General Equilibrium Models of Inflation and Interest Rates: Specification Considerations

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I. Introduction

The effect of expected inflation on interest rates continues to be a topic of substantial current interest. It is generally agreed that the seminal work on price expectations and interest rates was done almost ninety years ago by *Irving Fisher,* though his work was prompted by the 30-year post Civil War decline in prices and the attendant deflationary expectations, hence the title Appreciation and Interest. Earlier work on the same question can be found in *Douglass*.[1]

Fisher's framework is a partial equilibrium loanable funds model. Partial equilibrium models are very appealing. They are tractable and their workings generally have a strong intuitive appeal. They usually also are more amenable to econometric implementation. It, therefore, is not surprising that much of the empirical testing of the Fisherian proposition is based on his partial equilibrium framework, amended somewhat to accommodate other variables such as income and an inflation expectations adjustment mechanism. *Fisher* (1896) [(1930), Chapter XIX], *Gibson, Sargent* (1969), and *Steindl* (1980) are examples of this.

One nagging reservation about partial equilibrium models concerns the robustness of their conclusions in a general equilibrium world. General equilibrium models have the attraction of allowing for interaction among markets. As a result, partial equilibrium results are frequently modified,

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[1] A discussion of this issue by *William Douglass* [1970] anticipated *Fisher*'s work by one and a half centuries, though the AEA's publication of *Bullock*'s edition of *Douglass* was a year after its publication of Fisher's work.

even to the point of being reversed. Perhaps, the best known case of reversal is that of the effects of a minimum wage law (*Krauss* and *Johnson,* pp. 142 - 48).

Robert Mundell offered the first general equilibrium model of expected inflation and interest rates. His analysis modified *Fisher*'s results, concluding that inflation resulted in a decline in the real rate of interest, i.e., the nominal rate rose by less than the expected rate of inflation. Other general equilibrium models were subsequently presented. *Sargent*'s distributed lag IS-LM model rehabilitated Fisher's result, though the elapsed time before the nominal rate rose by the expected rate of inflation may well be considerably long. In the context of a *Patinkin* type model, *Steindl* (1973) showed that the behavior of the real rate of interest was ambiguous while *Obst* and *Rasche,* using the same framework, argued that it had to decrease. In each of the above models, inflationary expectations were exogenous.

Because of the complexity of general equilibrium models, *Walras*' Law is generally used to "eliminate" the excess demand equation in one market. No issue of principal is involved in deciding which market to "eliminate". Ironically, the usual procedure is to eliminate the bond market, the irony being that the fundamental work prompting all the attention to expected inflation and interest rates was done by *Fisher* in the context of a bond market. Yet, contemporary general equilibrium analysis of the issue almost without exception proceed on the basis of commodity and money markets.

In his Nobel Lecture, *James Tobin* (1982, p. 173) argues, as did *Patinkin* over a quarter century earlier, that "the best practice is to write down all the functions explicitly, even though one is redundant, and to put the same arguments in all the functions". It should be noted that this is the same argument as the one for use of mathematics in economics. Writing explicitly all the equations serves as a check both on the correctness of the analytical conclusions and the plausibility of the specification of the function which would have been eliminated. One example is provided in Tobin's lecture (p. 173) and another in Patinkin's analysis of the balanced budget case of an increase in income taxes to finance a higher level of government expenditures (pp. 268 - 69). The interesting specification restriction in the latter is that the demand for money must be a function of disposable income, a result which *Holmes* and *Smyth* (1972) later explore.

In this paper, the *Tobin / Patinkin* methodological predilection is used to explore specification and consistency considerations in four general equilibrium models of expected inflation and the real interest rate. Specifically, a bond market whose arguments in the demand and supply functions are price and constraint variables, and are isomorphic to those in the commodity and money markets, is formally included in each model.

I begin by analyzing the price-expectations augmented *Patinkin* model used in *Steindl* (1973). The bond market in that model is already explicitly specified and thus its arguments need not be inferred. The exercise turns out to be uneventful, but not uninteresting. The effect on the real rate of interest of a change in inflationary expectations is ambiguous. The *Obst / Rasche* model is of the same genre; it, however, concludes that the real rate falls when expected inflation increases. These are considered in the second section.

Next, I consider in each of the two subsequent sections the *Mundell* and *Sargent* (1972) models, respectively. Each uses *Walras*' Law, as does *Obst / Rasche,* to eliminate the bond market. Sargent finds that the real rate is unchanged whereas Mundell has it declining in response to an increase in inflationary expectations. Inferring a bond market, the arguments of which are the same as in the explicitly specified markets, the equilibrium vanishes. One method whereby a unique equilibrium can be (re)established is presented. This occurs when the expectation of inflation is an independent influence in each of the markets.[2] With this specification, the *Fisher* effect can prevail in the bond market. When inflationary expectations have an independent influence the real rate's behavior is ambiguous in Mundell, but rises in Sargent. When the Fisher effect holds in the bond market, the real rate rises in every model.

Macroeconomic models involving *Walras*' Law require explicit consideration of the asset equilibrium specification issue introduced by *Foley* (1975). He argues that beginning-of-period and end-of-period asset equilibrium specifications are generally inconsistent with each other. Further, Walras' Law holds only in the case of an end-of-period specification. As *Karni* (1979) shows, however, the Foley distinction between beginning- and end-of-period asset equilibrium rests crucially on the exchange restriction that for the former there be spot trading in assets only, whereas the latter involves solely futures trading in assets. When these strict transactions restrictions are removed, in either of at least two ways, Walras' Law holds in a macroeconomic model.

