

# The Simultaneous Determination of Spot and Forward Exchange Rates: An Asset Market Approach

By Jay H. Levin, Detroit/Michigan\*

The objective of this paper is to integrate the Modern Theory of Forward Exchange with the newly developed asset market approach to exchange rate determination. Such a synthesis produces a system that simultaneously determines the spot and forward rates on a currency in a floating exchange rate environment. It then becomes possible to analyze the short-run (or “impact”) effect on exchange rates of various disturbances that are exogenous to the system, including government intervention in the exchange markets. The limiting cases of speculative and arbitrage risk neutrality also can be considered in this context.

The organization of the paper is as follows. The structural model, in which the forward exchange market is integrated with a small country’s asset markets under conditions of imperfect substitutability between domestic and foreign securities, is developed in Section I. Section II isolates the effects of an exogenous change in exchange rate expectations on the spot and forward exchange rates. An exogenous change in the uncovered interest differential, due to a domestic monetary policy action, is discussed in Section III. In Section IV the exchange rate effects of government intervention are explored, including sterilized and non-sterilized official purchases in the spot market and government positions in the forward market. The major results of the paper are summarized in Section V.

## I. Equilibrium in the Asset and Forward Exchange Markets

The starting point of the model is the well-known Modern Theory of Forward Exchange, according to which equilibrium in the forward exchange market involves the matching of covered interest arbitrage and forward speculative positions,<sup>1</sup> or

---

\* Professor of Economics, Wayne State University. I wish to acknowledge the helpful comments of *Alan V. Deardorff* and *Robert M. Stern* but remain fully responsible for all remaining errors.

<sup>1</sup> The classic exposition is *Tsiang* [24]. See also *Grubel* [16], *McCallum* [20], and *Stoll* [23]. In *Tsiang*’s model, in contrast with *Stoll* and others, commercial traders are

$$(1) \quad A (\tau_1 - \tau_2 - p, \bar{W}_1, \bar{W}_2) = S \left( \frac{R_F^+ - R_S^*}{R_S^*} \right),$$

where  $A$  = foreign holdings of domestic securities ( $D_2$ ) minus domestic holdings of foreign securities ( $F_1$ );  $S$  = forward speculative position (+ for short position);  $r_1$  = domestic short-term interest rate;  $r_2$  = foreign short-term interest rate;  $p$  = forward premium on the foreign currency

$\left( = \frac{R_F - R_S}{R_S}, \text{ where } R_F \text{ denotes the forward rate and } R_S \text{ the spot rate on the foreign currency} \right)$ ;

$R_S^*$  = expected future spot rate on the foreign currency;  $W_1$  = domestic financial wealth; and  $W_2$  = foreign financial wealth. Parameter signs are shown above the arguments. The arbitrage position in forward exchange arises from net foreign holdings of domestic securities, which are covered in order to avoid exchange risk; and, because of exchange control risk (also known as political risk, as discussed in *Aliber* [3] and *Dooley and Isard* [6]) or default risk, this position depends on the covered interest differential between domestic and foreign short-term securities,  $r_1 - r_2 - p$ . Since net foreign asset positions are wealth constrained, domestic and foreign wealth are explicitly introduced into the arbitrage function for completeness. On the other hand, the speculative position in forward exchange depends on the percentage difference between the forward exchange rate and the uncertain expected future spot rate, since this difference measures the expected percentage profit on forward speculative positions.<sup>2</sup> The arbitrage and specula-

---

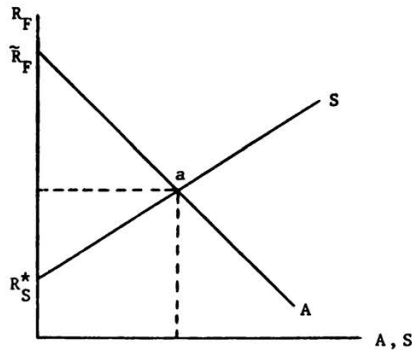
treated as separate transactors. However, it can be argued that, in the very short run, various well-known trade lags prevent movements in the forward rate from affecting commercial traders' positions in the forward market, so that their inclusion would contribute nothing to the analysis. Alternatively, some writers have suggested that commercial traders can be treated as either arbitrageurs or speculators. In the absence of any consensus here, I have assumed the simplest possible situation, namely that payments on all trade contracts are discharged through the spot market at the time of order. Thus, commercial traders do not participate in the forward market. See *Argy* [4, p. 217] for an interesting justification of this approach. Finally, for an alternative approach to the forward market, based on a portfolio choice model, see *Adler and Dumas* [2].

<sup>2</sup> In other words, speculators must be paid an exchange risk premium to undertake positions in the forward market. The determinants of this premium are discussed in *Dooley and Isard* [7, pp. 7 - 11]. See also *Eaton and Turnovsky* [11, p. 556]. *Frankel* [14] derives conditions under which the exchange risk premium disappears, despite the presence of exchange rate uncertainty and risk-aversion. In that event, the speculative schedule becomes perfectly elastic at  $R_S^*$ , and the situation becomes identical to the case of speculative risk neutrality discussed later in the text.

tive functions both imply that transactors in the forward market are risk averse.

Equilibrium in the forward market is shown in Fig. 1. At the interest parity forward rate,  $\tilde{R}_F$ , the arbitrage position is zero since domestic and foreign assets have the same covered yields.<sup>3</sup> As  $R_F$  declines below  $\tilde{R}_F$ , the covered yield on domestic securities held by foreigners,  $r_1 - p$ , rises above  $r_2$ , and foreign asset holders add domestic securities to their portfolios. Similarly, if the forward rate is equal to the expected future spot rate, the speculative position is zero because speculators anticipate no profit on such a position. As  $R_F$  rises above  $R_S^*$ , the expected profit on a short position becomes positive, and speculators assume such a position. Thus, the short-run equilibrium in the forward market occurs at point *a* at a forward rate that lies between  $\tilde{R}_F$  and  $R_S^*$ .

