A Schumpeter Model of Economic Growth and Innovation

By Karl W. Roskamp*, Michigan/USA

I. Introduction

Professor *Schumpeter*'s book, "Theorie der wirtschaftlichen Entwicklung" was published seventy-eight years ago.[1] Appearing in 1912, it presented a pathbreaking theory of innovation and growth which has not ceased to inspire economists. Today there is much renewed interest in the Schumpeterian economic growth process.[2]

The reasons for the continued attractiveness of *Schumpeter*'s theory are easily discernable. In a general sense it describes the very drama which unfolds in an innovating firm, that entity on which economic growth in a market economy crucially depends. Schumpeter's theory sets forth how such

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* Professor Emeritus of Economics at Wayne State University, Detroit, MI, 48202. I would like to thank Professors *Richard S. Eckaus, Manfred Neumann, Wolfgang F. Stolper* and *André Piatier* for helpful comments in the preparation of this paper.

1 *Joseph A. Schumpeter,* Theorie der wirtschaftlichen Entwicklung, Leipzig: Duncker & Humblot, 1912, pp. VII + 548 [preface dated Vienna, July 1911].

Joseph A. Schumpeter, second revised German edition, with subtitle: Eine Untersuchung über Unternehmergewinn, Kapital, Kredit, Zins und den Konjunkturzyklus, München und Leipzig: Duncker & Humblot, 1926, pp. XIV + 369.

Joseph A. Schumpeter, English translation, The Theory of Economic Development: An Inquiry into Profits, Capital, Credit, Interest and the Business Cycle, Cambridge, Mass.: Harvard University Press, 1934, pp. XII + 255, second printing – 1936, third printing – 1949.

Joseph A. Schumpeter, French translation, Théorie de l'Evolution Economique; Recherches sur le Profit, le Crédit, l'Interet et le Cycle de la Conjuncture, published as Vol. II in la Collection Scientifique d'Economie Politique, Librairie Dalloz, Paris, 1935, pp. XI + 589, with an introduction by *M. François Perroux.*

2 *Wolfgang F. Stolper,* „Schumpeters Theorie der wirtschaftlichen Entwicklung – Eine kritische Exegese," pp. 35 - 74, in *Horst Claus Recktenwald, Frederic M. Scherer* and *Wolfgang F. Stolper,* Über Schumpeters Theorie der wirtschaftlichen Entwicklung Vademecum zu einem genialen Klassiker, Verlag Wirtschaft und Finanzen GMBH, Düsseldorf, 1988.

F. M. Scherer, Innovation and Growth: Schumpetarian Perspectives Cambridge, Mass.: MIT Press, 1984.

a firm, striving for profits and capital accumulation, must in an uncertain world behave in order to be successful. The task is a difficult one and challenges to entrepreneurial abilities abound: uncertainties and risks arise because in a dynamic growth process productivity-changing innovations are difficult to foresee, it is not known how soon an existing market advantage might be eroded through the efforts of imitators and increases in wages, and other cost items might reduce profit margins sooner than expected. Faced with these factors, selections, readjustments and reoptimizations are necessary. There will be breaks and discontinuities and an ever present, close attention to changing conditions is needed.

Schumpeter's theory comes in many respects close to rationalizing what is observed in a complex reality. It is the theory of a great visionary, full of subtleties and intricate relationships. Yet, for reasons unknown to us, Schumpeter never formalized it. He did not leave us with an explicit mathematical model. The consequence is that in many places it is not clear how a model, based on his theory, would work. In the construction of such a model, a number of problems arise. The first one is how the various variables entering the model have to interact to generate a feasible, innovation-driven growth path. Then comes the difficult requirement of intertemporal maximization of profits and capital accumulation. This must be achieved in an intermittent adjustment process, and ideally with a distinction being made between real capital and financial capital. Because the Schumpeter growth process is a discontinuous, adaptive one, sensors, feedbacks and response mechanisms are evidently necessary to keep it rolling along, until some increasing cost variables and (or) random changes in productivity and prices ultimately stops it. The Schumpetarian innovation-growth process is a finite and bounded one. It does not possess an equilibrium. There is no steady state towards which it would converge.