[2] By having an independent influence in a behavioral equation is meant that price expectations affect the endogenous variables over and above its influence through the interest rate.

II. Inflationary-expectations Augmented Patinkin Model

The explicit model underlying the geometric presentation in *Steindl* (1973) is:

(1) $$i = r + \pi$$

(2) $$Y_0 = F(Y_0, r, m, \pi),\ 0 < F_1 < 1,\ F_2 < 0,\ 0 < F_3 < 1,\ F_4 > 0$$

(3) $$B\left(Y_0, \frac{1}{i}, m, \pi\right) = 0,\ B_1 \gtreqless 0,\ B_2 < 0,\ 0 < B_3 < 1,\ B_4 < 0$$

(4) $$m = L(Y_0, i, m, \pi),\ L_1 > 0,\ L_2 < 0,\ 0 < L_3 < 1,\ L_4 < 0$$

where the endogenous variables are

i = nominal rate of interest

r = real interest rate

m = real money balances, M/p.

Since the nominal stock of money is exogenous, determining m is equivalent to determining the price level p, which is *Patinkin*'s procedure. The exogenous variables are

Y_0 = full employment real income

π = fully anticipated rate of inflation.

π is exogenous in this and all of the models. Equation (1) is the expression for continuously compounded interest when there is no taxation of interest *(Darby)*; it is also used in all of the models. The equation permits translation of results concerned with the real rate of interest into conclusions about the behaviour of the nominal rate. Henceforth, all results are derived in terms of the real interest rate.

The first three partials in (2) - (4) come directly from *Patinkin*. The inflationary expectations augmentation in the model is represented here both by (1) and by π appearing as an independent argument in each market. $F_4 > 0$ indicates the negative influence of π on saving as individuals and firms attempt to beat the inflation. The Fisherian insight that inflationary expectations induce excess supply in the bond market gives $B_4 < 0$; real demand for money is negatively related to π; hence $L_4 < 0$.

By *Walras*' Law, only two of the markets are necessary to determine the equilibrium. Use the bond and money markets. Substitute (1) into (3) and (4), differentiate totally and solve.

(5) $$\frac{dr}{d\pi} = \frac{-\left[i^2 B_3 (L_2 + L_4) + (i^2 B_4 - B_2)(1 - L_3)\right]}{i^2 B_3 L_2 - B_2 (1 - L_3)} \gtreqless 0$$

The geometry of the adjustment is shown in figure 1. The initial equilibrium is at Q_0. An increase in π shifts the respective functions by

(6) $$CC\colon\ \frac{dr}{d\pi} = \frac{-F_4}{F_2} > 0$$

(7) $$LL\colon\ \frac{dr}{d\pi} = \frac{-(L_2 + L_4)}{L_2} < -1.$$

(8) $$BB\colon\ \frac{dr}{d\pi} = \frac{[B_4 i^2 - B_2]}{B_2} \gtreqless 0$$

The case where in (8) BB does not shift is *Fisher*'s partial equilibrium model in which the increase in π decreases the demand for and increases the supply of bonds by just the amounts so that the nominal interest rate rises by π and the real rate is unchanged, ceteris paribus, i.e., for a given level of real income, prices and wealth.

As (5) makes clear, the effect on r of an increase in π is ambiguous. In the case where the *Fisher* effect holds in the bond market, the new equilibrium is at Q_1. The real rate rises to r_1. The rising price level (to p_1) reduces real balances which in turn generates excess supply over and above that created by the individual borrower and lender actions Fisher emphasized in the bond market.

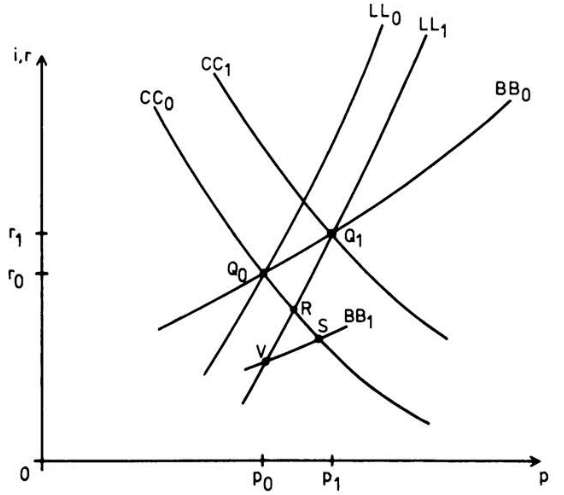

Figure 1

When inflationary expectations do not have an independent influence, changes in them operate in the model via equation (1) through the interest rate $r + \pi$ for markets written in terms of i and $i - \pi$ for markets specified in terms of r. Beginning with *Mundell*, this is the way general equilibrium models of expected inflation and interest rates are analyzed. These avenues also operate when π is an independent influence. When it has no independent influence, $F_4 = B_4 = L_4 = 0$; it is basically this price expectations augmented *Patinkin* model which *Obst / Rasche* analyze.

Their procedure is to use the commodity and money markets to determine the effect of an increase in π. They find that it declines. The geometry of their result can be seen in figure 1. The initial equilibrium is at Q_0. According to (6), CC does not shift, because $F_4 = 0$. The LL curve shifts down; in fact since $L_4 = 0$, it shifts down by exactly the increase in π. Thus, the effect is to move the equilibrium from Q_0 to R.

