Inflation Rates and Money Growth During High-Inflations

By Hans Gersbach, Basel*

I. Introduction

High-inflations, such as those experienced recently in Latin America, have received new attention in economic literature. A lot of issues are usually discussed concerning the reasons and stabilization of high inflations [e.g. *Dornbusch* and *Fischer* 1986, *Kharas* and *Pinto* 1988, *Helpman* and *Leiderman* 1988, *Fischer* 1988, and *Kiguel* 1989].

Many explanations of the connection between money growth and inflation for closed economies focus on money creation via an exogenous budget deficit [e.g. *Kiguel* 1989 or *Bernholz* and *Gersbach* 1991]. High budget deficits and strong monetary growth are observed in almost all high-inflation episodes [e.g. *Sargent* 1982 or *Siklos* 1990 a]. Additionally, as pointed out by many authors [e.g. *Bernholz* 1988 or *Kiguel* 1989], there are several cases in which inflation seems to be accelerating without bounds until stabilization occurs.

The usual analysis of high-inflation applies the standard monetary model under rational expectations by comparing steady states which exhibit a one to one relationship between inflation and money growth rates [e.g. *Bruno* 1989]. However, steady state analysis relies on the implicit assumption that the same money growth rate is achieved in each period which is inappropriate for periods with accelerating inflation [see e.g. *Helpman* and *Leiderman* 1988].

Moreover, there is empirical evidence that there is no very high correlation between money growth and inflation rates during accelerating inflation [e.g. *Siklos* 1990 b].

In this note we try to explain this low correlation by a dynamic version of the standard monetary framework. We analyze explicitly the transition

* I would like to thank *Peter Bernholz, Martin Hellwig* and *Harald Nedwed* for helpful conversations and suggestions.

from low to high inflation when people anticipate this development. We derive the implications of a period-to-period dynamic of the simple montary model in which agents anticipate systematic changes of the money growth rate.

We illustrate that anticipated changes of the money growth rates have different implications compared to those derived by the traditional steady state analysis. We show that a fully anticipated increasing inflation is only compatible with relatively low money growth rates. Namely, if nominal money would grow as strongly as inflation, there would be no possibility that real balances decrease which is implied by accelerating inflation and observed in many historical examples [e.g. *Bernholz* 1988, or *Siklos* 1990 a].

The analysis in this paper emphasizes that dynamic economic theory predicts no simple one-to-one relationship between the two time series money growth and inflation during accelerating inflations.

In this paper, we use the standard monetary framework under rational expectations. One may doubt whether such a model is at all adequate to explain reality. Several authors [e.g. *Bernholz* and *Gersbach* 1991] have given various reasons for its inadequacy. Thus, all additional features of such models, as derived in this paper, should be interpreted in the light of these inadequacies.

II. The Model

To develop our argument we use the standard monetary framework under rational expectations to analyze high-inflations [e.g. *Evans* and *Yarrow* 1981, and *Dornbusch* and *Fischer* 1986]. We present the model in discrete-time format. We assume a closed economy in which the demand for money is given by the standard formulation:

$$M_t = cP_t exp\,(-d\,\pi^e_{t+1}) \quad \text{for} \quad t = 0, 1, 2, \ldots \tag{1}$$

M_t represents the nominal money stock, c and d are constants, P_t denotes the price level, and π^e_t the expected rate of inflation plus one:

$$\pi^e_t := \frac{P^e_t}{P_{t-1}} \tag{2}$$

The exponential (semilogarithmic) specification of the money demand function is widely used in the literature [*Dornbusch* and *Fischer* 1986 and *Kiguel* 1989], and applied in many empirical studies [see *Siklos* 1990 a for a survey].

Two important assumptions are implicit in this formulation. Output and real interest rates are given and thus a part of the constant c.

Next we define the monetary growth rate μ_t by:

$$\mu_t = \frac{M_t - M_{t-1}}{M_{t-1}} \tag{3}$$

Under the assumptions of instantaneous market clearing and rational expectations, we get the relationship between money growth rate, actual and future inflation:

$$\mu_t = \frac{M_t}{M_{t-1}} - 1 = \frac{P_t}{P_{t-1}} \frac{exp\,(-\,d\,\pi_{t+1})}{exp\,(-\,d\,\pi_t)} - 1 \tag{4}$$

which leads to:

$$\mu_t = \pi_t exp\,(-\,d\,\pi_{t+1}) / \{\, exp\,(-\,d\,\pi_t)\,\} - 1 \tag{5}$$

Given a sequence of money growth rates $\mu_t\,(t = 1, 2, \ldots)$, equation (5) determines the evolution of inflation. By rearranging terms we get:

$$\begin{aligned} \pi_{t+1} &= -\,d^{-1} \ln \{\, (\mu_t + 1)\, exp\,(-\,d\,\pi_t)\, \pi_t^{-1} \} \\ &= -\,d^{-1} \ln \{\, (\mu_t + 1)\, \pi_t^{-1} \} + \pi_t \end{aligned} \tag{6}$$

