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Volatility Estimates of the Short-Term Interest Rate with an Application to German Data

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I. Introduction

The specification of the stochastic differential equation for the instantaneous rate of interest and its volatility, in particular, is fundamental for pricing contingent claims or bonds. In the past, the empirical literature on term structure models has lagged behind available theory but recently an impressive number of articles has emerged with the purpose of correctly specifying the short-term interest rate dynamics. This is especially true of those term structure models which *Jarrow* (1995) calls zero curve arbitrage models, i. e. term structure models which take the stochastic differential equation of the instantaneous risk-free rate of interest and a few bond prices as given in order to evaluate the remaining default-free zero coupon bond prices. The other class of models consists of contingent claim valuation models. Here, no measurement error is involved in calculating option prices because in addition to the stochastic differential equation for futures prices or the instantaneous risk free rate of interest, the entire zero coupon bond price curve is taken as given.

This paper focuses on zero curve arbitrage models. *Chan/Karolyi/Longstaff/Sanders* (1992) – referred to hereafter as CKLS – compare a number of zero curve arbitrage models by using an observable short-term interest rate as an approximation for the theoretical instantaneous

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rate of interest and by using a crude discretisation for the continuous time models. A much cited result of their study is the point estimate for the levels effect parameter γ of 1.5 (see Table 1 for a definition), which implies non-stationarity for the interest rate process, thereby violating the ergodicity assumption of the applied GMM estimator (*Bliss/Smith* (1997)).[1] The CKLS analysis has been extended in various ways. *Brenner/Harjes/Kroner* (1996), for instance, show that according to their data the volatility function incorporates both a levels effect and autoregressive conditional heteroskedasticity (ARCH). For monthly data, three proposed models yield point estimates of γ between 0.5 and 1.44. *Bliss/Smith* (1997) argue that the results derived in CKLS are invalid due to model misspecification. The Federal Reserve Board's monetary experiment in the period from October 1979 to September 1982 led to a structural break in the data generating process which is not accounted for in the CKLS analysis.

This paper also takes the CKLS model as a starting point for analysing German short-term interest rates.[2] The crude discretisation of the continuous time models is retained although estimation techniques exist, which attempt to eliminate the discretisation bias (e.g. *Duffie/Singleton* (1993), or *Gallant/Tauchen* (1996)). The justification is twofold: First, the continuous time models must be applied to discrete data regardless; the practitioner would probably like to know which of the zero curve arbitrage models fares best in this context. Second, both the efficient method of moments developed in *Gallant/Tauchen* (1996) and the indirect inference estimator of *Gouriéroux/Monfort/Renault* (1993) presuppose an auxiliary parametric model as a starting point for estimating the conditional density for the interest rate series. In this sense, the present paper might be considered a preliminary study for the application of either method.

The theory typically prescribes an AR(1) process for the short-term interest rate. Since this might not be sufficient from an econometrician's point of view, we employ the robust Lagrange Multiplier test (RB-LM test) developed in *Wooldridge* (1991) to identify the correct lag structure in the mean equation. Whereas the classical LM test is misspecified in the presence of ARCH effects in the residuals, the RB-LM test is not. According to our results the latter does not reject the AR(1) model.

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1 The non-stationarity of the interest rate process for $\gamma > 1$ is pointed out in *Dahlquist* (1996) and mentioned in both *Gouriéroux/Monfort* (1996) and *Broze/Scaillet/Zakoïan* (1995).

2 To the best of our knowledge no relevant study exists using German data.

We propose a consistent method for testing the restrictions of alternative zero curve arbitrage models. The test statistic used in CKLS does not have a standard distribution if the restrictions imply non-stationarity of the data generating process. Unlike *Andersen/Lund* (1997) and *Brenner/Harjes/Kroner* (1996), we find that for weekly data of the Euro-DM 3-Month rate its volatility depends either on the interest rate level or on information shocks but not on both. The results do not indicate a structural break in the data generating process for the time of the monetary experiment of the Federal Reserve Board. After testing various one-factor zero curve arbitrage models and econometric specifications, we derive a parsimonious continuous-time model with stochastic volatility for the short-term interest rate. Accordingly, two factors serve as the building blocks for a term structure model of interest rates in Germany.

The remainder of this article is organised as follows. Section 2 discusses the single-factor models as well as the data set to be studied and explains the econometric methodology to be employed. In Section 3 the empirical results are reported and a term structure model is derived. A summary and concluding remarks complete the paper.

II. Theory and Econometric Methodology

1. One-Factor Zero Curve Arbitrage Models

This section deals with term structure models which assume that a single stochastic factor causes the evolution of the entire zero coupon bond price curve. In other words all interest rates are perfectly correlated with one single state variable, the instantaneous risk free rate of interest, approximated by an observable short-term interest rate in practice. As in CKLS, the single-factor diffusion processes to be studied can be nested in the following stochastic differential equation for the instantaneous risk free rate of interest r:

$$dr = (a + b\,r)dt + \sigma r^{\gamma} dz \tag{1}$$

where dz denotes the standard Wiener process or Brownian motion $(dz = \epsilon\sqrt{t},\ \epsilon \sim N(0,1))$, and σr^{γ} the instantaneous standard deviation of interest rate changes which is often referred to as 'volatility'. The dependence of the instantaneous standard deviation on r^{γ} *is* known as the 'levels effect'. Within the models covered here, dz is the single factor driving the evolution of the entire term structure. Table 1 reports the term structure models included in (1). The specifications were chosen because of their analytical tractability and intuitiveness. The *Vasicek*, CIR-SR, and *Brennan/Schwartz* models assume 'mean reversion', i.e. the interest rate is drawn toward its long-term mean by the rate $|b|$.[3] Obviously, these models impose stationarity on the data generating process. The approximate discrete-time analog of the continuous-time model in equation (1) is (CKLS model)

