Kredit und Kapital, Heft 3/2000
Seiten 309–345

The Role of Foreign Exchange Intervention in a Chaotic Dornbusch Model

By Sergio Da Silva, Birmingham and Brasilia*

I. Introduction

A possible explanation for exchange rate movements away from the level consistent with macro fundamentals is given by destabilizing expectations of speculators who use supposedly recurring patterns in graphs to make forecasts. Such 'technical' or 'chart' analyses may be a source of nonlinearity leading to chaos in the *Dornbusch* (1976) model, as shown by *De Grauwe* and *Dewachter* (1992) and *De Grauwe*, *Dewachter*, and *Embrechts* ((1993), Chapter 5). These chaotic models are able to mimic the alleged random walk pattern of actual exchange rates despite the fact that the 'stochastic' behavior is produced by a deterministic solution.

The model presented here belongs to the same line of research. It is able to conciliate the two apparently divergent pieces of evidence that the nominal exchange rate appears to follow a random walk although it also seems to be explained by fundamentals. A martingale process (i. e. a random walk with heteroskedasticity) may be a solution of a chaotic version of the *Dornbusch* model in which fundamentals still matter.

In *De Grauwe* and *Vansanten* (1990) intervention could stabilize a chaotic exchange rate within a framework that was the forerunner of the models of *De Grauwe* and *Dewachter* (1992) and *De Grauwe*, *Dewachter*, and *Embrechts* (1993). However, nonlinearities were introduced in the *De Grauwe-Vansanten* model by assuming the existence of a J-curve. Such a restrictive assumption was dropped by *De Grauwe* and *Dewachter*, and

* This paper draws on chapter III of my PhD thesis at the University of Birmingham (Da Silva (1999a)). I am grateful to John Fender, Paul De Grauwe, Peter Sinclair, Nick Horsewood, David Kelsey, Somnath Sen, and an anonymous referee for comments on previous drafts. Any errors and shortcomings are my responsibility. Financial support from CAPES (Brazilian Post-Graduate Federal Agency) under grant BEX-118995-8 is acknowledged.

De Grauwe, *Dewachter*, and *Embrechts*, henceforth DD (1992) and DDE (1993) respectively.

This paper generalizes further the extension of the *Dornbusch* model accomplished by the DD (1992) and DDE (1993) models by rescuing the point made in the *De Grauwe* and *Vansanten* paper. A major novelty presented here refers to the introduction of a policy rule linking the nominal exchange rate to the nominal money supply. The aim is to show that massive interventions can remove chaos from the foreign exchange market in the DD (1992) and DDE (1993) models.

Section II. sets up the model, whose simulated solutions are presented in Section III.; the results are then contrasted with selected stylized facts and previous work (Section IV.); and Section V. concludes. Formal tests for chaos are relegated to an appendix.

II. The Model

1. The Building Blocks

The model is made up of equations (1)–(8) displayed in Table 1.

Variables and parameter χ are defined as follows. Variable S_t stands for the nominal exchange rate (the price of the foreign currency in units of the domestic currency) at time period t; S_t^* is the equilibrium nominal exchange rate at t; P_t represents the domestic price level at t; P_{t-1} is the domestic price level at $t-1$; P_t^* stands for the steady-state value of the domestic price level at t; and P_t^{f*} is the steady-state value of the foreign price level at t. Parameter $\chi \in (0, \infty)$ measures the actual speed of adjustment in the goods market and may be seen as a proxy for the degree of domestic price level flexibility, as explained below. Equation (1) gives the long-run equilibrium condition, which is defined as a situation in which purchasing power parity (PPP) holds. Since PPP is one of the long-run properties of the *Dornbusch* model, equation (1) states that explicitly. Equation (1) is employed by both the DD (1992, p. 28) and DDE (1993, p. 129) models.

Equation (2) is a substitute for the *Phillips* curve in describing the short-run price dynamics by linking domestic price level changes and nominal exchange rate deviations from equilibrium. It states that since $\chi > 0$ whenever the nominal exchange rate S_t exceeds its PPP-value S_t^* the domestic price level increases, i.e. $P_t > P_{t-1}$. So whenever the currency is undervalued an excess demand in the goods market follows,

Table 1

The Extended Nonlinear *Dornbusch* Model with Speculative Dynamics and Foreign Exchange Intervention

(1)
$$S_t^* = \frac{P_t^*}{P_t^{f*}}$$

(2)
$$\frac{P_t}{P_{t-1}} = \left(\frac{S_t}{S_t^*}\right)^{\chi}$$

(3)
$$\left(\frac{\bar{M}_t}{P_t}\right)\left(\frac{S_t}{\bar{S}_t}\right)^{\phi} = \frac{Y_t^{\delta}}{(1+i_t)^{\theta}}$$

(4)
$$\frac{S_{t+1}^e}{S_t} = \frac{1+i_t}{1+i_t^f}$$

(5)
$$\frac{S_{t+1}^e}{S_{t-1}} = \left(\frac{{}_C S_{t+1}^e}{S_{t-1}}\right)^{C_t}\left(\frac{{}_F S_{t+1}^e}{S_{t-1}}\right)^{1-C_t}$$

(6)
$$\frac{{}_C S_{t+1}^e}{S_{t-1}} = \left[\left(\frac{S_{t-1}}{S_{t-2}}\right)\left(\frac{S_{t-3}}{S_{t-2}}\right)\right]^{\nu}$$

(7)
$$\frac{{}_F S_{t+1}^e}{S_{t-1}} = \left(\frac{S_{t-1}^*}{S_{t-1}}\right)^{\lambda}$$

(8)
$$C_t = \frac{1}{1+\iota\,(S_{t-1}-S_{t-1}^*)^2}$$

causing the domestic price level to increase, and vice versa. Equation (2) is the same as the one employed in the DD ((1992), p. 28) model. Since parameter χ measures the speed of adjustment in the goods market, the value of χ may be interpreted as a proxy for the degree of domestic price level flexibility in the context of this model. Price rigidity occurs at the borderline case where $\chi \to 0$, whereas full price flexibility is represented by $\chi \to \infty$.

The money market equilibrium is given by equation (3). Variable Y_t stands for the domestic real income at time period t, which equals the exogenous level of domestic output by assumption; i_t represents the domestic nominal interest rate at t; $\bar{M}_t$ gives the central bank target to the domestic nominal money supply at t; and $\bar{S}_t$ is the nominal exchange-rate target at t.

Parameters $\delta \in (0, \infty)$ and $\theta \in (0, \infty)$ are proxies for the income elasticity of money demand and the absolute value of the interest elasticity of money demand respectively. As will be seen, parameter δ will not appear in the solution to this model (equation (14)). So the results presented in Section III should hold regardless of the value of the income elasticity of money demand. The central bank parameter ϕ captures the degree of official intervention in the foreign exchange market.

A major novelty in the model in Table 1 lies in equation (3); thus, it deserves a more detailed rationale. That equation is a standard LM such as $M_t/P_t = Y_t^{\delta}/(1+i_t)^{\theta}$, where M_t stands for the domestic nominal money supply at time period t, to which more structure is given by the introduction of the following policy rule:

$$\frac{M_t}{\bar{M}_t} = \left(\frac{S_t}{\bar{S}_t}\right)^{\phi}, \tag{9}$$

where parameter ϕ is zero under free float and approaches either plus or minus infinity to a fixed exchange rate; leaning-against-the-wind intervention is represented by $\phi \in (-\infty, 0)$, whereas leaning into the wind is given by $\phi \in (0, \infty)$. Policy rule (9) can be found, for instance, in *Marston* (1985, p. 910), who use its natural-logarithm version in other contexts.

The economy is under free float when $\phi = 0$ because in such a situation the central bank focuses exclusively on the target to the domestic nominal money supply abstaining from any intervention in the foreign exchange market ($\phi = 0$ produces $M_t = \bar{M}_t$ in (9)). When $M_t = \bar{M}_t$ and $\phi = 0$, equation (3) collapses to the standard LM referred to above, which makes up most versions of the *Dornbusch* model, including the DD (1992, p. 27) and DDE (1993, p. 128) models. Accordingly, it may be argued that the *Dornbusch* model implicitly presupposes free float. Thus, the DD (1992) and DDE (1993) models turn out to be particular cases of the model in Table 1 in that these implicitly assume $\phi = 0$. The fixed exchange-rate regime holds when $\phi \to \pm\infty$ because in such a situation the authorities focus exclusively on the nominal exchange-rate target without concern for the domestic nominal money supply ($\phi \to \pm\infty$ yields $S_t = \bar{S}_t$ in (9)). It might also be noted that, since credibility issues are not taken into account, $\phi \to \pm \infty$ can be thought of as a fixed exchange-rate regime with perfect credibility.

Leaning against the wind is the intervention operation that attempts to move the exchange rate in the opposite direction from its current trend. Leaning into the wind is the operation that is motivated by the central bank's desire to support the current exchange-rate trend. In the context of this model, both leaning-against-the-wind and leaning-into-the-wind interventions are carried out by changes in $\bar{M}_t$. Whether such changes are sterilized is not discussed here.

If $S_t > \bar{S}_t$ ($S_t < \bar{S}_t$) for any reason, the aim of leaning against the wind is thus to reduce (increase) the current nominal exchange rate S_t; in this model, that can be achieved by reducing (increasing) $\bar{M}_t$ in (9) when $\phi < 0$. By contrast, since leaning into the wind signifies supporting the current exchange-rate trend, if $S_t > \bar{S}_t$ ($S_t < \bar{S}_t$) such an intervention operation means increasing (reducing) $\bar{M}_t$ in (9) when $\phi > 0$. We will see in Section III that the degree of such intervention also matters in this model for a successful stabilization of a chaotic nominal exchange rate.

Equation (4) is the uncovered interest rate parity (UIP) hypothesis. Variable S^e_{t+1} stands for the forecast made at time period t for the nominal exchange rate at $t+1$; and i^f_t represents the foreign nominal interest rate at t. Both i_t and i^f_t are the nominal interest rates available on similar domestic and foreign securities respectively, with the same periods to maturity. UIP states that the expected foreign-exchange gain from holding one currency rather than another (the expected nominal exchange-rate change) must be just offset by the opportunity cost of holding funds in this currency rather than the other (the nominal interest rate differential). UIP is a basic ingredient in even the simplest versions of the *Dornbusch* model. Equation (4) is also used in both the DD (1992, p. 27) and DDE (1993, p. 128) models.

Equations (5)–(8) describe the speculative dynamics of the model by introducing chartist behavior among speculators. In equations (5)–(8), speculators are assumed to take positions in the market at time period t based on the forecasts they have made for $t+1$, and these forecasts were made by them using information available at $t-1$. That is the reason why S_{t-1} appears rather than S_t in these equations. Since S_t is the solution obtained when speculators have taken their market positions, S_t is not observable by these agents at the moment they make their forecasts. This point is observed by DDE (1993, p. 74).

