

# **Inflation Rates and Money Growth During High-Inflations**

By Hans Gersbach, Basel\*

## **I. Introduction**

High-inflations, such as those experienced recently in Latin America, have received new attention in economic literature. A lot of issues are usually discussed concerning the reasons and stabilization of high inflations [e.g. *Dornbusch and Fischer 1986, Kharas and Pinto 1988, Helpman and Leiderman 1988, Fischer 1988, and Kiguel 1989*].

Many explanations of the connection between money growth and inflation for closed economies focus on money creation via an exogenous budget deficit [e.g. *Kiguel 1989 or Bernholz and Gersbach 1991*]. High budget deficits and strong monetary growth are observed in almost all high-inflation episodes [e.g. *Sargent 1982 or Siklos 1990 a*]. Additionally, as pointed out by many authors [e.g. *Bernholz 1988 or Kiguel 1989*], there are several cases in which inflation seems to be accelerating without bounds until stabilization occurs.

The usual analysis of high-inflation applies the standard monetary model under rational expectations by comparing steady states which exhibit a one to one relationship between inflation and money growth rates [e.g. *Bruno 1989*]. However, steady state analysis relies on the implicit assumption that the same money growth rate is achieved in each period which is inappropriate for periods with accelerating inflation [see e.g. *Helpman and Leiderman 1988*].

Moreover, there is empirical evidence that there is no very high correlation between money growth and inflation rates during accelerating inflation [e.g. *Siklos 1990 b*].

In this note we try to explain this low correlation by a dynamic version of the standard monetary framework. We analyze explicitly the transition

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\* I would like to thank *Peter Bernholz, Martin Hellwig and Harald Nedwed* for helpful conversations and suggestions.

from low to high inflation when people anticipate this development. We derive the implications of a period-to-period dynamic of the simple monetary model in which agents anticipate systematic changes of the money growth rate.

We illustrate that anticipated changes of the money growth rates have different implications compared to those derived by the traditional steady state analysis. We show that a fully anticipated increasing inflation is only compatible with relatively low money growth rates. Namely, if nominal money would grow as strongly as inflation, there would be no possibility that real balances decrease which is implied by accelerating inflation and observed in many historical examples [e.g. *Bernholz* 1988, or *Siklos* 1990 a].

The analysis in this paper emphasizes that dynamic economic theory predicts no simple one-to-one relationship between the two time series money growth and inflation during accelerating inflations.

In this paper, we use the standard monetary framework under rational expectations. One may doubt whether such a model is at all adequate to explain reality. Several authors [e.g. *Bernholz* and *Gersbach* 1991] have given various reasons for its inadequacy. Thus, all additional features of such models, as derived in this paper, should be interpreted in the light of these inadequacies.

## II. The Model

To develop our argument we use the standard monetary framework under rational expectations to analyze high-inflations [e.g. *Evans* and *Yarrow* 1981, and *Dornbusch* and *Fischer* 1986]. We present the model in discrete-time format. We assume a closed economy in which the demand for money is given by the standard formulation:

$$(1) \quad M_t = c P_t \exp(-d \pi_{t+1}^e) \quad \text{for } t = 0, 1, 2, \dots$$

$M_t$  represents the nominal money stock,  $c$  and  $d$  are constants,  $P_t$  denotes the price level, and  $\pi_t^e$  the expected rate of inflation plus one:

$$(2) \quad \pi_t^e := \frac{P_t^e}{P_{t-1}}$$

The exponential (semilogarithmic) specification of the money demand function is widely used in the literature [*Dornbusch* and *Fischer* 1986 and *Kiguel* 1989], and applied in many empirical studies [see *Siklos* 1990 a for a survey].

Two important assumptions are implicit in this formulation. Output and real interest rates are given and thus a part of the constant  $c$ .