Had the analysis been conducted in terms of the bond and money markets, as was the analysis giving (5), the new equilibrium would not be at R. Rather, since (7) and (8) indicate that with $B_4 = L_4 = 0$ both the BB and LL curves shift down by exactly the same amount – namely the increase in π – the new equilibrium would be at V. As (5) makes clear, $dr/d\pi = -1$ when π has no independent influence. Both the price level and the nominal rate are unaffected by changes in π. If the bond and commodity markets are used, the new equilibrium would be at S. Thus with CC unchanged, BB shifting to BB_1, and LL to LL_1, three separate "equilibria" obtain, respectively at R, S, and V, which is to say that there is no new equilibrium in the *Obst / Rasche* version of a price expectations augmented *Patinkin* model, one which satisfies the *Tobin / Patinkin* methodological paradigm.

In the above analysis, augmenting the standard *Patinkin* model solely with (1) results in the equilibrium vanishing. That such a result occurs is a powerful argument in favor of the *Tobin / Patinkin* methodological paradigm. The paradigm holds that strange results – ones unintended by the model builder – may occur in its absence. Without formal consideration of the bond market, the *Obst / Rasche* result certainly appears reasonable. Yet, when the explicitly specified bond market is formally considered, the equilibrium is seen to vanish. One method whereby a new unique equilibrium can be reached involves specifying the model so that π is an independent argument in each market. This is the model given in (1) - (4), which is consistent with the *Tobin / Patinkin* methodological paradigm of having all markets depend on the same arguments; that is, in a general equilibrium world a change in a variable affecting one market in fact affects all markets.

There are, no doubt, other specifications that would similarly result in a determinate new equilibrium. One attraction of the above specification is that it corresponds directly with the *Fisher* hypothesis regarding the operation of inflationary expectations in the bond market; as such π then also appears in all the other markets.

III. The Mundell Model

The first general equilibrium model reporting theoretical results that support the notion that an increase in the expectation of inflation causes the real rate to fall was developed by *Mundell* in the context of a commodity and money market *Metzler*-type model. It differs from the models of the previous section in that it does not contain an explicit bond market. Rather, it consists of (1) and the following commodity and money market equations in the three unknowns i, r, and m,

$$I(r) = S(m) \qquad I_1 < 0, S_1 < 0 \tag{9}$$

$$m = L(i) \qquad L_1 < 0 \tag{10}$$

where

I = real investment,

S = real saving,

and the other variables are as defined earlier.

In the m, r plane, equation (9) is the *IS* curve; it is positively sloped. Equation (10) is the *LM* curve and is negatively sloped. To determine the effect of a change in inflationary expectations, substitute (1) into (10), differentiate the system totally and solve. This gives

$$\frac{dr}{d\pi} = \frac{L_1 S_1}{I_1 - L_1 S_1} < 0 \tag{11}$$

This is *Mundell*'s result, though he uses the geometric approach shown in figure 2. (Ignore for the moment the *HJ* curves.) The initial equilibrium is at Q where with $\pi = 0$, the nominal and real rates are the same. An increase in inflationary expectations of QV shifts LM down by the increase in the expected rate of inflation QV to LM_1; that is, in the LM relation, $dr/d\pi = -1$. Since *IS* drawn in terms of the real rate of interest, it does not shift in response to a change in π. The effect is to move the equilibrium to point T, with the real rate falling to r_1, and real money balances to m_1. The nominal interest rate rises to i_1 (associated with R).

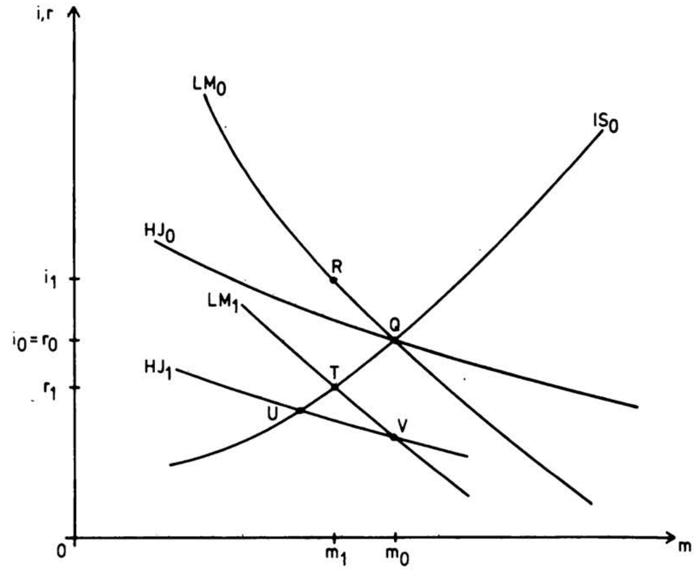

Figure 2

Consider now the question of specifying explicitly a bond market for the *Mundell* model, as is required by the *Tobin / Patinkin* paradigm. The market should have demand, supply and market clearing equations. The price of bonds, which is negatively related to the nominal interest rate, should be in both the supply and demand functions. Also, the constraint in both is real balances. Since π has no independent influence in either (9) or (10), it would not appear as a separate argument in the bond excess demand function. Inflationary expectations operate via (1) through the interest rate.