Figure 1  
*Equilibrium in the Forward Market*



The model now can be completed by considering the domestic asset markets.<sup>4</sup> Since the domestic private sector in the model can hold either domestic base money, domestic securities, or foreign securities, and since the interest rate on the latter is taken to be exogenously determined, it is only necessary to specify equilibrium conditions for the holdings of base money and domestic securities. With respect to the former, the equilibrium condition is simply given by

<sup>3</sup>  $\tilde{R}_F$  is the forward rate that produces a zero covered interest differential between domestic and foreign securities. Setting  $r_1 - r_2 - p$  to zero and solving for  $R_F$  yields  $R_S(1 + r_1 - r_2)$  for  $\tilde{R}_F$ .

<sup>4</sup> For a general treatment of asset markets in the open economy, but excluding forward market considerations, see *Branson and Henderson* [5].

$$(2) \quad Q = L(\bar{r}_1)$$

where  $Q$  denotes the supply of base money and  $L$  the demand for base money. Money is assumed to be held solely for transactions purposes by domestic residents, and furthermore the domestic security is taken to be the only other feasible asset that can be held as a transactions balance, possibly because of higher transactions costs involved in switching into and out of foreign securities, as well as capital controls risk on the latter. Consequently, the demand for base money is inversely related only to the domestic interest rate.<sup>5</sup>

Finally, the equilibrium condition for the domestic securities market is

$$(3) \quad U = D_1(\bar{r}_1, r_2 + p, \bar{W}_1) + D_2(r_1 - \bar{r}_2 - \bar{r}_2 - p, \bar{W}_2)$$

where  $U$  denotes the outstanding stock of domestic securities held by the public;  $D_1$  the stock demand for domestic securities by domestic residents; and, as previously noted,  $D_2$  the stock demand for domestic securities by foreign residents. It is easy to show from the domestic wealth constraint that  $D_1$  depends separately on  $r_1$  and  $r_2 + p$ , with an increase in  $r_1$  having a greater effect on  $D_1$  than an equal reduction in  $r_2 + p$ .<sup>6</sup> In contrast  $D_2$  depends on the covered interest differential since foreign asset holders consider domestic and foreign securities as the relevant alternatives for their non-transactions balances. Of course, foreign residents could also hold domestic securities on an uncovered basis. However, it is well known from *Tsiang's* early work [24] that an uncovered asset holding is equivalent to a covered asset holding and a simultaneous forward speculative position. Since desired forward speculative positions are already included in equation (1), net foreign asset holdings can simply be treated as covered interest arbitrage positions.

<sup>5</sup> Income, here omitted, not wealth, is the relevant scale variable since money is held only for transactions purposes. Income also is a relevant, but omitted, scale variable in the domestic demand function for domestic securities.

<sup>6</sup> The wealth constraint on domestic residents is given by

$$W_1 = L(\bar{r}_1) + D_1(\bar{r}_1, r_2 + p, \bar{W}_1) + F_1(r_1 - \bar{r}_2 - p, \bar{W}_1)$$

where, as previously noted,  $F_1$  denotes the domestic stock demand for foreign securities, which depends on the covered interest differential for the same reason that  $D_2$  does. The wealth constraint implies that  $D_1 W_1 + F_1 W_1 = 1$ ;  $L_{r_1} + D_{1r_1} + F_{1r_1} = 0$ ; and  $D_{1(r_2 + p)} + F_{1(r_2 + p)} = 0$ . Clearly,  $D_1$  depends separately on  $r_1$  and  $r_2 + p$ . An increase in  $r_1$  has a greater effect on  $D_1$  than does an equal reduction in  $r_2 + p$  because the former will cause domestic residents to reduce their holdings of both foreign securities and domestic money; whereas the latter will cause a switch only out of foreign securities, equal to the shift out of foreign securities produced by an equal rise in  $r_1$ .

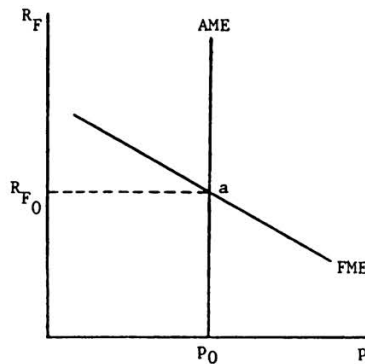
Equations (1) - (3) comprise a model of the forward market and asset markets under conditions of imperfect substitutability between domestic and foreign securities and exchange rate flexibility.<sup>7</sup> The model determines momentary values of  $R_F$ ,  $r_1$ , and  $R_S$ , given the exogenous variables  $r_2$ ,  $W_1$ ,  $W_2$ ,  $R_S^*$ ,  $Q$ , and  $U$ , and therefore constitutes an asset market model of spot and forward exchange rate determination. Furthermore, the model can be simplified for subsequent analysis by replacing equation (3) with the sum of equations (2) and (3), and noting that  $L + D_1$  coincides with  $W_1 - F_1$  from the domestic wealth constraint (as elaborated in footnote 4), to obtain

$$(4) \quad Q + U = W_1 + A(r_1 - \bar{r}_2 - p, \bar{W}_1, \bar{W}_2) .$$

This equation indicates that asset market equilibrium involves a specific level of net asset holdings,  $A$ , which must be consistent with the exogenous levels of  $Q$ ,  $U$ , and  $W_1$ . Furthermore, since the domestic interest rate,  $r_1$ , is determined in the money market in equation (2), equation (4) yields the value for the forward premium,  $p$ , that clears the domestic securities market. Given this level of  $p$ , equation (1) then determines the value of the forward rate that clears the forward market. These solutions for  $p$  and  $R_F$ , of course, imply the equilibrium value of the spot rate.