Equation (5) splits expectations between two components – the expectations based on charts ${}_C S^e_{t+1}$, and the expectations based on the fundamentals of the model ${}_F S^e_{t+1}$. Variable S_{t-1} is the nominal exchange rate at time period $t-1$, whereas $C_t \in (0,\ 1)$ stands for the weight given to charting at t. As far as the weight of chartist activity is concerned, if $C_t \in (0,\ 0.5)$ then there is less charting than forecasts based on fundamentals. If $C_t = 0.5$ then half of the speculators are involved in charting and the other half are making forecasts based on fundamentals. If $C_t \in (0.5,\ 1)$ then expectations are dominated by chartists. Equation (5) is the same as the one employed in the DDE (1993, p. 131) model.

The expectation rule for the forecasts based on charts is given by equation (6). Variables S_{t-2} and S_{t-3} are the nominal exchange rates at time periods $t-2$ and $t-3$ respectively; and parameter $\nu \in (0,\ \infty)$ stands for the degree of past extrapolation used in technical analysis. Since $\nu > 0$, the greater ν, the more the past will be extrapolated into the future in exchange rate forecasts, and chartists will expect the nominal exchange rate at time period $t+1$ to be less than the nominal exchange rate prevailing at $t-1$. Rule (6) is employed by DDE ((1993), Chapter 3, p. 80) in a simple chaotic model without money. Here it is used in the context of the *Dornbusch* monetary model. Doing so, no principle is apparently violated. *LeBaron* (1996) demonstrates empirically significant forecastability from a simple moving average trading rule (similar to (6)) for series of the US dollar against the mark and the yen that uses both weekly and daily data. Equation (6) may seem a little odd at first sight, but the further discussion presented below will help to clarify it.

The rationale to (6) is provided following DDE (1993, pp. 78–80). Speculators expect an increase in the nominal exchange rate whenever a short-run moving average of past exchange rates S^S_t crosses a long-run moving average of past exchange rates S^L_t from below (Figure 1). In such an event a buy order of the foreign currency is given by them. In turn, they expect a decline of the nominal exchange rate whenever S^S_t crosses S^L_t from above. In the latter case speculators order a selling of the foreign currency. This can be postulated as

$$\frac{{}_C S^e_{t+1}}{S_{t-1}} = \left(\frac{S^S_t}{S^L_t}\right)^{2\nu}. \tag{10}$$

Equation (10) states that since $\nu > 0$, whenever $S^S_t > S^L_t$ ($S^S_t < S^L_t$) chartists expect an increase (fall) of the nominal exchange rate relative to the most recently observed value S_{t-1}. By assumption, the short-run moving average S_t^S is based on a one-period change, i.e.

$$S_t^S = \frac{S_{t-1}}{S_{t-2}}, \tag{11}$$

and the long-run moving average S_t^L is based on a two-period change, i.e.

$$S_t^L = \left(\frac{S_{t-1}}{S_{t-2}}\right)^{\frac{1}{2}} \left(\frac{S_{t-2}}{S_{t-3}}\right)^{\frac{1}{2}}. \tag{12}$$

Rule (6) can be obtained by plugging (11) and (12) into (10). *De Grauwe* (1996, pp. 181–185) presents a microeconomic foundation for this chartist behavior.

While making forecasts based on the fundamentals of the model, speculators are assumed to use the rule given by equation (7). Variable S^*_{t-1} represents the equilibrium nominal exchange rate at time period $t-1$, whereas parameter $\lambda \in (0, \infty)$ gives the expected speed of return of the current nominal exchange rate toward its equilibrium value. According to (7), whenever fundamentalists observe a market rate above (below) the PPP-value, they will expect it to decline (increase) in the future. Since $\lambda > 0$, the greater λ, the higher the expected speed of return toward the fundamental rate. The greater λ, the faster fundamentalists will expect the nominal exchange rate to increase (fall) toward its equilibrium value if $S_{t-1} < S^*_{t-1}(S_{t-1} > S^*_{t-1})$. Values of λ greater than one mean that fundamentalists expect some sort of overshooting (DDE (1993), p. 111). Equation (7) is also employed in the DDE (1993, p. 131) model.

The weight of charting is endogenized by equation (8). The amount of technical analysis used by speculators is made dependent on the size of the deviation of the current nominal exchange rate from its equilibrium (fundamental) value. Equation (8) states that if $(S_{t-1} - S^*_{t-1})^2 \to \infty$ then $C_t \to 0$, i.e. whenever deviations from PPP increase, the expectations based on charts will be reduced. If $(S_{t-1} - S^*_{t-1})^2 \to 0$ then $C_t \to 1$, which means that whenever deviations from PPP tend to be eliminated, charting will grow in importance among speculators. Parameter $\iota \in (0, \infty)$ stands for the speed at which forecasts based on charts switch to forecasts based on fundamentals. The higher ι, the faster chartist activity will decrease, and vice versa. The same weighting function (8) is found in the DDE (1993, pp. 75–78) model. In accordance with (8), *LeBaron* (1994, p. 400) points out that predictability appears to be higher during periods of lower volatility, a phenomenon used by chartists to achieve some small out-of-sample improvements in forecasts. This completes the description of the model.

2. *Solution*

We can proceed toward the solution to the model. The eight endogenous variables are: S_t, P_t, i_t, S_t^*, S_{t+1}^e, ${}_C S_{t+1}^e$, ${}_F S_{t+1}^e$, and C_t. The model is recursive in that the block made up of equations (5)–(8) runs first.

An additional assumption beforehand helps to simplify matters. The rate at which speculators expect the nominal exchange rate to return toward its fundamental value is assumed to be the same as the speed at which prices in the goods market actually adjust, i.e.

$$\lambda = \chi \,. \tag{13}$$

One known property of the *Dornbusch* model is that after a possible overshooting of the nominal exchange rate in the impact period, it asymptotically moves back toward its equilibrium value at the same pace as the domestic price level movement (*Dornbusch* (1976), p. 1165). Therefore it is not unreasonable to think that such piece of information can be taken into account by fundamentalists. Assumption (13) is also employed in both the DD (1992, p. 34) and DDE (1993, p. 131) models.

Substituting (13) in (7) and then plugging the resulting equation along with (6) and (8) into (5) yields an expression for S_{t+1}^e. Next, inserting (1) into (2) obtains an expression for P_t; and inserting the expression for P_t into (3) gives an expression for $1+i_t$. Then, substituting the latter expression into (4) produces an expression for S_t. Without loss of generality we may assume additionally every appropriate exogenous (fundamental) variable to be constant and normalized to unity (and $i_t^f = 0$). Considering this assumption in the expression obtained earlier for S_t it becomes apparent that it depends only on a term for S_{t+1}^e. That assumption also implies $S_t^* = S_{t-1}^* = 1$ so that PPP holds in equilibrium. After inserting this result into the expression for S_{t+1}^e and substituting it in the expression for S_t we obtain the solution to the model for the nominal exchange rate given by the following weighted geometric moving average:

$$S_t = {S_{t-1}}^{f_1} \, {S_{t-2}}^{f_2} \, {S_{t-3}}^{f_3}, \tag{14}$$

where

(15)
$$f_1 \equiv \frac{\theta\left[1 + \nu + \iota(1-\chi)(S_{t-1}-1)^2\right]}{(\theta + \chi - \phi)\left[1 + \iota(S_{t-1}-1)^2\right]},$$

(16)
$$f_2 \equiv \frac{-2\theta\nu}{(\theta + \chi - \phi)\left[1 + \iota(S_{t-1}-1)^2\right]},$$

and

(16)
$$f_3 \equiv \frac{\theta\nu}{(\theta + \chi - \phi)\left[1 + \iota(S_{t-1}-1)^2\right]}.$$

Expression (14) is a nonlinear difference equation to which an analytical solution cannot be provided. It needs to be solved numerically. To do that initial conditions (values for S_{t-3}, S_{t-2}, and S_{t-1}) are required. Equation (14) has as many solutions as there are parameter combinations. In the intertemporal equilibrium given by $S_t^* = S_{t-1} = S_{t-2} = S_{t-3} = 1$, variable S_t also equals one in (14) regardless of parameter values. Due to that independence the characteristics of solution to the model can be evaluated in the neighborhood of point (1, 1, 1).

The nominal exchange rate is assumed to be in equilibrium at the starting period, i.e. $S_{t-3} = 1$. Then in the two subsequent periods there occur small deviations from that equilibrium. As in DDE (1993, pp. 132–133), here it is assumed that $S_{t-2} = 0.99$ and $S_{t-1} = 1.02$. As will be seen in the next section, the above initial set of conditions is sufficient to generate very complex dynamics in the *Dornbusch* model. The rich variety of solutions covers stability, cycles, chaos accompanied (or not) by crashes, and instability. This point is already made by DD (1992) and DDE (1993). However, it is shown further that massive interventions in the foreign exchange market are able to collapse chaotic, cyclical, and unstable motions to stable ones.

III. Simulation Results

1. Methodology

This section presents the numerical solutions to equation (14) in the (ν, ϕ), (ι, ϕ), (χ, ϕ), and (θ, ϕ) spaces. The simulation results are shown in Tables 2–5, where the private parameters ν, ι, χ, and θ respectively are combined with the policy parameter ϕ.

In Table 2 the degree of past extrapolation into the future taking place in forecasts based on charts ν is combined with the type of foreign exchange intervention ϕ. The other three parameters ι, χ, and θ are initially fixed. The speed at which forecasts based on charts switch to those based on fundamentals is chosen to be $\iota = 10^4$ in Table 2, the same value employed by DDE (1993, pp. 83, 132). In Table 3, however, other values for ι are considered. The actual speed of adjustment of the goods price χ – which is attached in (13) to the speed of exchange-rate return toward the fundamental value λ – is assumed to be $\chi = \lambda = 0.45$, as in DD (1992, p. 34 n7). Considering the range of possible values of $\chi \in (0, \infty)$, a strong price stickiness is assumed in Table 2, as might be expected in the context of the *Dornbusch* model. This assumption is relaxed in Table 4, however, where χ is allowed to vary. The proxy for the absolute value of the interest elasticity of money demand is initially picked as $\theta = 0.95$. Nevertheless, θ is allowed to vary between its theoretical range $\theta \in (0, \infty)$ in Table 5.

The conclusions about the nature of the solutions displayed in Tables 2–5 were reached after checking for the first 10000 datapoints, each one with eight decimal places. A cycle of periodicity above 10000 was considered as chaos for practical purposes.

