The Correct Economic Interpretation of Rational Expectations

Reply

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I. Introduction

In a paper I argued that accelerating inflation under perfect foresight requires money growth substantially below inflation rates, and for extremely high accelerating inflation rates may even lead to negative money growth rates. This shows that the standard monetary approach to hyperinflation has some serious inadequacies to explain empirical phenomena (see also *Buiter* 1990, *Dornbusch, Sturzenegger, Wolf* 1990, *Bernholz* and *Gersbach* 1992). In a comment on my article, *Wesche* and *Wierum,* henceforth WW, claim to show that in my model inflation and money growth can increase proportionally during accelerating inflations under perfect foresight. They write "Inflation and money growth both increase with factor k from period t onwards". However, their analysis has several flaws which invalidate their conclusions. A correct calculation and interpretation of their model fully supports the conclusions in my paper.

If real money demand is a decreasing function of the inflation rates, accelerating inflation under perfect foresight involves decreasing real money demand and hence decreasing equilibrium real balances. By simple algebra, a decrease in M/P is impossible unless the money stock M grows less quickly than the price level P (e.g. *Friedman* 1969 or *Gale* 1982). Strongly accelerating inflation requires substantially lower money growth rates. The familiar one-to-one relationship between money growth and inflation is limited to steady states. It is remarkable that WW want to apply it more generally to accelerating inflation rates.

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II. The Models

In order to show how the analysis of WW went wrong, we compare their approach to the one used in my paper. Equations referring to my model are labeled by *.

We begin with the money demand functions used in both models.

$$(1)^* \qquad M_t = cP_t e^{-d(\pi^e_{t+1})} \text{ or } m_t = \ln(c) + p_t - d(\pi^e_{t+1})$$

$$(1) \qquad M_t = cP_t(\pi^e_{t+1})^{-d} \text{ or } m_t = \ln(c) + p_t - d\ln(\pi^e_{t+1})$$

M_t represents the nominal money stock, P_t denotes the price level, and π^e_t the expected inflation rate plus one, expected at $t-1$, and c and d are constants. m, p denote logs. In order to keep the models comparable we always use π_t for the inflation rate plus one and $\ln(\pi_t)$ for the logarithm of the inflation rate plus one. (WW used the symbol π_t for the logarithm of the inflation rate plus one. They used c in stead of $\ln(c)$ as a description of the constant in the logarithmic version of the money demand function which obviously makes no difference).

Equations (1) and (1)* show that whereas in my paper I used an exponential money demand function WW used a power function. The latter is often used in empirical examinations to approximate the former, but it is well recognized that the approximation is substantially misleading for high inflation rates. Since high and accelerating inflation rates are the focus of both papers, their statement that "we begin by rewriting the model in a slightly different form" is not correct. The discrepancies between both models grow more and more when inflation accelerates.

III. Economic Interpretation of the Key Difference Equations

Since WW do not challenge the algebraic correctness of my paper we can jump directly to the key difference equation of the two models. Assuming perfect foresight we get:

$$(2)^* \qquad \pi_{t+1} = -d^{-1}\ln\{(\mu_t)\pi_t^{-1}\} + \pi_t$$

$$(2) \qquad \ln(\pi_{t+1}) = \frac{1+d}{d}\ln(\pi_t) - \frac{1}{d}\ln(\mu_t)$$

μ_t is defined as the money growth rate plus one. Equation (2)* corresponds to equation (6) in my original paper. Note, however, the difference in notation. In my original paper I used μ_t for the money growth rate only, and therefore $1 + \mu_t$ entered equation (2)*. Equation (2) corresponds to equation (8) in WW. Again WW use μ_t for the log of the money growth rate plus one.

How do we have to interpret these difference equations? Money growth rates are given exogenously so (2)* – and similarly (2) – is an equation of the endogenous variables π_t and π_{t+1}. However, one cannot interpret this equation in terms of causality. According to the standard interpretation of rational expectations equilibria (e.g. *Azariadis* 1993), this equation is only a consistency condition, requiring that agents forecast inflation correctly. As such it is one of a set of conditions – one for each period t – which help determine all inflation rates $\pi_0, \pi_1, \ldots$ *simultaneously* without any indication about causality.

In fact it is well-known that these consistency conditions are *not* sufficient to determine the sequence of inflation rates. Condition (2)* – and similarly (2) – gives only a difference equation for the inflation rates $\pi_0, \pi_1, \ldots$. Without a boundary condition, this difference equation typically has *many* solutions (e.g. *Samuelson* 1958, *Grandmont* 1983, *Farmer* 1993), and it is not even clear in what sense the sequence of money growth rates can be said to determine the sequence of inflation rates.

Without additional information about boundary behavior for $t = 0$ or for t approaching infinity, there is therefore no meaningful causality interpretation of equation (2)* and (2). Both interpretations, that the future determines the present, or that the present determines the future are incorrect. WW do not seem to be aware of this. Their conclusion "equation (4) determines current inflation" is false, since one equation is not enough to determine two endogenous variables π_t and π_{t+1}.