The general equilibrium of the system is depicted in Fig. 2. The AME (Asset Market Equilibrium) schedule shows the level of the forward pre-

Figure 2  
General Equilibrium in the Asset and Forward Markets



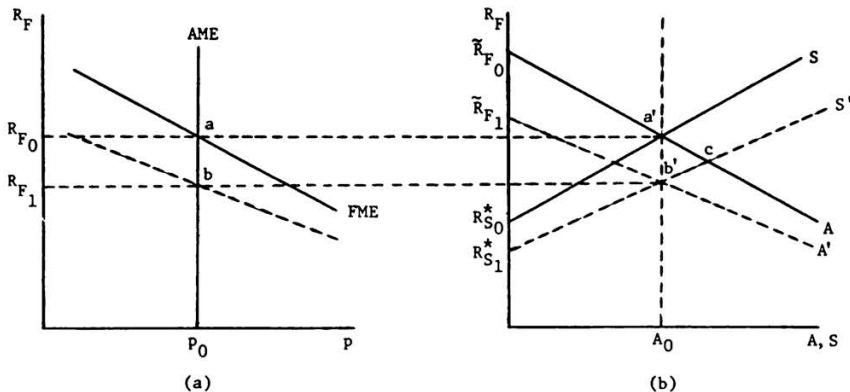
<sup>7</sup> The model is in the spirit of one developed by *Dornbusch* [8, pp. 118 - 121], in which the forward market is integrated with the asset markets under a fixed spot exchange rate system. See also *Kouri* [19], who develops a flexible exchange rate model with imperfect asset substitutability, but in which the forward market is not explicitly considered.

mium on the foreign currency from equation (4) that clears the domestic securities market, given the money market clearing domestic interest rate and the exogenous values of  $Q, U, r_2, W_1,$  and  $W_2$ . The FME (Forward Market Equilibrium) schedule shows, according to equation (1), that an increase in the forward premium, which reduces the arbitrage demand for forward exchange, requires a decline in the forward rate so as to reduce the forward speculative position and maintain equilibrium in the forward market. The general equilibrium of the system occurs at point  $a$ , with the short-run equilibrium premium  $p_0$  and forward rate  $R_{F_0}$ , respectively.

### II. Effects of a Change in Exchange Rate Expectations

According to the asset market approach, new information (“news”) that causes asset holders to revise their view of the long-run equilibrium exchange rate will be reflected in the current spot rate, through the impact on near term exchange rate expectations,  $R_s^*$ . To see if this result is obtained in the present model, and to see also how the forward rate will be affected by, say, a drop in  $R_s^*$ , consider Fig. 3, which shows the general equilibrium of the system along with the forward market. Notice first in panel (a) that the premium will remain unchanged since, as equation (4) indicates, exchange rate expectations do not directly affect desired net covered asset holdings. However, the FME schedule shifts downward in proportion to the change in  $R_s^*$  because, from equation (1), a drop in  $R_s^*$  requires, at a given premium, a proportionate drop in  $R_F$  to clear the forward market. It follows from the unchanged premium that the new equilibrium also involves a pro-

Figure 3  
Effects of a Decline in  $R_s^*$



portionate drop in the spot rate. Thus, new information which changes  $R_S^*$  will be reflected in proportionate changes in both spot and forward exchange rates.<sup>8</sup>

This conclusion can be interpreted in terms of the Modern Theory of Forward Exchange by considering panel (b). Here the drop in  $R_S^*$  causes the speculative schedule to shift proportionately downward. The resulting decline in the forward rate at  $c$  momentarily increases the yield differential between domestic and foreign securities, and covered interest arbitrageurs, attempt to shift funds into the home country. This action, however, simply causes the spot rate to decline sufficiently until the original covered yield differential is restored. In other words, the decline in  $R_S$  causes the interest parity forward rate,  $\tilde{R}_F$ , to decline and the arbitrage schedule to shift downward until a new equilibrium is reached at  $b'$ . Here the covered interest arbitrage position is restored to its original level, as required by equation (4), the condition for asset market equilibrium.<sup>9</sup>

Notice that the conclusion that changes in  $R_S^*$  produce proportionate changes in  $R_F$  does not require the assumption of speculative risk neutrality, under which the forward speculative schedule would be perfectly elastic and  $R_F$  and  $R_S^*$  would coincide. Similarly, if arbitrageurs were risk neutral, so that the arbitrage schedule was perfectly elastic and the interest parity theory prevailed (i. e.,  $p = r_1 - r_2$ ), a drop in  $R_S^*$  would necessarily leave the forward premium unchanged, and  $R_F$  and  $R_S$  would decline proportionately. But arbitrage risk neutrality is not required for this result. In general, then, both the spot and forward rates will fully reflect a change in exchange rate expectations, regardless of the degree of risk aversion of speculators and arbitrageurs; and the forward premium will be insensitive to these expectations.

### III. Effects of a Change in the Interest Differential

Now consider an open market operation that reduces the domestic monetary base and raises the domestic interest rate, with exchange rate expecta-

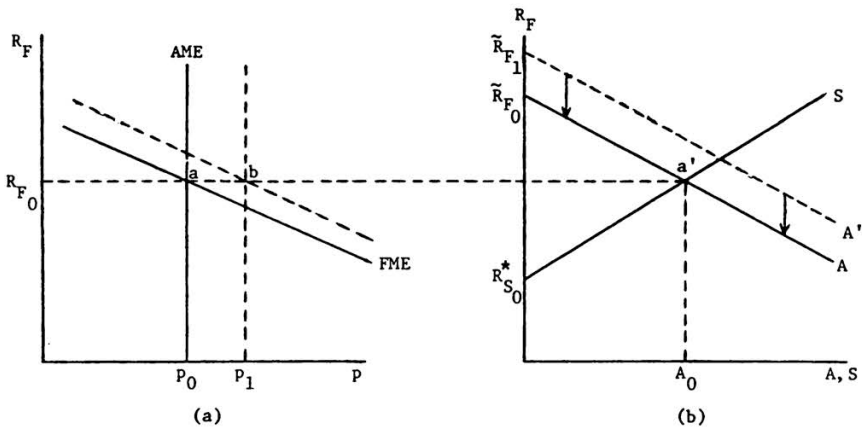
---

<sup>8</sup> This suggests that spot and forward exchange rates will have a tendency to move together, an empirical regularity that has been confirmed by *Mussa* [21, pp. 14 - 17]. See also *Flood* [13, pp. 230 - 231].