The interrelationship among the markets is taken into account by having variables which appear in the model's other behavioral equations included in the individual market demand and supply functions in a form that is standard for partial equilibrium analysis of that market. There are, therefore, prices and constraints in both the demand and supply functions. Let the bond market be specified then as

(12′) $$B^d = H\left(\frac{1}{i}, m\right), H_1 < 0, 0 < H_2 < 1$$

(12″) $$B^s = J\left(\frac{1}{i}, m\right), J_1 > 0, J_2 < 0$$

(12‴) $$B^d = B^s$$

where B^d and B^s are the demand and supply of inside consols paying \$ 1 per budget period. The signs of the partial derivatives need no explanation, with the possible exception of J_2. An increase in m decreases the supply of bonds because it allows expansion of firms' capital stock by substituting money for debt finance (*Infante* and *Stein*, p. 263; *Patinkin*, p. 217). The excess demand for the above bond market may then be written

$$B\left(\frac{1}{i}, m\right) = 0 \qquad B_1 < 0, B_2 > 0 \tag{12}$$

In figure 2, call the bond market curve *HJ*; its slope is $B_2(i^2/B_1) < 0$. From *Walras*' Law, *HJ* must lie below *LM* and above *IS* to the left of the equilibrium and similarly below *IS* and above *LM* to the right of equilibrium, as indicated in the figure.

If there are no inflationary expectations, the nominal and real rates of interest are equated at $i_0 = r_0$. According to the *Mundell* analysis, with inflationary expectations of $QV = RT = \pi_1 = i_1 - r_1$, the *LM* curve shifts by $dr/d\pi = -1$ to LM_1, intersecting IS_0 at *T*, thereby giving a nominal rate of i_1 and real rate of r_1. If, however, the analysis is conducted in terms of either of the other two markets or of the three markets jointly, the inconsistency of this system is readily apparent. For the analysis in terms of the bond and money markets, the shift of *HJ* is $dr/d\pi = -1$. The downward shift of *LM* is the same; consequently, the new "equilibrium" is at *V*.[3] The point *U* is the new "equilibrium" when the analysis is considered in terms of *HJ* and *IS*, the bond and commodity markets. In other words, *no* new equilibrium obtains.

Is the bond market given by (12) the one *Mundell* had in mind?[4] If it is, then the equilibrium vanishes. As before, one way to resolve this undesirable state of affairs is to include π as a separate argument in each of the bahavioral functions. In this manner, expectations of inflation influence behavior directly as well as by their (indirect) influence through the interest rate.

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[3] Inasmuch as $dr/d\pi = -1$ for both *HJ* and *LM*, inflationary expectations reduce the real rate by $QV = RT$, the full amount of the anticipated inflation. The nominal rate and real balances therefore are unchanged. Notice also that this specification precludes the *Fisher* effect from operating in the bond market.

[4] *Mundell* differs from *Obst / Rasche* in that their bond market comes directly from *Patinkin*. There is no uncertainty about its specification. It is well known that a specification of the bond market exists for an equilibrium at *T* for *Mundell*. This is the rationale for use of *Walras*' Law to eliminate the redundant equation. One such specification is $B(1/i, r, m) = 0$, where $\partial B/\partial r > 0$. The microeconomic rationale for such an excess demand function is difficult to see. Also, the *Fisher* effect could not hold.

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In equations (9), (10), and (12), insert π as a separate argument, the respective partials being $I_2 > 0$, $S_2 < 0$, $L_2 < 0$, $B_3 < 0$. Proceeding in the usual manner – using the commodity and money markets – the effect of a change in π is given by

(13) $$\frac{dr}{d\pi} = \frac{S_1(L_1 + L_2) + S_2 - I_2}{I_1 - S_1 L_1} \gtreqless 0.$$

For $L_2 - S_2 = I_2 = 0$, (13) reduces to (11). Graphically, the model operates as in figure 3. Anticipations of inflation shift the respective schedules by:

(14) $$IS\colon \quad \frac{dr}{d\pi} = \frac{S_2 - I_2}{I_1} > 0$$

(15) $$LM\colon \quad \frac{dr}{d\pi} = \frac{-(L_1 + L_2)}{L_1} < -1$$

(16) $$HJ\colon \quad \frac{dr}{d\pi} = \frac{[B_3(r + \pi)^2 - B_1]}{B_1} \gtreqless 0.$$ [5]

As indicated in the above shift derivatives, IS shifts up to IS_1, LM shifts down by more than the increase in π to LM_1. Assume behavior in the bond

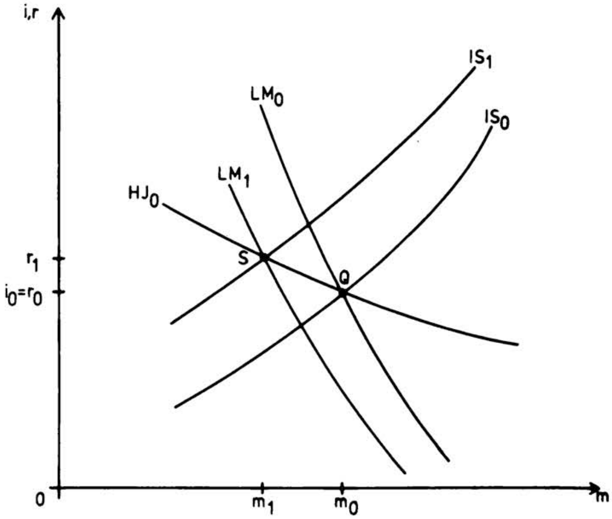

Figure 3

[5] The condition on HJ that $dr/d\pi = 0$ is the *Fisher* effect that borrowers and lenders act to drive up the nominal rate so as to leave the real rate unaffected by an increase in inflationary expectations.

