From Chaos to Antichaos and Back to Chaos: The Possibility of a Cycle

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The last years have seen a large number of publications on the topic of chaos and fractal geometry.[1] The subject has attracted wide attention. Yet, a quite new development in this area is now the possibility of antichaos. Chaos theory has shown that a complex non-linear system can generate greater and greater disorder and ultimately end up in a state of deterministic chaos. Antichaos theory on the other hand holds that, out of such chaos a high state of order may emerge, through spontaneous crystallization.[2] The question is what causes a chaos to transform into an antichaos. Furthermore, is this process a one way one or is it conceivable that the transformation involves several intermediate stages, with pulsating movements between chaos and antichaos? In order to study these problems we simulated a well-known chaos model, the so-called Mandelbrotbus.[3] The model is written as

$$X_t = X_{t-1}^2 - Y_{t-1}^2 + A \tag{1}$$

$$Y_t = 2\,(X_{t-1}\,Y_{t-1}) + B; \qquad t = 0, 1, 2, \ldots M \tag{2}$$

$$\textit{For} \quad t = 0 \quad : \quad X_t = A \;;\; Y_t = B^* = B \tag{3}$$

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1 See, for instance: *Heinz-Otto Peitjen* and *Peter H. Richter,* The Beauty of Fractals. Springer Verlag, 1986. *Benoit B. Mandelbrot,* The Fractal Geometry of Nature, W. H. Freeman Company, New York, 1983. *Michael Barnsley,* Fractals Everywhere. Academic Press Inc. Boston, 1988. *Robert L. Devaney* and *Linda Keen* (editors), Chaos and Fractals, The Mathematics behind Computer Graphics, Proceedings of Symposia in Applied Mathematics, Vol. 39. American Mathematical Society, Providence, Rhode Island.

2 See, *Stuart A. Kauffman,* "Antichaos and Adaptation", in Scientific American, August 1991, p. 78ff. *Elizabeth Corcoran,* "Ordering Chaos" in Scientific American, August 1991, p. 96ff. *Per Bak* and *Kan Chen,* "Self-Organized Criticality" in Scientific American, January 1991, p. 46ff.

3 See, *A. K. Dewdney,* "A Tour of the Mandelbrot Set aboard the Mandelbrotbus", in *Scientific American,* February 1989, p. 108 - 111.

It is a non-linear, non-homogeneous, system of difference equations. Through iteration the system generates, for each time period t, sets of (X_t, Y_t) which can be mapped. The parameters A and B specify initial conditions. They can be chosen freely. For certain values of these above system will produces impressive fractals.[4]

In order to make the model more general we introduce a disturbance term. We multiply the parameter B with a factor $1/(1 + K(t))$ and write equation (3) as

$$Y_t = 2(X_{t-1} Y_{t-1}) + (1/(1 + K(t)))B \tag{3}$$

Equation (3) replaces equation (2) in the conventional Mandelbrotbus. In (3) $K(t)$ is still to be specified.

The system (1) and (3) contains now a variable coefficient. Initial conditions can change at some point in time. As long as $K(t) = 0$ we have the special case of the Mandelbrotbus which can generate, for some given A and B values, a deterministic chaos. Yet, once $K(t) \neq 0$ things are different. We expect that, through $K(t) > 0$, a sudden change from chaos to antichaos will occur.

In our simulation, we begin with the conventional case, setting $K(t) = 0$. For our purpose we choose the value of the parameter A equal to 0.235535. For B we choose 0.513907910036. With these two values the system generates the deterministic chaos mapped in Diagram 1.

After the outer fractal "shoreline" is created, a ring suddenly forms in the center of the diagram. It encloses the inner, white area which is a sort of no man's land. The ring spreads outward and, as it approaches the outer fractal frontier, adjusts to the latter's shape. After a while the whole inner area gets filled in, except for the circle in the center. Following that, there is still some lingering on the outer fractal frontier but ultimately the system becomes unstable and computed (X_t, Y_t) values wander off into an electronic infinity.