<sup>9</sup> This disturbance demonstrates that the arbitrage and speculative schedules are not independent of each other under floating exchange rates. In contrast, *Adler* and *Dumas* [1, p. 958] criticize the Modern Theory for independence of the arbitrage and speculative schedules, due to the exogeneity of interest rates. However, their observation only applies to an adjustable peg or fixed exchange rate system.

tions held constant.<sup>10</sup> (Alternatively, one can consider an exogenous reduction in the foreign interest rate.) Of course, the open market operation has equal but opposite effects on the monetary base and the outstanding stock of domestic securities, leaving  $Q + U$  unchanged in equation (4), nor does it affect the level of domestic wealth. However, the increase in  $r_1 - r_2$  requires a corresponding increase in the forward premium in order to maintain asset market equilibrium; and the AME schedule therefore shifts to the right in panel (a) of Fig. 4. In addition, according to equation (1), the increase in  $r_1 - r_2$  requires, at a given forward rate, an offsetting increase in the forward rate to maintain equilibrium in the forward market. Thus, the FME schedule shifts to the right to the same extent as the AME schedule, and the new equilibrium occurs at  $b$  with an unchanged forward rate.<sup>11</sup> It follows from the increase in the forward premium that the full impact of the change in the interest differential is felt as a reduction in the spot rate.

Figure 4  
Effects of an Increase in  $r_1 - r_2$



In terms of the Modern Theory of Forward Exchange, the increase in the domestic interest rate momentarily raises the covered yield differential between domestic and foreign securities. Hence, covered interest arbitrageurs

<sup>10</sup> Of course, the change in the monetary base might directly alter exchange rate expectations, as in the *Dornbusch* model [9]. But our interest here is in isolating the interest rate effects from the exchange rate expectations effects. For another model of the forward market with exogenous exchange rate expectations, see *Kenen* [18, p. 657]. *Kenen* also considers the possibility of regressive expectations.

<sup>11</sup> Under a pegged exchange rate system, in contrast, it is well known that the forward rate would rise because of the resulting increase in arbitrage demand.



attempt to shift funds into the home country, as indicated by the momentary rightward shift of the arbitrage schedule in panel (b). But this action depresses the spot rate sufficiently to restore the original yield differential and maintain asset market equilibrium at the original covered interest arbitrage position. Thus, the decline in the spot rate causes the interest parity forward rate to return to its original level, the arbitrage schedule shifts back to its original position, and the forward rate consequently remains unchanged.

Once again, the effects on the spot and forward rates and the forward premium are independent of the degree of risk aversion of speculators and arbitrageurs. In the case of speculative risk neutrality, one would predict an unchanged forward rate as long as exchange rate expectations remained constant; and in the case of arbitrage risk neutrality, the interest rate change would produce an offsetting movement in the forward premium from the interest parity condition. Such findings clearly apply for any degree of arbitrage and speculative risk aversion.

#### **IV. Effects of Government Intervention in the Foreign Exchange Market**

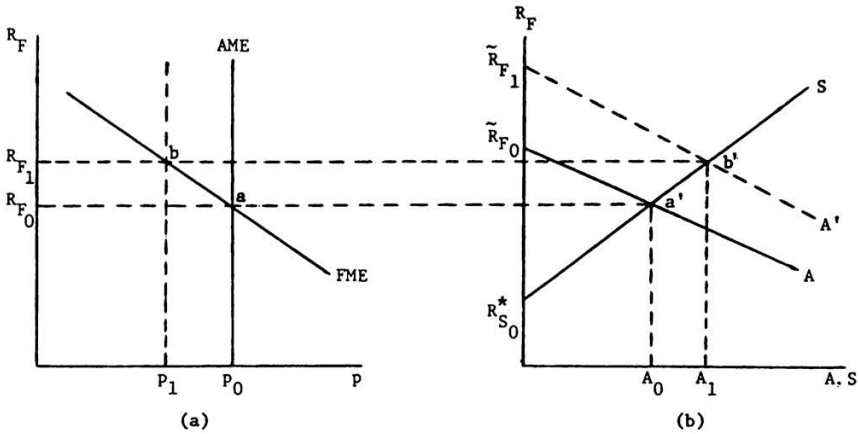
In this section we consider the effects of certain government operations in the spot and forward exchange markets that do not involve pegging of rates at specific levels. Rather, exchange rates are permitted to respond freely to such government intervention, and our interest is to determine whether and to what extent the intervention alters exchange rates, again holding exchange rate expectations constant.

Consider first a central bank purchase of foreign exchange on the spot market, the domestic monetary effects of which are sterilized by an open market sale of domestic securities. Such sterilized intervention is represented in equation (4) by an increase in  $U$ , the outstanding stock of domestic securities, while the monetary base and the level of domestic wealth remain unchanged.<sup>12</sup> Accordingly, asset market equilibrium requires an increase in net foreign holdings of domestic securities, and the forward premium consequently must decline, as shown in panel (a) of Fig. 5. In turn the forward rate must rise to keep the forward market in equilibrium; and it follows from the decline in the forward premium that the spot rate will rise proportionately more than the forward rate.

---

<sup>12</sup> The increase in  $U$  does not imply an increase in domestic wealth. To the extent that domestic residents are induced to shift from foreign securities into the newly available domestic securities by a new structure of asset yields, only the composition of their wealth changes. In essence, the central bank is trading domestic securities to the private sector in exchange for foreign securities.

Figure 5  
Effects of a Sterilized Central Bank Spot Purchase of Foreign Exchange



The easiest way to interpret these results is to hypothesize that the central bank purchase of foreign exchange initially drives up the spot rate, producing an incentive for arbitrageurs to increase their net holdings of domestic securities. This means that in panel (b) the interest parity forward rate rises, and the arbitrage schedule shifts to the right. The increased demand for forward exchange thus pushes up the forward rate, although the latter increases proportionately less than the spot rate since the forward premium must fall to induce larger holdings of domestic securities. Otherwise, the increased stock of domestic securities furnished by the central bank would not be absorbed. Of course, in the long run the spot rate would presumably return to its original steady state equilibrium level. But in the short run, as analyzed here, sterilized intervention can have an exchange rate impact.<sup>13</sup>

<sup>13</sup> A comparison between a sterilized purchase of foreign exchange and an open market purchase of domestic securities, in terms of their effects on the spot rate, at a given level of exchange rate expectations, can be easily accomplished by differentiating system (1), (2), and (4). The result for the sterilized purchase of foreign exchange is

$$dR_S = \left( \frac{A_r + S_S}{A_r S_S} \right) dU ,$$

where  $A_r$  denotes  $\partial A / \partial (\tau_1 - \tau_2 - p)$ ;  $S_S$  denotes  $\partial S / \partial \left( \frac{R_F + R_S^*}{R_S^*} \right)$ ; and  $R_F, R_S$

and  $R_S^*$  are initially normalized at unity. For the open market operation, holding  $Q + U$  constant, one obtains

Moreover, these exchange rate effects might be magnified if the intervention separately affected near term exchange rate expectations, which are treated here as exogenous.