In order to study all of this, a model which brings together the diverse elements in *Schumpeter's* theory, is therefore desirable. Such a model should be cast into the form of a computer model and thus be operative. With its aid one should be able to simulate, given suitable parameter values, optimal timepaths for the introduction of innovations, for profits and capital accumulation. To the knowledge of this author, a model of this type has not yet been constructed.[3]

[3] Pioneering work on a *Schumpeter* model, albeit a non-maximization one based on an Evolutionary Theory, was done by Professors *Richard R. Nelson* and *Sidney G. Winter,* see:

II. The Model

We assume that the Schumpetarian entrepreneur maximizes over time the discounted utility of profits. The latter we define as profits over and above "normal profits," that is profits which are conventional and customary in the line of activity the firm is engaged in and obtainable even by non-innovating enterprises.

The utility function of the firm is assumed to be a constant elasticity of substitution one. We write it as

$$(1) \qquad U(E_i) = [1/(1-b)]\, E_i^{1-b}, \quad \mathrm{b} > 0$$

In the literature one finds that this function is a useful tool of analysis, now frequently employed.[4]

The firm maximizes profits, subject to constraints. It does this over a finite number of i maximization periods, each spanning N years. The problem can be stated as follows,

$$(2) \qquad \max \int_{T(0,\,i)}^{T(1,\,i)} U(E_i)\, e^{-\lambda t}\, dt$$

subject to

$$(3) \qquad \dot{K} = R_i - \omega_{0,\,i}\, e^{\sigma_{0,i} t} L_i - K_{1,\,i}\, e^{\sigma_{1,i} t} - \mathrm{K}_{2,\,i} - \mathrm{E}_i$$

and the boundary conditions

$$(4) \qquad K(T(0,i)) = K(T(1,i-1)) + B(E_{i-1})$$

Richard R. Nelson and *Sidney G. Winter,* "Neoclassical vs. Evolutionary Theories of Economic Growth: Critique and Prospectus," Economic Journal, No. 336, Vol. 84, December 1974, p. 886 - 905.

Richard R. Nelson and *Sidney G. Winter,* An Evolutionary Theory of Economic Change, Cambridge, Mass. and London England: The Belknap Press of Harward University Press, 1982.

For a recent mathematical model of economic growth, along neoclassical lines inspired by *Schumpeter's* theory of creative destruction, see:

Philippe Aghion and *Peter Howitt,* "A Model of Growth through Creative Destruction", unpublished Working Paper No. 527, Department of Economics, Massachusetts Institute of Technology, Cambridge, MA, May 1989.

[4] See, for instance, *L. H. Summers,* "Capital Taxation and Accumulation in a Life Cycle Growth Model", The American Economic Review, Sept. 1981. In this case, a constant elasticity of substitution utility function is used for intertemporal maximization of consumption.

$$(5) \quad K(T(1, i)) = K(T(0, i))\,(1 + r_w)^{(T(1, i) + T(0, i))}$$

$$\text{for } i = 1\text{: } K(T(1, i - 1)) = K_0;\ E_{i-1} = 0$$

$$\text{for } i = 2, 3, \ldots, m; \quad \text{if} \quad \text{at } m + 1\text{: } \frac{E_{m+i}}{K(T(1, m + 1)} < r_w$$

In (1) to (5) the symbols have the following meaning:

E_i	profits
U	utility function
b	utility coefficient
$T(0, i)$	initial time in maximization period i
$T(1, i)$	terminal time in maximization period i
e	basis of natural logarithm
λ	time preference coefficient
t	time
i	number of maximization period
K	capital
R_i	revenue from product sales
$\omega_{0, i}$	initial wage rate
$\sigma_{0, i}$	annual increase in the wage rate
L_i	labor input
$K_{1, i}$	cost item for "anti-imitator" expenditures
$\sigma_{1, i}$	annual increase in "anti-imitator" expenditures
$K_{2, i}$	jump cost item for intermittent innovation expenditures
K_0	initial capital
$K_{0, i}$	initial capital in period i
$K_{1, i}$	terminal capital in period i
r_w	minimun rate of return on assets
B	a feedback factor indicating what percentage of excess profits from period i is reinvested in period $i + 1$.

