

# **A Rational Expectations Model of Inflation for West Germany: Evidence from Quarterly Data**

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## **I. Introduction**

In this paper a rational expectations model of inflation is developed by using the quarterly data for West Germany. The theoretical model consists of a demand for money function and a *Lucas* supply function. This relatively simple model which assumes monetary neutrality, is able to provide a satisfactory description of inflation in West Germany. The findings reveal that there is an almost one-to-one positive relation between inflation and the anticipated growth in the money supply, while the unanticipated growth in the money supply has a negative effect on the rate of inflation.

In a recent paper, *Demery et al.* (1984) provide evidence in favor of monetary neutrality and a rational expectations explanation of real variables for West Germany using quarterly data. The findings reported here for a nominal variable, inflation, complement those of *Demery et al.* The findings also corroborate with those of *Atesoglu* (1988) who has provided a rational expectations account of price and wage inflation for West Germany using annual data.

The proposed model of inflation is detailed in Section II. The empirical results are discussed in Section III, and the concluding remarks are in Section IV.

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## II. A Model of Inflation

### 1. Inflation Equation

The inflation equation of the model is similar to the price level equations of *Barro* (1978, 1981, pp. 154 - 168) and *Barro and Rush* (1980) for the United States, and to the inflation equation of Atesoglu for West Germany, and it is:

$$(1) \quad p_t = \alpha_0 + \alpha_1 m_t + \alpha_2 m_t^u + \alpha_3 r_t^e + \alpha_4 c_{t-1} + v_t$$

$$\alpha_1 = 1; \alpha_2, \alpha_4 < 0; \alpha_3 > 0,$$

here and elsewhere in the paper  $t$  stands for the quarter in time period  $t$ . All variables above are measured at annual rates of change – percentage change between the current quarter and the corresponding quarter in the previous year. The variables are:

$p$  = the rate of inflation,  
 $m$  = growth in the money stock,  
 $m^u$  = unanticipated growth in the money stock,  
 $r^e$  = expected growth in the interested rate,  
 $c$  = growth in real cash balances,  
 and  $v$  is the random disturbance term.

The derivation of equation (1) resembles that of *Sheffrin* (1983, p. 45) and begins with a demand for money function:

$$(2) \quad \log M_t - \log P_t = \beta_1 \log Y_t + \beta_2 \log R_t + \beta_3 \log C_{t-1}$$

$$\beta_1, \beta_3 > 0; \beta_2 < 0,$$

where:  $M$  = the stock of money,  $P$  = the price level,  $Y$  = real income,  $R$  = the interest rate,  $C$  = real cash balances.

In equation (2), the lagged real cash balances variable allows for the partial adjustment process. The recent findings of *Buscher* (1984) indicate that the above demand for money function is likely to be an appropriate specification for West Germany.

Substituting an exogenously determined supply of money – in logarithms – into equation (2), and solving for the price level, and differentiating this price level equation with respect to time, yields an inflation equation:

$$(3) \quad p_t = \alpha_1 m_t + \gamma_2 y_t + \gamma_3 r_t + \gamma_4 c_{t-1}$$

$$\alpha_1 = 1, \gamma_3 > 0; \gamma_2, \gamma_4 < 0,$$

here,  $y$  is the growth in real income,  $r$  is the growth in the interest rate, and the other variables are the same as defined above.

The rate of growth in real income is given by a *Lucas* supply function:

$$(4) \quad y_t = \varnothing_0 + \varnothing_1 (p_t - p_t^e) \quad \varnothing_1 > 0$$

where  $p^e$  is the expected inflation and  $\varnothing_0$  is the trend growth in natural output.

Similar to the approach of Barro,  $r^e$  is substituted as an instrument variable for  $r$  in equation (3), and then substituting (4) into (3):

$$(5) \quad p_t = \alpha_1 m_t + \gamma_2 [\varnothing_0 + \varnothing_1 (p_t - p_t^e)] + \gamma_3 r_t^e + \gamma_4 c_{t-1}$$

and taking the mathematical expectation of (5):

$$(6) \quad p_t^e = \gamma_2 \varnothing_0 + \alpha_1 m_t^e + \gamma_3 r_t^e + \gamma_4 c_{t-1}$$

here,  $m^e$  is the expected rate of growth in the money supply. Substituting identity  $m^e = m - m^u$  and equation (6) into (5), gives the inflation equation, (1).