Next we define the monetary growth rate  $\mu_t$  by:

$$(3) \quad \mu_t = \frac{M_t - M_{t-1}}{M_{t-1}}$$

Under the assumptions of instantaneous market clearing and rational expectations, we get the relationship between money growth rate, actual and future inflation:

$$(4) \quad \mu_t = \frac{M_t}{M_{t-1}} - 1 = \frac{P_t}{P_{t-1}} \frac{\exp(-d \pi_{t+1})}{\exp(-d \pi_t)} - 1$$

which leads to:

$$(5) \quad \mu_t = \pi_t \exp(-d \pi_{t+1}) / \{ \exp(-d \pi_t) \} - 1$$

Given a sequence of money growth rates  $\mu_t$  ( $t = 1, 2, \dots$ ), equation (5) determines the evolution of inflation. By rearranging terms we get:

$$(6) \quad \begin{aligned} \pi_{t+1} &= -d^{-1} \ln \{ (\mu_t + 1) \exp(-d \pi_t) \pi_t^{-1} \} \\ &= -d^{-1} \ln \{ (\mu_t + 1) \pi_t^{-1} \} + \pi_t \end{aligned}$$

Equation (6) determines all possible inflation paths according to the money growth rate sequence  $\mu_1, \mu_2, \mu_3, \dots$ . In general, there exists a continuum of solutions, depending on initial conditions. The usual analysis is based on the comparative statics of steady states. A steady state is determined by  $\pi_{t+1} = \pi_t = \pi^*$ . Hence via equation (6), we get

$$(7) \quad \pi^* - 1 = \mu_t = \mu^*$$

Comparative statics of steady states exhibits a total uniformity between nominal money growth rates and inflation since in every steady state money growth rate  $\mu^*$  corresponds to inflation  $\pi^* - 1$ . Thus, different money growth rates are usually associated with exactly the same different inflation rates. However, the steady state analysis relies on the implicit assumption that the same money growth rate is achieved in each period. As already pointed out by *Helpman and Leiderman* [1988], the steady state analysis is quite inappropriate for high inflation processes. The likelihood of future policies being unanticipated is very small. In the following, we derive the implications of a period-to-period dynamic of the monetary model in which agents anticipate systematic changes of the money growth.

### III. Sustaining Accelerating Inflations

Let us consider an accelerating inflation to study the connection with money growth rates. We consider all inflation processes in which the growth of the rate of inflation has some lower bound (at least for a certain time). That is, we will analyze all inflation paths for which there exists some value  $k > 1$ , so that

$$(8) \quad \pi_{t+1} \geq k \pi_t, k > 1$$

We ask which money growth rates can sustain such accelerating inflation processes. That is, we try to determine the bounds accelerating inflation rates implies for money growth rates.

We first observe that equation (6) implies that for any inflation rate  $\pi_t$ , the lower  $\mu_t$ , the higher becomes  $\pi_{t+1}$ . Thus, if we focus on the inflation process  $\pi_{t+1} = k \pi_t$ , we will obtain an upper bound for money growth rates sustaining accelerating inflation processes restricted by equation (8).

For  $\pi_{t+1} = k \pi_t$ , equation (5) implies:

$$(9) \quad \mu_t = \pi_t \exp \{-d(k-1)\pi_t\} - 1$$

Equation (9) determines the upper bound for money growth rates which can sustain accelerating inflations of the described form. We call equation (9) the sustaining function which links money growth rates to actual inflation in order to let evolve an accelerating inflationary process given by  $\pi_{t+1} = k \pi_t$ .

Note that for  $k = 1$ , we obtain  $\mu_t = \pi_t - 1$  and thus the steady state solution. For  $k > 1$ , however, equation (9) immediately implies:

$$(10) \quad \mu_t < \pi_t - 1$$

The relationship between money growth rates and inflation, embodied by equation (10), rests on the connection between real and nominal money balances during accelerating inflations. Namely, accelerating inflation rates require decreasing real balances. But, this will be only possible, if nominal money growth does not offset the decrease of real money balances which sets an upper bound for the money growth rates. For any actual inflation rate  $\pi_t$ , a higher nominal money growth leads to an increased real money stock, implying lower future inflation rates.