In the limiting case of a perfectly elastic arbitrage schedule, the interest parity condition indicates that the forward premium would not change at all; and the spot and forward rates therefore would rise in the same proportion.<sup>14</sup> On the other hand, if the speculative schedule were perfectly elastic, the forward rate would remain unchanged, but the required decline in the forward premium would still necessitate an increase in the spot rate.<sup>15</sup> The extreme situation occurs where both arbitrage and speculative schedules are perfectly elastic, so that neither the spot nor forward rate changes in response to the intervention. The increased stock of domestic securities is absorbed without any exchange rate movements at all.<sup>16</sup>

The case of non-sterilized intervention in the spot market can now be analyzed. A central bank purchase of foreign exchange now has the effect in equation (4) of increasing the monetary base without changing the outstanding stock of domestic securities or the level of domestic wealth. Once again,

$$dR_s = \left( -\frac{1}{L_{r_1}} \right) dQ .$$

So sterilized intervention has a greater effect on the spot rate than an equal open market operation if and only  $|L_{r_1}|$  exceeds

$$A_r \left( \frac{S_s}{A_r + S_s} \right) .$$

<sup>14</sup> From footnote 13, the spot rate would rise by only  $\left( \frac{1}{S_s} \right) dU$  when  $A_r \rightarrow \infty$ .

The reason is as follows. The forward rate would rise to  $R_{F_1}$ , regardless of the arbitrage elasticity, since the increase in  $A$  required for asset market equilibrium necessitates a given rise in the forward rate to clear the forward market. On the other hand, as the arbitrage elasticity increases, the required change in the premium declines, and therefore so does the change in the spot rate.

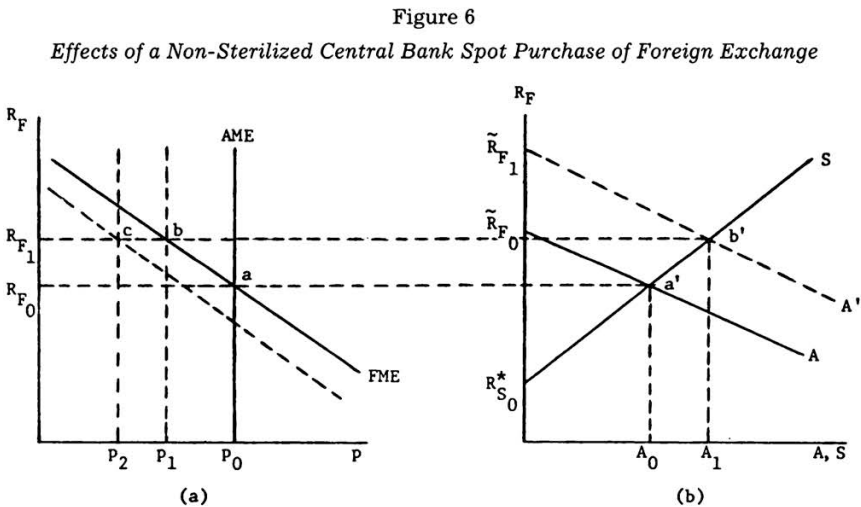
<sup>15</sup> However, from footnote 13, the spot rate would rise by only  $\left( \frac{1}{A_r} \right) dU$ . The

reason is as follows. As the speculative elasticity increases, the forward rate rises by a smaller amount to clear the forward market. However, the drop in the forward premium required for asset market equilibrium does not depend on the speculative elasticity. Consequently, the spot rate also rises by a smaller amount as the speculative elasticity increases.

<sup>16</sup> A similar conclusion with respect to the spot rate has been reached by several authors. See, for example, Kenen [17, p. 67].

a corresponding increase in net foreign holdings of domestic securities is required for asset market equilibrium.<sup>17</sup> But in contrast to the case of sterilized intervention, it is obvious from equation (2) that the domestic interest rate must fall. Consequently, as shown in panel (a) of Fig. 6, a given increase in net foreign holdings of domestic securities requires an even larger decline in the forward premium (to  $p_2$ ) than in the case of sterilized intervention (to  $p_1$ ). In addition, the decline in the domestic interest rate causes the FME schedule to shift to the left because, for any given forward rate, an offsetting reduction in the forward premium is necessary to maintain forward market equilibrium. Such a reduction, however, would simply keep the level of net foreign asset holdings unchanged, whereas asset market equilibrium requires an increase in these holdings. Therefore, the AME schedule shifts by more than the FME schedule, and the new equilibrium again involves an increase in the forward rate. Also, the decline in the forward premium again implies an even greater proportionate rise in the spot rate.

In terms of panel (b), one can think of two separate impacts on exchange rates. First, as in the case of sterilized intervention, the central bank pur-



<sup>17</sup> One might wonder how such an increase is possible when the outstanding stock of domestic securities has not changed. But domestic residents are liquidating domestic securities to increase their money holdings as domestic interest rates decline; and foreign residents will have an incentive to purchase these domestic securities because, as shown below, their covered yield advantage over foreign securities will rise. For the same reason, domestic residents will also want to reduce their holdings of foreign securities.

chase of foreign exchange drives up the spot rate, causing the arbitrage schedule to shift to  $A'$ . In fact, for equal amounts of sterilized and non-sterilized intervention, the increase in the arbitrage position (to  $A_1$ ) must be the same because of the asset market equilibrium requirement. Thus, the forward rate must rise by the same amount in both cases. Second, the decline in the domestic interest rate has only a transitory effect on the arbitrage schedule, as discussed in Section III; but since it also puts upward pressure on the spot rate, the latter rises more than in the case of a corresponding sterilized purchase of foreign exchange. This conclusion is consistent with the finding in panel (a), that the premium falls by more than in the case of sterilized intervention. To summarize, non-sterilized intervention has the same effect on the forward rate as does an equal amount of sterilized intervention. But it has a greater effect on the spot rate than does sterilized intervention because of its impact on domestic interest rates.