The novelty in the above model is the $K_{2, i}$ variable, signifying innovation expenditures. It is a jump variable whose size is determined anew at the beginning of each maximization period. The value of $K_{2, i}$ for a particular period's maximization problem can be either zero or positive, depending on the amount of profits obtained in the previous period. If the latter are over and above a certain stipulated level, the firm shall, as a rule of behavior, consider this a satisfactory state of affairs and it shall not make innovation expenditures. On the other hand, if previous profits were below the stipulated level, this adversity shall induce the firm to opt for innovation expenditures. The process can be formalized through the introduction of a sensor variable X_{i-1}. We write

$$X_{i-1} = \int_{T(0,\, i-1)}^{T(1,\, i-1)} E_{i-1}\, e^{(r_{i-1}/b)t}\, dt - G_{i-1} \tag{6}$$

$$K_{2,\, i} = 0 \quad \text{if} \quad X_{i-1} \geq 0 \tag{7}$$

$$K_{2,\, i} = Z_{2,\, i} \quad \text{if} \quad X_{i-1} < 0 \tag{8}$$

$$Z_{2,\, i} = X_0 \,(1 + \sigma_{3,\, i})^{(i-1)} \tag{9}$$

The R_i variable signifies revenues from the sale of a product Q_i which is produced, in line with the production function (10). The chosen production function is a *Leontieff* fixed factor proportion one, with capital and labor as inputs. We write it as

$$Q_i = \min \left[\frac{K_i}{a_K}, \frac{L_i}{a_L} \right], \; a_K > a_L \tag{10}$$

The jump variable $K_{2,\, i}$ changes the production function. The innovations which it introduces reduce the size of the labor coefficient a_L in equation (10) in a manner described below in Section III. To what extent the firm can benefit from the introduction of innovations depends however on feasible productivity increases, given the state-of-the-art in a particular period. To simplify the model we assume that productivity increases occur in a random fashion. The value of a random productive increase in maximization period i is denoted by q_i. Changes in the production function are therefore due to changes in technology, which, in *Schumpeter*'s spirit, trace back to innovations.

The product price may also change during the various maximization periods. The first possibility is that it rises continuously,

$$P_i = P_{i-1} \,(1 + \sigma_4)^{(i-1)} \tag{11}$$

An alternative is to let product price increase in a random fashion,

$$P_i = P_{i-1} + p_i \tag{11a}$$

where p_i is the random price change in period i.

From (10) and (11), respectively (11a), will be determined each period's sales revenue as

$$R_i = P_i Q_i \tag{12}$$

The additional symbols in equations (6) to (14) have the following meaning:

X_{i-1}	a sensor variable
G_{i-1}	minimum excess profits in a maximization period
$Z_{2,i}$	value of the $K_{2,i}$ jump variable in each maximization period
X_0	initial value of jump variable $K_{2,i}$
$\sigma_{3,i}$	rate of increase in jump variable $K_{2,i}$
q_i	random increase in productivity in period i
Q_i	product produced in period i
P_i	product price in maximization in period i
σ_4	increase in product price during period i
R_i	revenue from sale of product in period i

This is the mathematical version of our model. In order to find solutions to it and compute optimal timepaths, it is necessary to create a computer model.

III. The Computer Model

The above model consists of two parts. The first contains the maximization model in equations (1) to (5); the second specifies inputs in equations (6) to (14b). These two sets of equations are interrelated and the computational sequence is as follows.

To start with, initial values are given for all variables. Subsequently, an optimal solution vector is found for the first maximization period with the aid of the Pontryagin Maximum Principle. For the next (second) maximization period some of the needed inputs are computed from the first period's optimal solution vector. Thereafter the model is solved again with the Maximum Principle. The optimal values in this solution are then used for the computation of inputs for the ensuing period, and so on. Through this recursive method time paths for the innovation jump variable $K_{2,i}$, profits and total asset accumulation can be determined.