## 2. The Money Growth and Interest Rate Growth Forecasting Equations

The unanticipated growth in the money stock,  $m^u$ , is given as the residuals of a money growth forecasting equation similar to those of *Barro* (1977) for the United States, and *Hoffman* et al. (1982) and *Atesoglu* for West Germany. The expected growth in the interest rate,  $r^e$ , is obtained as the predictive values of an interest rate growth forecasting equation. Following *Mishkin* (1982, a, b, 1983, pp. 21 - 22), these forecasting equations are specified by an atheoretical statistical procedure, since there is no agreement concerning the correct model of the economy and the interest rate growth and the money growth forecasting equations. In this approach, the forecasting equation at the current quarter  $t$ , should include all available relevant information in the previous quarters and should be free of serial correlation for making a rational forecast.

## III. Empirical Results

### 1. The Method of Estimation

Each equation in this paper is estimated by using the ordinary least squares method. The use of residuals and predicted values as regressors in

the inflation equation, (1), however may not be satisfactory. The statistical difficulties associated with this type of “two step” estimation procedure are discussed by *Pagan* (1984). The findings of *Demery et al.* for West Germany obtained by using the “two step” procedure and alternatively a joint estimation procedure are very similar. Therefore no attempt was made to estimate the forecasting equations and the inflation equation jointly as a system while taking into account the cross equation restrictions.

The specific procedure adopted for specifying the forecasting equations is the same as used by *Hoffman et al.*, which is based on a multivariate *Granger* test for various variables, which are likely to be useful for predicting the money growth and the interest rate growth. The *Granger* tests were conducted by regressing the money growth on a constant and four lagged values of the money growth – to obtain white noise residuals – and four lagged values of a variable considered to be a potential predictor of the money growth. The four lagged values of the predictor variable were included in the final forecasting equation if they were significant as a group at the 5 percent level. This procedure was also repeated for the growth in the interest rate variable.

The variables used in the *Granger* tests as potential predictors for the money growth were: the change in the interest rate,  $\Delta R$ , the change in the unemployment rate,  $\Delta U$ , the rate of inflation, the growth in real income, the growth in the exchange rate,  $e$ , and the growth in real public consumption,  $g$ . The potential predictors tested for the growth in the interest rate included the money growth, and all variables mentioned above for the money growth, except the change in the interest rate.

The change in the potential predictors were measured between the current quarter and the corresponding quarter in the previous year. The rate of growth of potential predictors and the interest rate growth were measured as percentage change between the current quarter and corresponding quarter in the previous year.

## 2. The Results for the Forecasting Equations

The *F*-statistics for the significance of the four lagged values of each potential predictor variable as a group is given in Table 1. It is seen that the change in the interest rate and the growth in real income can be included as predictors for the money growth. For the interest rate growth, the real income growth should be considered as a predictor.

*Table 1*  
**F-Statistics for Money Growth and the Interest Rate Growth**  
**(1962: 1 – 1987: 3)**

Dependent Variable	Potential Predictor						
	$\Delta R$	$\Delta U$	$p$	$y$	$e$	$g$	$m$
$m$	14.225*	0.628	1.642	7.601*	1.384	0.483	n. a.
$r$	n. a.	2.147	0.691	3.055*	1.511	0.870	2.149

\* Significant at the 5 percent level.

The forecasting equations estimated are detailed in Table 2. These equations contain a constant term and the four lagged values of the dependent variable to insure that disturbances are white noise. A test for one to fourth order autocorrelation was made using *Durbin's m* test discussion by *Kmenta* (1986, p. 333).

For *Durbin's m* test the residuals from the money growth forecasting equation were regressed on a constant and four lagged values of the residuals as well as all other explanatory variables in the money growth forecasting equation. A test for the significance of the four lagged residuals as a group yields  $F = 1.746$  indicating absence of serial correlation at the 5 percent level. *Durbin's m* test was also applied to the interest rate growth forecasting equation. The four lagged values of the residuals were again found not to be significant as a group at the 5 percent level, with an  $F = 2.048$ .

