A Note on Tax Multiplier*

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I. Introduction

It is well-known in the standard IS-LM macromodel that national income will decrease (increase) as government undertakes a tax surcharge (cut) action. However, in an interesting paper *Holmes* and *Smyth* (1972) propose that the transactions demand for money should be a function of disposable income (national income after tax) rather than national income based on theoretical and empirical viewpoints, and challenge the conventional wisdom. As a result, they find that the tax multiplier may be positive depending on the relative shifts in the IS and LM curves. The question is, are their results robust enough to sustain changes in the specification of the macroeconomic equations?

Consider the consumption and the investment function. *Weber* (1970) argues that consumption is positively related to the interest rate, and *Yarrow* (1975) claims that for the growth-maximizing firms, as opposed to the profit-maximizing firms, investment may be an increasing function of the interest rate. *Cebula* (1976), *Tavlas* (1980, 1982), and *Jaeger* (1981) put these two findings into an IS-LM framework and obtain some interesting results which are contrast with the conventional wisdom[1]. In addition, *Yan-*

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[1] To obtain the conclusions reached by *Cebula,* it is sufficient that an increase in the interest rate raises the sum of consumption and investment; it is not required that the increased interest rate will raise both. There are a few empirical studies which show that an increase in the interest rate may at least raise the sum of the two, that is, raise the total expenditure. For the positive impact that interest rate has on consumption, see *Boskin* (1978) and *Carlino* (1982); and for the insensitivity of investment with respect to interest rate, see *White* (1970) and *Clark* (1979).

nacopoulos (1982) and *Lai* and *Chang* (1987) apply the main characteristics of Cebula (1976) model to the open economy, and investigate the effects of monetary and fiscal policies on the balance of payments and the impact of currency devaluation on domestic output in the context of fixed exchange rates, respectively. In line with these studies, this paper attempts to reevaluate the conclusion reached by *Holmes* and *Smyth* (1972) by adopting Cebula's (1976) specification of the consumption and investment functions. We find that the tax multiplier is always negative, which runs in sharp contrast with the Holmes and Smyth (1972) assertion.

The remainder of the paper is arranged as follows. Section II presents the analytical framework. The tax multiplier under alternative specifications is discussed in section III. Finally, the concluding remarks are presented in section IV.

II. The Model

Except for the fact that the money demand function incorporates the specific feature emphasized by *Holmes* and *Smyth* and that the expenditure function embodies the characteristics of the *Cebula* model, the analytical framework is basically that of standard IS-LM model. The model consists of the following two equations:

$$y = E(y - \tau y, r) + G; \tag{1}$$

$$M = L(y - \tau y, r); \tag{2}$$

where y = national income; E = expenditure on consumption and investment; τ = a proportional tax rate; $y_d \equiv (y - \tau y)$ = disposable income; r = interest rate; G = government expenditure; M = supply of money; L = demand for money. Using subscripts with the relevant variables to indicate partial derivatives, we will impose the following restrictions on the behavior function: $1 > E_{y_d} > 0$, $E_r \gtrless 0$, $L_{y_d} > 0$, $L_r < 0$. It is worth mentioning that the assumption $E_r > 0$ is conformable with the findings of *Weber* (1970) and *Yarrow* (1975).

Equations (1) and (2) are the equilibrium conditions for the commodity market and money market, respectively. The demand for money is specified to be a function of disposable income, which is the main argument of *Holmes* and *Smyth* (1972).

III. The Impact of Tax Policy

We first examine the effects of a change in tax policy. Totally differentiating the system with respect to τ, we have

$$\begin{bmatrix} E_{y_d}(1-\tau) - 1 & E_r \\ L_{y_d}(1-\tau) & L_r \end{bmatrix} \begin{bmatrix} dy \\ dr \end{bmatrix} = \begin{bmatrix} yE_{y_d}\,d\tau \\ yL_{y_d}\,d\tau \end{bmatrix} \tag{3}$$

By *Cramer*'s rule, it implies that

$$\frac{dy}{d\tau} = \frac{y\,[L_r E_{y_d} - E_r L_{y_d}]}{\Delta} \gtreqless 0 \tag{4}$$

where $\Delta = L_r[E_{y_d}(1-\tau) - 1] - E_r L_{y_d}(1-\tau) > 0$ according to *Routh-Hurwitz* stability condition. It is worth pointing out that the stability condition imposes a restriction related to the relative steepness between the IS curve and the LM curve.[2] We will mention it in due time.