market reflects the Fisherian view that inflationary expectations cause the demand for bonds to decrease and the supply of bonds to increase so that for any given level of wealth the nominal rate in the bond market rises exactly by the increase in π, that is, $dr/d\pi = 0$ in (16). In figure 3, this corresponds to the bond market curve staying at $H5_0$. As the figure makes clear, the real rate of interest must rise. The increase in inflationary expectations moves the equilibrium from Q to S. Suppliers and demanders of bonds each undertake actions in the bond market to maintain the real rate at its initial level. Their simultaneous actions in the commodity and money markets drive up the price level and thereby lower the stock of real balances. The decrease in real balances generates excess supply in the bond market ($B_2 > 0$), thereby pushing the interest rate up further to r_1.

As (13) indicates, the real rate could decline. For this to happen, however, the real excess demand for bonds must increase – $dr/d\pi < 0$ in (16) above. This is difficult to accept. Why should bondholders be willing to accept, and bondsellers not be willing to take advantage of, lower real returns when they expect greater inflation? *Fisher*'s insight may, therefore, be the most plausible – inflationary expectations do not change the real excess demand for bonds.

This section considered the *Mundell* model. The *Tobin / Patinkin* methodological predilection of (a) explicitly specifying all markets and (b) having the same variables appear in each market was followed. For a bond market whose specification is orthodox for an individual market and whose prices and constraint variables also are the same as in the explicitly specified commodity and money markets, an increase in π resulted in the equilibrium vanishing.

Respecifying all markets to include π as a separate argument in each market results in a unique determinate equilibrium, but one in which the effect of a change in π on r is ambiguous. Though other specifications are possible, one attraction of this specification is that the *Fisher* effect can hold in the bond market. That is, demanders and suppliers of bonds adjust their respective demand and supply prices, ceteris paribus, so that the real rate is unchanged and the nominal rate exceeds it by π. Should the *Fisher* effect hold in the bond market, the real rate rises in the economy.

IV. The Sargent Model

Sargent uses an income expenditures model to demonstrate that the nominal rate of interest would rise exactly by the change in the expected rate of inflation π, though full adjustment to the new equilibrium may well be slow due to the interaction of lags in each of the behavioral equations. For example, the average mean lag is 8.25 years in the eighteen models with plausible parameter values he presents.

The model is linear *IS–LM* with flexible prices. The real side consists of consumption and investment. Consumption is a function of permanent income

$$C_t = c_o + c_1 \sum_{i=0}^{\infty} c_2^i \, Y_{t-i}, \; 0 \le c_2 < 1, \; 0 < c_1 < 1 - c_2 \tag{17}$$

where C_t = real consumption in period t and Y = real income in t. Using lag operator notation, this can be written as[6]

$$C_t = c_o + \frac{c_1}{1 - c_2 L} Y_t \tag{17'}$$

Real investment in period t, I_t, is given by the flexible accelerator

$$I_t = a_o + a_1 \sum_{i=0}^{\infty} a_3^i \, \Delta Y_{t-i} + a_2 \sum_{i=0}^{\infty} a_3^i \, (i_{t-i} - \pi_{t-i}), \; a_1 > 0, \; a_2 < 0, \; 0 \le a_3 < 1 \tag{18}$$

or

$$I_t = a_o + \frac{a_1 (I - L)}{1 - a_3 L} Y_t + \frac{a_2}{1 - a_3 L} (i_t - \pi_t). \tag{18'}$$

Note that equation (1) is used in writing the real rate of interest in equations (18) and (18′). Combining equations (17′) and (18′) gives the *IS* curve

$$\frac{N(L)}{D(L)} Y_t = a_2 (i_t - \pi_t) + (1 - a_3 L)(a_o + c_o) \tag{19}$$

where $N(L) = (1 - c_2 L)(1 - a_3 L) - c_1 (1 - a_3 L) - a_1 (1 - c_2 L)(I - L)$

and $D(L) = 1 - c_2 L$

The nominal stock of money M_t^s is exogenous. Equating this with the distributed lag nominal demand for money gives the *LM* curve

$$(I - b_4 L) M_t^s = b_o + b_1 i_t + b_2 Y_t + b_3 p_t \tag{20}$$

$$b_1 \le 0, \; b_2 > 0, \; b_3 > 0.$$

where p_t is the price level in t.

[6] The lag operator L is defined by $L^n x_t = x_{t-n}$. For a discussion of lag operators, see *Sargent* (pp. 171 - 77).

A *Phillips* curve for the price adjustment equation is

$$p_t = p_{t-1} + \beta(Y_{t-1} - Y_F), \beta > 0 \tag{21}$$

where Y_F is the noninflationary full employment level of real income, comparable to Y_0 in the *Patinkin* model.