$$
\begin{aligned}
& r_t - r_{t-1} = \alpha + \beta r_{t-1} + u_t \\
& E\left[u_t \mid F_{t-1}\right] = 0, \quad E\left[u_t^2 \mid F_{t-1}\right] = h_t \\
& h_t = \sigma^2 r_{t-1}^{2\gamma}
\end{aligned} \tag{2}
$$

where F_t denotes the information set at time t, and $\sigma^2 r_{t-1}^{2\gamma}$ the (conditional) variance of interest rate changes. The restrictions $\beta = 0$ as well as $\gamma = 1.5$ yield a non-stationary data generating process (see e.g. *Dahlquist* (1996)). Restricting the parameters to these values leads to a test statistic with a nonstandard distribution and, consequently, to unknown critical values. Therefore we propose to employ stationarity tests first. These in combination with volatility estimates can determine whether interest rates should be assumed to be mean-reverting in linear parametric models. In case of stationarity, mean reversion and $\gamma \leq 1$ follow whereas non-stationarity could be due to $\gamma > 1$ and/or a non-mean reverting data generating process. Only if γ is estimated to be less than one and the restriction $\gamma = 1$ is rejected is the test result for non-stationarity unambiguous.[4]

As pointed out by *Bliss/Smith* (1997), this model might be misspecified with regard to the probable change in the process during the late 1970s and early 1980s. As Figure 1 on page 13 suggests, both the level as well as the volatility appear elevated. Since this period coincides with the temporary monetary targeting experiment of the Federal Reserve Board, it is to be concluded that the U.S. market strongly influenced German rates. Following *Bliss/Smith* (1997), a dummy variable is introduced for this period:

$$
\begin{aligned}
& r_t - r_{t-1} = (\alpha + \delta_1 D_t) + (\beta + \delta_2 D_t) r_{t-1} + u_t \\
& E\left[u_t \mid F_{t-1}\right] = 0, \quad E\left[u_t^2 \mid F_{t-1}\right] = h_t \\
& h_t = (\sigma^2 + \delta_3 D_t)\, r_{t-1}^{2(\gamma + \delta_4 D_t)}
\end{aligned} \tag{3}
$$

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3 This can clearly be seen if (1) is written as $dr = -b(-a/b - r)dt + \sigma r^{\gamma} dz$ (with $b < 0$) where $|a/b|$ is the long-term mean of r.

4 We restrict ourselves to the case where r follows a finite AR process. *Backus/Zin* (1993) propose a one-factor term structure model with fractional integration where r is non-stationary and yet mean reverting.

Table 1

Single-Factor Term Structure Models

Alternative single-factor zero curve arbitrage models are nested in

$$dr = (a + b\,r)dt + \sigma r^{\gamma} dz$$

Model		Restrictions			
		a	b	γ	σ
Merton[a]	$dr = a dt + \sigma dz$	–	0	0	–
GBM[b]	$dr = b\,r dt + \sigma r dz$	0	–	1	–
Dothan[c]	$dr = \sigma r dz$	0	0	1	–
Vasicek[d]	$dr = (a + b\,r)dt + \sigma dz$	–	–	0	–
CIR-SR[e]	$dr = (a + b\,r)dt + \sigma\sqrt{r} dz$	–	–	0.5	–
BSch[f]	$dr = (a + b\,r)dt + \sigma r dz$	–	–	1	–
CIR-VR[g]	$dr = \sigma r^{1.5} dz$	0	0	1.5	–
CEV[h]	$dr = b\,r dt + \sigma r^{\gamma} dz$	0	–	–	–

[a] *Merton* (1973).
[b] Geometric Brownian Motion as used in *Rendleman/Bartter* (1980).
[c] *Dothan* (1978).
[d] *Vasicek* (1977).
[e] The CIR square-root model (*Cox/Ingersoll/Ross* (1985)).
[f] *Brennan/Schwartz* (1980).
[g] The CIR variable rate model (*Cox/Ingersoll/Ross* (1980)).
[h] Constant Elasticity of Variance model as discussed in *Cox* (1975) and *Cox/Ross* (1976).

where

$$D_t = \begin{cases} 1 & \text{for } t \in \text{ (Oct. 1979 until Sept. 1982} \\ 0 & \text{other} \end{cases}$$

Moreover, *Brenner* et al. (1996) show that for U.S. data the volatility of the short-term interest rate must be modeled as a function of both the level and of information shocks. The former is included in (3) because the lagged interest rate level directly affects its conditional variance. Information shocks are introduced into the volatility function by specifying an ARCH model.[5] We follow *Brenner* et al. (1996) in using their

[5] *Lamoureux/Lastrapes* (1990) argue that ARCH effects arise when information shocks are serially correlated.

AR(1)-GARCH(1,1)-X model which is an extension of the GARCH model as developed in *Bollerslev* (1986):[6]

$$
\begin{aligned}
&r_t - r_{t-1} = \alpha + \beta r_{t-1} + u_t \\
(4) \qquad &E\left[u_t \,|\, F_{t-1}\right] = 0, \quad E\left[u_t^2 \,|\, F_{t-1}\right] = h_t \\
&h_t = c_0 + c_1\, u_{t-1}^2 + c_2\, h_{t-1} + c_3\, r_{t-1}^{2\gamma}
\end{aligned}
$$

Alternatively, we adopt the EGARCH model (*Nelson* (1991)) because *Andersen/Lund* (1997) have shown that it fits their interest rate data best. However, we modify it to obtain a specification (AR(1)-EGARCH(1,1)-X) which is comparable to the GARCH-X model:

$$
\begin{aligned}
&r_t - r_{t-1} = \alpha + \beta r_{t-1} + u_t \\
&u_t = \eta_t \sqrt{h_t}, \quad \eta_t \sim i.i.d.\ N(0,1) \\
(5) \qquad &\log(h_t) = \omega_0 + \omega_1 g(\eta_{t-1}) + \omega_2(\log(h_{t-1})) + \omega_3 r_{t-1}^{2\gamma} \\
&g(\eta_t) = \Theta\eta_t + \vartheta\left[\,|\eta_t| - E\left[\eta_t\right]\right]
\end{aligned}
$$