Let us derive additional features of the sustaining function  $\mu_t(\pi_t)$ . Consider the range of inflation rates for which the nominal money growth rate



is positive. The critical values are defined by  $\mu_t(\pi_t) = 0$  which leads via equation (9) to:

$$(11) \quad \ln \pi_t = d(k-1)\pi_t \quad \text{or} \quad \ln \pi_t / \pi_t = d(k-1)$$

Equation (11) defines two critical values  $1 < \pi^1 < \pi^2$ , depending on the parameters  $d$  and  $k$ . Under suitable conditions  $\pi^1, \pi^2$  will exist.<sup>1</sup> Thus, only within the range  $[\pi^1, \pi^2]$  the nominal money growth rate can be positive, but has always smaller to be than  $\pi_t$ .

For low inflation rates (smaller than  $\pi_1$ ), and high inflation rates ( $\pi_t > \pi_2$ ), accelerating inflation will be only possible, if the nominal money stock decreases. For instance, we get  $\mu_t(1) = \exp\{-d(k-1)\} - 1 < 0$ . An accelerating inflation process, starting from low inflation will be only compatible with negative money growth rates, if agents anticipate the evolution of money growth rates [see also Gale 1982 or Bernholz and Jaksch 1988].

Very high inflation requires strongly negative nominal money growth rates since  $\mu_t(\pi_t)$  approaches  $-1$  for huge inflation rates.

Note that the higher the lower bound for the inflation process (the higher  $k$ ), the smaller is the range which allows positive money growth rates.

Additionally, we derive from equation (9):

$$(12) \quad d\mu_t/d\pi_t = \exp\{-d(k-1)\pi_t\} \{1 - d(k-1)\pi_t\}$$

$$(13) \quad d\mu_t/d\pi_t = 0 \quad \text{for} \quad \pi_t = \{d(k-1)\}^{-1}$$

Equation (13) implies that the function  $\mu_t(\pi_t)$  reaches its maximum at  $\pi_t = \{d(k-1)\}^{-1}$ .

Hence, only for inflation rates lower than  $\{d(k-1)\}^{-1}$ , money growth rates rise with increasing actual inflation to sustain accelerating inflation.

But always  $d\mu_t/d\pi_t < 1$  since both factors in equation (12) are smaller than one.

To sum up, accelerating inflation implies money growth rates which rise less than inflation. They money growth rates are only positive within a certain range.

These results are caused by the connection between lower real balances, which are implied by future higher inflation, and higher real balances in the

<sup>1</sup> We neglect the case for which there is no range associated with positive money growth rates, which would require very high values  $d$  and  $k$ .

past. Such an evolution will be only possible, if nominal money growth is not too strong and substantially below actual inflation rates. For a certain range of inflation rates, decreasing real balances require even a decrease of nominal money supply.

Our analysis is built on the standard assumptions of the monetary approach, but we have dropped the usual comparative statics of steady states. The foregoing discussion shows that there will be nothing odd about finding no one to one relationship between inflation and money growth during accelerating inflations, if such an analysis is based upon standard monetary models.

Additionally, as e.g. shown in *Drazen and Helpman* [1990], the correlation between budget deficits and the rate of monetary growth on the one hand, and inflation on the other, depends on the public's expectations about stabilization policies. This, in general, predicts no simple correlation structure, even though budget deficits are the primary source which generates inflation.

These findings are related to recent empirical investigations whether inflation and money growth are highly correlated during hyperinflations and whether inflation displays bubble behavior [see *Casella* 1989 and *Siklos* 1990 a].

For instance, *Siklos* [1990 b] reports that money growth and inflation correlations were low for the entire period of the Hungarian hyperinflation of 1945 - 1946. For identifiable shorter periods the correlation may be much higher.

Our analysis emphasizes that economic theory predicts no simple relationship between the two time series for money growth and inflation during accelerating inflations. As shown in this paper, a variable deviation from a one to one relationship between inflation and money growth is an implication of every anticipated acceleration of inflation in traditional monetary models. Thus, even though inflation processes are caused, in a sense, by monetary policy, the lack of any contemporaneous correlation between monetary growth and inflation will arise.