At the beginning of the first maximization period potential productivity increases are determined in a random process. Depending on excess profits

– whether they were in the previous period larger or smaller than a desired amount – the innovation expenditure $K_{2,i}$ is triggered. If the latter is positive, this permits that then existing, potential productivity increasing innovations can be made. This changes production cost and increases profits. Yet, innovations do not come without cost, especially if they require substantial research and development expenditures. Over time the cost of the jump variable $K_{2,i}$ can be expected to increase and perhaps do so at a fairly rapid rate. Because of rising research and development cost, early innovations are therefore less costly than later ones. As a consequence, if $K_{2,i}$ expenditures are made too late, it is possible that the firm may not be able to afford them anymore. The firm also has to use a certain amount of resources to maintain its market position. This is reflected in the variable $K_{1,i}$. Because imitators will, in a *Schumpetar*ian fashion, continue to erode an existing market advantage, the cost of holding on to the latter, through $K_{1,i}$ expenditures, shall rise. The innovating firm shall in all of these efforts never lose sight of its basic objective, namely intertemporal capital accumulation. In each maximization period it shall therefore insist on a minimum increase of its total capital. Doing this it shall orient itself on a sort of natural interest rate. Thus for each maximization period the terminal assets are in our model always recalculated to assure that this objective is fulfilled. If the stipulated minimum increase in capital cannot be achieved, the firm stops its present line of activity. It can, in such a case, presumably do better in other ones.

Our model maximizes profits over and above a pre-determined minimum asset accumulation. It should be noted that these profits can be withdrawn from the firm, for instance for consumption purposes, if this is desired. They may also be used to increase the capital stock for the next maximization period. Because we are interested in maximum growth and asset accumulation through the *Schumpetar*ian process, we assume that all profits are reinvested.

Before we present our simulation results, the following transformation in constraint (3) should be indicated. Because of (12) we have with a fixed proportion production function (10)

$$(13) \qquad R_i = P_i \left\{ \min \left[\frac{K_i}{a_K}, \frac{L_i}{a_L} \right] \right\}, a_K > a_L$$

Thus K_i is the limiting input. Furthermore, because the wage rate

$$(14a) \qquad \omega_i = \omega_o \qquad \text{for: } i = 1$$

(14b) $\omega_i = \omega_{i-1} e^{\sigma_{0,i} T_{1,i}}$ for: $i = 2, 3, \ldots m$

is given for each maximization period, the constraint (3) can be written as

(15) $$\dot{K} = \left[\frac{P_i}{a_K} - \frac{a_L}{a_K} \; \omega_i\right] K_i - K_{1,i}\, e^{\sigma_{1,i} t} - K_{2,i} - E_i$$

The bracketed expression in (15) signifies the after-labor cost, gross rate of return on capital. This rate varies with changes in P_I, ω_i, a_K and (or) a_L. Because it is assumed that all innovations are labor-saving, a_K is kept constant but a_L decreases once innovations are introduced in the production process.

If the chosen set of initial conditions and parameters is such that a feasible solution exists, the model begins to generate a sequence of optimal solution vectors. From these can be derived optimal time paths for the introduction of innovations, for profits and capital accumulation. However, because productivity increases occur in the present model in a random manner, a degree of indeterminacy attaches to these paths. An identical set of initial conditions and parameters does not necessarily lead to the same optimal time paths. Various time path patterns may arise. Yet, these variations in time paths notwithstanding, if it should happen that innovation expenditures $K_{2,i}$ are triggered rather early, productivity and production may increase at a rapid rate in the first years already. This may lead to an early sustained, high-level of profits, giving the firm a desirable headstart.