IV. Sustaining Accelerating Inflation

The key question in both models concerns the relationship between inflation and money growth rates in perfect foresight equilibria with accelerating inflation. The rate of acceleration is assumed to be constant, and we are concerned with rational expectations equilibria (if any exist) for which:

$$\pi_{t+1} = k\pi_t \quad \text{with} \quad k > 1$$

Note that the authors again mix up logarithms and absolute values since they use $\ln(\pi_{t+1}) = k \ln(\pi_t)$ instead of the correct form $\ln(\pi_{t+1}) = \ln(k) + \ln(\pi_t)$. In the following we use $\ln(\pi_{t+1}) = k \ln(\pi_t)$ for the model of WW since their arguments are based on this specification.[1]

Without suggesting a causality interpretation we can directly insert the requirement $\pi_{t+1} = k\pi_t$ (or $\ln(\pi_{t+1}) = k \ln(\pi_t)$ as used by WW) in equations (2)* and (2) and get:

$$(3)^* \qquad \mu_t = \pi_t e^{-d(k-1)\pi_t}$$

$$(3) \qquad \ln(\mu_t) = \{1 + d(1 - k)\} \ln(\pi_t)$$

Equation (3)* corresponds to equation (9) in my original paper, again μ_t stands now for $\mu_t + 1$. Equation (3) corresponds to equation (14) in the paper of WW. WW claim that their equation (14) can only be obtained by forward substitution. This is obviously wrong since we just obtained the desired result by substituting $\pi_{t+1} = k\pi_t$ directly into equation (2)* and $\ln(\pi_{t+1}) = k \ln(\pi_t)$ into equation (2). For the final result of WW no forward substitution is necessary (see also *Azariadis* 1993). Their claim to the contrary is based on a fundamental misunderstanding of the economic content of rational expectations.

How should equations (3)* and (3) be interpreted? Both equation (3) and (3)* are derived from combining the rational expectation condition (2) and (2)* with the constant acceleration requirement $\pi_{t+1} = k\pi_t$. The sequence of conditions (3) and (3)* for $t = 1, 2, \ldots$ must therefore be interpreted as a joint requirement of the sequence of inflation rates and the sequence of money growth rates. The sequence of money growth rates is now endogenous. Whereas in section 3 we looked at money growth rates as exogenous and asked what sequences of inflation rates are compatible with, we now impose the requirement $\pi_{t+1} = k\pi_t$ which is not compatible with most configurations of money growth rates.

The conclusion from (3)* are stated in my original paper and are obviously still true. First, the money growth rate is lower than the inflation rate. Second, for extremely high accelerating inflation rates, money

[1] If we would use the correct specification $\ln(\pi_{t+1}) = \ln(k) + \ln(\pi_t)$ for the model used by WW then money growth would still be substantially lower than inflation, but it would be always positive and evolve as a constant fraction of the inflation rate.

growth rates even can become negative which is obviously at odds with empirical phenomena.

The authors conclude from equation (3) that if inflation increases with factor k then also money growth does increase with factor k. However, they forget that this is still an equation in logs. In absolute terms equation (3) becomes:

$$(3)' \qquad \mu_t = \pi_t^{\{1+d(1-k)\}}$$

Hence, with the specifications used by WW, money growth is below the growth of inflation and increases less than with factor k since the exponent of inflation $1 + d(1 - k)$ is smaller than 1. With the specifications used by WW one could also obtain negative money growth rates if k becomes very large which, however, is obviously at odds with empirical phenomena (e.g. *Bernholz* and *Gersbach* 1992, *Sargent* 1982).

To sum up, an economically correct version of the exercise of WW supports fully the results obtained in my paper. The remaining differences in results of both models are due to different specifications. However, they have nothing to do with whether or not the solution is obtained by forward substitution or the direct method applied in my paper.

The main conclusion of the analysis may be at odds with fundamental beliefs about the relationship between money and inflation. They are also at odds with empirical findings. However, as stressed in my paper and elsewhere (e.g. *Dornbusch, Sturzenegger, Wolf,* 1990, *Bernholz* and *Gersbach* 1992), this inadequacy cannot be removed by changes in solution methods, provided they are correctly done.

References

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Summary

The Correct Economic Interpretation of Rational Expectations – Reply

In a comment on my article, *Wesche* and *Wierum* criticize the derived relationship between money growth and inflation during accelerating inflation for a model with exponential money demand. However, their analysis has several flaws and the authors seem not to be aware of the economic content of rational expectations equilibria. A correct calculation and interpretation of their exercise fully supports the conclusions in my earlier paper.

Zusammenfassung

Die richtige wirtschaftspolitische Interpretation von rationalen Erwartungen – Erwiderung

In einer Stellungnahme zu meinem Artikel kritisieren *Wesche* und *Wierum* die abgeleitete Beziehung zwischen Geldmengenwachstum und Inflation während eines Zeitraums sich beschleunigender Geldentwertung bei einem Modell mit exponentiell fallender Geldnachfrage. Die Analyse von Wesche und Wierum weist jedoch mehrere Fehler auf, und die Autoren scheinen sich der ökonomischen Interpretation einer Gleichung mit rationalen Erwartungen nicht bewußt zu sein. Eine korrekte Berechnung und Interpretation des Aufsatzes der beiden Autoren stützt die in einer meiner früheren Ausarbeitungen gezogenen Schlußfolgerungen vollinhaltlich.

Résumé

L'interprétation économique des anticipations rationnelles – Réponse

Dans un commentaire de mon article, *Wesche* et *Wierum* critiquent la relation dérivée entre la croissance monétaire et l'inflation en périodes d'inflation accélérée pour un modèle avec une demande monétaire exponentielle. C'ependant, leur analyse a plusieurs vices de forme et les auteurs ne semblent guère être conscients du contenu économique de l'équilibre des anticipations rationnelles. Un calcul et une interprétation corrects de leur exercise soutiennent entièrement les conclusions lirées de mon article précédent.