporate exposure*

major aims of the New Capital Accord to leave the average economy-wide requirement unchanged at the current rate of 8%.

The motivation was, and still is to make capital rules risk-sensitive, but not to change the economy-wide average.

So, the committee reacted upon the critique and the results of the Quantitative Impact Study and made the changes described above.

With these modifications another Quantitative Impact Study is currently carried out and it might be expected that in the light of the forthcoming results some additional changes will be made.

This process of: calibration → impact study → re-calibration → ⋯, gives the impression that the committee is following some kind of "trial and error"-approach.

The model parameter, notably the parameters of the risk weight-functions, are repeatedly adjusted so as to fit the stated goal of achieving an economy-wide average capital requirement of 8%.[19] While the IRB-approach suggests that regulatory capital requirements are calculated applying a given model-based method, the calibration and re-calibration

19 It should be noted that in economic downturns the *PDs* will generally increase, which leads ceteris paribus to a capital requirement above 8%. In general, the capital requirement, given a specific risk weight, exhibits a high correlation with the business cycle, which in turn makes calibration difficult.

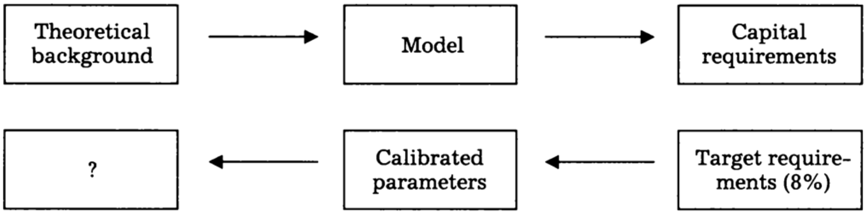


*Fig. 3: Reversal of the chain of causality*

of parameters exhibit the converse relationship: The model is adjusted so as to produce the desired capital requirement. Figure 3 visualizes this point.

In the upper half, the starting point has been the theoretical background upon which a concrete model has been built. Given this model and given the input data, a certain capital requirement will ensue. In the lower half of figure 3, the point of departure is the target level of the average capital requirement. To reach this target, the model parameters were adjusted correspondingly.

But if we agree with this approach, the question will arise: Do we need the theoretical background at all?

If we do not like the model-based results, why building a model at all? In particular, why assuming a theoretically-advanced model with formulas that the majority of users will not fully understand, when the theoretical background is diluted through calibration to an extent, where it becomes negligible?

To avoid misunderstandings: We are well aware of the fact, that in order to make a model operable, it is necessary to calibrate the corresponding parameters. But, in the present context the committee seems to attribute such a paramount importance to the target of 8% on average, that the theoretical foundation is only playing a minor role.

This is also reflected in the seemingly arbitrary choice of parameter values. Consider the *correlation coefficient*. In the initial proposal, the $\rho$ has been fixed at 20% for corporate exposures. Under the potential modifications, $\rho$ is modelled as being a decreasing function of the *PD*s and varies for corporate exposures between 12% and 23.8%. The committee argues that these figures are "broadly consistent with industry practice and research carried out"[20], but it also states that it "does not have any explicit information about asset correlations"[21].

Also, it is not clear why $\rho$ should be negatively related to the *PD*. The Committee seems to follow results due to Lopez[22], who concluded from his empirical study, that average asset correlations are a decreasing function of the *PD*s. However, this seems to confront the findings of Zhou, who found that "the high credit quality of firms not only generates a low default probability of each firm, but also implies a low default correlation between firms for typical time horizons."[23] Other authors, i.e. Erlenmaier/Gersbach, Gersbach/Lipponer, confirm these finding[24].

As a whole, the choice of $\rho$ is not convincing, and one might indeed recognize some "creativity" on the part of the committee, as it has been stated by Wilkens et al.[25]

Altogether, these calibrations and parameterizations give the impression that the committee has abandoned the model-driven approach to capital rules in favor of maintenance of the 8% on average rule, as it has been described above.

But then it seems to be more than reasonable to ask why we need a sophisticated model at all. We can have the same result much easier.

The possibility to simplify the IRB-approach was already recognized by Gersbach/Wehrspohn[26], who proposed a somewhat simpler model, but nevertheless retained the basic structure of the IRB-approach. They derived the following risk weight function for corporate exposure:

$$RW_{GW}^{c} = 12.5 \cdot LGD \cdot N\big(2.283 + 1.336 \cdot N^{-1}(PD)\big) \tag{18}$$

This function conforms well with the initial Basel function and was calibrated to the stated aim of the committee to assign a risk weight of 100% to an obligor with a *PD* of 0.7%. The key parameter for the calibration was once again the asset correlation $\rho$, which turned out to be 44%.