Of course, the dummy variable as defined for the CKLS model would have to be added to the AR(1)-GARCH(1,1)-X as well as to the AR(1)EGARCH(1,1)-X model. For tractability, these versions are not stated. In (5), the conditional variance is a function of the lagged absolute disturbance instead of the lagged squared disturbance. In addition, η_t enters directly the conditional variance equation, which is known as a representation of the leverage effect. Negative shocks with respect to the expected bond prices are likely to be followed by increased volatility whereas positive shocks should lead to a reduced volatility. Given the relationship between interest rates and bond prices one would expect the opposite to hold in the above model, i.e. $\omega_1\Theta$ is expected to be positive. The AR(1)-EGARCH(1,1)-X model allows the interest rate level to influence its conditional variance in two ways: Through the leverage effect just described and through the levels effect which is measured by the parameter ω_3.

Apart from the inclusion of asymmetry, this specification has two significant advantages. First, it ensures a positive correlation between the conditional variance and its lagged values, and lagged squared disturbances. Negative parameter estimates cannot *a priori* be ruled out in the GARCH-X model although theoretically the model is defined only for positive parameter values. Second, for $c_1 + c_2 = 1$ in the GARCH specifi-

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6 *Bollerslev/Chou/Kroner* (1992) and *Bera/Higgins* (1993) give an overview of ARCH models.

cation the interest rate process would be covariance non-stationary and yet possibly strictly stationary. Such a conflict does not arise within the EGARCH framework (see e.g. *Andersen/Lund* (1997) and the literature cited therein).

2. Econometric Methodology

We start the analysis with the CKLS model specification as given in equation (3) and continue with the GARCH and EGARCH models. Unlike CKLS and *Bliss/Smith* (1997), we estimate all models by Maximum Likelihood assuming normally distributed residuals. The Student-t distribution might have been employed instead but for reasons of consistency we prefer the former. This is the same approach used in *Andersen/Lund* (1997).[7] The log-likelihood function to be maximised is

$$\log L(\boldsymbol{p}) = -\frac{1}{2}\left(\log(h_t)\frac{u_t^2}{h_t}\right) \tag{6}$$

where $\boldsymbol{p}$ is the vector of parameters of the model to be estimated. *Engle* (1982) argues in his seminal paper that a consistent and efficient ML estimation presupposes a consistent initial estimate of the mean equation parameters. Therefore, we first estimate the mean equation by least squares and use its parameter estimates and residuals as initial values for the ML estimation. The log-likelihood function is maximised by the Broyden/Fletcher/Goldfarb/Shanno (BFGS) algorithm.

Apart from testing various volatility specifications, we test for the correct lag structure in the mean equation. In *Brenner* et al. (1996) as well as in *Bliss/Smith* (1997), misspecification tests are an issue only when they are discussing the volatility function. This is especially surprising since *Brenner* et al. (1996) report Ljung-Box Q statistics which indicate the presence of serial correlation in all models. One explanation may be that the theory prescribes an AR(1) process for the instantaneous risk free rate of interest. But in practice this assumption does not necessarily hold with respect to an observable short rate (an exception is *Andersen/Lund* (1997): none of their two-factor models exhibits serial correlation in the residuals of the mean equation). The argument in *Engle* (1982) gives a justification for neglecting serial correlation in the conditional mean for ARCH models with a block diagonal information matrix.

7 For properties of the Quasi Maximum Likelihood approach see also *Weiss* (1986) and *Bollerslev/Wooldridge* (1992).

Accordingly, conditional mean and conditional variance can be estimated independently without a loss of asymptotic efficiency. But this argument does not hold for asymmetric ARCH models such as the EGARCH specification.

Diebold (1986) points out that the Ljung-Box test for serial correlation is misspecified in the presence of ARCH effects because they invalidate the standard asymptotic distribution theory. Therefore, the robust LM test (RB-LM test) developed in *Wooldridge* (1991) is employed (*Brenner* et al. (1996) use this kind of test for diagnostics of the volatility function). The terminology refers to the fact that the test statistic is robust with regard to a possibly misspecified volatility function. The following paragraph briefly discusses the RB-LM test.[8]

The first step involves a standardisation of the estimated residuals ($\hat{u}_t$) which are to be tested for serial correlation:

(7) $$\tilde{x}_t = x_t\left(\sqrt{\hat{h}_t}\right)^{-1}, \quad \tilde{\hat{u}}_{t-i} = \hat{u}_{t-i}\left(\sqrt{\hat{h}_t}\right)^{-1}, \quad i = 0, \ldots, k$$

where x_t denotes the vector of regressors used in the mean equation and k is the lag order which is to be used in the test for serial correlation. Next, the effect of the regressors on lagged residuals is eliminated by means of the following linear regressions:

(8) $$\tilde{\hat{u}}_{t-i} = \tilde{x}_t' b + \tilde{\hat{u}}^*_{t-i} \quad i = 1, \ldots, k.$$

This would give the following test regression:

(9) $$\tilde{\hat{u}}_t = \lambda_1 \tilde{\hat{u}}^*_{t-1} + \ldots + \lambda_k \tilde{\hat{u}}^*_{t-k} + v_t$$

Instead, *Wooldridge* (1991) proposes to multiply (9) by $\tilde{\hat{u}}_t$ and take the conditional expectation which gives

(10) $$1 = \rho_1 \tilde{\hat{u}}^*_{t-1} \tilde{\hat{u}}_t + \ldots + \rho_k \tilde{\hat{u}}^*_{t-k} \tilde{\hat{u}}_t + \omega_t$$

where w_t denotes the expectation error. The test statistic is the number of observations (T) minus the sum of squared residuals (SSR) of (10) with $T - SSR \sim \chi^2(k)$ under the null hypothesis. This test is called the *robust* Lagrange Multiplier (RB-LM) test because the estimation of the covariance matrix of $\tilde{\hat{u}}^*_{t-i} \tilde{\hat{u}}_t$ is not affected by the specification of the function for h_t.