The DD (1992) and DDE (1993) models may be thought of as being represented by the column for $\phi = 0$ in Tables 2–5 because these models implicitly assume free float, as argued in Section II. The information shown in the $\phi = 0$ columns reveals that this model is able to replicate the same rich variety of solutions to the DD (1992) and DDE (1993) models. The pictures in Figure 2 display some selected chaotic solutions to the model.

2. *Charting versus Intervention*

As far as Table 2 is concerned, at the borderline case in which $\nu \to 0$ the model is stable for all ϕ values, except $\phi = 1$. The very high degree of past extrapolation in chartist activity represented by $\nu = 10^5$ on the other hand, makes the model unstable with free float ($\phi = 0$). Chaos mostly accompanied by crashes may also occur under free float. A currency crash is defined in this model as a situation in which the nominal exchange rate suddenly depreciates by more than two digits.

Looking at both the first and the last columns in Table 2 one can see that the model is, in most cases, stable in the presence of massive foreign exchange intervention when ϕ is very large (an exception occurs for $\nu = 10^5$).

Table 2 also shows that chaotic solutions may emerge out of free float for small amounts of intervention in the foreign exchange market. Cases of chaos mostly accompanied by crashes for different degrees of past extrapolation in charting may appear with leaning-against-the-wind ($\phi < 0$) and leaning-into-the-wind ($\phi > 0$) interventions. The more chartists extrapolate the past into the future (the greater ν), the larger the variance of chaotic nominal exchange-rate movements. Simple chaotic series become accompanied by slight crashes which turn violent soon after. This enables us to conclude that the more chartists extrapolate the past in forecasts, the greater the crashes. It may be noted that currency crashes always come to an end in this chaotic model due to the presence of centripetal forces operating for large deviations of the nominal exchange rate from equilibrium.

Figure 3 provides an illustration of the property of extreme sensitivity to initial conditions with past extrapolation in charting ($\nu = 15$) and free float ($\phi = 0$). An exogenous shock of 1% is introduced at time period 9950; as a result, the new (dotted) series follows an entirely different trajectory. This is the famous 'butterfly effect' of the chaos literature. Massive intervention is able to stabilize the same series, however. Figure 4 displays the same degree of past extrapolation into the future by chartists as the one considered in Figure 3 (i.e. $\nu = 15$) being neutralized by massive leaning-into-the-wind intervention ($\phi = 10^4$).

The most striking discovery obtained in the simulations presented in Tables 2–5 concerns the clear pattern emerging as far as stability is concerned. As a rule, stable solutions can be recognized in both the left and right hand sides in Tables 2–5. An exception occurs for the top of Table 5, as discussed below. This reveals that massive central bank intervention has the ability to reverse chaotic, cyclical, and unstable series to stable ones. In particular, the 'steps' of stable solutions shown in Table 2 indicate that the higher the past extrapolation by chartists, the larger must be the volume of intervention aiming at stabilizing the nominal exchange rate.

3. Change of Forecast Rule and Intervention

The speed at which forecasts based on charts switch to forecasts based on fundamentals is now allowed to vary along with the policy parameter in Table 3, where the solutions to the model in the (ι, ϕ) space are presented. The other fixed parameter values are $\nu = 500$, $\chi = \lambda = 0.45$, and $\theta = 0.95$. Thus the row for $\iota = 10^4$ in Table 3 matches with the row for $\nu = 500$ in Table 2 (and with the row for $\chi = 0.45$ in Table 4, and the row for $\theta = 0.95$ in Table 5).

DDE (1993, p. 109 n3) report that the size of parameter ι does not affect their results in that if a large ι produces chaos then a smaller ι also does. The results displayed in Table 3 do confirm that if a large ι generates chaos then a smaller value for this parameter also does. Looking at the column for $\phi = 0$ one can note the presence of chaos for both high and low speeds at which chartist activity takes place. The striking discovery that massive interventions in the foreign exchange market are capable of stabilizing chaotic, cyclical, and unstable nominal exchange rates is also robust regarding the results in Table 3, as can be seen in the columns on both the left and right hand sides.

4. *Price Flexibility and Intervention*

Table 4 displays the solutions to equation (14) focusing on the relationship between goods price flexibility (parameter χ) and central bank intervention (parameter ϕ). Since the assumption of fixed prices is ad hoc in the context of the *Dornbusch* model, it is reasonable to relax it allowing for increasing goods price flexibility. As a result, nominal exchange rate variability no longer necessarily means real volatility.

The solutions to the model in the $(\chi,\ \phi)$ space are obtained regarding $\nu = 500$ as given, and the other parameters and initial conditions are the same as in Table 2, namely $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\iota = 10^4$, and $\theta = 0.95$. The same fundamental discovery shown in Tables 2 and 3 can be seen in Table 4, namely that massive foreign exchange intervention has the ability to stabilize the nominal exchange rate. This can be checked in both the first three columns on the left hand side and the last two columns on the right hand side.

5. *Money Demand and Intervention*

The interest elasticity of money demand is now related to the policy parameter in Table 5, which shows the solutions to the model in the $(\theta,\ \phi)$ space. The fixed parameters are $\nu = 500$, $\iota = 10^4$, and $\chi = 0.45$, and the initial values for the nominal exchange rate are the same as those assumed so far.

If the interest elasticity of money demand approaches minus infinity the economy is in the so-called liquidity trap. Since θ refers to the absolute value of the interest elasticity, the liquidity trap is defined in this model as $\theta \to \infty$. As long as figure 10^5 is considered as a proxy to plus infinity, the first row at the top of Table 5 displays the liquidity-trap situation.

Two major patterns can be recognized in Table 5. The first row at the bottom for $\theta \to 0$ shows that the system is stable irrespective of the type of intervention if the demand for money is not influenced by the nominal interest rate. Taking the $\phi = 0$ column as a benchmark, it can be noted – from the row for $\theta = 0.1$ to around the row for $\theta = 10$ – that chaotic, cyclical, and unstable series can be stabilized by massive interventions. Thus, these sensible values for the interest elasticity of money demand reproduce the critical discovery shown in Tables 2–4 concerning the stabilizing power of intervention. However, commencing at the row for $\theta = 10^2$ until very high interest elasticities of money demand, it can be realized that intervention plays no role. These results are in line with a textbook wisdom which states that monetary policy becomes impotent under the liquidity trap.

IV. Contrast with Stylized Facts and Previous Work

The above findings are now contrasted with some stylized facts and previous results in literature. DD (1992, pp. 45–47) and DDE (1993, pp. 150–151) apply the augmented *Dickey-Fuller* test to show that their simulated chaotic data cannot be used to reject the hypothesis that the nominal exchange rate exhibits unit roots. Since it is a stylized fact that actual exchange-rate series are nonstationary, these authors conclude that there is indirect empirical support for their models. Such a test is not repeated here. It should be noted that most chaotic series displayed in Tables 2–5 show that chaos is accompanied by crashes, which means that not only does the nominal exchange rate exhibit a random-like behavior but also heteroskedasticity is present (e.g. panel *b* in Figure 2). Therefore, the stylized fact that a martingale process is more likely to describe nominal exchange rate behavior is replicated in chaotic models.

Using simulated chaotic data, DDE (1993, pp. 150–156) also test for and find evidence supportive of the forward premium as a biased predictor of future nominal exchange-rate movements. Since this mimics another feature of actual data, the authors interpret that as one more piece of indirect evidence in favor of their model.

Some studies adopt the modeling strategy of reducing all structure of a model to only one single variable intending to focus the analysis on the effect of 'news', i.e. unexpected changes in the nominal exchange rate resulting from changes in the fundamentals that come as a surprise. The news approach thus relies on the existence of an unexpected shock to explain every nominal exchange-rate movement. However, only a small proportion of spot movements of the nominal exchange rate seem to be caused by news (*Goodhart* (1990)). As in DD (1992) and DDE (1993), the results presented in Section III are consistent with the fact that large variations in the nominal exchange rate may occur without it being possible to identify the cause in any shock. Violent currency crashes may emerge in chaotic series with no change in the exogenous variables of the model. Crashes in the foreign exchange market may be caused by dynamic chaos without random external influences. Hence an advantage of chaotic models is that they do not rely on random shocks to explain swings in the nominal exchange rate.

For a given foreign price level, since the domestic price level is rigid in the impact period, the real exchange rate follows the nominal rate in the *Dornbusch* model. This feature along with the circumstance that free float is implicit in that model makes it consistent with the stylized fact that the volatility of the real exchange rate is much higher under flexible exchange rates than under fixed rates. It has been suggested that a factor explaining the bad empirical performance of the *Dornbusch* model is that actual data after the *Bretton Woods* era are managed-floating data rather than the pure-float data which that model addresses (e.g. *Gärtner* (1993), p. 196). The chaotic model presented here is consistent with the stylized fact that real exchange rates are more volatile under free float, as can be appreciated in Tables 2, 3, and 5.

For those who are not satisfied with the mechanistic trading rule of this model, one way out is to take into account the lines suggested by *Jeanne* and *Masson* (1998). These authors present a second-generation nonlinear model of currency crises that assumes rational expectations and still gives rise to multiple equilibria and chaotic dynamics in the expectation of devaluation. The model presented here also provides a case for the importance of macromodels – in which fundamentals play a role – to explain nominal exchange-rate behavior. This is thus in line with recent attempts to revive explanations based on fundamentals to beat the simple random-walk model (e.g. *Mark* (1995)). Here fundamentals matter because the interaction between speculative private behavior and foreign exchange intervention can give rise to chaos and thus mimic a random walk. Fundalmentals also matter because massive foreign exchange interventions are able to stabilize these chaotic motions and thus influence the nominal exchange rate.

There is a piece of indirect evidence addressing the implication that intervention may stabilize a chaotic nominal exchange rate. In accordance with that, *Dominguez* (1993) presents evidence that foreign exchange intervention actually reduces the volatility of the nominal exchange rate. There is a puzzling piece of evidence, too. Using both weekly and daily data of foreign exchange intervention and foreign exchange series of the US dollar against the mark and the yen, *LeBaron* (1996) shows that after removing periods in which the *Federal Reserve* is active, the ability to predict future exchange rates coming from technical trading rules is dramatically reduced. This suggests that central bank intervention may introduce noticeable trends into the evolution of the nominal exchange rate and thus create profit opportunities coming from speculation against the central bank. In this connection, *Taylor* (1982) and *Leahy* (1995) find evidence that central banks make money on their foreign exchange intervention operations; and *Silber* (1994) presents evidence in a cross sectional context that technical rules have value whenever governments are present as major players. More strikingly, *Szpiro* (1994) argues that an intervening central bank may even induce chaos in the nominal exchange rate. Thus, these results suggest that the more central banks intervene, the more they give incentives to chartists to enter the market, thereby increasing the chance of chaos.