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20 Basel (2001b), section 172.

21 Basel (2001b), section 302.

22 *Lopez* (2002), we thank an anonymous referee for drawing our attention to this work.

23 *Zhou* (1997), p. 10 – Clearly $\rho$ represents the *asset* correlation and not *default* correlation, but since both have the same sign, the argument carries over.

24 *Erlenmaier/Gersbach* (2001), *Gersbach/Lipponer* (2000).

25 See *Wilkens, Entrop, Scholz* (2002), p. 144.

26 *Gersbach, Wehrspohn* (2001).

This again shows, the quite arbitrary nature of $\rho$.

## VI. Proposed Simple Risk Weight Function

Given the model-inherent difficulties concerning the underlying assumptions and input data and – more importantly – the "trial and error" approach of the committee in fitting the model to the targeted 8% on average, we think that we can arrive at the same goal – notably providing a risk-sensitive capital rule – with much less model-theoretical burden.[27]

As it has been already pointed out, the committee has stated some basic target values for the risk weights. For example, in the initial proposal a *PD* of 0,7% should generate a risk weight of 100%. In the potential modification paper the benchmark risk weight of 100% was shifted to a *PD* of 1%. Another benchmark seems to be given by a *PD* of 20%, for which a risk weight of 375% is chosen, or the minimum *PD* of 0.03%, for which a risk weight of roughly 20% is chosen.

The point we want to make is the following: Given the fact, that the determination of these benchmarks is for the most part a political decision and only partly based on model results, why not simply fix these values and construct an "appropriate" risk weight function around them. Given these anchor values, it is easy to find a corresponding function through regression techniques, which fits well to the data.

For example, by using power regression and taking the above benchmarks as input data, we arrive at the following relationship:

$$RW_{reg} = 0.645 \cdot PD^{0.556} + 0.034 \tag{19}$$

Figure 4 compares this risk weight function with the current Basel IRB-risk weight function (see (1)).

Obviously, and not surprisingly, the $RW_{reg}$ conforms well with the IRB-risk weight function ($RW_{Nov}$), yielding comparable risk weights.

A more meaningful test how the regressed function performs, may be done by calculating the average capital requirements.

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[27] Since at the time when the following calculations have been performed, the third Consultative Document has not yet been published, the following section uses the risk weight function ($RW_{Nov}$) as put forward in the Potential Modification paper (Basel (2001c)). But since $RW_{CD3}$ and $RW_{Nov}$ do not differ significantly, the argument remains valid.

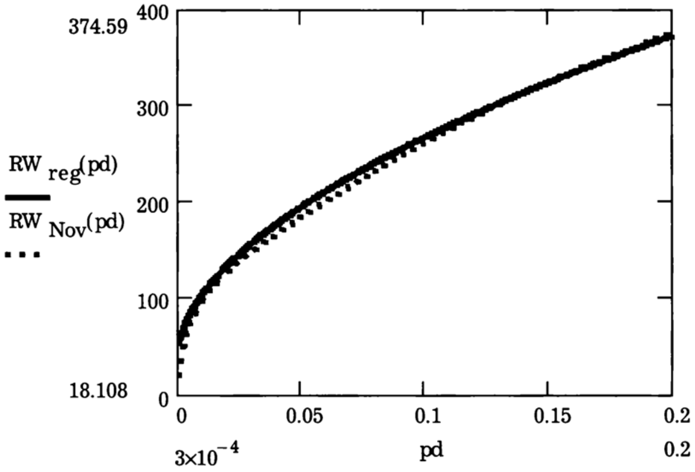


*Fig. 4: Risk weight function from regression*

In order to do this, we rely on the data provided in the study by Schwaiger, Lawrenz (2002), where the average capital requirement was calculated based upon a hypothetical (unsecured) credit portfolio of roughly 2.5 million obligors, representing the overall German economy.[28]

For this credit portfolio the authors arrived at an average capital requirement of 9.41 % using the modified IRB-risk weight function.