[8] An application and description can also be found in *Dankenbring/Missong* (1997).

3. The Data

This study uses the Euro-DM 3-Month rate (London market, $R_{tD}^{(3m)}$) which is based on weekly observations supplied by Datastream. The data covers the period from February 1975 to early April 1998, i.e. 1210 observations in total. With respect to U.S. data, *Duffee* (1996) argues that the 3-Month rate is better suited than the 1-Month rate as a proxy for the theoretical instantaneous risk free rate of interest. Data sampled weekly are likely to lead to a smaller discretisation bias than monthly data. Figure 1 shows the series as well as the absolute changes.

The econometric concept of stationarity corresponds to the theoretical concept of mean reversion for finite AR processes. Therefore stationarity

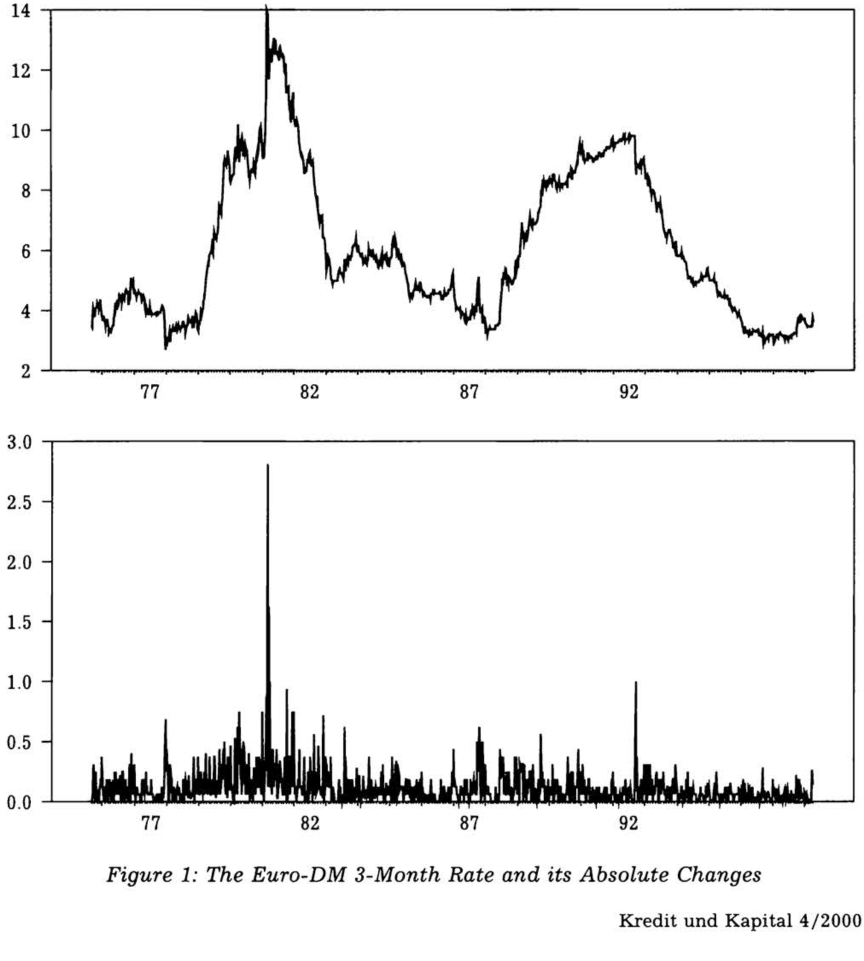

Figure 1: The Euro-DM 3-Month Rate and its Absolute Changes

tests are an important tool for detecting the correct model specification. For this purpose we employ the KPSS test, derived in *Kwiatkowski/Phillips/Schmidt/Shin* (1992), as well as the augmented Dickey/Fuller (ADF) and the Phillips/Perron (PP) test. In contrast to the latter two, the first tests the null hypothesis of stationarity against the alternative of a unit root. The following paragraph briefly introduces the test. Since the data and interest rates in general do not show a deterministic time trend for a long enough sample period, we confine ourselves to testing for level stationarity.

First, the variable z_t to be tested is regressed on an intercept and the corresponding residuals e_t are computed (i.e. $e_t = z_t - \bar{z}$, $t = 1, \ldots, T$). Next, the partial sum process of e_t, S_t, is defined as

$$S_t = \sum_{i=1}^{t} e_i \quad t = 1, \ldots, T. \tag{11}$$

The test statistic is

$$\eta = T^{-2} \sum_{t=1}^{T} S_t^2 / \sigma^2 \tag{12}$$

where σ^2 is the long-run variance defined as

$$\sigma^2 = \lim_{T \to \infty} T^{-1} E\left[S_T^2\right]. \tag{13}$$

Of course, σ^2 is not observable. A consistent estimator denoted by $s^2(l)$ is constructed from the residuals e_t in the following way:

$$s^2(l) = \frac{1}{T} \sum_{t=1}^{T} e_t^2 + \frac{2}{T} \sum_{g=1}^{l} \left(1 - \frac{g}{l+1}\right) \sum_{t=g+1}^{T} e_t e_{t-g}. \tag{14}$$

Finally, the estimated test statistic denoted by $\hat{\eta}$ is

$$\hat{\eta} = T^{-2} \sum_{t=1}^{T} s_t^2 / s^2(l). \tag{15}$$

Unfortunately, the test statistic depends on the value of the lag truncation parameter l chosen. A considerable size distortion might arise for small values due to significant serial correlation in the residuals e_t. Conversely the power under the alternative decreases as l increases because $s^2(l)$ increases and consequently the test statistic decreases as l increases by construction. *Kwiatkowski* et al. (1992) argue that a good compromise between large size distortions and small power under the alternative is given for $l = 8$. However, Table 2 shows the test statistics for $l = 0, \dots 12$.