Solving equations (19), (20) and (21) for the nominal interest rate gives

$$i = \alpha + H(L)\, M_t^s + K(L)\, \pi_t \tag{22}$$

where α is a complicated expression containing Y_F which is of no concern for the present analysis and

$$H(L) = \frac{(I - b_4 L)\, N(L)\, (I - L)}{a_2\, D(L)\, [b_2 + (b_3 \beta - b_2) L] + b_1 (I - L)\, N(L)}$$

$$K(L) = \frac{a_2\, D(L)\, [b_2 + (b_3 \beta - b_2) L]}{\text{denominator of } H(L)}\,.$$

The long-run (or to use *Sargent*'s terminology, the stationary) equilibrium level of real income is Y_F. Two long-run propositions are contained in equation (22). These are, first, the nominal interest rate is independent of the money supply and, second, the nominal interest rate increases by the amount of the increase in the expected rate of inflation. These conclusions are abtained by setting $L = 1$ and evaluating $H(1)$ and $K(1)$. For $H(1)$, $\partial\, i_t / \partial\, M_t^s = 0$ and for $K(1)$, $\partial\, i_t / \partial\, \pi_t = 1$.

The operation of the model is presented in figure 4. Ignore for the moment what will be the bond market *FG* curves. With $\pi = 0$, the initial equilibrium is at Q and has $i_0 = r_0$ and $Y = Y_F$. With an increase in inflationary expectations to $QA = \pi_1 = i_1 - r_1$, the *LM* curve shifts down by the increase in the expected rate of inflation to LM_1. That is, on the *LM* curve, $dr_t / d\pi_t = -1$. The rise in income above its full employment level to Y_1 causes prices to rise. The rise in the price level increases the nominal demand for money, thereby shifting the *LM* curve back to the left. The process continues until the new equilibrium is reached, at which point the real rate is again r_0, the nominal rate rises by the increase in the expected rate of inflation to i_2 and income is back at its noninflationary full employment level Y_F.

An explicit specification of the bond market has a demand, supply and market clearing equation. In addition, in line with the *Tobin / Patinkin* methodological paradigm, since the model emphasizes lags in adjustment,

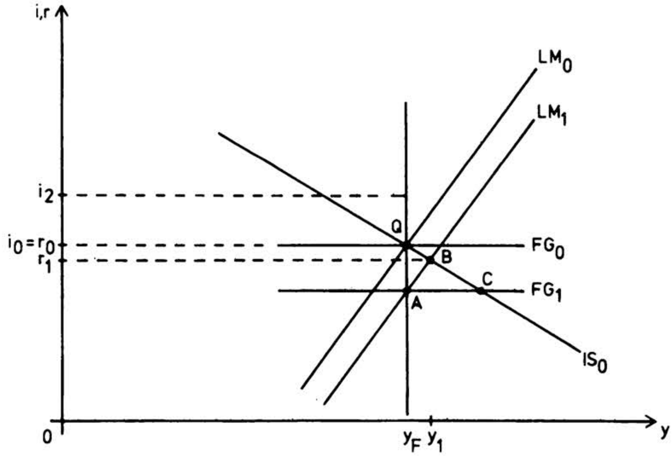

Figure 4

the demand and supply equation are distributed lag functions (modeled on the money market) of the price of bonds, real income, and the price level. Specifically, the bond market is

(23) $$(I - f_4\,L)\,B_t^d = f_0 + \frac{f_1}{i_t} + f_2\,Y_t + f_3\,p_t, f_1 < 0, f_2, > 0,\ 0 < f_3 \leq 1.$$

(24) $$(I - g_4\,L)\,B_t^s = g_o + \frac{g_1}{i_t} + g_2\,Y_t + g_3\,p_t, g_1, g_2 > 0,\ 0 < g_3 \leq 1$$

where B_t^d and B_t^s are the nominal demand and supply of inside consols paying \$ 1 interest.

Equate demand with supply to get equilibrium in the bond market. Call the curve *FG*. The slope of *FG* in the r, y plane cannot be signed.[7] The effect of a change in inflationary expectations on the bond market is to shift the bond equilibrium curve down by the increase in π, i.e., $dr_t/d\pi_t = -1$ in the bond market.

In figure 4, the initial equilibrium is again at Q. Assume that *FG* is horizontal, that is, an increase in real income increases the demand and supply of bonds by the same amount. An increase in π_t shifts both *FG* and *LM* down by the increase in expected rate of inflation. Three equilibria "exist", respectively at points A (which results when the analysis is conducted in

[7] Note that though positively, zero or negatively sloped as $[g_2/(I - g_4L] \gtreqless (f_2/(I - f_4L)]$, it must pass below the *IS* and above the *LM* curve in the region to the left of the equilibrium income (*Patinkin*, pp. 258 - 60).

terms of the money and bond markets), B (which occurs when the analysis is in terms of money and commodity markets), and C (the results when the commodity and bond markets are used).[8] To put it more correctly, no new general equilibrium position exists.