Diagram 2 gives more details of this chaos. (For space reasons its lower part is not shown; the same will be the case in all following diagrams).

[4] See *Dewdney* op. cit. p. 109 to 110. A special exposition was devoted to "Fractal Geometry" in the Musée de la Découverte, Grand Palais, Paris France, May 1991. There detailed features of various Mandelbrot sets could be seen.

Diagram 1

We now come to the effects of parametric changes in $K(t)$ on the nature of above chaos. We begin with the case of a single switch in $K(t)$. For the latter we now require

(4a) $K(t) = 0 \quad \text{for} \quad t = 1, 2, \ldots, 1000$

and

(4b) $K(t) = 0.05 \quad \text{for} \quad t > 1000$

The simulation result is shown in Diagram 3. As before there is at first the creation of a fractal shoreline. Yet, for $t > 1000$, there is suddenly an inward movement, coming simultaneously from four different points on the fractal frontier. Four lines converge towards an equilibrium point and stay there. Instead of chaos there is now order and determinateness. A chaos became transformed into an antichaos.

If one sets in (4b) $K(t) = 0.01$, ceteris paribus, the paths towards the antichaos consists of four spirals, as shown in Diagram 4. It turns out that the size of $K(t)$ determines the shape of the paths towards an anti-chaos. Still more important, the size of $K(t)$ determines whether an anti-