### 3. The Results for the Inflation Equation

The persistence effects discussed by *McCallum* (1980), are found to be significant in West Germany by *Demery* et al. who have used quarterly data in their study. Accordingly, in estimating the inflation equation (1), four lagged values of the real income growth were included as additional explanatory variables representing the persistence effects. The estimated inflation equation is:

$$\begin{aligned}
 p_t = & 0.426 + 0.837 m_t - 0.671 m_t^u - 0.746 c_{t-1} + 0.034 r_t^e \\
 & (1.80) \quad (19.63) \quad (-8.09) \quad (-17.70) \quad (11.65) \\
 & - 0.034 y_{t-1} + 0.082 y_{t-2} + 0.024 y_{t-3} - 0.195 y_{t-4} \\
 & (-1.17) \quad (1.44) \quad (0.43) \quad (-4.18)
 \end{aligned}$$

period: 1962: 1 - 1987: 3       $R^2 = 0.836$        $SE = 0.771$        $DW = 2.066$

*Table 2*  
**Forecasting Equations for Money Growth and the Interest Rate Growth**  
**(1962: 1 – 1987: 3)**

Dependent Variable	$m_t$	$r_t$
Constant	0.682 ( 1.89)	– 7.120 (– 1.94)
$m_{t-1}$	0.862 ( 7.57)	
$m_{t-2}$	0.095 ( 0.60)	
$m_{t-3}$	– 0.027 (– 0.18)	
$m_{t-4}$	– 0.094 (– 1.00)	
$\Delta R_{t-1}$	– 0.935 (– 7.15)	
$\Delta R_{t-2}$	0.643 ( 3.07)	
$\Delta R_{t-3}$	– 0.026 (– 0.12)	
$\Delta R_{t-4}$	– 0.046 ( 0.32)	
$y_{t-1}$	0.115 ( 1.60)	1.642 ( 1.37)
$y_{t-2}$	– 0.128 (– 1.47)	1.879 ( 1.26)
$y_{t-3}$	0.096 ( 1.10)	– 1.292 (– 0.88)
$y_{t-4}$	0.097 ( 1.33)	0.955 ( 0.81)
$r_{t-1}$		1.148 ( 11.26)
$r_{t-2}$		– 0.308 (– 2.01)
$r_{t-3}$		– 0.275 (– 1.78)
$r_{t-4}$		0.121 ( 1.22)
$R^2$	0.924	0.829
$SE$	1.162	19.990
$DW$	1.943	2.017

Note: Figures in parantheses are  $t$  statistic.

A joint test for the significance of the four lagged values of the real income growth yields  $F = 6.124$ , which is significant at the 5 percent level. The persistence effects which can be attributed to the slow adjustment of employment in the labor markets appear to be an important feature of the West German economy.

The signs of the coefficient estimates for the unanticipated money growth, growth in real cash balances and expected growth in the interest rate are as required theoretically and they are highly significant.



The negative effect of the unanticipated money growth on inflation corroborate with the findings of *Demery* et al. who have found a positive relation between the unanticipated money growth and real output variables in West Germany.

The coefficient estimate for money growth is significantly different than unity at the 5 percent level, contrary to the theoretical requirement. This may very well be due to the much discussed use of the West German Mark as money by other countries. Nevertheless, the coefficient estimate is positive and highly significant and its value is very close to unity. This result indicates that the changes in  $m$  measuring anticipated growth in the money – while holding other explanatory variables of the inflation equation fixed – has almost one-to-one effect on inflation. A test for serial correlation using *Durbin's m* test mentioned above indicates no serial correlation at the 5 percent level with an  $F = 1.468$  for the four lagged values of the residuals of the inflation equation. This result raises the confidence that can be placed on the estimates, and the explanatory power of the inflation equation measured by the relatively high coefficient of determination.

#### 4. *The Data and the Seasonality*

In the regressions reported above, the variables are measured at annual rates of change or annual change – between the current quarter and the corresponding quarter in the previous year. This procedure should eliminate seasonality in the data. The empirical concepts employed and the sources of the data are discussed in the Appendix.