A unique equilibrium can be determined by respecifying the behavioral equations to include π_t as a separate variable in each of the behavioral equations. The simplest way to do this is to add $c_3\,\pi_t$ $(0 < c_3)$ to equation (17), $a_4\,\pi_t$ $(a_4 > 0)$ to (18), $b_5\,\pi_t$ $(b_5 < 0)$ to (20), $f_5\,\pi_t$ $(f_5 < 0)$ to (23) and $g_5\,\pi_t$ $(g_5 > 0)$ to (24). Incorporating these into the respective equations, totally differentiating and solving for the influence in each market of a change in inflationary expectations gives the shift in the respective curves

$$(25) \qquad IS\colon\ \frac{dr_t}{d\pi_t} = \frac{-(a_4 + c_3)(1 - a_3 L)}{a_2} > 0$$

$$(26) \qquad LM\colon\ \frac{dr_t}{d\pi_t} = \frac{-(b_1 + b_5)}{b_1} < -1$$

$$(27) \qquad FG\colon\ \frac{dr_t}{d\pi_t} = \frac{(I - g_4 L)(f_1 - i^2 f_5) - (I - f_4 L)(g_1 - i^2 g_5)}{g_1 (I - f_4 L) - f_1 (I - g_4 L)} \gtreqless 0.$$

In (27), as before, $dr/d\pi = 0$ reflects the *Fisher* effect. The operation of the model is shown in figure 5. Here, as in the previous figure, the initial equilibrium is at Q. The increase in π_t shifts the *IS* and *LM* curves via the above expressions to R initially.[9] With income now above Y_F, prices rise, thereby causing *FG* and *LM* to shift up until they reach FG_2 and LM_2 at which point the new stationary equilibrium is at S.

Of particular importance in this specification of *Sargent*'s model is that the real rate of interest must increase.[10] Thus, the nominal interest rate must

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[8] Notice that the problem of absence of a new equilibrium cannot be glossed over by arguing that it is a transitory phenomenon in that points B and C have income rising above Y_F; prices, therefore rise, thus increasing the demand for money as well as the supply and demand for bonds. The *LM* curve accordingly shifts to the left until it again reaches LM_0. Assuming the supply of bonds increases by more than the demand for bonds, *FG* similarly shifts up until it reaches FG_0. At point A, the equilibrium relevant to the bond and money markets, income remains at Y_F and there therefore are no forces causing either market to be in disequilibrium.

[9] In this analysis I follow *Sargent* in that the first period (short-run) equilibrium at R has r declining. This need not be. If the *Fisher* effect holds in the bond market, then the *IS* and *LM* curves would shift according to (25) and (26) along FG_0. From here, the analysis would be identical with that in the text.

[10] The increase in the real rate from the initial equilibrium is $[-(a_4 + c_3)(1 - a_3)/a_2] > 0$.

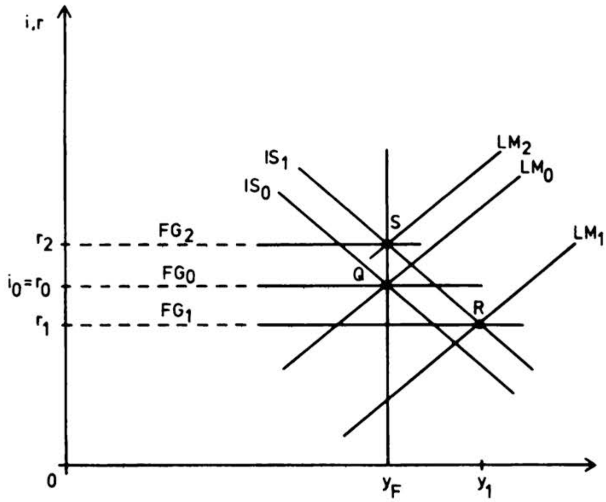

Figure 5

rise by more than the increase in π_t, contrary to *Sargent*'s finding that it increases by just the increase in inflationary expectations.

V. Concluding Remarks

This paper takes up the cudgel for the *Tobin / Patinkin* methodological paradigm in the context of general equilibrium models of expected inflation and interest rates. *Tobin* states explicitly what *Patinkin* earlier practiced, namely that in a general equilibrium model one should (a) explicitly consider all markets and (b) include in each market the same arguments as in the other markets.

The usual procedure in general equilibrium models is to invoke *Walras'* Law to eliminate a market. Ironically, at least from the perspective of the Fisherian insight stimulating the work on the price expectations effect, it is generally the bond market which is not explicitly considered. Standard practice, then, analyzes the effects of π and r by using the commodity and money markets.

The bond market that was introduced in each model is based on the standard specification for a partial equilibrium model, e.g., the demand for bonds depends negatively on the price of bonds and positively on a constraint variable such as income or wealth. In addition, the *Tobin / Patinkin* specification requirements are met. With this specification, the models were solved for $dr/d\pi$. No new equilibrium is reached when the initial (assumed) equilibrium is disturbed.

This does not imply that the original models are necessarily lacking. What it does imply is that the model in which a bond market that on the face of it is orthodox from a partial equilibrium perspective and also satisfies the *Tobin / Patinkin* specification requirements in fact behaves peculiarly. That, of course, is precisely the point of the *Tobin / Patinkin* methodological paradigm, namely that explicit consideration of all markets forces one to face and deal with a model generating strange, peculiar results.

One appealing specification for each of the models is to have π as an independent argument in each of the excess demand functions. In that way, changes in π have both a direct influence on behavior as well as an indirect influence via (1) through the interest rate. Further, this specification does not prohibit the *Fisher* effect from holding in the bond market. That is, on the basis of initially fixed income or wealth, agents can take action in the bond market to keep r unchanged. Their simultaneous actions in other markets, however, drive up the price level, thereby reducing their wealth, leading to further excess supply in the bond market.