We took the same data base and calculated correspondingly the average capital requirement using our regression-risk weight function and arrived at a figure of 9.88 %, showing that results are comparable.[29]

The proposed risk weight function not only fits well in the IRB risk weight function, but is also easily adapted to the committees goals – notably achieving a 8 % average.

Using the distribution of corporate exposures, according to Schwaiger, Lawrenz (2002) and applying a numerical approximation algorithm, we arrive at the adjusted risk weight function:

$$RW^{*}_{reg} = 0.645 \cdot PD^{0.641} + 0.034 \tag{20}$$

28 See *Schwaiger, Lawrenz* (2002); The data is due to Creditreform.

29 For a complete assessment of the result, one has to take account of the assumptions underlying the calculation. For a full discussion see *Lawrenz, Schwaiger* (2002).

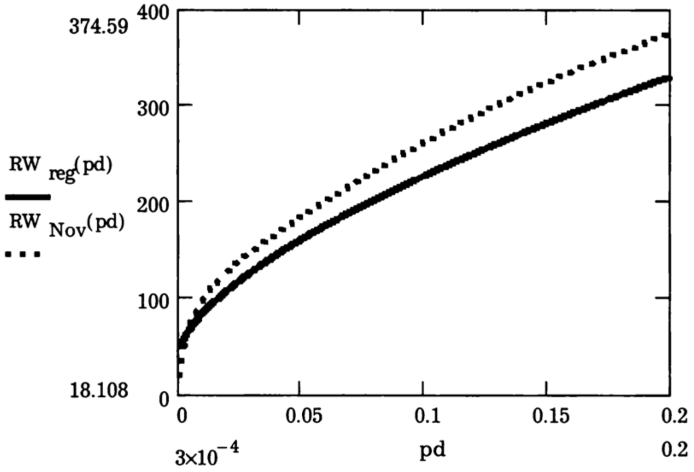


*Fig. 5: Approximated risk weight function*

where only the exponent has changed. The graph of the function, together with the IRB risk weight function is shown, in figure 5.

The average regulatory capital requirement, assuming the same credit portfolio as previously described and the risk weight function ($RW^{*}_{reg}$) derived form the approximation, now turns out to be 8,08 % and thus pretty close to the desired level.

The careful reader will certainly have noticed, that the proposed regression risk weight function lies above the IRB risk weight function for very small $PD$-values (below 25 basispoints). We admit that this is the case, but we consider this not to be a major drawback of the proposal, since firstly a function that fits closely this lower part can easily be found, and then we might simply put the two regression risk weight functions together. The resulting, composed function then might look like:

$$RW^{**}_{reg} = \begin{cases} 0.38 \cdot PD^{0.283} - 0.026 & \text{if} \quad PD < 0.01 \\ 0.645 \cdot PD^{0.556} + 0.034 & \text{if} \quad PD \geq 0.01 \end{cases} \tag{21}$$

The resulting graph in the range of 0.0003 to 0.02 is shown in figure 6, and one recognizes that this function fits the lower part very closely.

Calculations show that the overall capital requirement average is practically not affected. With the same data as above, the average capital requirement changes from 8,08 % to 8,07 %.

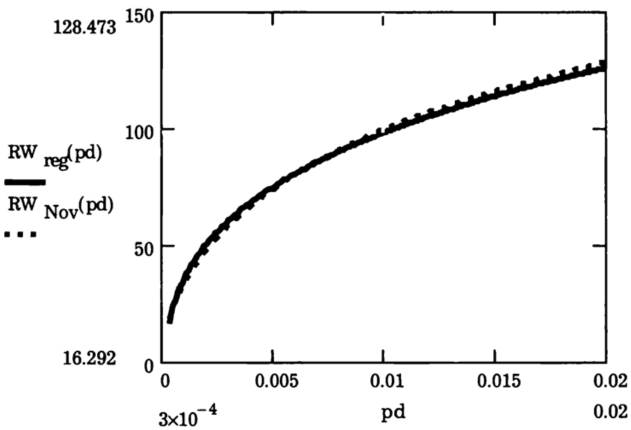


*Fig. 6: Risk weight function from regression for low PDs*

## VII. Conclusion

A careful analysis of the IRB risk weight function, as it has been provided in the first part of the present paper, shows that the theoretical background is quite complex, relying on formal results which may not be obvious to all practitioners and supervisors. This fact alone is certainly not an argument against using theoretical models in regulatory capital rules, but on the other hand, we want to stress that the application of models should neither be a goal per se. This is particularly true for the use in a regulatory context. However, the acting of the committee gives the impression, that it tries to maintain a model-based approach only for the model's sake.