We now use the result stated in equation (4) to discuss the performance of a tax policy under alternative specifications of the expenditure function.

1. *The Holmes-Smyth Specification*

Following conventional analysis, Holmes-Smyth specifies $E_r < 0$; then the stability condition $\Delta > 0$ is always met. So from equation (4), we obtain

$$\frac{dy}{d\tau} \gtreqless 0 \text{ as } (L_r E_{y_d} - E_r L_{y_d}) \gtreqless 0 \tag{5}$$

Moreover, from equation (3) we have

$$\left.\frac{\partial r}{\partial \tau}\right|_{IS} = \frac{yE_{y_d}}{E_r} < 0 \tag{6}$$

$$\left.\frac{\partial r}{\partial \tau}\right|_{LM} = \frac{yL_{y_d}}{L_r} < 0 \tag{7}$$

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[2] From equation (3) the slopes of the IS and LM curves are given by

$$\left.\frac{\partial r}{\partial y}\right|_{IS} = -\frac{[E_{y_d}(1-\tau) - 1]}{E_r} \gtreqless 0$$

$$\left.\frac{\partial r}{\partial y}\right|_{LM} = -\frac{L_{y_d}(1-\tau)}{L_r} > 0$$

which imply that a tax cut will make IS curve and LM curve shift upwards, respectively. According to equations (6) and (7), we can express (5) in the following manner

$$\frac{dy}{d\tau} \gtreqless 0 \quad \text{as} \quad \left| \frac{yE_{y_d}}{E_\tau} \right| \lesseqgtr \left| \frac{yL_{y_d}}{L_\tau} \right| \tag{8}$$

Equation (8) is exactly the conclusion of the *Holmes* and *Smyth* analysis. It indicates that the larger the shift in LM curve is relative to the shift in IS curve due to a change in tax rate, the more likely the perverse tax multiplier will prevail.

2. *The Cebula Specification*

Alternatively, if we adopt *Cebula*'s specification, $E_\tau > 0$, the stability condition ($\Delta > 0$) requires that the slope of the IS curve be greater than that of the LM curve. Then from (4) we have

$$\frac{dy}{d\tau} < 0 \tag{9}$$

Equation (9) reveals that, in the presence of the *Holmes* and *Smyth* consideration, the national income will always be expansionary as government undertakes a tax cut policy. This result contrasts with the Holmes and Smyth assertion.

These contrast results can be illustrated clearly by using the IS-LM apparatus. As is evident, the IS schedule is negatively sloped under the Holmes-Smyth specification, while a positively sloped IS schedule prevails under the Cebula specification.

In figure 1, the initial equilibrium is at E_0; initial output is y_0. Under the *Holmes-Smyth* situation, in response to a tax cut from τ_0 to τ_1, both $\mathrm{IS}_{HS}(\tau_0)$ and $\mathrm{LM}(\tau_0)$ shift upward to $\mathrm{IS}_{HS}(\tau_1)$ and $\mathrm{LM}(\tau_1)$, respectively. As indicated in figure 1, the domestic output will depress from y_0 to y_{HS} provided that LM shifts upwards by a greater distance than IS does. Next, we turn to examine *Cebula*'s circumstance. Following a tax cut from τ_0 to τ_1, $\mathrm{IS}_C(\tau_0)$ shifts downward to $\mathrm{IS}_C(\tau_1)$, while $\mathrm{LM}(\tau_0)$ again shifts upward to $\mathrm{LM}(\tau_1)$. Obviously, the domestic output will increase from y_0 to y_C in figure 1.[3]

[3] Equations (6) and (7) tell us that the shifts of the IS and LM curves depend on y. Therefore, in figure 1, the shifts upward and downward from $\mathrm{IS}_{HS}(\tau_0)$, $\mathrm{IS}_C(\tau_0)$ and $\mathrm{LM}(\tau_0)$ should not be parallel.