Equation (6) determines all possible inflation paths according to the money growth rate sequence $\mu_1, \mu_2, \mu_3, \ldots$. In general, there exists a continuum of solutions, depending on initial conditions. The usual analysis is based on the comparative statics of steady states. A steady state is determined by $\pi_{t+1} = \pi_t = \pi^*$. Hence via equation (6), we get

$$\pi^* - 1 = \mu_t = \mu^* \tag{7}$$

Comparative statics of steady states exhibits a total uniformity between nominal money growth rates and inflation since in every steady state money growth rate μ^* corresponds to inflation $\pi^* - 1$. Thus, different money growth rates are usually associated with exactly the same different inflation rates. However, the steady state analysis relies on the implicit assumption that the same money growth rate is achieved in each period. As already pointed out by *Helpman* and *Leiderman* [1988], the steady state analysis is quite inappropriate for high inflation processes. The likelihood of future policies being unanticipated is very small. In the following, we derive the implications of a period-to-period dynamic of the monetary model in which agents anticipate systematic changes of the money growth.

III. Sustaining Accelerating Inflations

Let us consider an accelerating inflation to study the connection with money growth rates. We consider all inflation processes in which the growth of the rate of inflation has some lower bound (at least for a certain time). That is, we will analyze all inflation paths for which there exists some value $k > 1$, so that

$$\pi_{t+1} \geq k\,\pi_t, k > 1 \tag{8}$$

We ask which money growth rates can sustain such accelerating inflation processes. That is, we try to determine the bounds accelerating inflation rates implies for money growth rates.

We first observe that equation (6) implies that for any inflation rate π_t, the lower μ_t , the higher becomes π_{t+1}. Thus, if we focus on the inflation process $\pi_{t+1} = k\,\pi_t$, we will obtain an upper bound for money growth rates sustaining accelerating inflation processes restricted by equation (8).

For $\pi_{t+1} = k\,\pi_t$, equation (5) implies:

$$\mu_t = \pi_t \exp\{-d\,(k-1)\,\pi_t\} - 1 \tag{9}$$

Equation (9) determines the upper bound for money growth rates which can sustain accelerating inflations of the described form. We call equation (9) the sustaining function which links money growth rates to actual inflation in order to let evolve an accelerating inflationary process given by $\pi_{t+1} = k\,\pi_t$.

Note that for $k = 1$, we obtain $\mu_t = \pi_t - 1$ and thus the steady state solution. For $k > 1$, however, equation (9) immediately implies:

$$\mu_t < \pi_t - 1 \tag{10}$$

The relationship between money growth rates and inflation, embodied by equation (10), rests on the connection between real and nominal money balances during accelerating inflations. Namely, accelerating inflation rates require decreasing real balances. But, this will be only possible, if nominal money growth does not offset the decrease of real money balances which sets an upper bound for the money growth rates. For any actual inflation rate π_t, a higher nominal money growth leads to an increased real money stock, implying lower future inflation rates.

Let us derive additional features of the sustaining function $\mu_t(\pi_t)$. Consider the range of inflation rates for which the nominal money growth rate

is positive. The critical values are defined by $\mu_t(\pi_t) = 0$ which leads via equation (9) to:

$$\ln \pi_t = d(k-1)\pi_t \quad \text{or} \quad \ln \pi_t / \pi_t = d(k-1) \tag{11}$$

Equation (11) defines two critical values $1 < \pi^1 < \pi^2$, depending on the parameters d and k. Under suitable conditions π^1, π^2 will exist.[1] Thus, only within the range $[\pi^1, \pi^2]$ the nominal money growth rate can be positive, but has always smaller to be than π_t.

For low inflation rates (smaller than π_1), and high inflation rates ($\pi_t > \pi_2$), accelerating inflation will be only possible, if the nominal money stock decreases. For instance, we get $\mu_t(1) = exp\{-d(k-1)\} - 1 < 0$. An accelerating inflation process, starting from low inflation will be only compatible with negative money growth rates, if agents anticipate the evolution of money growth rates [see also *Gale* 1982 or *Bernholz* and *Jaksch* 1988].

Very high inflation requires strongly negative nominal money growth rates since $\mu_t(\pi_t)$ approaches -1 for huge inflation rates.

Note that the higher the lower bound for the inflation process (the higher k), the smaller is the range which allows positive money growth rates.