Such an early success may, however, induce a self-satisfied, complacent attitude within the firm. One may feel that one can do without costly innovation expenditures, for some time at least. This might continue until adversity strikes. Ultimately, because of rising cost, profits will come under pressure. It may then trigger a whole series of frantic innovation expenditures towards the end of the maximization sequence in an effort to stem, and hopefully reverse, the downward trend. Because the cost of introducing innovations are then rather high, this may well be too little, too late. The model stops once profits fall below the stipulated minimum level. The next section presents the results of a simulation run with the above model.

IV. Simulation Results

For our simulation we assume that a firm in manufacturing has an initial capital stock equal to \$ 6,000,000. Its initial labor input shall be 150,000 man hours per year. The production function is a fixed factor proportion one with a capital coefficient a_K equal to 5. The initial labor coefficient a_L is equal to 0.25. Each maximization period spans two years. At the end of each of these, all profits are reinvested, increasing the capital stock in production; there are thus no consumption drains. Because of the fixed factor proportion production function, labor inputs increase pari passu with capital accumulation. Yet, how large the increases in labor input will have to be depends on the introduction of new methods of production, the introduction of labor-saving innovations. If the firm chooses to avail itself of the latter, the labor coefficient a_L decreases. The innovation-induced reduction of the labor coefficient is a function of general, random productivity increases. In our model the latter translate into a reduction of the labor coefficient a_L through a feedback factor. Several assumptions can be made about the magnitude of the latter. We assumed that for our particular firm a one-percent general increase in productivity leads to a four-percent reduction of the labor coefficient. The wage rate, initially assumed to be \$ 1.20 an hour, shall increase at an annual rate of 3%. Thus wage costs are rising secularly, but these increases can be moderated through the introduction of labor-saving innovations. The product price, initially at \$ 1.00, shall not change.

In our model, there are two action thresholds for the firm which cause it to respond to an existing situation. The first is an insistence on a minimum 2% annual increase in capital – a sort of natural rate of return – as a precondition for continued operations. The second is an amount of \$ 650,000 in profits for each bi-annual maximization period. If profits fall below this level, innovation expenditures are made. We assume that initially the latter amount to \$ 150,000. They rise at an annual rate of 4%. To defend its market position against imitators, the firm shall have to make expenditures initially equal to \$ 110,000. Because the pressure from imitators rises as time goes on, these expenditures shall rise steeply, namely 7.5% per annum. With respect to other inputs, we assume that the time preference of the firm is equal to 4%. Its utility coefficient is 0.999.

The generated optimal time paths are shown in Diagrams 1 to 5. To start with, Diagram 1 shows annual random productivity increases. These range from 0% to 4.5% and reflect what the state-of-the art can offer at a particular point in time. A look at Diagram 3 then shows that at the end of the first, biannual, maximization period excess profits fall short of the stipulated threshold amount of \$ 650,000. Therefore, the first innovation expenditures occur, costing \$ 168,700, as can be seen from Diagram 2. A look at Diagram 4 reveals that this innovation outlay keeps labor cost from rising. As a consequence, profits increase, as can be seen in Diagram 3. The latter are now over and above the threshold level which triggers innovation expenditures.

This is a welcome state of affairs and for the next three maximization periods nothing is done (see Diagrams 2 and 3). However, labor cost and anti-imitator cost rise and there is therefore the first, decline in profits. When the 6th maximization period (12th year) arrives, profits have fallen so far that innovation expenditures are once more triggered. This time they actually lead to a temporary decrease in labor cost, as can be seen from Diagram 4. Profits get a boost, but this does not prevent their ensuing decline (see Diagram 3).

When the 12th maximization period arrives (24th year), excess profits have again fallen below the threshold level and new innovation expenditures are made, now costing $ 358,380. Yet, in this case the timing was wrong. The expenditures were made at a moment when general productivity increases amounted to a mere 0.8% only. It was a costly mistake: labor cost continues to rise (see Diagram 4) and profits fall further (see Diagram 3). Thus, already in the next maximization period, additional innovation expenditures are made. This time there is a substantial productivity increase of nearly 4%, causing a sizeable decrease in labor cost (see Diagram 4) and profits soar (see Diagram 3). As a consequence, profits remain again over the threshold level for three maximization periods, yet because of rapidly rising anti-imitator expenditures (see Diagram 5) and rising wage cost, they decrease. In the 18th maximization period (36th year), there is finally a last innovation outlay which costs now, however, the hefty sum of $ 593,000. It can once more brake the increase in labor cost temporarily. Yet, after the 19th maximization period, it is no longer possible to assure a minimum annual increase in capital of 2% and the program stops. For one line of activity the *Schumpeter* process has come to an end.