Table 2

KPSS Test for Stationarity

l^a	0	1	2	3	4	5	6
Test stat. for r_t^b	8.02	4.02	2.68	2.02	1.62	1.35	1.16
Test stat. for Δr_t^c	0.33	0.32	0.32	0.32	0.32	0.31	0.30
l	7	8	9	10	11	12	
Test stat. for r_t	1.01	0.91	0.82	0.75	0.69	0.63	
Test stat. for Δr_t	0.30	0.30	0.30	0.29	0.28	0.28	

[a] l denotes the lag truncation parameter of the long-run variance estimator. The critical values derived in *Kwiatkowski* et al. (1992) for a significance level of 5% (1%) are 0.463 (0.739).
[b] This row gives the test statistics for r_t.
[c] This row shows the test statistics for $\Delta r_t = r_t - r_{t-1}$.

For $l = 0, \dots, 10$ the null of stationarity is rejected at the 1% level, for $1 = 11$, 12 at the 5% level whereas the null of difference stationarity clearly cannot be rejected.

Table 3 gives the results of the more standard ADF and PP test for stationarity. First, the ADF test regression was run with a constant, i.e. under the hypothesis of a deterministic linear time trend in the level. This gives a test for trend stationarity. Since the intercept proved to be insignificant in each case here, too, only the test results for level stationarity are reported. The tests imply that the Euro-DM 3-Month rate is a random variable of a data generating process which is integrated of order one.

We conclude that the German short rate does not exhibit a deterministic time trend and is to be modeled as a variable of an integrated process of order 1.[9] Consequently, the short-term interest rate does not mean revert

[9] *Ball/Torous* (1996) perform simulation studies which show that neglecting non-stationarity yields misleading results for zero curve arbitrage models. This holds regardless of the estimation technique used.

Table 3

ADF and PP Test for Stationarity

lag[b]	ADF test[a] Δr_t	r_t	PP test Δr_t	r_t
1	–25.39	–1.39	–32.23	–1.37
2	–21.48	–1.27	–32.18	–1.36
3	–16.66	–1.19	–32.15	–1.34
4	–14.89	–1.33	–32.15	–1.35
5	–13.90	–1.36	–32.15	–1.37
6	–12.83	–1.34	–32.16	–1.38

[a] These columns show the ADF test statistics. Within this model without an intercept (i.e. the time series does not contain a deterministic time trend), the critical value of the 1% significance level for both tests is –2.57 (cf. *Davidson/McKinnon* (1992)).

[b] In connection with the ADF test, 'lag' denotes the maximum lag order; in connection with the PP test, the truncation parameter for the Bartlett window.

in our framework unless the volatility function causes non-stationarity. Although this poses conceptual difficulties since the model cannot rule out negative values in the future, linear parametric empirical analyses have to be carried out within the econometric framework for non-stationary data.[10] There are simply too few observations for which the process mean reverts. Nor does the series exhibit a deterministic time trend.

III. Model Estimations

First, we estimate the CKLS model as given in equation (3) with dummy variables for the period of the monetary experiment of the Federal Reserve Board.[11] The RB-LM(1) test statistic amounts to 0.11 with a marginal significance level of 0.74. The autocorrelation function (not given) indicates serial correlation to be present in the residuals whereas the RB-LM test does not. Therefore the estimations are carried out with lagged interest rate differences as well as without. The coefficients of interest hardly alter at all and the additional coefficients are insignifi-

10 *Stock/Watson* (1993) also mention these conceptual difficulties but nevertheless accept their test results and assume interest rates to be $I(1)$.

11 The results are available from the author upon request.

cant. Moreover, a test for joint significance, i.e. $H_0 : \phi_1 = \phi_2 = \phi_3 = 0$, does not allow for a rejection of the null hypothesis (the $\chi^2(3)$ distributed test statistic is 2.658, with a marginal significance level of 0.447).

According to these estimates the data generating process is also non-stationary. The levels effect parameter γ equals 0.12 and is insignificant for both econometric models.[12] The dummy variables in the conditional variance equation are significant which implies that the model cannot explain the increased interest rate volatility during the early eighties. However, since the data plot exhibits two significant outliers for February/March 1981 the model is re-estimated without these observations.[13] Table 4 gives the results. Here, the importance of the Bliss/Smith dummies becomes apparent. Only within the specification without special treatment for the period of elevated conditional variance does the levels effect parameter γ become significant.

The parameter δ_3 remains significant but a joint test with $\delta_3 = \delta_4 = 0$ under the null hypothesis gives a $\chi^2(2)$ distributed test statistic of 4.68 with a marginal significance level of 0.096. Therefore we conclude that there is no structural break in the data generating process. The one-factor zero curve arbitrage models are to be tested within the traditional CKLS framework. The Hannan-Quinn information criterion also favours the model without any dummies. As shown in Table 4, the CKLS model gives a point estimate of the levels effect parameter γ which is close to 0.5 and highly significant. The CIR-SR model assumes this particular value. Moreover, if the outliers in February/March 1981 were eliminated from the sample the CKLS model would yield this result (not given).[14]

CKLS chose this framework for testing the restrictions of alternative term structure models. However, the test statistic is not standardly distributed if a non-stationary DGP is assumed under the null. This is the case for $\beta = 0$ as well as $\gamma > 1$. We avoid such difficulties by first determining the characteristics of the mean equation and second analysing the properties of the conditional variance. The only testable restrictions are those on the levels effect parameter with $\gamma \leq 1$ under the null. Table 5 gives the results.

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12 The t-statistics of γ and its dummy parameter will only be valid if the true value is less than one. The estimates do not point to a violation of this assumption.

13 In *Deutsche Bundesbank* (1981) these values are explained by a temporary abandonment of the Bank's short-term loan instrument, which is called the *Sonderlombard*.

14 This conflicts with previous estimates using monthly data. There one single outlier significantly influenced the results as in *Bliss/Smith* (1997).