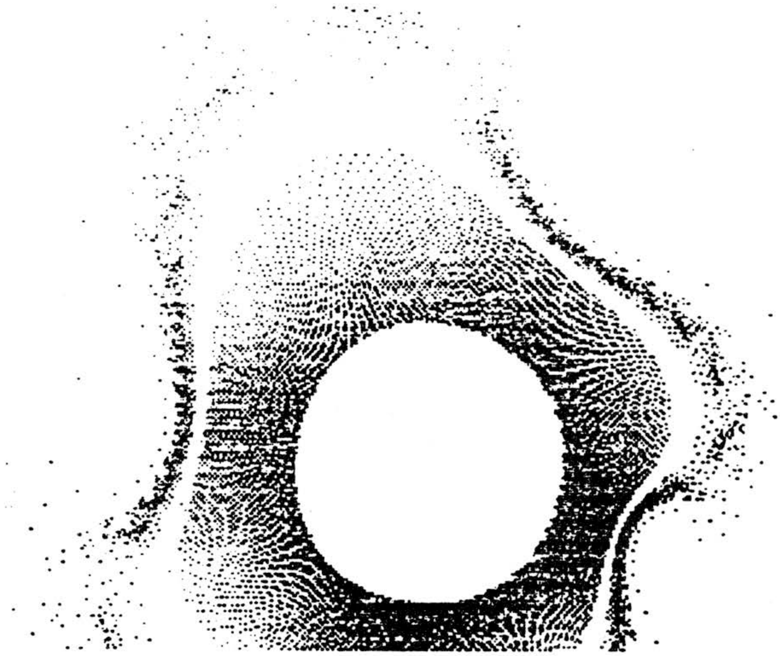

Diagram 2

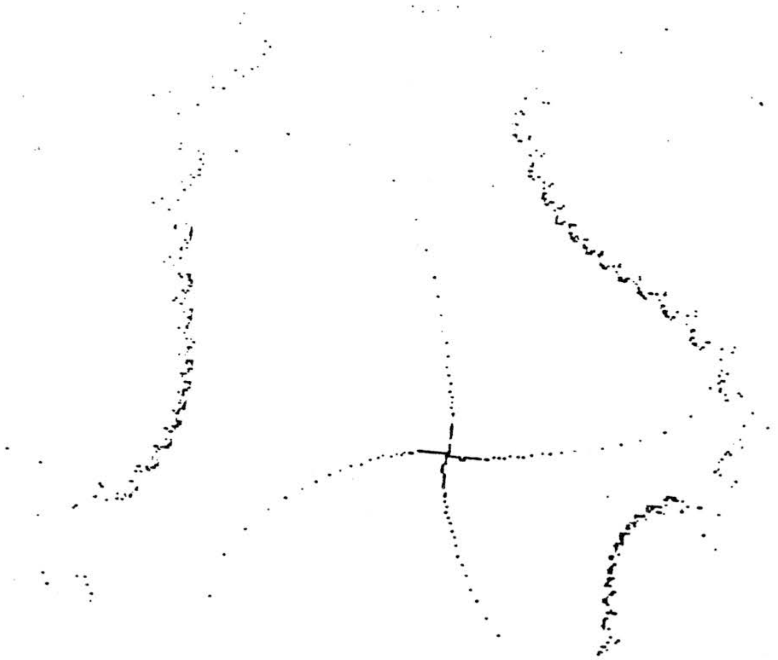

Diagram 3

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chaos can be reached at all. If $K(t)$ is too small there will be no inward movement towards an antichaos. If $K(t)$ is too large it leads to an immediate instability and explosion of the system. The implication is that disturbances in the present chaos model must be of a certain magnitude, if a transition from chaos to antichaos shall occur. An infinitesimally small disturbance, like the much talked about "flip of the wings of a butterfly", will not suffice.

As described above, for some values of $K(t)$ the chaos will be transformed into an antichaos. There will be a movement towards an equilibrium and that equilibrium is a stable one. It is however possible that the time dependent nature of the disturbance $K(t)$ is more complicated. $K(t)$ may continue to change its magnitude as time elapses.

To explore this we choose a simple case. We set

$$
\text{(5a)} \qquad K(t) = hV \qquad
\begin{array}{lll}
V = 0 & \text{for} & t = 0,1, \ldots, m \\
V = 1 & \text{for} & t = m + 1, \ldots, 2m \\
V = 2 & \text{for} & t = 2m + 1, \ldots, 3m \\
\vdots & & \\
V = n & \text{for} & t = nm + 1, \ldots, (n + 1)m
\end{array}
$$

In (5a) h is the magnitude of the basic disturbance. It is multiplied with the number of the period V. Thus over time the disturbance $K(t)$ increases. The length of each V period itself is equal to m time units, where the number of m can be chosen freely. In this way $K(t)$ will be zero in the first V period ($V = 0$). For this value of V initially a fractal shoreline, like in the Mandelbrotbus case, is computed. The next V period ($V = 1$) will then see a sudden departure from the fractal frontier. A movement towards an antichaos equilibrium occurs. Yet, once the latter is reached, there will be another increase in the $K(t)$ value because the next V period arrives ($V = 2$). With a larger disturbance the original antichaos equilibrium must be given up. There appear now new equilibrium points. These move along a narrow path, away from the original equilibrium, towards the initial fractal frontier. Each time there is subsequently another change in V a new set of $(X_t\, Y_t)$ values is created which, once it is mapped, can be seen to form arches. These are rushing towards an *attractor*, the first equilibrium. With increasing V values the outward move towards the fractal frontier continues. Yet, the moment the line of equilibria loci touches the fractal frontier, the power of the attractor ceases and there are no longer any inward bending arches. A new set of points appears which clearly belongs to the fractal frontier