One can also compare non-sterilized intervention and an open market operation that have identical effects on the monetary base and hence domestic interest rates. From the analysis in Section III, we can infer that an open market purchase will raise the spot rate but leave the forward rate unchanged. In contrast, a non-sterilized spot purchase has an additional impact in raising both the spot and forward rates because of the government's purchase of spot exchange. In other words, non-sterilized intervention is simply a combination of an open market operation and sterilized intervention.

In the limiting case of a perfectly elastic arbitrage schedule, the interest parity condition indicates that the forward premium will still drop in response to a non-sterilized purchase of foreign exchange because of the decline in the domestic interest rate; and since asset market equilibrium still requires an increase in net foreign holdings of domestic securities, the forward rate must rise as before to maintain forward market equilibrium. So the decline in the forward premium again implies an even greater rise in the spot rate.<sup>18</sup> In the case of a perfectly elastic speculative schedule, the forward rate would remain unchanged, but the required decline in the forward premium would still imply an increase in the spot rate.<sup>19</sup> Finally, if both

<sup>18</sup> Perfectly elastic arbitrage, however, reduces the rise in the spot rate. From system (1), (2), and (4), holding  $U$  constant, one obtains

$$dR_s = \left( \frac{1}{S_s} + \frac{1}{A_r} - \frac{1}{L_{r_1}} \right) dQ ,$$

where  $S_s$  and  $A_r$  are defined in footnote 13.  $dR_s$  obviously declines as  $A_r$  increases. The reason is the same as that given in footnote 14 for the case of sterilized intervention.

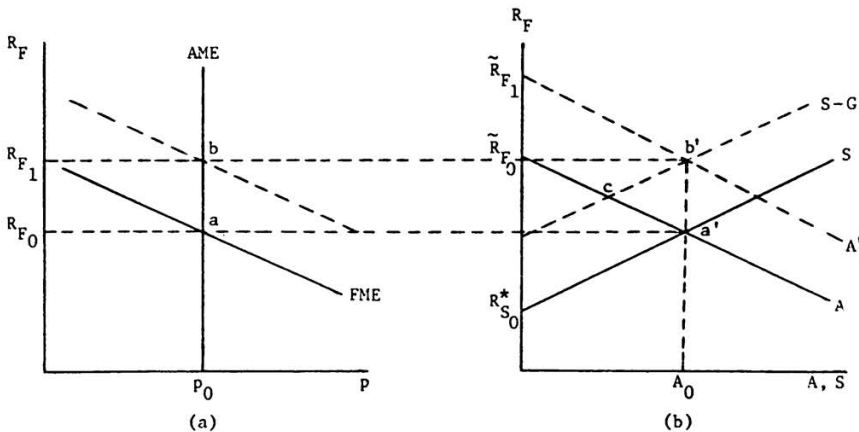
arbitrage and speculation are perfectly elastic, the forward rate again remains unchanged, but from the interest parity condition, the decline in the domestic interest rate still requires a drop in the forward premium and hence a rise in the spot rate. Thus, non-sterilized intervention always alters the spot rate, regardless of the degree of risk aversion of arbitrageurs and speculators.

Finally, instead of intervening in the spot market to influence exchange rates, the government could take a position in the forward market. The forward market equilibrium condition then becomes

$$(1') \quad A(r_1 - r_2 - p, W_1, W_2) = S \left( \frac{R_F - R_S^*}{R_S^*} \right) - G$$

where  $G$  denotes the central bank's forward position (+ if long, - if short). The effects of an official long position in this market, at a given level of exchange rate expectations, are shown in Fig. 7. In panel (a), the FME schedule shifts upward because, for a given forward premium, the forward rate must rise to keep the forward market in equilibrium. Since the premium in fact remains fixed as a requirement for asset market equilibrium, the spot rate rises in the same proportion.

Figure 7  
Effects of a Central Bank Long Position in the Forward Market



<sup>19</sup> From the solution in the previous footnote, it is obvious that perfectly elastic speculation cushions the rise in the spot rate. The reason is the same as that given in footnote 15 for the case of sterilized intervention.

In panel (b), the central bank's forward market position can be thought of as first putting upward pressure on the forward rate, as at point *c*. As arbitrageurs then attempt to shift funds abroad, the spot rate increases until the original covered yield differential is restored. The new equilibrium occurs at *b'*, where the arbitrage position returns to its original level, as required for asset market equilibrium; and the spot and forward rates have both risen in the same proportion. These conclusions also hold for the case of arbitrage risk neutrality. However, in the case of speculative risk neutrality, the forward intervention has no effect since the government's position is fully absorbed by speculators at the going forward rate.

Finally, if one compares the effects of forward intervention with an equal amount of sterilized spot intervention, both require the same changes in speculative forward positions and thus the forward rate. To see this, recall that a government forward purchase leaves the arbitrage position unchanged and thus entails an equal increase in the speculative forward position. On the other hand, the sterilized government spot purchase requires an equal increase in the arbitrage position for asset market equilibrium; but this necessitates a similar increase in the speculative forward position for forward market equilibrium. It follows that the sterilized spot purchase has a greater effect on the spot rate than does the forward intervention since the spot purchase also requires a decline in the forward premium.<sup>20</sup>

## V. Conclusions

The model developed in this paper integrates the Modern Theory of Forward Exchange with asset market equilibrium conditions for a small country, thereby determining the short-run spot and forward exchange rates for a given level of exchange rate expectations. In this setting, new information that alters the expected future spot rate produces proportionate changes in

<sup>20</sup> For an exact comparison, one may use (1') with *A* and *p* unchanged to obtain

$$dR_s = \left( \frac{1}{S_s} \right) dG .$$

In contrast, from footnote 13, the effect of sterilized intervention can be written

$$dR_s = \left( \frac{1}{S_s} + \frac{1}{A_r} \right) dU .$$

In the case of arbitrage risk neutrality, however, these operations have identical exchange rate effects, a conclusion reached earlier by *Girton* and *Henderson* [15, p. 178].

the spot and forward rates, regardless of the degree of speculative and arbitrage risk aversion. On the other hand, an increase in the uncovered interest differential in favor of the home country, due, say, to a domestic open market operation, leaves the forward rate unchanged (with unchanged exchange rate expectations) but causes the spot rate on the foreign currency to fall. The result is an exactly offsetting increase in the forward premium on the foreign currency. The last disturbance analyzed in the paper is government intervention in the spot and forward exchange markets, which causes spot and forward rates to move in the same direction as long as speculators are risk averse. However, if speculators are risk neutral, forward intervention has no effect on exchange rates; and if arbitrageurs are also risk neutral, sterilized intervention in the spot market has no exchange rate effects. Nevertheless, non-sterilized intervention in the spot market still alters the spot rate because of its impact on the domestic interest rate.