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Zusammenfassung

Allgemeine Gleichgewichtsmodelle von Inflationsrate und Zinssätzen: Überlegungen zur Spezifikation

Bei allgemeinen Gleichgewichtsmodellen wird üblicherweise das *Walras*'sche Gesetz zugrunde gelegt, um einen Markt zu „eliminieren". Methodologisch gesehen, besteht die *Tobin-Patinkin*-Methode darin, das entsprechende allgemeine Gleichgewichtsmodell so zu gestalten, daß formal alle Märkte betrachtet werden. In diesem Aufsatz werden vier Gleichgewichtsmodelle der erwarteten Inflationsrate und der realen Zinssätze untersucht, und zwar im Hinblick auf ihre Spezifikation und Konsistenz, wobei das methodologische Verfahren von Tobin-Patinkin verwendet wird.

Es wird gezeigt, daß ein um Preiserwartungen erweitertes *Patinkin*-Modell, über das der Verfasser früher berichtet hat, konsistent ist. Eine Zunahme der Inflationserwartungen kann einen Anstieg oder ein Sinken des realen Zinses verursachen. Wenn der *Fisher*-Effekt am Bodenmarkt gilt, muß der reale Zins steigen.

In den Modellen von *Mundell, Sargent* und *Obst-Rasche* fehlt der Bondmarkt. Wendet man das methodologische Verfahren von *Tobin-Patinkin* an und führt einen Bondmarkt ein, der alle Variablen der Güter- und Geldmärkte berücksichtigt, so ergibt sich für keines dieser Modelle ein Gleichgewicht. Das heißt, die Modelle sind inkonsistent, sie sind falsch spezifiziert. Wenn jeder Markt neu spezifiziert wird, indem die Inflationserwartungen als unabhängiger Einflußfaktor einbezogen werden, ergibt sich ein eindeutiges Gleichgewicht für jedes Modell.

Für die neu spezifizierten Modelle von *Mundell* und *Obst-Rasche* zeigt sich, daß mit zunehmenden Inflationserwartungen der reale Zinssatz sowohl steigen als auch sinken kann. Sofern der *Fisher*-Effekt am Bondmarkt gilt, *muß* der reale Zinssatz steigen. Im neu spezifizierten *Sargent*-Modell steigt der reale Zinssatz, wenn die Inflationserwartungen steigen.

Summary

General Equilibrium Models of Inflation and Interest Rates: Specification Considerations

The usual procedure in general equilibrium models is to invoke *Walras*'s Law to "eliminate" one market. As a methodological matter, the *Tobin / Patinkin* method is to model their respective general equilibrium frameworks by formally considering all markets. In this paper, four general equilibrium models of expected inflation and real interest rates are investigates in terms of their specification and consistency using the Tobin / Patinkin methodological procedure.

A price expectations augmented *Patinkin* model reported earlier by the author is shown to be consistent. An increase in inflationary expectations may cause the real rate to rise or fall. When the *Fisher* effect holds in the bond market, the real rate must rise.

The models of *Mundell, Sargent,* and *Obst / Rasche* do not include a bond market. Using the *Tobin / Patinkin* methodological procedure, if a bond market incorporating all of the variables in the goods and money markets is included, no equilibrium occurs in any of those models. That is, the models are inconsistent; they are misspecified. Respecifying each market to include inflationary expectations as an independent influence results in a unique equilibrium in each model.

For both the respecified *Mundell* and *Obst / Rasche* models, the real rate may rise or fall when expectations of inflation increase. If the *Fisher* effect holds in the bond market, the real rate must rise. For the respecified *Sargent* model, the real rate increases whenever inflationary expectations increase.

Résumé

Modèles d'équilibre général d'inflation et taux d'intérêt: quelques réflexions de spécification

Dans les modèles d'équilibre général, la procédure usuelle est d'invoquer la loi de *Walras* pour «éliminer» un marché. D'une manière méthodologique, la méthode de *Tobin-Patinkin* est de modeler leurs systèmes d'équilibre général respectifs en considérant formellement tous les marchés. L'auteur de cet article analyse la spécification et la constance de quatre modèles d'équilibre général d'inflation attendue et de taux d'intérêt réels, en utilisant la procédure méthodologique de Tobin-Patinkin.

L'auteur montre qu'un modèle de *Patinkin* qu'il a expliqué précédemment, augmenté d'attentes de prix, est constant. Une augmentation des attentes inflationnistes peut entraîner la montée ou la chute des taux réels. Si l'effet de *Fisher* apparaît sur le marché des obligations, le taux réel doit augmenter.

Les modèles de *Mundell, Sargent* et *Obst-Rasche* n'incluent pas de marché des obligations. En utilisant la procédure méthodologique de *Tobin-Patinkin,* si un marché des obligations incorporant toutes les variables des marchés des biens et de l'argent est inclu, il ne se produit d'équilibre dans aucun de ces modèles. Les modèles sont alors inconstants; ils sont mal spécifiés. Si l'on respécifie chaque marché pour y inclure des attentes inflationnistes comme une influence indépendante, on obtient un équilibre unique dans chaque modèle.

Pour les modèles respécifiés de *Mundell* et d'*Obst-Rasche,* les taux d'intérêt réel peuvent augmenter ou chuter si les attentes d'inflation augmentent. Si l'effet de *Fisher* apparaît sur le marché des obligations, les taux d'intérêt réels doivent augmenter. Pour le modèle respécifié de *Sargent,* le taux réel augmente lorsque les attentes inflationnistes augmentent.