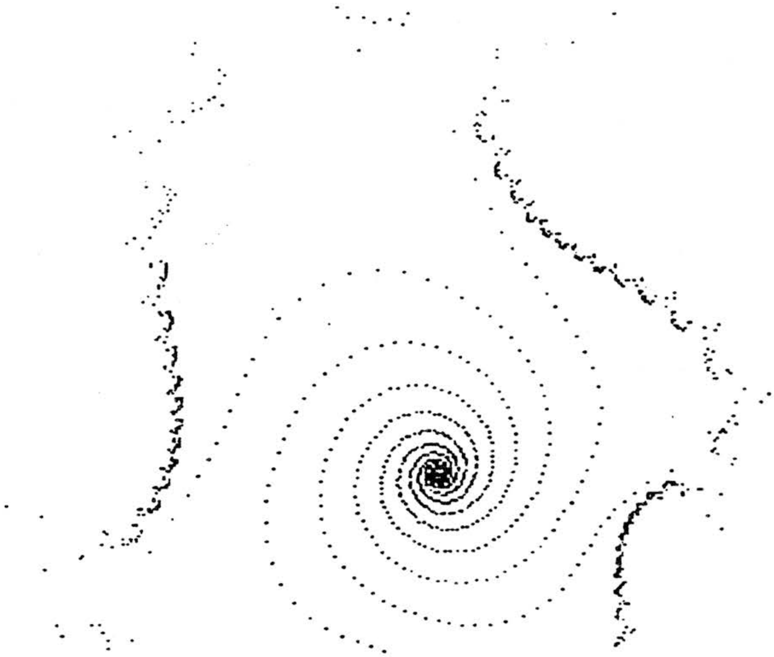

Diagram 4

itself. Sometimes this set contains just a few points, sometimes there are many. The old chaos has been reached again. If nothing is done, the line of equilibria will cross the fractal frontier and move on. With that the system enters an area of instability and heads for disintegration.

Diagram 5 shows these movements. It was generated with the following additional parameter values:

$$h = 0.0001 \tag{6a}$$

$$m = 1000 \tag{6b}$$

$$V = 0, 1, 2 \ldots, 50 \tag{6c}$$

In order to complete the cycle, there must now be an additional steering mechanism which switches the system back to the initial chaos. Here the following possibility offers itself.

To the original antichaos equilibrium as well as to each point on the straight line emanating from this equilibrium belongs a certain value for the last term on the right side in (3). For the sake of shortness we shall denote it as B^*

(7) $$B^* = (1/(1 + K(t))B$$

These B^* values can be used to steer the system back towards the initial chaos. We can require that once the fractal frontier is reached – B^* attaining a certain value at that instant –, the whole process to start over again. The rational for this is, that because a state of disorder prevails again, one might as well switch back to the first chaos. As a result, all points on the fractal frontier are recomputed and after a while a new movement towards a second antichaos occurs. A new line of equilibria loci heads towards the north-east for another encounter with the fractal frontier. In this way the process can go on and on. It is a complete cycle: from chaos to antichaos and back to chaos.

It is however not necessary to redo exactly the initial chaos. Through proper choice of the B^* value, one can continue the first chaos at that point where the first departure for an antichaos occurred. Another possibility is to trace out successive segments of the chaos only (each a ring or a similar shape). This process involves a number of cycles, as described above. The difference is that the critical B^* values, which trigger the