Chaos is also shown to be possible in a microfounded sticky-price model by *Da Silva* (1999b), where the model developed recently by *Obstfeld* and *Rogoff* (1995; 1996) is extended to encompass the speculative dynamics and the modeling of foreign exchange intervention discussed in this paper. In the *Da Silva* (1999b) model chaos is possible under a non-zero amount of foreign exchange intervention, which is in line with the 'puzzling' findings referred to in the paragraph above.

By contrast, another effect of intervention is analyzed in the present paper: the more central banks intervene, the less the likelihood of chaos. Most precisely, *massive* intervention can remove chaos from the foreign exchange market. However, it might be noted that the 'puzzling' effect of intervention generating chaos also appears in the results displayed in Tables 2–5, where *low* volumes of foreign exchange intervention can also lead to chaos. Here low amount of intervention can be interpreted as either intervention toward profitability on the part of the central bank or non-credible attempts at stabilization of the nominal exchange rate.

V. Conclusion

This paper examines whether a result obtained in *De Grauwe* and *Dewachter* (1992) and *De Grauwe*, *Dewachter*, and *Embrechts* (1993, Chapter 5), that chaos can be generated in a *Dornbusch* style model if some traders are chartists, will continue to hold when central banks engage in foreign exchange intervention. The answer seems to be a qualified no, that is, generally massive interventions are able to stabilize the chaos.

Central bank intervention is introduced to represent anti-chartist behavior through a rule connecting the nominal exchange rate to the nominal money supply. The analysis of both leaning-against-the-wind and leaning-into-the-wind interventions becomes possible and the *Dornbusch* model is reduced to the particular case of free float. Chaotic, cyclical, and unstable nominal exchange-rate series under free float and low amount of intervention are shown to collapse to stable ones as long as massive central bank interventions are carried out.

The results in this paper show that past extrapolation into the future in chartists' forecasts can produce very complex dynamics in the *Dornbusch* model. The possible solutions to the model range from stability and cycles of different periodicities to chaos (with or without crashes) and instability. Under both free float and low volumes of intervention, the more chartists extrapolate the past into the future, the larger the variance of the chaotic nominal exchange rate, and the greater the currency crashes. The higher the past extrapolation by chartists, the larger must be intervention to stabilize the nominal exchange rate. Also, the emergence of chaos does not show dependence on the speed at which forecasts based on charts switch to forecasts based on fundamentals.

Massive interventions can also remove chaotic, cyclical, and destabilizing movements even if the assumption that prices are sticky is relaxed. Chaos, crashes, cycles, and instability emerge with both free float and low volumes of intervention, when the interest elasticity of money demand assumes sensible values. Massive interventions can again produce stability in such a scenario. However, intervention plays no role in the context of the liquidity trap.

This study shows consistency with a number of stylized facts and previous results in the literature on exchange rates and foreign exchange intervention. Most remarkably, the model presented here is able to conciliate the two apparently divergent pieces of evidence that the nominal exchange rate appears to follow a martingale process although it also seems to be explained by fundamentals. The random-like behavior of the nominal exchange rate in which crashes crop up is generated by deterministic solutions to the model, and since massive foreign exchange interventions are able to stabilize these chaotic motions, fundamentals – most precisely, exchange rate policy – also influence the nominal exchange rate.

Appendix

The decision that a given solution to the model is chaotic is made in Tables 2–5 on the grounds that no datapoint repeats itself in the range of 10000 periods. A problem with simulations is that there is no formal guarantee that a reached conclusion still applies for the simulation range plus one. For that reason, formal tests for chaos are carried out in this Appendix for the obtained chaotic solution with parameters $\nu = 15$, $\phi = 0$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, and initial values 1.00000000, 0.99000000, and 1.02000000. A data record of 15000 points is here taken, and the first 100 values are skipped to allow for the nominal exchange rate to settle into its final behavior. The program employed for data analysis was Chaos Data Analyzer: The Professional Version 2.1® by J. C. *Sprott*, copyright © 1995 by the *American Institute of Physics*. The pictures in Figure 5 were obtained using such a software. Information regarding description of statistics as well as suggestions for analysis strategy were taken from the PC user's manual of that software by *Sprott* and *Rowlands* (SR) (1995).

Panel *a* in Figure 5 suggests that the nominal exchange rate exhibits a random-like behavior similar to the graph of data of a white (uncorrelated) noise (e.g. SR (1995), p. 51). No obvious structure is seen. However, even in well-known chaotic systems – such as the logistic and *Hénon* maps – this sort of graphing of data usually reveals no structure (e.g. SR (1995), pp. 50–51). But a discernible structure emerges in a three-dimensional embedding plotting (panel *b* in Figure 5), i.e. there is a 'strange attractor'. Strange attractors are suggestive pictures that can be plotted from chaotic series showing some order in fake randomness. The shape of the attractor in panel *b* in Figure 5 is very similar to the one displayed in DD (1992, p. 39), which is generated by a plotting of 6000 observations in a two-dimensional diagram. It also retains a certain resemblance to the two-dimensional phase diagram shown in DDE (1993, p. 136). A blow-up showing that no datapoint repeats itself in this attractor is given by DD (1992, p. 39). If the data were random, a formless cloud of dots ('random dust') would have emerged, whereas a cycle and a nearly periodic system would have displayed a closed and a fuzzy loop respectively (SR (1995), p. 17). Thus, since the data are aperiodic but not random, they are chaotic. Indeed, chaos is defined as apparently stochastic behavior occurring in deterministic systems (*Stewart* (1997), p. 12).

Panel *c* in Figure 5 shows a logarithmic plot of the probability distribution of the data record, using 32 bins into which the datapoints are sorted according to their value. The bins all have the same width and are spread uniformly between the lowest and highest data value. Looking at the left-hand side in panel *c* in Figure 5, the distribution might look Gaussian like the one emerging in genuinely random data (e.g. SR (1995), pp. 51–52). However, as the right-hand side is also taken into account, a 'fractal' shape associated with chaos can be recognized. If the data were periodic, a simple histogram with sharp edges would have appeared (SR (1995), p. 18). Table 6 provides a summary of the standard statistics obtained from the data set as well as the other statistics presented below.

A calculated *Hurst* exponent of about 0.04 indicates that the data are not actually random. The *Hurst* exponent gives a measure of the extent to which the data can be represented by a random walk (or a fractional Brownian motion). White (uncorrelated) noise has a *Hurst* exponent of 0.5 (SR (1995), pp. 22, 37). Also, the IFS clumpiness test (panel *d* in Figure 5) shows that the data cannot be represented by a random walk. In a picture displaying the IFS clumpiness test, white noise fills it uniformly whereas chaos or colored (correlated) noise generates localized clumps (SR (1995), p. 18).

Relative LZ complexity gives a measure of the algorithmic complexity of a time series. Maximal complexity (randomness) has a value of 1.0, whereas perfect predictability (cycles) has a value of 0 (SR (1995), p. 36). The data are not periodic either, because the calculated LZ complexity is around 0.6. It might be noted that this is the same value as the one the program calculates for the *Hénon* map.

The fact that the data are not periodic or quasi-periodic is confirmed, too, as their power spectrum is visualized. The power spectrum is reasonably broad (panel *e* in Figure 5) if calculated by a fast-*Fourier* transform on the data record (SR (1995), pp. 8–9 provide a technical discussion of this method). Panel *e* in Figure 5 displays the log of the power spectrum versus frequency, using 32 frequency intervals. Chaos and random data give rise to broad spectra, while cycles and quasi-cycles generate a few dominant peaks in the spectrum (SR (1995), p. 20). The dominant frequency refers to the frequency at which the power spectrum has its maximum value (SR (1995), p. 37). Calculated by the fast-*Fourier* transform method, the dominant frequency is about 0.02. This suggests that the data are more likely to be chaotic or random, not periodic or quasi-periodic. Using the maximum-entropy method, there is virtually no dominant frequency (panel *f* in Figure 5) (SR (1995), p. 21 give a technical account for this method). Panel *f* in Figure 5 is a log-linear view of dominant frequencies, using 2 complex poles of discrete frequency. It suggests that the data are neither period nor quasi-periodic.

An attractor can be quantified by measures of its dimension and its *Lyapunov* exponents. The dimension evaluates the complexity of the attractor, whereas the *Lyapunov* exponent measures the sensitivity to initial conditions, i.e. the famous 'butterfly effect' of chaotic series. Capacity dimension and correlation dimension are major measures of dimension of a chaotic attractor (a description of these statistics is presented by SR (1995), pp. 8, 24, 25). Values greater than about 5 for these measures give an indication of randomness, whereas values less than 5 provide further evidence of chaos (SR (1995), p. 25). Extreme sensitivity to tiny changes in initial conditions and therefore evidence of chaos is obtained as long as the largest *Lyapunov* exponent is positive. A zero exponent occurs near a bifurcation; periodicity is associated with a negative *Lyapunov* exponent; and white (uncorrelated) noise is related to an exponent approaching infinity (SR (1995), pp. 23, 37).

The correlation dimension calculated from the data is about 1.7. This gives evidence of chaos. Panel g in Figure 5 shows a saturation in the calculated correlation dimension as the embedding dimension is increased. Such a well-defined plateau indicates an appropriate embedding dimension in which to reconstruct the attractor. Another indication of proper embedding is the minimum dimension for which the number of false nearest neighbors falls to zero (panel h in Figure 5). Such a picture suggests the proper embedding as given by 3. Thus, panels g and h in Figure 5 provide an indication of low-dimensional chaos, albeit some quasi-periodic data (such as the one generated from the so-called two incommensurate sine waves) may also exhibit the properties shown in these pictures (SR (1995), p. 50).

The capacity dimension calculated from the data is about 1.9. Similar to the correlation dimension, a capacity dimension greater than about 5 would have implied random data. So the calculated capacity dimension is compatible with the presence of chaos in the data.

The largest *Lyapunov* exponent calculated from the data is about 0.3, whereas the largest *Lyapunov* exponent to the base e is about 0.2. These calculations considered the proper embedding dimension as given by 3, using 3 time steps and accuracy of 10^{-4}. Such positive values for the *Lyapunov* exponents give further evidence of chaos.

Entropy is a measure of disorder in the data. It is given by the sum of the positive *Lyapunov* exponents to the base e (SR (1995), p. 37). Its reciprocal gives roughly the time over which meaningful prediction is possible (SR (1995), p. 25). The approximated entropy calculated from the data set is about 0.3. It might be noted that this is exactly the same value as the one the software calculates for the *Lorenz* attractor. Also, the calculated entropies of the *Hénon* and logistic maps are about 0.4 and 0.5 respectively. Thus, the entropy of this chaotic solution to the *Dornbusch* model is very similar to those expected for well-known chaotic systems.

The BDS statistic is a measure of deviation of the data from pure randomness (SR (1995), p. 37). The BDS statistic calculated from the data record is around 1. It is worth pointing out that the program calculates exactly the same BDS statistic to the logistic map.