The portfolio model underlying the IRB risk weight function is based on four crucial assumptions, namely (i) one single systematic factor, (ii) asymptotically fine-grained portfolios, (iii) non-decreasing conditional loss functions and (iv) normally distributed variables. We have pointed out, that each assumption can be challenged with reasonable arguments, implying that the model generates at best rough approximations of real conditions. Especially, the single systematic factor assumption is difficult to justify, given the fact that the Basel Capital Accord is intended to apply for large international banks. The problem of homogeneity has been encountered by introducing the granularity adjustment, but since the adjustment was cancelled from the third consultative paper, these assumptions become critical again.

We argued, that this model-inherent problems come along with the vagueness of input data quality. Taken together, these problems imply, that the model may only be able to generate quite rough results.

Therefore, the committee seems to have followed a rather "trail and error" approach, whereby the paramount aim was to achieve an economy-wide average of 8%. Given this political goal, we think that it is neither necessary nor desirable to maintain a model-driven approach, since the model insinuates a theoretically founded accurateness of results that is not justified. Instead, we consider it more appropriate to admit that the determination of the risk weight function is mainly a political question. Then, given the politically determined benchmarks, it is easy to construct a function that fulfills the same task, notably providing a risk-sensitive capital rule and is easily implemented and understood. What is more, the function can be scaled to any given target level by simple approximation algorithms.

If the committees aim is to provide risk-sensitive capital rules, together with an given average level of capital requirement, this can be achieved more easily.

## VIII. Appendix

To see who accounts for the portfolio variance, we can use the standard decomposition of the variance of any random variable that is:

$$Var(\psi) = Var(E(\psi|\xi)) + E(Var(\psi|\xi)) \tag{22}$$

In words: The variance of a random variable can be decomposed in the sum of the variance of the conditional expectation and the expectation of the conditional variance.[30]

Applying this to our example, we find for the conditional expectation of the portfolio value: $1 - p(PF_n|X)$.[31] Taking the variance of this is just the variance of $p(PF_n|X)$.

$$\begin{aligned} Var(p(PF_n|X)) &= E\big(p(PF_n|X)^2\big) - E\big(p(PF_n|X)\big)^2 \\ &= E\big(p(PF_n|X)^2\big) - p^2 \end{aligned} \tag{23}$$

30 See e.g. *Mood, Graybill, Boes* (1974) p. 159.

31 Note that the conditional expectation on a single obligor is $(1/N)(1 - p(i|X))$ and we have $N$ such obligors. Given the conditional independence, we are allowed to sum over $p(i|X)$ and we are left with $1 - p(PF_n|X)$.

Evaluating the first term in (23) means finding the expected value of a random variable in a conditional distribution and results in the present context in a bivariate normal cumulative distribution function[32], denoted by $N_2(\cdot,\cdot,\rho)$, with correlation of $w^2$.

$$E\big(p(PF_n|X)^2\big) = \int_{-\infty}^{\infty} n(x) \cdot p\big(PF_n|x\big)^2\, dx = N_2(\alpha,\alpha,w^2) \tag{24}$$

where $n(x)$ denotes the density function of the normal distribution.

Putting the result together, the variance of the conditional expectation of the portfolio value equals: $N_2(\alpha,\alpha,w^2) - p^2$.

To evaluate the expectation of the conditional variance of the portfolio value, we first note that the conditional variance of the value of *individual* loans is $p(i|X)\cdot(1-p(i|X))/N^2$. Since the loan values are conditionally independent, the conditional *portfolio* variance is the sum of the individual conditional variances: $p(PF_n|X)\cdot(1-p(PF_n|X))/N$.

Taking the expectation and using the previous result for $E\big(p(PF_n|X)^2\big)$, we get for the second term: $(p - N_2(\alpha,\alpha,w^2))/N$.

Putting both parts together, we obtain for the portfolio variance:

$$Var_{PF} = \underbrace{(N_2(\alpha,\alpha,w^2) - p^2)}_{\text{market variance}} + \underbrace{(p - N_2(\alpha,\alpha,w^2))/N)}_{\text{idiosyncratic variance}} \tag{25}$$

Note, that since the first term has been calculated as the variance of the conditional expectation, we can think of this term as the part of the portfolio variance, that is due to market movements. Equivalently, the second term, represented as the expectation of conditional variance can be thought of as the part of portfolio variance, that is attributable to idiosyncratic variance.