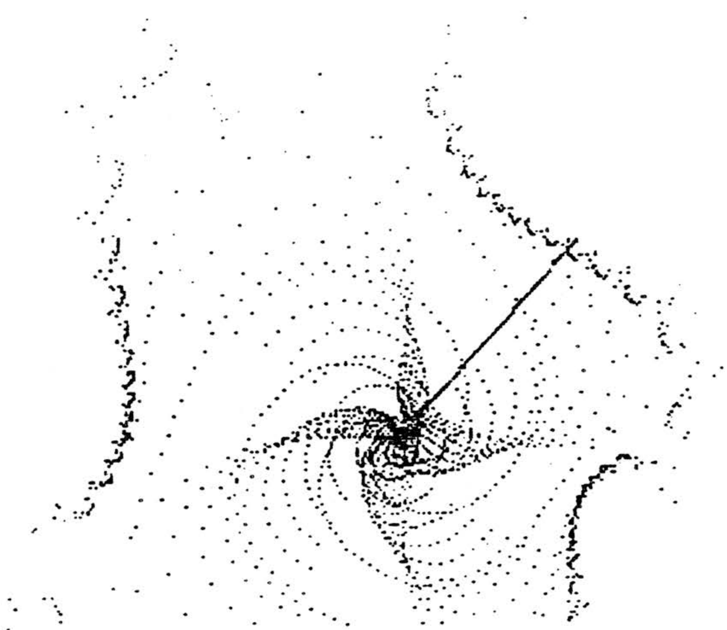

Diagram 5

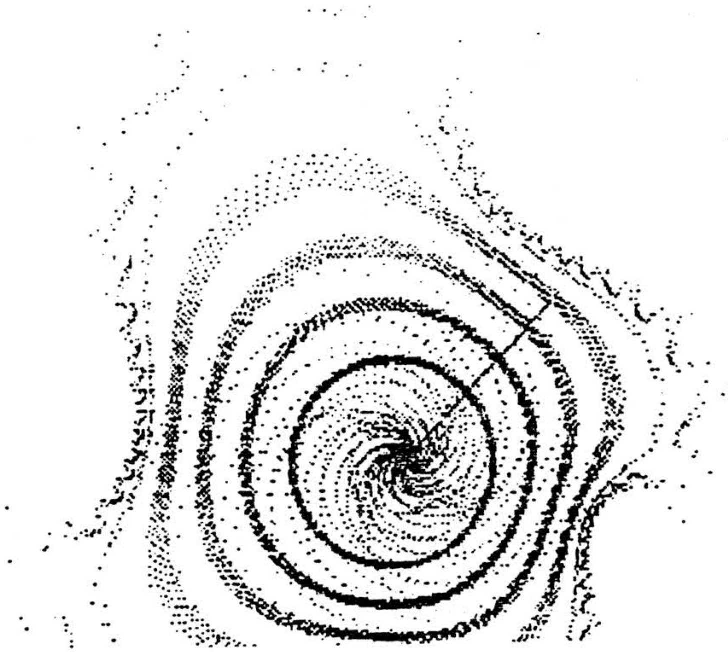

Diagram 6

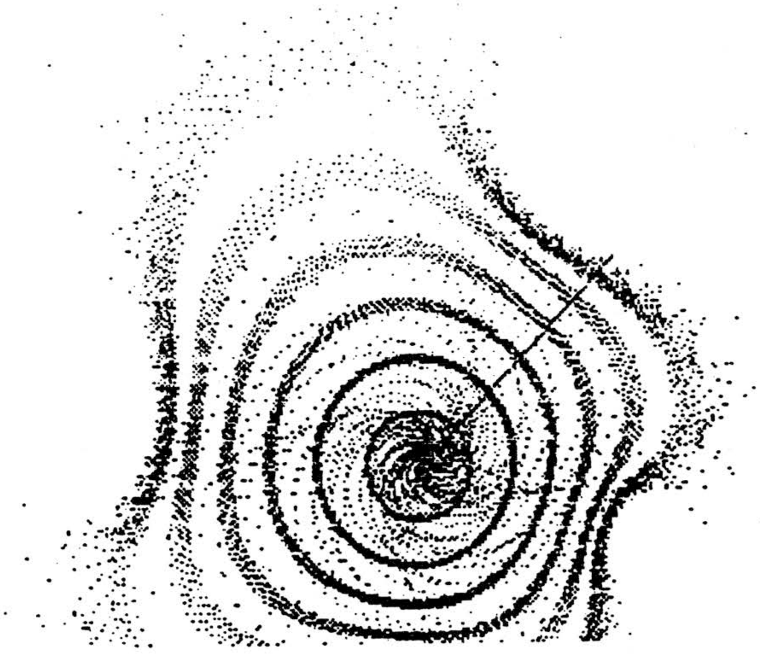

Diagram 7

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repeats, now successively increase. Diagrams 6 and 7 illustrate what is possible.