To test whether the evidence of hidden determinism in the data is robust, it is prudent to repeat the calculations of the quantitative measures of the attractor using surrogate data that resemble the original data but with the determinism removed. Robustness implies that analysis of these surrogate data should provide values that are statistically distinct from those calculated from the original data (SR (1995), pp. 14–15). As observed, "this test is a very important one and is rarely included in papers claiming observation of low-dimensional chaos in experimental data" (SR (1995), p. 15). The most useful method is to *Fourier*-transform the data, randomize the phases, and then inverse *Fourier*-transform the result to get a new time series with the same spectral properties as the original but lacking determinism (SR (1995), p. 14); this implies a different probability distribution. Panel *i* in Figure 5 shows a bell-shaped distribution (indicating lack of determinism) which differs from the distribution of the original chaotic data in panel *c*.

The major quantitative measures of the surrogate data are indeed different from those of the original data. The largest *Lyapunov* exponent and the largest *Lyapunov* exponent to the base *e* are 0.847 ± 0.011 and 0.587 ± 0.008 respectively. The correlation dimension is 4.505 ± 0.089, and the calculated capacity dimension is 2.238 ± 0.128. To know whether this difference is statistically significant, ideally one should generate many surrogate-data sets and see whether the results from the original data lie within the range of values corresponding to the surrogates. If they do, then the difference is not statistically significant and the original data are indistinguishable from colored noise (SR (1995), p. 15). By repeating the above test twice, the calculated *Lyapunov* exponents were found to be greater than the values shown above. Another surrogate-data set was generated simply by shuffling the original data values, as one shuffles a deck of cards. The calculated *Lyapunov* exponents were still greater than the ones obtained from the original data. Thus, the conclusion that the data obtained from the *Dornbusch* model are chaotic and distinguishable from colored noise seems to be robust.

Table 2

Solutions to the Model in the (ν, ϕ) Space: Degree of Past Extrapolation in Charting versus Foreign Exchange Intervention

ν \ ϕ	-10^5	-10^4	-10^3	-10^2	-10	-1	-0.5	0	0.5	1	10	10^2	10^3	10^4	10^5
10^5	C4	C4	CH*	C8	U	CH*	U	U	U	U	U	U	C8	C2	U
10^4	ST	C4	C4	C4	C4	U	U	U	U	U	C2	C2	C2	C2	ST
10^3	ST	ST	C8	C4	U	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
500	ST	ST	ST	CH*	U	CH	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
250	ST	ST	ST	C4	U	CH*	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
225	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
200	ST	ST	ST	C4	CH*	CH	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
175	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
150	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
125	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
10^2	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
75	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
50	ST	ST	ST	ST	CH	C8	CH*	CH*	CH*	U	C2	C2	ST	ST	ST
25	ST	ST	ST	ST	C4	CH*	CH*	CH*	CH	U	C2	ST	ST	ST	ST
15	ST	ST	ST	ST	C4	CH	CH	CH	CH	U	C2	ST	ST	ST	ST
10	ST	ST	ST	ST	C4	CH	CH	CH	CH	U	C2	ST	ST	ST	ST
5	ST	ST	ST	ST	ST	C4	C4	CH	CH	U	C2	ST	ST	ST	ST
0	ST	ST	ST	ST	ST	ST	ST	ST	ST	U	ST	ST	ST	ST	ST

ST = stable solution; Ci = cycle of periodicity i; CH = chaotic solution (* denotes occurrence of crashes); U = unstable solution.

The greater ν, the more the past is extrapolated into the future in nominal exchange-rate forecasts.
$\phi = 0$ represents free float; intervention increases as one moves to both the left and right hand sides of the $\phi = 0$ column.
$S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$.

Table 3

Solutions to the Model in the (ι, ϕ) Space: Speed at which Forecasts Based on Charts Switch to Forecasts Based on Fundamentals versus Foreign Exchange Intervention

ι \ ϕ	-10^5	-10^4	-10^3	-10^2	-10	-1	-0.5	0	0.5	1	10	10^2	10^3	10^4	10^5
10^5	ST	ST	ST	C4	U	CH*	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
10^4	ST	ST	ST	CH*	U	CH	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
10^3	ST	ST	ST	C4	C4	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
500	ST	ST	ST	C4	C4	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
300	ST	ST	ST	C4	C4	U	U	U	CH*	U	C4	C2	C2	ST	ST
275	ST	ST	ST	C4	C4	U	U	CH*	CH*	U	C6	C2	C2	ST	ST
250	ST	ST	ST	C4	U	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
225	ST	ST	ST	C4	U	U	U	CH*	CH*	U	C2	C2	C2	ST	ST
200	ST	ST	ST	C4	C4	U	U	U	CH*	U	C2	C2	C2	ST	ST
10^2	ST	ST	ST	C4	U	U	U	CH*	CH*	U	U	C2	C2	ST	ST
75	ST	ST	ST	C16	U	U	U	CH*	CH*	U	U	C2	C2	ST	ST
50	ST	ST	ST	C4	U	U	U	CH*	CH*	U	U	C2	C2	ST	ST
25	ST	ST	ST	C4	U	CH*	U	CH*	U	U	U	C2	C2	ST	ST
20	ST	ST	ST	C4	U	CH*	U	CH*	CH*	U	U	C2	C2	ST	ST
10	ST	ST	ST	U	CH	CH*	CH*	CH*	CH*	U	U	U	C2	ST	ST
5	ST	ST	ST	U	CH	CH*	CH*	CH*	CH*	U	U	U	C2	ST	ST
1	ST	ST	ST	CH*	CH*	U	U	CH*	U	U	U	U	C2	ST	ST
0	ST	ST	ST	CH*	U	U	U	U	U	U	CH*	CH*	U	ST	ST

ST = stable solution; Ci = cycle of periodicity i; CH = chaotic solution (* denotes occurrence of crashes); U = unstable solution.

The greater ι, the faster chartist activity decreases.
$\phi = 0$ represents free float; intervention increases as one moves to both the left and right hand sides of the $\phi = 0$ column.
$S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\nu = 500$, $\chi = \lambda = 0.45$, $\theta = 0.95$.

Table 4

Solutions to the Model in the (χ, ϕ) Space: Price Flexibility versus Foreign Exchange Intervention

χ \ ϕ	-10^5	-10^4	-10^3	-10^2	-10	-1	-0.5	0	0.5	1	10	10^2	10^3	10^4	10^5
10^5	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST
10^4	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST
10^3	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	C3	ST	ST	ST
900	ST	ST	ST	ST	C3	C3	C3	C3	C3	C3	C3	C6	U	ST	ST
800	ST	ST	ST	C10	C6	C6	C6	C6	C6	C6	C6	C6	U	ST	ST
700	ST	ST	ST	C4	C12	C12	C12	C12	C12	C12	C12	U	U	ST	ST
600	ST	ST	ST	C12	CH	CH	CH	CH	CH	CH	CH	U	U	ST	ST
500	ST	ST	ST	CH	CH	CH	CH	C24	C24	C12	C6	U	CH	ST	ST
250	ST	ST	ST	CH	C3	C3	C3	C3	CH	CH	CH	CH*	C2	ST	ST
200	ST	ST	ST	CH	C12	CH	CH	CH	CH	CH	CH	U	C2	ST	ST
10^2	ST	ST	ST	CH	C6	CH	CH	CH	CH	CH	U	CH*	C2	ST	ST
50	ST	ST	ST	CH	CH*	CH*	CH*	CH*	CH*	CH*	U	CH	C2	ST	ST
25	ST	ST	ST	C4	CH*	CH*	CH*	CH*	CH*	CH*	U	CH	C2	ST	ST
10	ST	ST	ST	C4	CH*	CH*	CH*	C6	C3	CH*	CH*	CH	C2	ST	ST
5	ST	ST	ST	C4	C4	CH*	CH*	CH*	CH*	C3	C2	CH	C2	ST	ST
1	ST	ST	ST	CH*	CH*	CH*	CH*	CH	CH*	CH*	C8	C2	C2	ST	ST
.45	ST	ST	ST	CH*	U	CH	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
0	ST	ST	ST	C16	CH	CH*	CH*	ST	U	CH*	C2	C2	C2	ST	ST

ST = stable solution; Ci = cycle of periodicity i; CH = chaotic solution (* denotes occurrence of crashes); U = unstable solution.

$\chi = 0$ means price rigidity; $\chi \to \infty$ signifies full price flexibility.
$\phi = 0$ represents free float; intervention increases as one moves to both the left and right hand sides of the $\phi = 0$ column.
$S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\nu = 500$, $\iota = 10^4$, $\theta = 0.95$.

Table 5

Solutions to the Model in the (θ, ϕ) Space: Interest Elasticity of Money Demand (Absolute Value) versus Foreign Exchange Intervention

θ \ ϕ	-10^5	-10^4	-10^3	-10^2	-10	-1	-0.5	0	0.5	1	10	10^2	10^3	10^4	10^5
10^5	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	U
10^4	U	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	U	C2
10^3	CH*	U	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	U	C2	C2
10^2	CH	C24	U	CH*	CH*	CH*	CH*	CH*	CH*	CH*	CH*	U	C2	C2	CH
50	ST	C4	U	U	CH*	CH*	CH*	CH*	CH*	CH*	CH*	C2	C2	C2	C2
10	ST	CH	CH*	U	CH*	CH*	CH*	CH*	CH*	CH*	U	C2	C2	CH	ST
1.5	ST	ST	C4	CH*	U	CH*	C4	CH*	CH*	CH*	C2	C2	C2	ST	ST
1	ST	ST	CH	CH*	U	CH*	CH*	CH	CH*	U	C2	C2	CH	ST	ST
.95	ST	ST	ST	CH*	U	CH	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
.8	ST	ST	ST	C68	U	CH*	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
.7	ST	ST	ST	C4	U	CH*	CH*	C4	CH*	U	C2	C2	C2	ST	ST
.6	ST	ST	ST	C4	U	C3	CH*	CH*	CH*	U	C2	C2	C2	ST	ST
.5	ST	ST	ST	C4	U	C12	U	CH*	CH*	CH*	C2	C2	C2	ST	ST
.4	ST	ST	ST	C4	CH*	U	C3	CH*	CH*	CH*	C2	C2	ST	ST	ST
.3	ST	ST	ST	C4	CH*	U	U	CH	CH*	C2	C2	C2	ST	ST	ST
.2	ST	ST	ST	C4	CH*	U	U	C6	CH*	C2	C2	C2	ST	ST	ST
.1	ST	ST	ST	ST	CH*	U	U	U	U	C2	C2	C2	ST	ST	ST
0	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST	ST

ST = stable solution; Ci = cycle of periodicity i; CH = chaotic solution (* denotes occurrence of crashes); U = unstable solution.