Table 4

Estimates of the Levels Effect Model

The model estimated with weekly data for the DM 3-Month rate is

$$r_t - r_{t-1} = \alpha + \delta_1 D_{2/81} + \delta_2 D_{3/81} + \beta r_{t-1} + u_t$$
$$E\left[u_t \,|\, F_{t-1}\right] = 0, \quad E\left[u_t^2 \,|\, F_{t-1}\right] = h_t, \quad h_t = (\sigma^2 + \delta_3 D_t) r_{t-1}^{2(\gamma + \delta_4 D_t)}$$
$$D_t = \left[1 \text{ for } t \in (5/10/1979 - 24/9/1982),\ 0 \text{ other}\right]$$
$$D_{2/81} = \left[1 \text{ for } t = 27/2/1981,\ 0 \text{ other}\right]$$
$$D_{3/81} = \left[1 \text{ for } t = 6/3/1981,\ 0 \text{ other}\right].$$

	CKLS model		Model without outl. and with *Bliss/Smith* dummies	
α[a]	0.0146	(0.488)	0.0163	(0.641)
δ_1			2.8322	(5.235)
δ_2			–0.0964	(–0.140)
β	–0.0027	(–0.508)	–0.0032	(–0.688)
σ^2	0.0343	(3.308)	0.0980	(2.872)
δ_2			–0.0909	(–2.664)
γ	0.4671	(5.906)	0.1229	(1.158)
δ_4			0.6942	(1.483)
HQ[b]	–2.129		–1.973	
RB-LM(1) Test[c]	0.015	(0.903)	0.0278	(0.868)
RB-LM(11) Test	3.887	(0.973)	11.67	(0.389)

[a] t-values are in brackets.
[b] Hannan-Quinn information criterion.
[c] Marginal significance levels are in brackets.

Not surprisingly, the only restriction which is not rejected is $\gamma = 0.5$. Consequently, the stationarity tests analysed in the last section taken together with these results imply a model without mean reversion. The zero curve arbitrage model suggested by the data thus far is

$$dr = \sigma r^{0.5} dz \tag{16}$$

i.e. a generalized Wiener process without a drift but with an instantaneous standard deviation which depends on the interest rate level. With respect to Table 1 the restrictions are $\alpha = 0$, $\beta = 0$ und $\gamma = 1/2$. However,

Table 5

Test of Alternative Zero Curve Arbitrage Models

Model	Testable Restrictions	Test statistic[a]
Merton, Vasicek	$\gamma = 0$	35.14 (<0.001)
CIR-SR	$\gamma = 1/2$	0.151 (0.728)
GBM, Dothan, BSch	$\gamma = 1$	44.20 (<0.001)

[a] The test statistic is distributed as χ^2 with one degree of freedom. Marginal significance levels are in brackets. The unrestricted econometric model is $\Delta r_t = \alpha + \beta r_{t-1} + u_t,\ E[u_t \mid F_{t-1}] = 0,\ E\left[u_t^2 \mid F_{t-1}\right] = h_t,\ h_t = \sigma^2 r_{t-1}^{2\gamma}$.

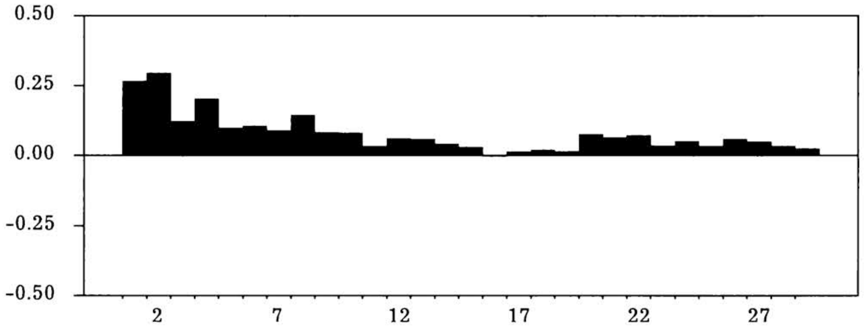

Figure 2: Absolute CKLS Residuals Autocorrelation Function

the autocorrelation function of the absolute standardised residuals of the unrestricted CKLS model (i.e. the autocorrelation function of u_t/h_t with u_t and h_t as given in equation (2), Figure 2) suggests that the conditional variance time dependence is not adequately modeled. Accordingly, a GARCH specification is to be preferred.

We follow the general-to-specific approach and therefore analyse the most general GARCH-X specification which includes dummies for the monetary experiment period first.[15] The model with an exact fit for the two outliers does not indicate a structural break. Consequently, the GARCH model is clearly able to explain the period of increased vola-

[15] These results are also available from the author upon request.

tility and outperforms the CKLS model on these grounds as well. However, the conditional variance shows a negative intercept whereas GARCH models are only defined for $c_0,\ c_1,\ c_2 > 0$ but this result depends on the inclusion of dummies. The estimates without any dummies yield a levels effect parameter γ in the GARCH-X model which is nearly equal to 0.5, as the CIR-SR model predicts, but it remains insignificant. In the GARCH model, moreover, the ARCH and GARCH parameters sum up to more than one and thus violate the definition. Therefore we re-estimate the model with a dummy in the conditional variance equation which is equal to one on February 27, 1981 and March 6, 1981. Now, $c_1 + c_2$ is strictly less than one, as required. Taken together, the traditional GARCH(1,1) model fares best on all counts and we report only these estimates in Table 6. Another advantage over the GARCH-X model worth mentioning is that its asymptotics are well known.