#### 5. *Tests for Parameter Stability*

The difficulty of volatile coefficients that appear in time series models, especially those covering long periods of time, can be examined by applying the *Chow* (1960) test. This was done by *Mishkin* (1983, p. 84) for his forecasting equations for the United States. Despite the findings of *Buscher*, there still seem to be doubts in West Germany concerning the stability of the demand for Money.

Accordingly, the money growth, the interest rate growth, and the inflation equations were tested for parameter stability using the *Chow* test. For this purpose the sample period was divided into two almost equal parts, 1962:1 - 1974:4 and 1975:1 - 1987:3. The appropriate  $F$ -statistics calculated concerning the money growth, interest rate growth and inflation equations

were: 0.980, 0.932 and 2.235. None of these could reject the hypothesis that the coefficients of the regressions are equal in two sub-samples at the 5 percent level.

#### IV. Concluding Remarks

A plausible account of inflation has been provided for West Germany with a rational expectation model, without recourse to ad hoc explanations, such as union power or imported inflation. The results reported here and those of *Demery et al.* and *Atesoglu* suggest that the rational expectation approach is likely to be useful for interpreting and predicting the fluctuations in real and nominal variables in West Germany.

#### Appendix

##### *The Empirical Concepts and the Data Sources*

The money stock concept used is M1 – adjusted for changes in the reporting requirements of the banks and the adjustment was done by the Institut für Weltwirtschaft. The 3-month money market rate was used as the measure of the interest rate. The source of the original M1 data and the source of the 3-month money market rate is Deutsche Bundesbank, Monatsberichte, Verschiedene Jahrgänge.

The GNP deflator was employed as the measure of prices. The GNP in 1980 prices was used as the measure of real income. The real public consumption was measured by public consumption in 1980 prices. The source of these national accounts data prior to 1968:1 is Deutsches Institut für Weltwirtschaftsforschung, Berlin, Sozialprodukt und Einkommenskreislauf, and the source starting with 1968:1 is Statistisches Bundesamt, Wiesbaden, Konten und Standardtabellen, Reihe 1, 1986.

The source of the unemployment rate data is Statistisches Bundesamt, Wiesbaden, Wirtschaft und Statistik, Verschiedene Jahrgänge.

The exchange rate measure used is the inverse of Deutsche Mark per U.S. Dollar, series RF. The source of the series is IFS Data Fund/87, International Monetary Fund, Washington D. C.



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## Zusammenfassung

### Ein Modell rationaler Erwartungen zur Erklärung der Inflation in Westdeutschland: Evidenz aufgrund von Vierteljahresdaten

Für Westdeutschland wird ein Modell rationaler Erwartungen anhand von Vierteljahresdaten aufgestellt. Dieses theoretische Modell besteht aus einer Geldnachfragefunktion und einer *Lucas*-Angebotsfunktion. Es ermöglicht eine zufriedenstellende Beschreibung der Inflation in Deutschland. Die Ergebnisse zeigen eine positive Elastizität von nahezu eins zwischen der Inflation und dem antizipierten Geldmengenwachstum sowie eine negative Beziehung zwischen nicht-antizipiertem Geldmengenwachstum und Inflation.

### Summary

#### **A Rational Expectations Model of Inflation for West Germany: Evidence from Quarterly Data**

A rational expectations model is developed by using the quarterly data for West Germany. The theoretical model consists of a demand for money function and a *Lucas* supply function. This model is able to provide a satisfactory description of inflation in West Germany. The results indicate an almost one-to-one positive relation between inflation and anticipated growth in the money supply, and a negative relation between the unanticipated growth in the money supply and inflation.

### Résumé

#### **Un modèle d'inflation à attentes rationnelles pour la République fédérale d'Allemagne: preuve à l'appui de données trimestrielles**

Un modèle d'attentes rationnelles est développé en utilisant les données trimestrielles pour la République fédérale d'Allemagne. Le modèle théorique consiste en une fonction de demande monétaire et une fonction d'offre de *Lucas*. Ce modèle peut décrire de façon satisfaisante l'inflation en République fédérale d'Allemagne. Les résultats indiquent une relation positive presque d'un à un entre l'inflation et la croissance anticipée de l'offre monétaire et une relation négative entre la croissance non-anticipée de l'offre monétaire et de l'inflation.