#### IV. Conclusion

Standard steady state analysis exhibits a complete uniformity between inflation and money growth rate. It is, however, difficult to justify the vehicle of comparative statics of steady states to explain the behavior of inflation and money growth rates during hyperinflations, since the government

follows unstable monetary and fiscal policies during a longer episode. Under conditions of accelerating inflation, the likelihood of future policies being unanticipated is so small as to be irrelevant [see also *Siklos* 1990 a].

In this paper it was shown that fully anticipated accelerating inflation is only compatible with substantially lower rates of growth of the money supply. This result rests on the fact that accelerating inflations will be only sustainable in the analyzed model, if real balances decrease which, however, sets an upper bound for the growth rate of nominal money supply. A very strong acceleration of inflation is only compatible with relatively low money growth rates.

Of course, there will be many reasonable criticisms of rational expectations, especially if money growth rates vary. Moreover, the monetary framework for analyzing hyperinflations shows several inadequacies to explain reality [see *Bernholz* and *Gersbach* 1991]. In this paper we have shown the implications of the traditional monetary model for inflation dynamics. The derived pattern shows that the view based on a one to one relationship between inflation and money growth rate neglects the transitions effects of an evolution from low to high rates of inflation.

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## **Zusammenfassung**

### **Inflationsraten und Geldmengenwachstum in Hochzinsphasen**

Üblicherweise werden den Analysen von Hochzinsphasen das auf der Theorie der rationalen Erwartungen beruhende Standardmodell für das Geldmengenwachstum zugrunde gelegt und stabile Zustände verglichen, bei denen eine Beziehung zwischen Inflationsrate und Geldmengenwachstumsrate von eins zu eins unterstellt wird. In diesem Beitrag wird die bei sich beschleunigender Inflation empirisch festgestellte schwache Korrelation zwischen Geldmengenwachstumsrate und Inflationsrate auf der Grundlage einer dynamisierten Version des Standardmodells für das Geldmengenwachstum erläutert. Dabei werden die Implikationen des durch Periodenvergleich dynamisierten einfachen Modells für das Geldmengenwachstum, bei dem durch Faktoren systematische Veränderungen der Geldmengenwachstumsrate vorweggenommen werden, abgeleitet. Es wird gezeigt, daß eine im Modell voll berücksichtigte steigende Inflationsrate nur mit relativ niedrigen Geldmengenwachstumsraten korreliert. Wenn das nominale Geldmengenwachstum so stark expandierte wie die Inflation zunimmt, wäre ein Rückgang der Realvermögen ausgeschlossen, was von einer sich beschleunigenden Inflation impliziert und von vielen Beispielen belegt wird.

## **Summary**

### **Inflation Rates and Money Growth During High-Inflations**

In this paper, the relatively low correlation between money growth and inflation rates during accelerating inflation is explained by a dynamic version of the standard monetary framework. Whereas steady state analysis exhibits a strong uniformity between inflation and money growth rate, a dynamic analysis shows a weaker relationship. This result rests on the fact that anticipated accelerating inflation is only sustainable if real balances decrease which, however, sets an upper bound for the growth rate of the nominal money supply.



**Résumé****Les taux d'inflation et la croissance monétaire  
durant les périodes de forte inflation**

L'analyse habituelle de la forte inflation applique le modèle monétaire standard sous des attentes rationnelles en comparant des conditions stables qui montrent une relation directement proportionnelle entre les taux d'inflation et de croissance monétaire. Dans cet article, l'auteur explique par une version dynamique du modèle monétaire standard la faible corrélation observée empiriquement entre le taux de croissance monétaire et le taux d'inflation lors de l'accélération de l'inflation. Il dérive les implications d'une dynamique de période à période du modèle monétaire simple dans lequel les agents anticipent des changements systématiques du taux de la croissance monétaire. Il montre qu'une inflation croissante entièrement anticipée est uniquement compatible avec des taux de croissance monétaire relativement bas. Si la monnaie nominale croît aussi fort que l'inflation, il ne serait pas possible que les encaisses réelles décroissent, ce qui est impliqué par l'inflation croissante et observé dans de nombreux exemples historiques.