Now, the following result is obvious. If $N$, the number of loans in the portfolio is very large, the second term – which is the only one depending on $N$ – gets very small and vanishes eventually, so that the variance of the portfolio is completely determined by the variance of the market.

32 See *Finger* (1999).

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## Summary


**Why Simple, When it Can Be Difficult?**
**Some Remarks on the Basel IRB Approach**

One of the major innovations in the New Basel Capital Accord (Basel II) is represented by the Internal Ratings Based (IRB) Approach. It can be considered as a *conceptually* new approach to capital rules, since the IRB risk weight function is derived from a simple portfolio model. A careful analysis of the underlying model reveals, that it is based on a quite complex theoretical background and depends on some critical assumptions. Beside this inherent vagueness of model-based results, the committee shows a politically determined will to calibrate the parameters of the model so as to obtain an economy-wide average of 8% – implying an unchanged level of regulatory capital requirements on average.

Taken together, this suggests that the model-driven approach is more like a pretext to disguise politically determined decisions, and pretend, an accurateness, that is not given. We argue, that if it is the committees aim to provide risk-sensitive capital rules, together with some target level of average capital requirements, this can be achieved much easier. (JEL G21, G28)


## Zusammenfassung


**Warum einfach, wenn's auch umständlich geht?**
**Anmerkungen zum Basler IRB-Ansatz**

Eine der wesentlichen Neuerungen im Konsultationspapier zur Neuen Basler Eigenkapitalverordnung (Basel II) stellt der auf internen Ratings basierte Ansatz (IRB-Ansatz) dar. Er kann als eine konzeptionelle Innovation betrachtet werden, da die Risikogewichtungsfunktion aus einem einfachen Portfoliomodell abgeleitet ist.

Eine sorgfältige Analyse des zugrunde liegenden Modells zeigt jedoch, dass die Ergebnisse auf nicht ganz „harmlosen“ Annahmen beruhen und eine Genauigkeit vorspiegeln, die in dieser Form nicht gerechtfertigt ist. Andererseits war und ist es das erklärte Ziel des Basler Ausschusses für Bankenaufsicht, die Eigenmittelquote

im volkswirtschaftlichen Durchschnitt unverändert bei 8% zu belassen. Insgesamt drängt sich damit der Eindruck auf, dass hier der Ursache-Wirkungs-Zusammenhang umgekehrt wird: Statt mit dem zugrunde gelegten Modell die Höhe der Eigenmittelanforderungen zu bestimmen, wird ein politisch bestimmtes Niveau vorgegeben und das Modell entsprechend diesen Anforderungen „kalibriert". Damit drängt sich allerdings die Frage auf, ob dann ein modelltheoretischer Hintergrund überhaupt noch notwendig und sinnvoll ist.

Im vorliegenden Artikel wird argumentiert, dass das Ziel, risikosensitive Eigenkapitalvorschriften zu schaffen und dabei ein gewünschtes durchschnittliches Eigenmittelniveau zu erreichen, auch wesentlich einfacher erreicht werden könnte.


## Résumé

### Pourquoi simple quand cela peut être difficile? Quelques remarques sur l'approche IRB du Nouvel Accord de Bâle sur l'adéquation des capitaux

L'approche IRB représente une des innovations majeures du Nouvel Accord de Bâle sur l'adéquation des capitaux (Bâle II). Elle peut être considérée comme une nouvelle approche conceptuelle de la régulation des capitaux puisque la fonction de risque IRB est dérivée d'un simple modèle de portefeuille. Une analyse minutieuse du modèle sous-jacent révèle que l'approche est basée sur une théorie assez complexe et dépend de certaines hypothèses douteuses. A ce manque de précision inhérent des résultats basés sur le modèle s'ajoute la volonté politique du Comité d'étalonner les paramètres du modèle de sorte à obtenir une moyenne économique de 8% – impliquant un niveau inchangé des exigences régulatrices de capitaux.

Ceci suggère que l'approche dérivée du modèle ressemble plus à un prétexte pour camoufler des décisions politiques et qu'elle prétend à une exactitude qui n'est pas donnée. Nous affirmons que, si le but du Comité est de fournir des régulations des capitaux sensibles aux risques, avec certains niveaux d'exigences de capitaux moyens, ceci peut être obtenu beaucoup plus facilement.




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