Diagram 1

Annual Random Productivity Increases in %

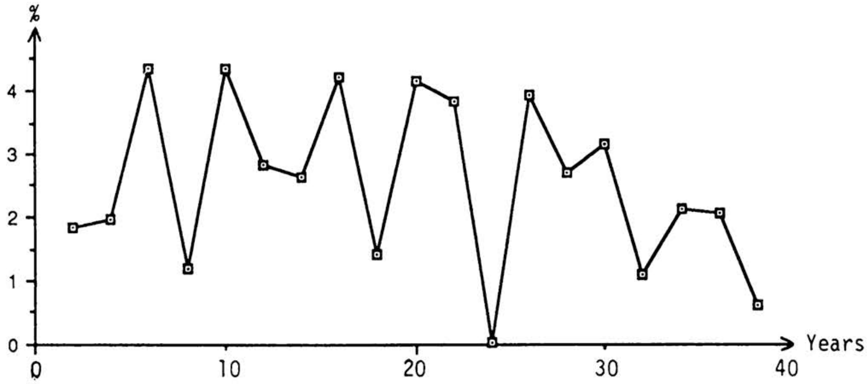


Diagram 2

Innovation Expenditures in 1000 Dollars

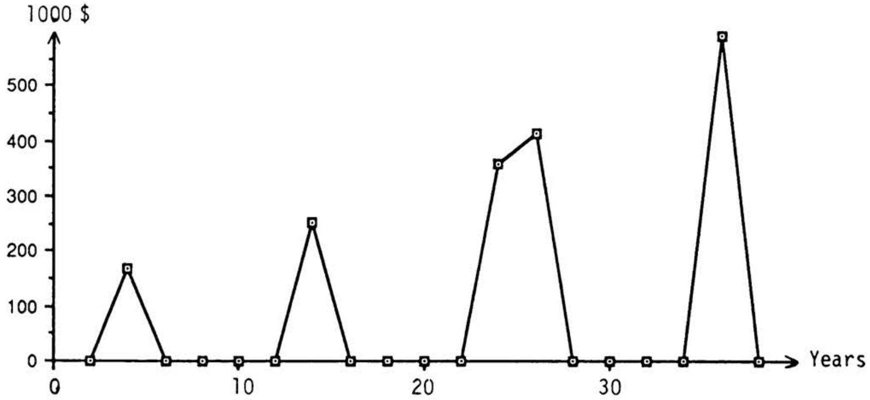


Diagram 3

Actual Profits in 1000 Dollars

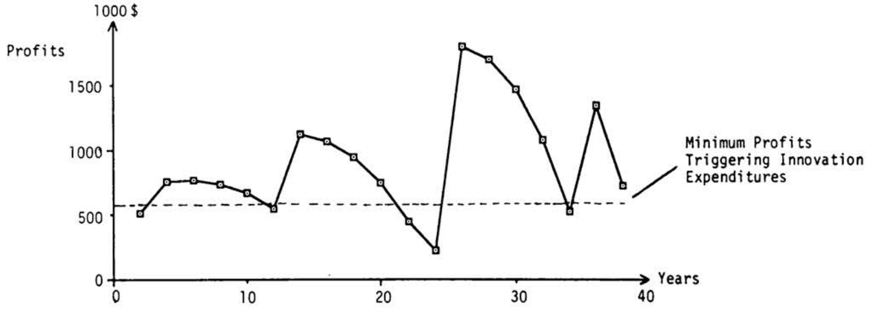


Diagram 4

Total Labor Cost in 1000 Dollars

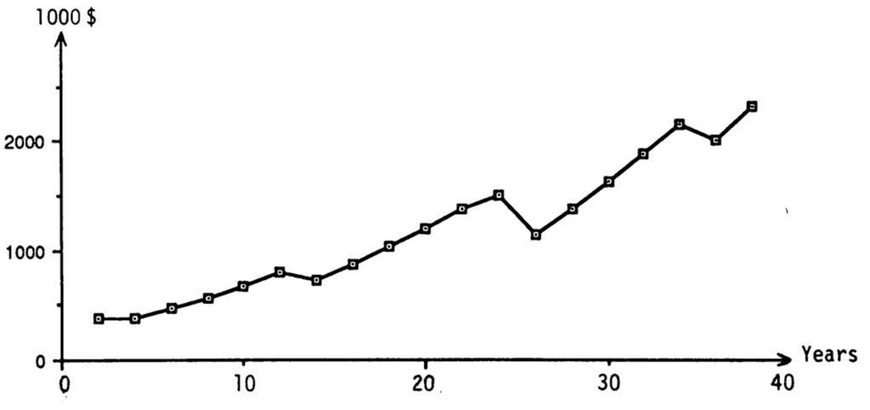
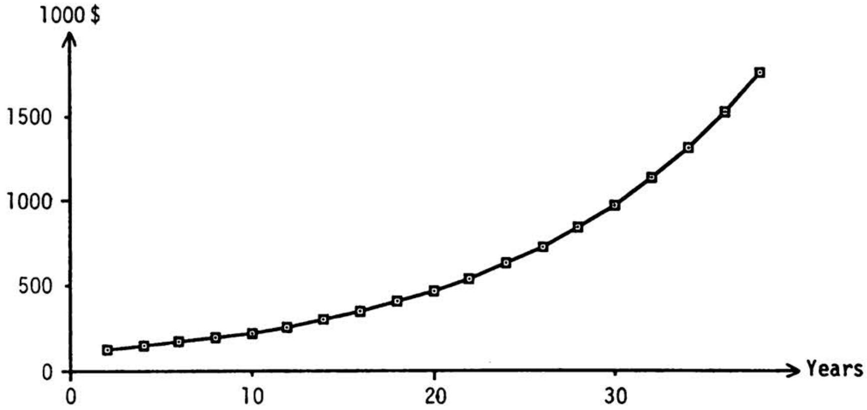


Diagram 5

Anti-Imitator Expenditures in 1000 Dollars



Summary

A Schumpeter Model of Economic Growth and Innovation

It is possible to formalize the *Schumpeter* model of innovation and economic growth. An adaptive, discontinuous maximization model, which allows for random technological change and increases in productivity, can be shown to generate optimal time paths for the introduction of innovations, the capital stock, labor inputs and excess profits as described in Schumpeter's work. In order to demonstrate how the model works, we assumed in this paper rather short maximization periods and therefore frequent possibilities of readjustments. The maximization periods can however be of any lengths.

With respect to innovations we assumed in this paper, for the sake of simplicity, that they occur in a random fashion and are thus exogenous. This is one way of introducing innovations. We are aware that there are also many other ones, e. g., innovations induced by one or several endogenous factors and (or) innovations occurring in a bunched fashion. As far as the difficult problem of the effect of innovation and technological change on production cost is concerned, we assumed that the former are labor-saving. Other assumptions can be made about these features of the model. The various parameters and coefficients entering the model can be changed freely, but each variation will generate optimal time paths of different shape, and possibly, different length. Repeated simulation runs indicate however that they all do have important Schumpetarian traits in common. They all belong to a discontinuous, innovation-driven process of capital accumulation and growth, in the presence of uncertainties with respect to the speed of technological progress, rising wage cost and rising cost to hold ones own against encroachments by competitors and imitators.