One extension is the EGARCH model. On the one hand it incorporates the leverage effect (unexpected interest rate hikes are typically followed by an increased conditional variance) while ensuring positive values for the conditional variance, on the other. Table 7 gives the results for the

Table 6

Estimates of the GARCH(1,1) Model

The model estimated with weekly data for the Euro-DM 3-Month rate is

$$r_t - r_{t-1} = \alpha + \beta r_{t-1} + u_t, \quad E\left[u_t \,|\, F_{t-1}\right] = 0, \quad E\left[u_t^2 \,|\, F_{t-1}\right] = h_t,$$
$$h_t = c_0 + \delta D_t + c_1 u_{t-1}^2 + c_2 h_{t-1}, \quad D_t = \left[1 \text{ for } t = 27/2 \text{ \& } 6/3/1981,\ 0 \text{ other}\right].$$

Parameters			Diagnostics
α[a]	0.0179	(0.649)	RB-LM(1) test[b]
β	–0.0036	(–0.793)	0.707 (0.400)
c_0	0.0720	(2.032)	RB-LM(11) test:
δ	1.1407	(1.560)	10.57 (0.480).
c_1	0.3175	(2.037)	Hannan-Quinn[c]
c_2	0.4980	(2.213)	–1.782

[a] t-values are in brackets.
[b] Marginal significance levels are in brackets.
[c] Hannan-Quinn information criterion.

Table 7

Estimates of the EGARCH-X and EGARCH Model

The model estimated with weekly data for the Euro-DM 3-Month rate is

$$r_t - r_{t-1} = \alpha + \beta r_{t-1} + u_t, \quad u_t = \eta_t \sqrt{h_t},\ \eta_t \sim\ i.i.d.\ N(0,1)$$
$$\ln(h_t) = \omega_0 + \omega_1 g(\eta_{t-1}) + \omega_2(\ln(h_{t-1})) + \omega_3 r_{t-1}^{2\gamma}$$
$$g(\eta_t) = \Theta\eta_t + \vartheta\big[\,|\eta_t| - E[\eta_t]\big]$$

	EGARCH-X Model		EGARCH Model	
α[a]	0.0194	(0.744)	0.0375	(1.387)
β	–0.0030	(–0.680)	–0.0066	(–1.583)
ω_0	0.0218	(1.497)	0.0160	(1.734)
ω_1	0.2980	(5.254)	0.2756	(6.087)
ω_2	0.6260	(5.365)	0.7447	(8.568)
Θ	0.3791	(1.909)	0.4000	(2.208)
ϑ	0.9511	(4.071)	0.9186	(5.814)
ω_3	0.0001	(0.144)		
γ	1.3390	(0.932)		
$\omega_1\Theta$[b]	0.1130	(1.893)	0.1103	(2.180)
$\omega_1\vartheta$[c]	0.2834	(3.560)	0.2532	(3.020)
HQ[d]	0.1965		0.1826	
RB-LM(1) test[e]	0.005	(0.941)	0.004	(0.949)
RB-LM(11) test	6.802	(0.815)	6.857	(0.811)

[a] t-values are in brackets.

[b] $\omega_1\Theta$ denotes the leverage effect parameter. Its variance is computed as $\mathrm{Var}(\omega_1\Theta) = \Theta^2\mathrm{Var}(\omega_1) + \omega_1^2\mathrm{Var}(\Theta) + 2\Theta\omega_1\,\mathrm{Cov}(\omega_1, \Theta)$.

[c] $\omega_1\vartheta$ denotes the ARCH effect parameter. Its variance is computed accordingly.

[d] Hannan-Quinn information criterion.

[e] Marginal significance levels are in brackets.

EGARCH-X model as well as for the traditional EGARCH model, with the latter outperforming the former. These estimates also reveal that the asymptotic characteristics of the estimators in conditional variance models with levels and ARCH effects are quite problematic. γ reaches an implausibly large (but insignificant) value. Nevertheless, it may be inferred that leverage and levels effects are not significant in the model including both whereas the traditional EGARCH model yields a positive and significant estimate for the leverage effect parameter, as expected.

The conclusion to be drawn thus far is that a model with ARCH and levels effect is overparametrised with respect to the Euro-DM 3-Month rate. Within the traditional GARCH and EGARCH models no structural break is detected. The autocorrelation function of the absolute standardised residuals of the GARCH(1,1) model which assumes the mean to be generated by a random walk as in (17) is given in Figure 3. The model also outperforms the CKLS model on these counts, although many autocorrelation coefficients are significant.

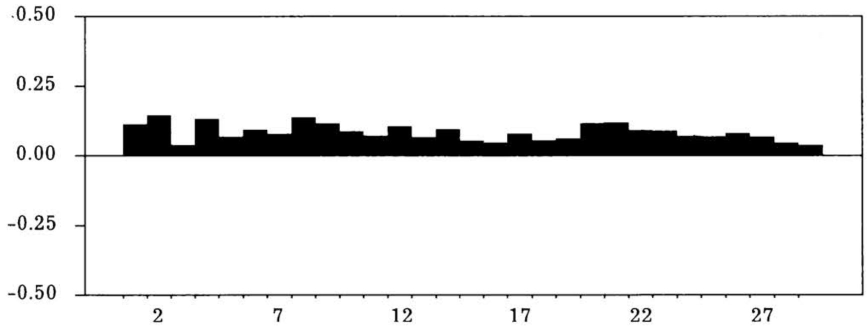

Figure 3: Absolute GARCH Residuals Autocorrelation Function

Due to the significant asymmetry parameter in the EGARCH model the GARCH model appears to be misspecified. However, applying the formula given in *Drost/Werker* (1996) the latter can easily be translated into a linear two-factor term structure model unlike the former, which would require auxiliary simulations.