Sensible values of θ fall into the interval between zero and one; very high values of θ give the situation of liquidity trap.
$\phi = 0$ represents free float; intervention increases as one moves to both the left and right hand sides of the $\phi = 0$ column.
$S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\nu = 500$, $\iota = 10^4$, $\chi = \lambda = 0.45$.

Table 6

Summary of Statistics for the Chaotic Solution with $\nu = 15$, $\phi = 0$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, and Initial Values 1.00000000, 0.99000000, and 1.02000000

Number of Datapoints	15000
Minimum Value	0.2553084
Average Value	1.013214
Maximum Value	3.703084
Range of Data	3.447776
Resolution	1.168847×10^{-4}
Lower Quartile	0.9455401
Median Value	1.000472
Upper Quartile	1.055588
Mode (i.e. Maximum Probability)	1.015625
Average Deviation	9.975048×10^{-2}
Standard Deviation	0.1925547
Variance	0.0370773
Skewness	4.356391
Kurtosis	44.13611
Pearson's Correlation	0.3072745
Estimated Fixed Point	1.011292
Relative LZ Complexity	0.6168716
Dominant Frequency (Calculated by the Fast Fourier Transform Method)	0.015625
Dominant Frequency (Calculated by the Maximum-Entropy Method)	0
Dominant Period (Calculated by the Fast Fourier Transform Method)	64
Dominant Period (Calculated by the Maximum-Entropy Method)	–
Hurst Exponent	3.936994×10^{-2}

Table 6 (continued)

Largest Lyapunov Exponent	0.353 ± 0.026
Largest Lyapunov Exponent to the Base *e*	0.245 ± 0.018
Capacity Dimension	1.877 ± 0.107
Correlation Dimension	1.656 ± 0.045
Entropy	0.334
BDS Statistic	1.036

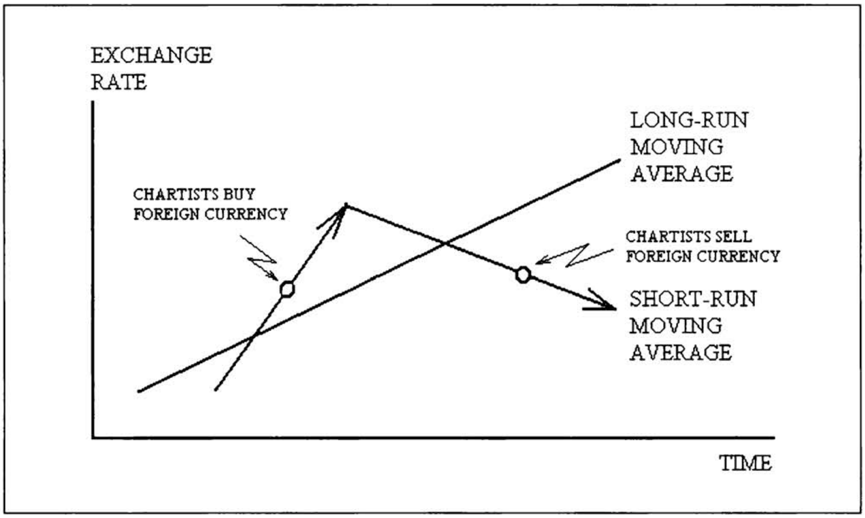
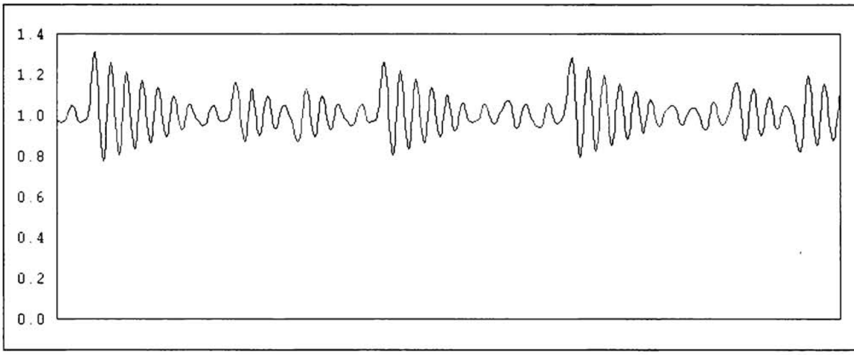


Figure 1: The Chart Used in the Model Forecasts.

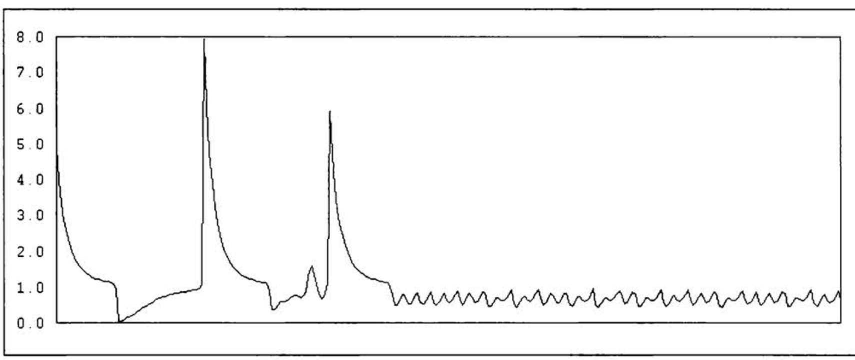
Speculators expect an increase in the nominal exchange rate whenever a short-run moving average of past exchange rates S_t^S crosses a long-run moving average of past exchange rates S_t^L from below; in such an event they give a buy order for the foreign currency.

By contrast, they expect a decline of the nominal exchange rate whenever S_t^S crosses S_t^L from above; in the latter case speculators order a selling of the foreign currency.

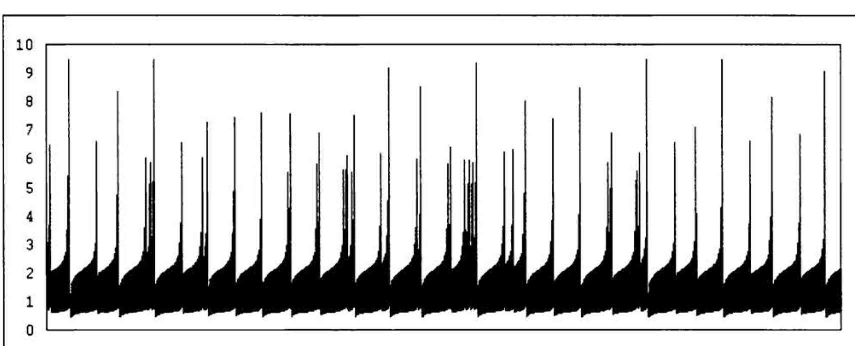
Source: DDE (1993, p. 73) with minor modifications.



a) $\phi = 0$, $\nu = 500$, $\iota = 10^4$, $\chi = \lambda = 200$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: $9900 - 10000$.

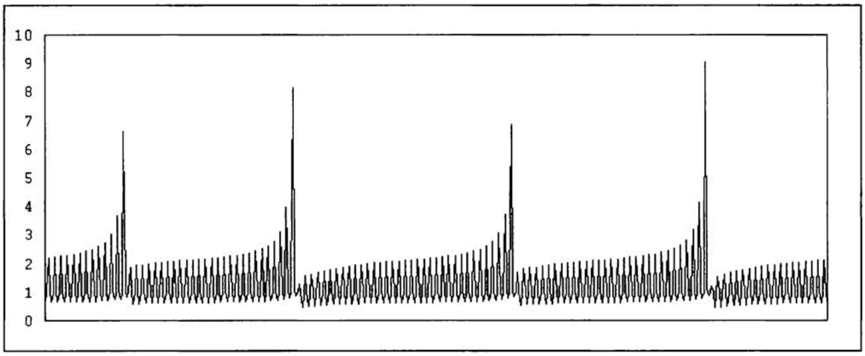


b) $\phi = 1$, $\nu = 500$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 1.5$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: $9700 - 10000$.

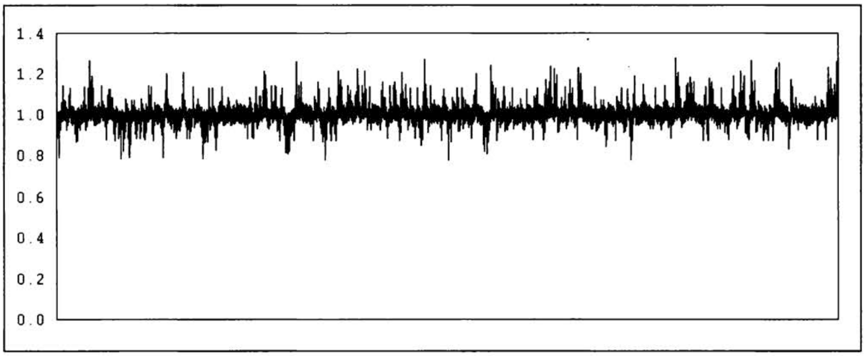


c) $\phi = -10$, $\nu = 50$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: $6001 - 10000$.

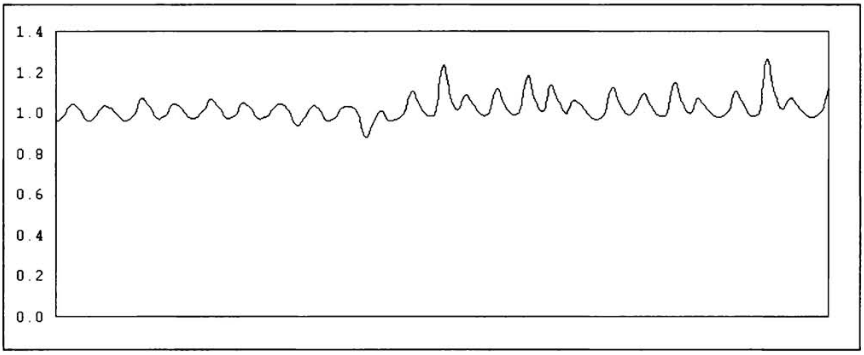
Figure 2: Display of Selected Chaotic Solutions.



d) $\phi = -10$, $\nu = 50$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range $9500 - 10000$.

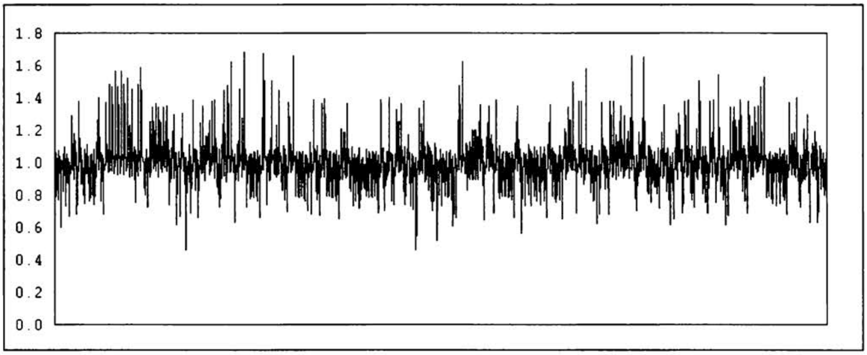


e) $\phi = 0$, $\nu = 5$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: $6001 - 10000$.