IV. Concluding Remarks

Based upon a standard IS-LM macromodel, this paper reexamines the effects of tax policy on domestic output. It is shown that the specification of the expenditure function plays an important role in determining the performance of tax policy. Under the *Holmes-Smyth* specification, a tax cut policy will depress domestic output provided that LM shifts upwards by a greater distance than IS does. However, if we adopt *Cebula*'s specification, a tax cut will definitely stimulate domestic output, rather than depress it, even if the Holmes-Smyth claim that the money demand is a function of disposable income rather than national income, is taken into consideration.

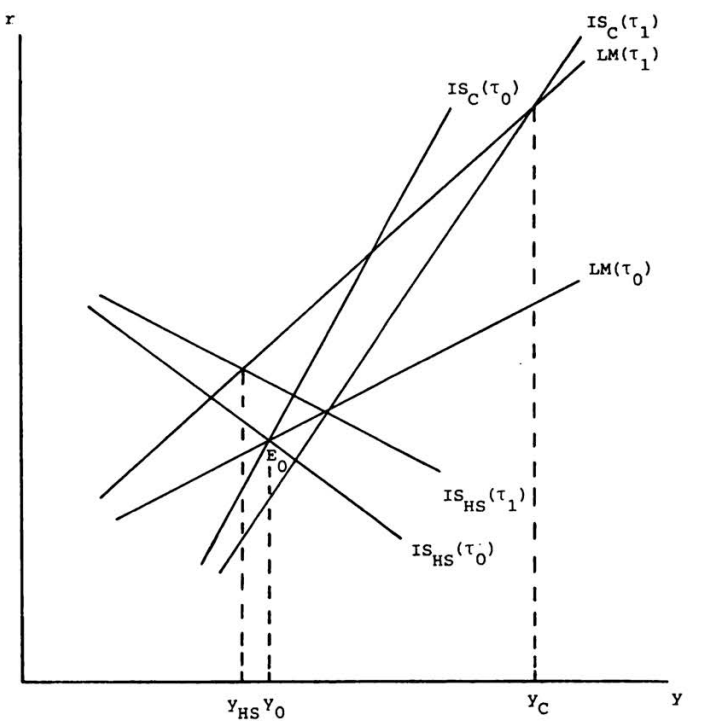

Figure 1

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Zusammenfassung

Bemerkungen zum Steuermultiplikator

Die Schlußfolgerung von *Holmes* und *Smyth* (1972), die in der Literatur starke Beachtung fand, zeigt, daß eine Politik der Steuersenkung die Inlandsproduktion senken kann, sofern die Geldnachfragefunktion mehr durch das verfügbare Einkommen als durch das Volkseinkommen determiniert wird. Dieser Aufsatz übernimmt die Spezifikation von *Cebula* (1976) für die Konsum- und Investitionsfunktionen und kommt zu dem Ergebnis, daß die Holmes-Smyth-Behauptung in diesem verbesserten Modell keine Gültigkeit hat.

Summary

A Note on Tax Multiplier*

The conclusion of *Holmes* and *Smyth* (1972), which has received wide attention in the literature, indicates that a tax cut policy may depress domestic output if the money demand function is determined by disposable income rather than national

income. This paper adopts the *Cebula* (1976) specification of the consumption and investment functions, and finds that the Holmes-Smyth assertion is not valid under this amended framework.

Résumé

Une remarque sur le multiplicateur fiscal

La conclusion de *Holmes* et *Smyth* (1972) qui a fortement attiré l'attention dans la littérature, indique qu'une politique de réduction fiscale peut faire diminuer l'output national si la fonction de demande monétaire est déterminée par le revenu disponible plutôt que par le revenu national. Ce travail adopte la description de *Cebula* (1976) des fonctions de consommation et d'investissement et montre que l'assertion d'Holmes-Smyth n'est pas valide dans ce contexte modifié.