Finally, this paper has had the limited objective of analyzing the short-run (or "impact") effects of various disturbances with exchange rate expectations treated exogenously, so as to isolate their separate influences on exchange rates. An extension of the model would be to integrate it with a goods market sector, and then permit exchange rate expectations to be determined endogenously.<sup>21</sup> One could then undertake a dynamic analysis that would involve the simultaneous movement over time of spot and forward rates as well as domestic variables.<sup>22</sup> A second kind of dynamic analysis would incorporate the delayed effects of spot transactions that are undertaken by speculators when their forward contracts mature.<sup>23</sup>

## References

1. *Adler*, Michael and Bernard *Dumas*: "International Portfolio Choice and Corporation Finance," *Journal of Finance* (June, 1983). – 2. *Adler*, Michael and Bernard *Dumas*: "Portfolio Choice and the Demand for Forward Exchange," *American Economic Review* (May, 1976). – 3. *Aliber*, Robert Z.: "The Interest Rate Parity Theorem: A Reinterpretation," *Journal of Political Economy* (November/December, 1973). – 4. *Argy*, Victor: *The Postwar International Money Crisis* (George Allen and Unwin, 1981). – 5. *Branson*, William H. and Dale W. *Henderson*: "The Specification and Influence of Asset Markets," in Ronald W. Jones and Peter B. Kenen, eds., *Handbook of International Economics* (North-Holland, 1985). – 6. *Dooley*, Michael P. and Peter *Isard*: "Capital Controls, Political Risk, and Deviations from Interest-Rate Parity," *Journal of Political Economy* (April, 1980). – 7. *Dooley*, Michael P. and Peter

---

<sup>21</sup> For an example of this kind of work, see *Eaton* and *Turnovsky* [12].

<sup>22</sup> *Stein* [22] integrates a portfolio model of the forward market with a dynamic adjustment mechanism involving the basic balance.

<sup>23</sup> See, for example, *Driskill* and *McCafferty* [10].



*Isard*: "The Portfolio Balance Model of Exchange Rates," International Finance Division Paper No. 141 (Board of Governors of the Federal Reserve System, May, 1979). – 8. *Dornbusch*, Rudiger: "Capital Mobility and Portfolio Balance," in Robert Z. *Aliber*, ed., *The Political Economy of Monetary Reform* (Allanheld, Osmun & Co., 1977). – 9. *Dornbusch*, Rudiger: "Expectations and Exchange Rate Dynamics," *Journal of Political Economy* (December, 1976). – 10. *Driskill*, Robert and Stephen *McCafferty*: "Spot and Forward Rates in a Stochastic Model of the Foreign Exchange Market," *Journal of International Economics* (May, 1982). – 11. *Eaton*, Jonathan and Stephen J. *Turnovsky*: "Covered Interest Parity, Uncovered Interest Parity, and Exchange Rate Dynamics," *Economic Journal* (September, 1983). – 12. *Eaton*, Jonathan and Stephen J. *Turnovsky*: "Exchange Risk, Political Risk, and Macroeconomic Equilibrium," *American Economic Review* (March, 1983). – 13. *Flood*, Robert P.: "Explanations of Exchange Rate Volatility and Other Empirical Regularities in Some Popular Models of the Foreign Exchange Market," *Carnegie-Rochester Conference Series on Public Policy*, Volume 15 (Autumn, 1981). – 14. *Frankel*, Jeffrey A.: "The Diversifiability of Exchange Risk," *Journal of International Economics* (August, 1979). – 15. *Girton*, Lance and Dale W. *Hendeson*: "Central Bank Operations in Foreign and Domestic Assets Under Fixed and Flexible Exchange Rates," in Peter B. *Clark*, Dennis E. *Logue*, and Richard L. *Sweeney*, eds., *The Effects of Exchange Rate Adjustments* (U.S. Government Printing Office, 1977). – 16. *Grubel*, Herbert G.: *Forward Exchange, Speculation, and the International Flow of Capital* (Stanford University Press, 1966). – 17. *Kenen*, Peter B.: "Effects of Intervention and Sterilization in the Short Run and the Long Run," in Richard N. *Cooper*, et. al., eds., *The International Monetary System Under Flexible Exchange Rates* (Ballinger Publishing Company, 1982). – 18. *Kenen*, Peter B.: "Forward Rates, Interest Rates, and Expectations under Alternative Exchange Rate Regimes," *Economic Record* (September, 1985). – 19. *Kouri*, Pentti J. K.: "Monetary Policy, the Balance of Payments, and the Exchange Rate," in D. *Bigman* and T. *Taya*, eds., *The Functioning of Floating Exchange Rates: Theory, Evidence, and Policy Implications* (Ballinger Publishing Company, 1980). – 20. *McCallum*, Bennett T.: "The Role of Speculation in the Canadian Forward Exchange Market: Some Estimates Assuming Rational Expectations," *Review of Economics and Statistics* (May, 1977). – 21. *Mussa*, Michael: "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market," *Carnegie-Rochester Conference Series on Public Policy*, Volume 11 (1979). – 22. *Stein*, Jerome L.: "The Dynamics of Spot and Forward Prices in an Efficient Foreign Exchange Market," *American Economic Review* (September, 1980). – 23. *Stoll*, Hans R.: "An Empirical Study of the Forward Exchange Market Under Fixed and Flexible Exchange Rate Systems," *Canadian Journal of Economics* (February, 1968). – 24. *Tsiang*, S. C.: "The Theory of Forward Exchange and Effects of Government Intervention on the Forward Exchange Market," *I.M.F. Staff Papers* (April, 1959).