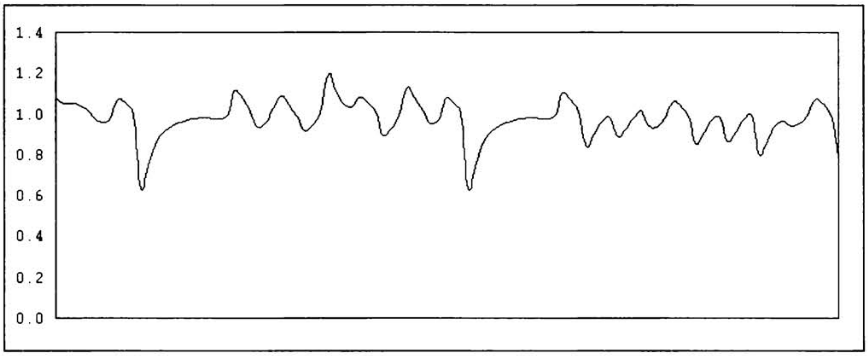


f) $\phi = 0$, $\nu = 5$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: $9900 - 10000$.

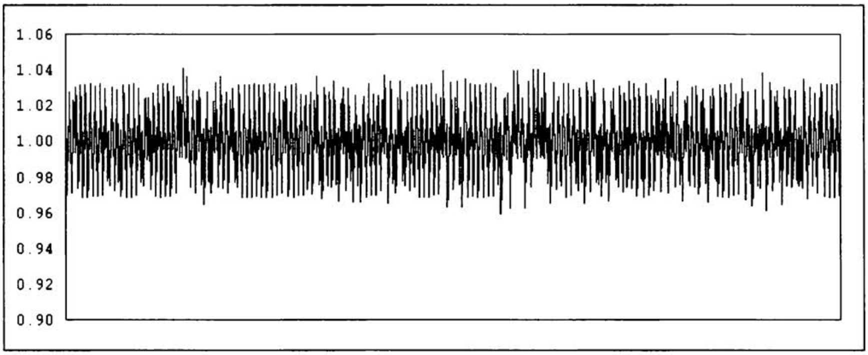
Figure 2 (continued)



g) $\phi = 0.5$, $\nu = 10$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: $6001 - 10000$.



h) $\phi = 0.5$, $\nu = 10$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: $9900 - 10000$.



i) $\phi = 10^3$, $\nu = 500$, $\iota = 10^4$, $\chi = \lambda = 500$, $\theta = 0.95$, $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, and $S_{t-1} = 1.02000000$. Data range: $6001 - 10000$.

Figure 2 (continued)

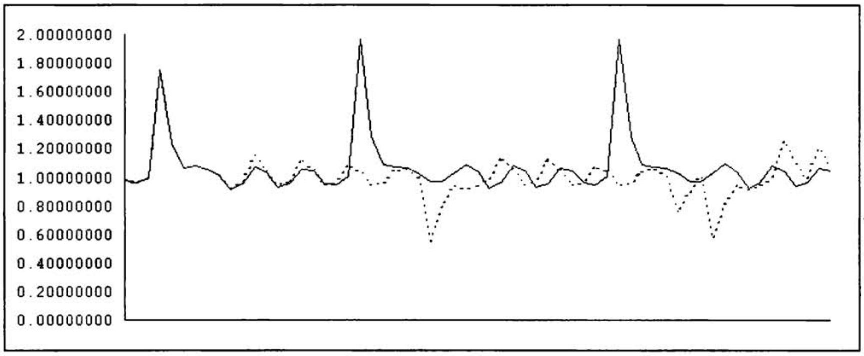


Figure 3: Extreme Sensitivity to Changes in Initial Conditions ('Butterfly Effect') with Past Extrapolation into the Future in Charting ($v = 15$) and Free Float ($\phi = 0$).

Other values are: $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\iota = 10^4$, $\chi = \lambda = 0.45$, and $\theta = 0.95$. Range: $9940 - 10000$. A shock (increase) of 1% is introduced at time period 9950; as a result, the new (dotted) series follows an entirely different trajectory, showing extreme sensitivity to changes in initial conditions.

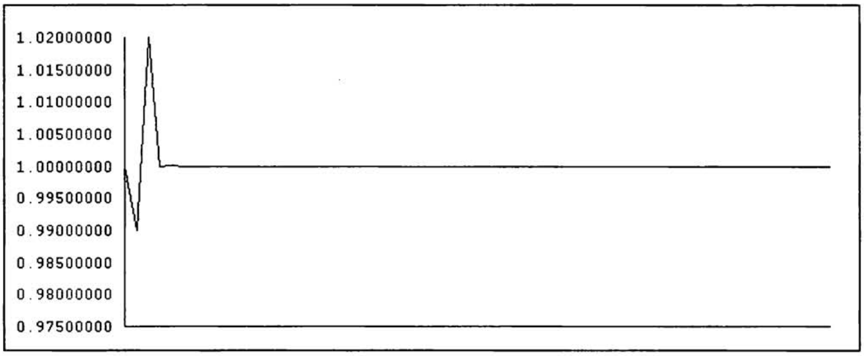
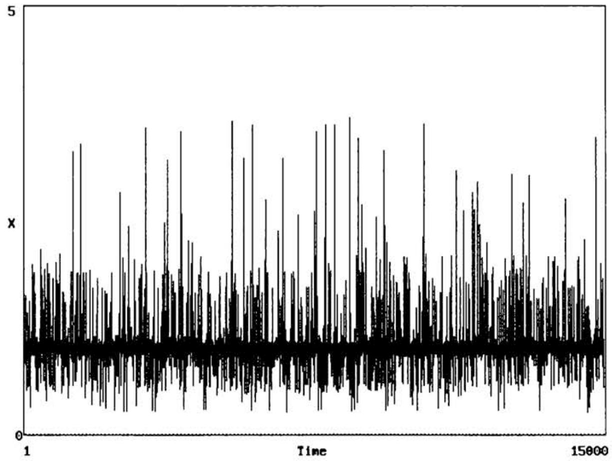


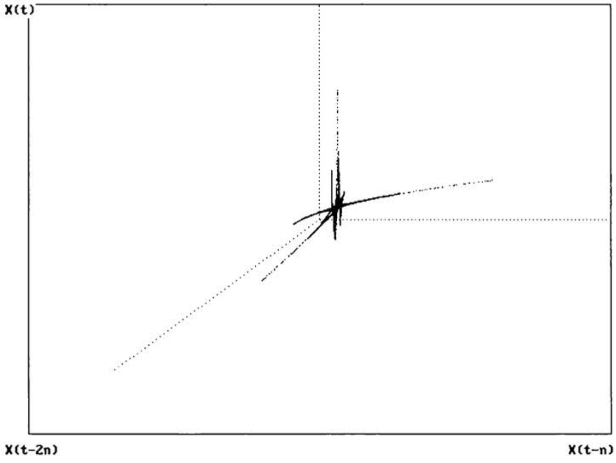
Figure 4: Massive Foreign Exchange Intervention is Capable of Stabilizing Chaos and Currency Crashes in the Extended Dornbusch Model of the Foreign Exchange Market.

Other values are: $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, $S_{t-1} = 1.02000000$, $\iota = 10^4$, $\chi = \lambda = 0.45$, and $\theta = 0.95$. This picture shows the first 60 datapoints for $\nu = 15$ and $\phi = 10^4$.



a) Graph of Data

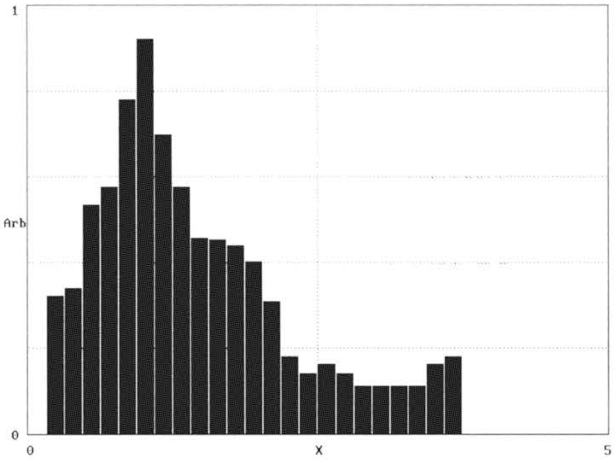
This graph of data looks like white (uncorrelated) noise and shows no obvious structure.



b) Strange Attractor

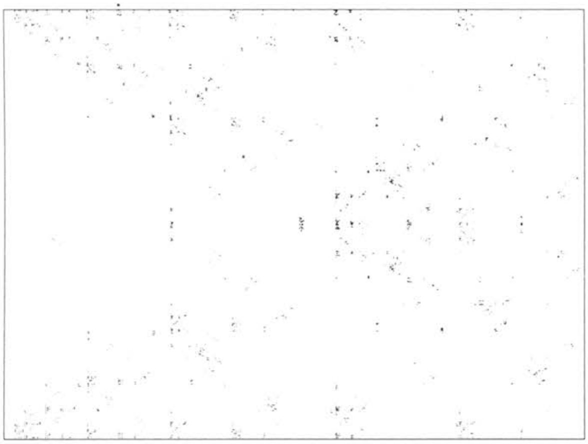
This plot in a three-dimensional embedding reveals a clear structure in the data: a strange attractor can be recognized.