Drost/Werker (1996) derive a continuous-time model which is equivalent to a GARCH model in discrete time.[16] Accordingly the model

$$r_t - r_{t-1} = u_t,\ u_t \sim i.i.d.\ N(0,\ h_t),\ h_t = c_0 + c_1 u_{t-1}^2 + c_2 h_{t-1} \tag{17}$$

with $c_0,\ c_1,\ c_2 > 0,\ c_1 + c_2 < 1$ and a finite fourth moment can be translated directly into a continuous-time model of the form

[16] *Drost/Werker* (1996) define a so called *weak* GARCH discrete time model which is closed under time aggregation. Its definition of the unconditional variance differs from that to be found in the traditional GARCH model.

$$dr_t = \sigma_{t-} dz_1,\ d\sigma_t^2 = \kappa(\theta - \sigma_{t-}^2)dt + \sqrt{2\lambda\kappa}\,\sigma_{t-}^2 dz_2 \tag{18}$$

where z_1 and z_2 are independent Brownian motions (i.e. $E[dz_1] = 0$, $E[dz_2] = 0$, $E[dz_1 dz_2] = 0$) and its parameters are determined by c_0, c_1 and c_2 (the distance between two observations Δ is assumed to approach zero):

$$\frac{c_o/\Delta}{1 - c_1 - c_2} \to \theta \qquad \frac{1 - c_1 - c_2}{\Delta} \to \kappa \qquad \frac{c_1^2}{1 - c_1 - c_2} \to \lambda$$

The discrete time estimates are[17]

$$\begin{gathered} r_t - r_{t-1} = u_t \quad u_t \sim i.i.d.\ N(0,\ h_t) \\ h_t = 0.0727 + 0.3180 u_{t-1}^2 + 0.4947 h_{t-1}. \end{gathered} \tag{19}$$

With Δ set to 1 we obtain the following short-term interest rate dynamics

$$\begin{aligned} dr_t &= \sigma_{t-} dz_1 \\ d\sigma_t^2 &= 0.19(0.39 - \sigma_{t-}^2)dt + 0.20\sigma_{t-}^2 dz_2 \end{aligned} \tag{20}$$

As shown in e.g. *Cox/Ingersoll/Ross* (1985) or more generally for exponentially affine term structure models in *Duffie/Kan* (1996) such stochastic differential equations lead to second-order partial differential equations for zero coupon bond prices. Consequently, the factor dynamics of (20) determine the entire zero coupon term structure.

A prerequisite for the two-factor model in (20) being able to explain the stochastics of all interest rates is that a multivariate vector error correction analysis yields one stochastic trend, i.e. all interest rates need to be cointegrated with r_t since the other factor is stationary by definition.

With respect to U.S. data, *Johnson* (1994), *Engsted/Tanggaard* (1994), *Hall/Anderson/Granger* (1992) and *Pagan/Hall/Martin* (1995) do indeed find one stochastic trend whereas *Wolters* (1998) finds two stochastic trends for German yields. However, if our second (stationary) factor has a stronger influence on short-term interest rates than on long-term ones, this model can explain the relatively high volatility of short-term interest rates vis-à-vis long-term ones.[18]

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17 The model also incorporates a dummy variable for the extreme interest rate values in February and March 1981, as shown in Table 6, because these values are due to institutional irregularities (see footnote 13 as well).

18 *Pfann/Schotman/Tschernig* (1996) propose a non-linear two-regime model in order to explain this phenomenon as well as mean reversion for double-digit interest rate values.

IV. Summary and Conclusion

We presented a procedure for testing the restrictions of alternative zero curve arbitrage models which does not lead to invalid distributions of the test statistic. It was shown that within a framework of linear parametric models the data-generating process of the Euro-DM 3-Month rate does not exhibit mean reversion. The simplification of the zero curve arbitrage models obtained by assuming an AR(1) process cannot be rejected by the RB-LM test. In contrast to previous studies for U.S. data, the volatility depends on either information shocks or the interest rate level but not on both. However, the GARCH model outperforms the levels effect model. Finally, we propose a two-factor model of the term structure in Germany, where one factor is the short-term interest rate level and the second its conditional variance.

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Summary

Volatility Estimates of the Short-Term Interest Rate with an Application to German Data

This paper proposes a procedure for testing alternative specifications of the short-term interest rate's dynamics which takes into account the non-stationarity of the interest rate process for certain restrictions, i.e. the traditional test statistic has a non-standard distribution. Moreover, we do not take the specification of the mean equation as given by the theory but rather base the choice of the lag structure on a robust Lagrange Multiplier test. In contrast to U.S. data we find that the volatility depends either on the interest rate level or on information shocks but not on both. Finally, we propose to describe the short-term interest rate's dynamics by means of an AR(1) model with stochastic volatility. (JEL C2, E4, G1)

Zusammenfassung

Schätzungen von Volatilitätsfunktionen des kurzfristigen Zinssatzes unter Verwendung von deutschen Daten

Der vorliegende Artikel testet verschiedene Spezifikationen zur Modellierung des Euro-DM-3-Monats-Zinssatzes, wobei die Hypothese einiger Modelle, der zufolge der Daten generierende Prozeß nicht-stationär und somit die Teststatistik nicht standardmäßig verteilt ist, Berücksichtigung findet. Die theoretisch vorgegebene Anzahl von einer eigenen Verzögerung in der Mittelwertgleichung wird durch den robusten Lagrange-Multiplikator-Test bestätigt. Im Gegensatz zu Untersuchungen auf der Basis von Daten des US-amerikanischen Geldmarktes zeigt sich, daß die Volatilität entweder in Abhängigkeit vom Zinsniveau oder von Informationsschocks zu modellieren ist. Eine Berücksichtigung beider Effekte führt zu einer Überparametrisierung des Modells. Abschließend wird für den Euro-DM-3-Monats-Zinssatz ein AR(1)-Modell mit stochastischer Volatilität vorgeschlagen.

Résumé

Evaluations des fonctions de volatilité du taux d'intérêt à court terme à l'aide de données allemandes

Cet article teste différentes spécifications pour modeler le taux d'intérêt euro-DM à trois mois. On y considère l'hypothèse de certains modèles selon laquelle le processus généré n'est pas stationnaire et ainsi les statistiques de tests sont non standardisées. Le nombre fixé théoriquement d'un retard propre dans l'équation de la valeur moyenne est confirmé par le solide test du multiplicateur de Lagrange. A l'encontre des analyses basées sur des données du marché monétaire américain, on voit que la volatilité est à modeler soit en fonction du niveau des taux d'intérêt soit en fonction des chocs d'information. La considération des deux effets conduit à une surparamétrisation du modèle. Finalement, un modèle AR(1) avec une volatilité stochastique est proposé pour le taux d'intérêt euro-DM à trois mois.