Zusammenfassung

Schumpeter-Modell für Wirtschaftswachstum und Innovation

Es ist möglich, *Schumpeters* Innovations- und Wachstumsmodell zu formalisieren. Ein anpassungsfähiges diskontinuierliches Maximierungsmodell, welches zufallsbedingten technologischen Wandel und zufallsbedingte Produktivitätszuwächse zu berücksichtigen gestattet, erweist sich für die Bestimmung des optimalen Innovationsverlaufs, den Kapitalstock, Arbeitsvorleistungen und Überschußgewinne, wie von Schumpeter selbst dargestellt, als geeignet. Um die Funktionsweise des Modells zu demonstrieren, geht diese Arbeit von eher kürzeren Maximierungszeiträumen und somit von häufigen Anpassungsmöglichkeiten aus. Die Maximierungszeiträume können jedoch von beliebiger Länge sein.

Für Innovation unterstellt diese Arbeit aus Gründen der Vereinfachung, daß sie zufallsbedingt und somit exogener Natur ist. Dies ist eine Darstellungsweise im Modell. Es gibt unbestritten jedoch auch weitere, zum Beispiel Innovation auf Grund eines bzw. mehrerer endogener Faktoren, welche gebündelt auftreten. Was das schwierige Problem der Auswirkungen von Innovation und technologischem Wandel auf die Produktionskosten anbetrifft, wird in dieser Arbeit angenommen, daß Innovation arbeitskräftesparend ist. Hierzu gestattet das Modell jedoch auch andere Annahmen. Die verschiedenen in das Modell eingehenden Parameter und Koeffizienten sind frei veränderbar; doch jede Veränderung führt zu optimalen unterschiedlichen Abläufen im Zeitdiagramm und möglicherweise auch zu unterschiedlichen Zeitlängen. Wiederholte Simulationsläufe lassen jedoch wichtige gemeinsame *Schumpeter*sche Merkmale und die Zugehörigkeit zu einem diskontinuierlichen innovationsgetriebenen Prozeß von Kapitalbildung und Wachstum vor dem Hintergrund von Unsicherheiten hinsichtlich der Geschwindigkeit des technologischen Fortschritts, steigenden Lohnkosten und zunehmenden Kosten der Unternehmen im Kampf mit Konkurrenten und Imitatoren erkennen.

Résumé

Un modèle de Schumpeter de la croissance économique et de l'innovation

Il est possible de formaliser le modèle d'innovation et de croissance économique de *Schumpeter*. Un modèle discontinu de maximisation adaptable, qui tient compte du changement technologique fortuit et des accroissements de productivité, peut générer des structures d'évolution optimales pour l'introduction d'innovations, les fonds, les inputs de main d'oeuvre et les profits supplémentaires, comme le décrit le travail de Schumpeter. Pour démontrer comment le modèle fonctionne, nous assumons dans cet article des périodes de maximisation plutôt brèves et par conséquent, de fréquentes possibilités de réajustements. Les périodes de maximisation peuvent cependant être de n'importe quelle longueur.

En ce qui concerne les innovations, nous assumons ici, en guise de simplification, qu'elles se produisent au hasard et qu'elles sont donc exogènes. Ceci est une façon

d'introduire des innovations. Nous savons qu'il y a beaucoup d'autres façons de le faire, par exemple des innovations peuvent être induites par un ou plusieurs facteurs endogènes et (ou) elles peuvent survenir de façon groupée. Pour ce qui est du problème complexe des effets de l'innovation et du changement technologique sur les coûts de production, nous assumons que ceux-ci sont des économies de la main-d'oeuvre. On peut faire d'autres hypothèses sur ces caractéristiques du modèle. Les divers paramèt-res et les coéfficients considérés dans le modèle peuvent être modifiés librement. Mais chaque variation entraînera des structures d'évolution optimales de différente forme et, probablement, de différente longueur. Des simulations répétées indiquent cependant qu'elles ont toutes en commun des traits importants de modèle de *Schumpeter*. Elles appartiennent toutes à un processus discontinu, poussé par des innovations, d'accumulation de capital et de croissance, en présence d'incertitudes quant à la vitesse du progrès technologique, au coût des salaires croissant et au coût croissant pour conserver sa propriété face aux atteintes des concurrents et des imitateurs.