Additionally, we derive from equation (9):

$$d\mu_t / d\pi_t = exp\{-d(k-1)\pi_t\}\{1 - d(k-1)\pi_t\} \tag{12}$$

$$d\mu_t / d\pi_t = 0 \quad \text{for} \quad \pi_t = \{d(k-1)\}^{-1} \tag{13}$$

Equation (13) implies that the function $\mu_t(\pi_t)$ reaches its maximum at $\pi_t = \{d(k-1)\}^{-1}$.

Hence, only for inflation rates lower than $\{d(k-1)\}^{-1}$, money growth rates rise with increasing actual inflation to sustain accelerating inflation.

But always $d\mu_t / d\pi_t < 1$ since both factors in equation (12) are smaller than one.

To sum up, accelerating inflation implies money growth rates which rise less than inflation. They money growth rates are only positive within a certain range.

These results are caused by the connection between lower real balances, which are implied by future higher inflation, and higher real balances in the

[1] We neglect the case for which there is no range associated with positive money growth rates, which would require very high values d and k.

past. Such an evolution will be only possible, if nominal money growth is not too strong and substantially below actual inflation rates. For a certain range of inflation rates, decreasing real balances require even a decrease of nominal money supply.

Our analysis is built on the standard assumptions of the monetary approach, but we have dropped the usual comparative statics of steady states. The foregoing discussion shows that there will be nothing odd about finding no one to one relationship between inflation and money growth during accelerating inflations, if such an analysis is based upon standard monetary models.

Additionally, as e. g. shown in *Drazen* and *Helpman* [1990], the correlation between budget deficits and the rate of monetary growth on the one hand, and inflation on the other, depends on the public's expectations about stabilization policies. This, in general, predicts no simple correlation structure, even though budget deficits are the primary source which generates inflation.

These findings are related to recent empirical investigations whether inflation and money growth are highly correlated during hyperinflations and whether inflation displays bubble behavior [see *Casella* 1989 and *Siklos* 1990 a].

For instance, *Siklos* [1990 b] reports that money growth and inflation correlations were low for the entire period of the Hungarian hyperinflation of 1945 - 1946. For identifiable shorter periods the correlation may be much higher.

Our analysis emphasizes that economic theory predicts no simple relationship between the two time series for money growth and inflation during accelerating inflations. As shown in this paper, a variable deviation from a one to one relationship between inflation and money growth is an implication of every anticipated acceleration of inflation in traditional monetary models. Thus, even though inflation processes are caused, in a sense, by monetary policy, the lack of any contemporaneous correlation between monetary growth and inflation will arise.

IV. Conclusion

Standard steady state analysis exhibits a complete uniformity between inflation and money growth rate. It is, however, difficult to justify the vehicle of comparative statics of steady states to explain the behavior of inflation and money growth rates during hyperinflations, since the government

follows unstable monetary and fiscal policies during a longer episode. Under conditions of accelerating inflation, the likelihood of future policies being unanticipated is so small as to be irrelevant [see also *Siklos* 1990a].

In this paper it was shown that fully anticipated accelerating inflation is only compatible with substantially lower rates of growth of the money supply. This result rests on the fact that accelerating inflations will be only sustainable in the analyzed model, if real balances decrease which, however, sets an upper bound for the growth rate of nominal money supply. A very strong acceleration of inflation is only compatible with relatively low money growth rates.

Of course, there will be many reasonable criticisms of rational expectations, especially if money growth rates vary. Moreover, the monetary framework for analyzing hyperinflations shows several inadequacies to explain reality [see *Bernholz* and *Gersbach* 1991]. In this paper we have shown the implications of the traditional monetary model for inflation dynamics. The derived pattern shows that the view based on a one to one relationship between inflation and money growth rate neglects the transitions effects of an evolution from low to high rates of inflation.

References

Bernholz, P., and *Jaksch,* H. J., 1989: "An Implausible Theory of Inflation", Weltwirtschaftliches Archiv, Vol. 125, No. 2, 359 - 365. – *Bernholz,* P., 1988: "Inflation, Monetary Regime and the Financial Asset Theory of Money", Kyklos, Vol. 41, 5 - 34. – *Bernholz,* P., and *Gersbach,* H., 1991: "The Present Monetary Theory of Advanced Inflation: A Failure?", University of Basel. – *Bruno,* M., 1988: "Econometrics and the Design of Economic Reform", Econometrica, Vol. 57, No. 2, 275 - 306. – *Cagan,* P., 1956: "The Monetary Dynamics of Hyperinflation", in: Friedman, M. (Eds.): Studies in the Quantity Theory of Money, University of Chicago Press, Chicago. – *Casella,* A., 1989: "Testing for Rational Bubbles with Exogenous or Endogenous Fundamentals, Journal of Monetary Economics, Vol. 24. – *Drazen,* A. and *Helpman,* E., 1990: "Inflationary Consequences of Anticipated Macroeconomic Policies", Review of Economic Studies, Vol. 57, 147 - 166. – *Dornbusch,* R. and *Fischer,* S., 1986: "Stopping Hyperinflation Past and Present", Weltwirtschaftliches Archiv, 122, 1 - 47. – *Evans,* I. L., and *Yarrow,* G. K., 1981: "Some Implications of Alternative Expectations Hypotheses in the Monetary Analysis of Hyperinflation." Oxford Economic Papers 33 (March 1981), 61 - 80. – *Fischer,* S., 1987: "The Israeli Stabilization Program, 1985 - 1986, American Economic Review (Papers and Proceedings), Vol. 77, 275 - 278. – *Gale,* D., 1982: "Money in Equilibrium". Cambridge University Press, Cambridge, New York. – *Helpman,* E., *Leiderman,* L., 1988: "Stabilization in High Inflation Countries: Analytical Foundations and Recent Experiences", Carnegie Rochester Conference Series on Public Policy, Vol. 28, 9 - 84. – *Kharas,* H. and *Pinto,* B., 1989: "Exchange Rate Rules, Black Market Premia and Fiscal Deficits: The Bolivian