Zusammenfassung

Von Chaos zu Antichaos und zurück zum Chaos: ein möglicherweise bestehender Zyklus

Chaotische Dynamik ist in der Volkswirtschaft sowie auf vielen anderen Gebieten zu einer wichtigen Größe geworden. Bekanntlich kann ein komplexes nichtlineares System zu Unordnung und letztlich zu einem deterministischen Chaos führen. Antichaos ist nunmehr möglicherweise eine neue Entwicklung. Aus einem Chaos kann sich plötzlich spontan ein stabiler Ordnungszustand herauskristallisieren.

In diesem Beitrag wird gezeigt, wie im Zustand chaotischer Dynamik die sensible Abhängigkeit von Randbedingungen einen Übergang von Chaos zu Antichaos bewirken kann. Eine geringfügige parametrische Störung der Randbedingungen reicht aus, um einen solchen Übergang zu bewirken. Dennoch darf die Störung weder zu klein noch zu groß sein. Sie muß einen genügend großen Leverage-Effekt haben und muß zum richtigen Zeitpunkt kommen.

Es läßt sich spekulieren, daß Elemente von Ordnung und Determiniertheit in die Struktur des deterministischen Chaos eingebettet sind. Diese Elemente sind anscheinend bis zu dem Zeitpunkt inaktiv, an dem geeignete Störungen mit impliziten Steuerungsmerkmalen auftreten.

Es ist möglich, daß eine spezifische Sequenz von zeitabhängigen Störungen einen Zyklus entstehen läßt, nämlich eine Bewegung von Chaos zu Antichaos und wieder zurück zum Chaos.

Summary

From Chaos to Antichaos and Back to Chaos: The Possibility of a Cycle

Chaotic dynamics has become an important field in economics and in many other areas. It is known that a complex non-linear system can generate disorder, and ultimately end up in a state of deterministic chaos. A new development is now the possibility of antichaos. Out of a chaos may suddenly arise a high state of order. Involved is a kind of spontaneous crystallization.

In this paper it is shown how in chaos dynamics the sensitive dependence on initial conditions can explain a transition from chaos to antichaos. A small parametric disturbance in the initial conditions suffices to cause such a switch. Yet, the disturbance must be neither too small, nor too large. It must have a sufficient leverage effect and it must come at the right time.

One may speculate that imbedded in the structure of deterministic chaos are elements of order and determinateness. These are seemingly dormant, until appropriate disturbances, with implicit steering characteristics, occur.

It is possible that a specific sequence of time-dependent disturbances generates a cycle which moves from chaos to antichaos and back to chaos.

Résumé

Un cycle probablement existant: chaos – antichaos – chaos

En économie politique de même que dans beaucoup d'autres secteurs, la dynamique chaotique est devenue une dimension importante. Comme on le sait, un système complexe non-linéaire peut engendrer du désordre et finalement un chaos déterministe. Aujourd'hui, l'antichaos est probablement une nouvelle évolution. D'un chaos, il peut tout à coup se cristalliser spontanément un ordre stable.

Dans cet article, il est montré comment, dans une situation de dynamique chaotique, la dépendance sensible de conditions marginales peut faire passer du chaos à l'anti-chaos. Il suffit d'une perturbation paramétrique minimale des conditions marginales pour provoquer une telle transition. Cependant, la perturbation ne doit être ni trop faible ni trop forte. Elle doit avoir un effet de levier suffisamment important et survenir au bon moment.

On suppose qu'il y a des éléments d'ordre et de déterminisme incrustés dans la structure du chaos déterministe. Ces éléments sont, semble-t-il, inactifs jusqu'au moment où se présentent des perturbations adéquates avec des régulations implicites.

Il est possible qu'une séquence spécifique de perturbations en fonction du temps laissent naître un cycle, à savoir un mouvement du chaos à l'anti-chaos, puis un retour au chaos.