## Zusammenfassung

### Simultane Wechselkursbestimmung an Kassa- und Terminmärkten: Ein Aktivmarkt-Ansatz

Dieser Aufsatz will die moderne Theorie der Terminkurse in den Aktivmarkt-Ansatz der Wechselkursbestimmung integrieren, und zwar unter Bedingungen

unvollständiger Substituierbarkeit von inländischen durch ausländische Wertpapiere. Eine solche Synthese ermöglicht eine simultane Bestimmung der Kassa- und Terminkurse einer Währung in einem System flexibler Wechselkurse für gegebene Erwartungen über das Wechselkursniveau. Auf diese Weise können die kurzfristigen (oder „unmittelbaren“) Wirkungen verschiedener exogener Störungen, einschließlich staatlicher Interventionen an den Devisenmärkten, auf die Wechselkurse analysiert werden. Auch können in diesem Zusammenhang die Grenzfälle risikoneutraler Spekulationen und Arbitrage betrachtet werden.

Als wichtigste Ergebnisse wären zu nennen: Erstens, neue Informationen, die die Erwartungen über die zukünftigen Kassakurse ändern, führen zu proportionalen Änderungen der Kassa- und Terminkurse, unabhängig von dem Grad der Risikoaversion bei Spekulation und Arbitrage. Zweitens, eine Zunahme der ungedeckten Zinsdifferenz zugunsten des Inlands, etwa aufgrund einer heimischen Offenmarktoperation, läßt den Terminkurs unverändert (bei unveränderten Wechselkurserwartungen), aber verursacht einen Fall des Kassakurses der ausländischen Währung. Das Ergebnis ist ein exakt ausgleichender Anstieg des Swapsatzes. Schließlich führen staatliche Interventionen an den Kassa- und Terminmärkten zu gleichlaufenden Bewegungen der Kassa- und Terminkurse, solange die Spekulanten risikoscheu sind. Allerdings haben Terminmarktinterventionen keinen Einfluß auf die Wechselkurse, sofern die Spekulanten risikoscheu sind. Wenn auch die Arbitrageure risikoneutral sind, haben Interventionen am Kassamarkt keine Wechselkurswirkung. Dennoch führen nicht-sterilisierte Interventionen am Kassamarkt immer zu Änderungen des Kassakurses, und zwar wegen ihrer Wirkung auf das heimische Zinsniveau.

## Summary

### **The Simultaneous Determination of Spot and Forward Exchange Rates: An Asset Market Approach**

The objective of this paper is to integrate the modern theory of forward exchange with the asset market approach to exchange rate determination, under conditions of imperfect substitutability between domestic and foreign securities. Such a synthesis produces a system that simultaneously determines the spot and forward exchange rates on a currency in a floating exchange rate environment, for a given level of exchange rate expectations. It then becomes possible to analyze the short-run (or “impact”) effect on exchange rates of various disturbances that are exogenous to the system, including government intervention in the exchange markets. The limiting cases of speculative and arbitrage risk neutrality also can be considered in this context.

The major results of the paper are as follows. First, new information that alters the expected future spot rate produces proportionate changes in the spot and forward rates, regardless of the degree of speculative and arbitrage risk aversion. Second, an increase in the uncovered interest differential in favor of the home country, due, say, to a domestic open market operation, leaves the forward rate unchanged (with unchanged exchange rate expectations) but causes the spot rate on the foreign currency to fall. The result is an exactly offsetting increase in the forward premium on the foreign currency. Finally, government intervention in the spot or forward exchange markets causes spot and forward rates to move in the same direction as long as

speculators are risk averse. However, if speculators are risk neutral, forward intervention has no effect on exchange rates; and if arbitrageurs are also risk neutral, sterilized intervention in the spot market has no exchange rate effects. Nevertheless, non-sterilized intervention in the spot market still alters the spot rate because of its impact on the domestic interest rate.

## Résumé

### **La détermination simultanée des cours des changes au comptant et à terme: une approche du marché des valeurs**

Cet article vise à intégrer la théorie moderne des changes à terme à l'approche du marché des valeurs, pour déterminer les taux des changes, lorsque les titres nationaux et étrangers ne sont pas parfaitement substituables. Une telle synthèse produit un système qui détermine simultanément les cours des changes au comptant et à terme d'une monnaie, lorsque les taux de change sont flottants, et pour un niveau donné d'attentes de cours des changes. Il est alors possible d'analyser l'effet (ou l'«impact») à court terme sur les cours des changes de divers troubles, exogènes au système, y compris l'intervention gouvernementale sur les marchés des changes. On peut donc considérer dans ce contexte les cas limitants de neutralité de risques spéculatifs et d'arbitrage.

Les résultats principaux de cet article sont les suivants: tout d'abord, de nouvelles informations, qui modifient le cours futurs des changes au comptant prévus, modifient proportionnellement les cours des changes au comptant et à terme, sans tenir compte du degré d'aversion au risque spéculatif et d'arbitrage. En second lieu, une augmentation des disparités des niveaux d'intérêts, non couvertes, en faveur de l'intérieur, dues par exemple à une opération nationale de marché libre, ne change pas les cours à terme (avec des attentes inchangées de cours des changes), mais fait chuter les cours au comptant de monnaies étrangères. Il en résulte une augmentation exactement compensée de la prime à terme sur la monnaie étrangère.

Finalement, lorsque le gouvernement intervient sur les marchés des changes au comptant ou à terme, les cours au comptant et à terme se déplacent dans la même direction, aussi longtemps que les spéculateurs sont averses au risque. Cependant, si les spéculateurs ont une attitude neutre envers le risque, une intervention à terme n'a pas d'effet sur les cours des changes. Si les arbitrageurs sont aussi indifférents au risque, une intervention stérilisée sur le marché au comptant n'a pas d'effets sur les cours des changes. Néanmoins, une intervention non stérilisée sur le marché au comptant modifie encore les cours au comptant, à cause de son impact sur les taux d'intérêt nationaux.