Hyperinflation", The Review of Economic Studies, 56, 435 - 448. – *Kiguel,* M. A., 1989: "Budget Deficits, Stability, and the Monetary Dynamics of Hyperinflation", Journal of Money, Credit and Banking, 21, 148 - 157. – *Sargent,* T. J., 1982: "The Ends of Four Big Inflations", in: Hall, R. E. (Eds.), "Inflation", University of Chicago Press for the National Bureau of Economic Research, Chicago. – *Siklos,* P. L., 1990a: "Hyperinflations: Their Origins, Development, and Termination", Journal of Economic Surveys, Vol. 4 (3), 225 - 248. – *Siklos,* P. L., 1990b: "The Link between Money and Prices under Different Policy Regimes: The Postwar Hungarian Experience", Explorations in Economic History, Vol. 27, pp. 468 - 482.

Zusammenfassung

Inflationsraten und Geldmengenwachstum in Hochzinsphasen

Üblicherweise werden den Analysen von Hochzinsphasen das auf der Theorie der rationalen Erwartungen beruhende Standardmodell für das Geldmengenwachstum zugrunde gelegt und stabile Zustände verglichen, bei denen eine Beziehung zwischen Inflationsrate und Geldmengenwachstumsrate von eins zu eins unterstellt wird. In diesem Beitrag wird die bei sich beschleunigender Inflation empirisch festgestellte schwache Korrelation zwischen Geldmengenwachstumsrate und Inflationsrate auf der Grundlage einer dynamisierten Version des Standardmodells für das Geldmengenwachstum erläutert. Dabei werden die Implikationen des durch Periodenvergleich dynamisierten einfachen Modells für das Geldmengenwachstum, bei dem durch Faktoren systematische Veränderungen der Geldmengenwachstumsrate vorweggenommen werden, abgeleitet. Es wird gezeigt, daß eine im Modell voll berücksichtigte steigende Inflationsrate nur mit relativ niedrigen Geldmengenwachstumsraten korreliert. Wenn das nominale Geldmengenwachstum so stark expandierte wie die Inflation zunimmt, wäre ein Rückgang der Realvermögen ausgeschlossen, was von einer sich beschleunigenden Inflation impliziert und von vielen Beispielen belegt wird.

Summary

Inflation Rates and Money Growth During High-Inflations

In this paper, the relatively low correlation between money growth and inflation rates during accelerating inflation is explained by a dynamic version of the standard monetary framework. Whereas steady state analysis exhibits a strong uniformity between inflation and money growth rate, a dynamic analysis shows a weaker relationship. This result rests on the fact that anticipated accelerating inflation is only sustainable if real balances decrease which, however, sets an upper bound for the growth rate of the nominal money supply.

Résumé

Les taux d'inflation et la croissance monétaire durant les périodes de forte inflation

L'analyse habituelle de la forte inflation applique le modèle monétaire standard sous des attentes rationelles en comparant des conditions stables qui montrent une relation directement proportionnelle entre les taux d'inflation et de croissance monétaire. Dans cet article, l'auteur explique par une version dynamique du modèle monétaire standard la faible corrélation observée empiriquement entre le taux de croissance monétaire et le taux d'inflation lors de l'accélération de l'inflation. Il dérive les implications d'une dynamique de période à période du modèle monétaire simple dans lequel les agents anticipent des changements systématiques du taux de la croissance monétaire. Il montre qu'une inflation croissante entièrement anticipée est uniquement compatible avec des taux de croissance monétaire relativement bas. Si la monnaie nominale croît aussi fort que l'inflation, il ne serait pas possible que les encaisses réelles décroissent, ce qui est impliqué par l'inflation croissante et observé dans de nombreux exemples historiques.