Figure 5: Chaotic Solution with Parameters $\nu = 15$, $\phi = 0$, $\iota = 10^4$, $\chi = \lambda = 0.45$, $\theta = 0.95$, *and Initial Values* $S_{t-3} = 1.00000000$, $S_{t-2} = 0.99000000$, *and* $S_{t-1} = 1.02000000$.



c) Probability Distribution

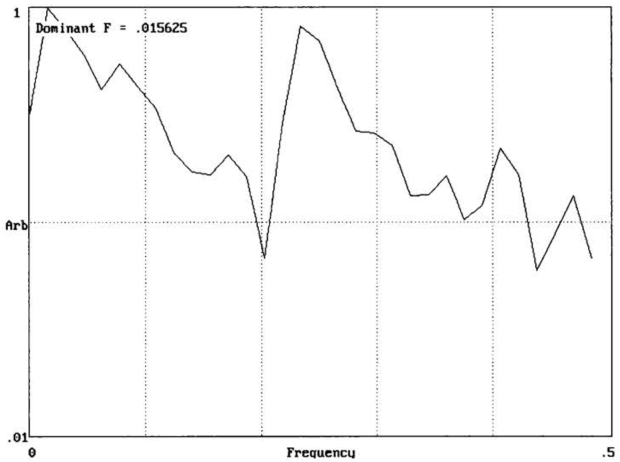
Logarithmic plot of the probability using 32 bins into which the datapoints are sorted according to their value. This distribution looks Gaussian (as in random data) if only the left-hand side is considered; however, a 'fractal' shape associated with chaotic data emerges when the right-hand side is also taken into account.



d) IFS Clumpiness Test

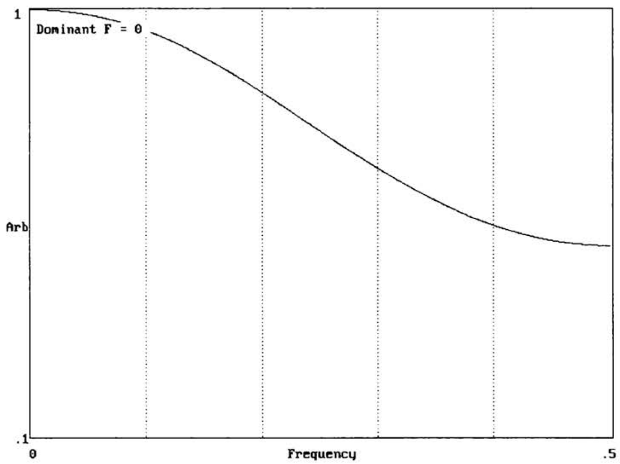
This picture shows localized clumps indicating chaos or colored (correlated) noise.

Figure 5 (continued)



e) Power Spectrum (Fast-Fourier)

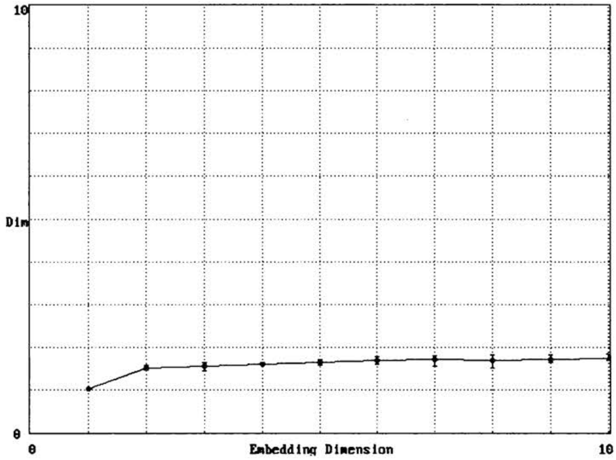
Log of the power spectrum versus frequency, using 32 frequency intervals. Since this picture shows a reasonably broad power spectrum, the data are more likely to be chaotic or random, and not periodic or quasi-periodic.



f) Power Spectrum (Entropy)

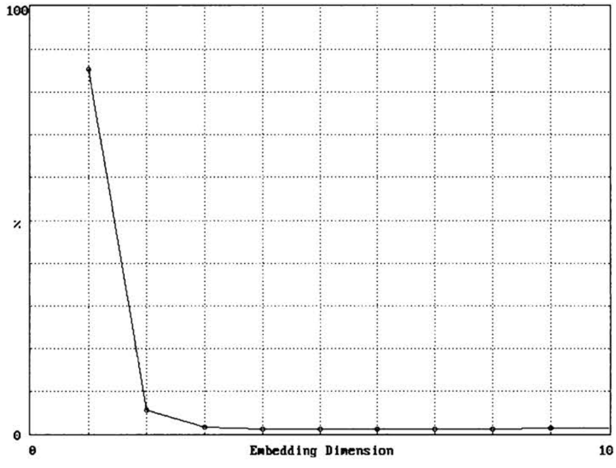
Log-linear view of dominant frequencies in the power spectrum, using 2 complex poles of discrete frequency. This picture shows no dominant frequency, which indicates that the data are neither periodic nor quasi-periodic.

Figure 5 (continued)



g) Correlation Dimension: Saturation

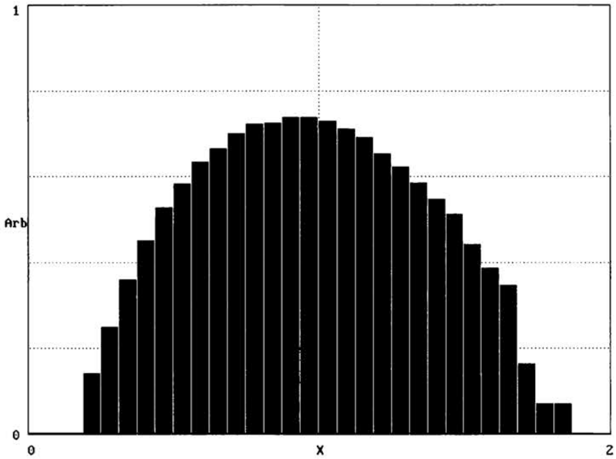
This picture shows a clear saturation in the calculated correlation dimension as the embedding dimension is increased; the well-defined plateau gives an indication of the proper embedding associated with chaos, although some quasi-periodic data may also exhibit such a property.



h) Correlation Dimension: Neighbors

This picture gives another indication of the proper embedding (associated with chaos) as the minimum dimension for which the number of nearest false neighbors falls to zero.

Figure 5 (continued)



i) Surrogate Data

Logarithmic plot of the probability distribution of the surrogate data using 32 bins into which the datapoints are sorted according to their value. This picture shows a bell-shaped distribution (indicating lack of determinism) which differs from the distribution of the original chaotic data in panel *c*.

References

Da Silva, S.: Exchange Rate Models, Chaos, and Foreign Exchange Intervention, PhD Thesis, Birmingham University Department of Economics, unpublished, 1999a. – *Da Silva*, S.: Exchange Rate Dynamics Redux and Chaos, Birmingham University Department of Economics Discussion Paper, No. 99–08, 1999b. – *De Grauwe*, P.: International Money: Postwar Trends and Theories, Oxford: Oxford University Press, Second Edition, 1996. – *De Grauwe*, P. and *Dewachter*, H.: Chaos in the Dornbusch Model of the Exchange Rate, Kredit und Kapital 25(1), pp. 26–54, 1992. – *De Grauwe*, P., *Dewachter*, H. and *Embrechts*, M.: Exchange Rate Theory: Chaotic Models of Foreign Exchange Markets, Oxford and Cambridge, MA: Blackwell, 1993. – *De Grauwe*, P. and *Vansanten*, K.: Deterministic Chaos in the Foreign Exchange Market, CEPR Discussion Paper, No. 370, January 1990. – *Dominguez*, K. M.: Does Central Bank Intervention Increase the Volatility of Foreign Exchange Rates?, NBER Working Paper, No. 4532, November 1993. – *Dornbusch*, R.: Expectations and Exchange Rate Dynamics, Journal of Political Economy 84(6), pp. 1161–1176, 1976. – *Gärtner*, M.: Macroeconomics under Flexible Exchange Rates, New York: Harvester Wheatsheaf, 1993. – *Goodhart*, C. A. E.: 'News' and the Foreign Exchange Market, LSE Financial Markets Group Discussion Paper, No. 71, January 1990. – *Jeanne*, O. and *Masson*, P.: Currency Crises, Sunspots and Markov-Switching Regimes, CEPR Discussion Paper, No. 1990, October 1998. – *Leahy*, M. P.: The Profitability of US Intervention in the Foreign Exchange Markets, Journal of International Money and Finance 14(6), pp. 823–844, 1995. – *LeBaron*, B.: Chaos and Nonlinear Forecastability in Economics and Finance, Philosophical Transactions of the Royal Society of London A 348 (1686), pp. 397–404, 1994. – *LeBaron*, B.: Technical Trading Rule Profitability and Foreign Exchange Intervention, NBER Working Paper, No. 5505, March 1996. – *Mark*, N. C.: Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability, The American Economic Review 85(1), pp. 201–218, 1995. – *Marston*, R.: Stabilization Policies in Open Economies, in *Jones*, R. W. and *Kenen*, P. B., eds., Handbook of International Economics, Amsterdam: North-Holland, Volume 2, pp. 859–916, 1985. – *Obstfeld*, M. and *Rogoff*, K.: Exchange Rate Dynamics Redux, Journal of Political Economy 103(3), pp. 624–660, 1995. – *Obstfeld*, M. and *Rogoff*, K.: Foundations of International Macroeconomics, Cambridge, MA and London: The MIT Press, 1996. – *Silber*, W. L.: Technical Trading: When It Works and When It Doesn't, The Chartered Financial Analyst Digest 24(3), pp. 66–68, 1994. – *Sprott*, J. C.: Chaos Data Analyzer: The Professional Version 2.1, University of Wisconsin, Madison, WI 53706, 1995. – *Sprott*, J. C. and *Rowlands*, G.: PC User's Manual of Chaos Data Analyzer: The Professional Version, New York: American Institute of Physics, 1995. – *Stewart*, I.: Does God Play Dice? The New Mathematics of Chaos, London: Penguin Books, Second Edition, 1997. – *Szpiro*, G. G.: Exchange Rate Speculation and Chaos Inducing Intervention, Journal of Economic Behavior and Organization 24(3), pp. 363–368, 1994. – *Taylor*, D.: Official Intervention in the Foreign Exchange Market, or, Bet Against the Central Bank, Journal of Political Economy 90(2), pp. 356–368, 1982.

Summary

The Role of Foreign Exchange Intervention in a Chaotic Dornbusch Model

Massive foreign exchange interventions are shown to remove chaos, cycles, and instability in the models of De Grauwe and Dewachter (1992) and De Grauwe, Dewachter, and Embrechts (1993), where the exchange rate can behave chaotically in the framework of the Dornbusch model. (JEL F31, F41, F47)

Zusammenfassung

Die Rolle der Devisenmarktintervention im Chaotischen Dornbusch-Modell

Modelle von De Grauwe und Dewachter (1992) und De Grauwe, Dewachter und Embrechts (1993), in denen der Wechselkurs im Rahmen des Dornbusch-Modells chaotischen Tendenzen unterliegen kann, zeigen, daß starke Devisenmarktinterventionen Chaos, Zyklen und Instabilität eliminieren.

Résumé

Le Rôle de l'Intervention dans les Marchés du Cours du Change dans la Version Chaotique de la Modèle du Dornbusch

L'intervention massif dans les marchés du cours du change a prové d'emmener des chaos, des cercles et de l'instabilité dans la modèle du De Grauwe et Dewachter (1992) et dans la modèle du De Grauwe, Dewachter et Embrechts (1993). Dans ces modèles le cours du change peux s'amener chaotiquement en la version de la modèle du Dornbusch.