A "Textbook"-Model of Inflation and Unemployment

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A simple "textbook" model of inflation and unemployment was suggested by *J. Vanderkamp*[1], *R. Dornbusch*[2] and estimated for the German economy by *H. König*[3]. The model consists of three building blocks: (1) The quantity theory, (2) the "expectations-augmented" *Phillips* curve and (3) *Okun's* Law. This model allows an analysis of the interaction of the rate of growth of money supply, the rate of growth of real income and the rate of inflation. The model is "monetarist" since the rate of monetary expansion determines (in the long run) the rate of inflation, nevertheless it is not exclusively "monetarist" because the quantity equation is linked with the real sector through the *Phillips* curve and *Okun's* Law. Despite its simplicity this model summarizes some important results of the so-called "monetarist debate."[4]

This paper is a further investigation into the details and the working of this model. It is mainly concerned with the discussion of the short-run and long-run adjustment process implicit in the model. We show that the short-run adjustment process (with no revisions of the inflationary expectations) can be represented by a system of difference-equations of the first order, the long-run process through a second order difference equations' system.

The main results, important for the understanding of the dynamics of the model are:

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[1] *J. Vanderkamp*, Inflation: A Simple Friedman Theory with a Phillips Twist, JME, 1975 (1) p. 117 - 122.

[2] *R. Dornbusch*, Inflation, Growth and Unemployment: An Expository Framework, Center for Mathematical Studies in Business and Economics, Report 7529 (1975) University of Chicago.

[3] *H. König*, Ein Monetaristisches Modell zur Erklärung von Arbeitslosigkeit und Inflation: Modellprobleme und -implikationen für die BRD, ZfN, 1978, 38 (1 - 2) p. 85 - 104. — A similar version of this model is presented in: *Dornbusch, R.*, and *Fischer, St.*: Macroeconomics, Ch. 13, McGraw Hill, 1978.

[4] *L. J. Stein* (ed.): Monetarism, North Holland Pub. Co., Amsterdam; New York, Oxford 1975. — *R. J. Gordon* (ed.): Milton Friedman's Monetary Framework. The University of Chicago Press, 1974. — *T. Mayer* (ed.): The Structure of Monetarism. W. W. Norton & Co., New York, London 1979.

(1) The short-run adjustment process is exclusively determined by the operation of *Okun*'s Law.

(2) The long-run adjustment process is determined by both the shift in the *Phillips* curve and in the *Okun* curve.

(3) It has been proven that the short-run adjustment process approaches the equilibrium position monotonically whereas the long-run process is characterized by dampened oscillations (under most plausible assumptions).

This paper differs from *J. Vanderkamp*'s graphical exposition in that it makes the formal structure of the model explicit. It differs from the presentation by *R. Dornbusch* mainly by the separate (analytical) treatment of the *Phillips* curve and the *Okun* curve in the adjustment process, whereas *Dornbusch* "merges" both curves into one equation.

I. The Model and Short-Run Analysis

The Model has three equations and three unknowns, namely x_t, π_t, and u_t.

(1) $\bar{m}_t = x_t + \pi_t$ Quantity-theory

(2) $\pi_t = \pi_t^* - b\,(u_t - u^*)$ *Phillips* curve

(3) $u_t - u_{t-1} = -\,a\,(x_t - x^*)$ *Okun*'s Law

List of Variables

Endogenous Variables

x_t rate of growth of real income

π_t rate of inflation

u_t rate of unemployment in percent

Exogenous Variables

$\bar{m}_t$ rate of growth of money supply

π_t^* anticipated rate of inflation

x^* equilibrium rate of real growth

u^* natural rate of unemployment

u_{t-1} rate of unemployment lagged one period

Equation (1) is the quantity theory equation in percentage rates of growth, the (exogenous) rate of money supply equals the sum of the rate of real growth and the rate of inflation. Equation (2) is a linearized

version of the *Phillips* curve with the natural rate hypothesis, i.e. if $u_t = u^*$, the rate of inflation equals the anticipated rate of inflation. *Okun's* Law in equation (3) says that a deviation of the rate of real growth from its trend rate leads to a change in the rate of unemployment.

Vanderkamp provides a geometric solution of the model, which is reproduced in Figure 1. The two upper quadrants show *Vanderkamp's* presentation, the lower half is added in order to show *Okun's* Law and its role in the adjustment process.

In the upper right hand quadrant, in the (π, u) space the function R_0 is a linear *Phillips* curve for given $\pi^* = 0$. The intersection of R_0 with the abscissa denotes the natural rate of unemployment u^*. In the upper left hand quadrant the rate of real growth x is on the abscissa and the rate of inflation π on the ordinate (Note that x increases along the abscissa). The lines FF_0 and FF_1 show equation (1) for two different rates of growth of the money supply $\bar{m}_0$ and $\bar{m}_1$. Each point on the FF-line represents a possible combination of x and π for a given rate of nominal expenditure. Finally the norm x^* denotes the (exogenous) equilibrium rate of growth of real income explained by the rate of growth of employment and labor productivity.

The lower right hand quadrant contains a graphical representation of *Okun's* Law. The *OK*-line shows the *Okun* curve in the initial position. It denotes the rates of unemployment which correspond to alternative rates of real growth. Since the *Okun* curve brings out a first order difference equation, it shifts (to the left in this graphical representation) when the rate of unemployment declines, due to the occurrence of an unanticipated increase in the rate of real growth (For example the distance $A''\ x^*$ in Fig. 1). The lower left hand quadrant serves only to mirror x values from the first to the fourth quadrant.

In this model we discuss the effects of an increase in the rate of growth of money supply from $\bar{m}_0$ to $\bar{m}_1$. The following analysis describes the short-run adjustments which occur in this system through an interaction of the change in money supply, the *Phillips* curve and *Okun's* Law. "Short-run" means that during this adjustment process *no* change in inflationary expectations takes place.
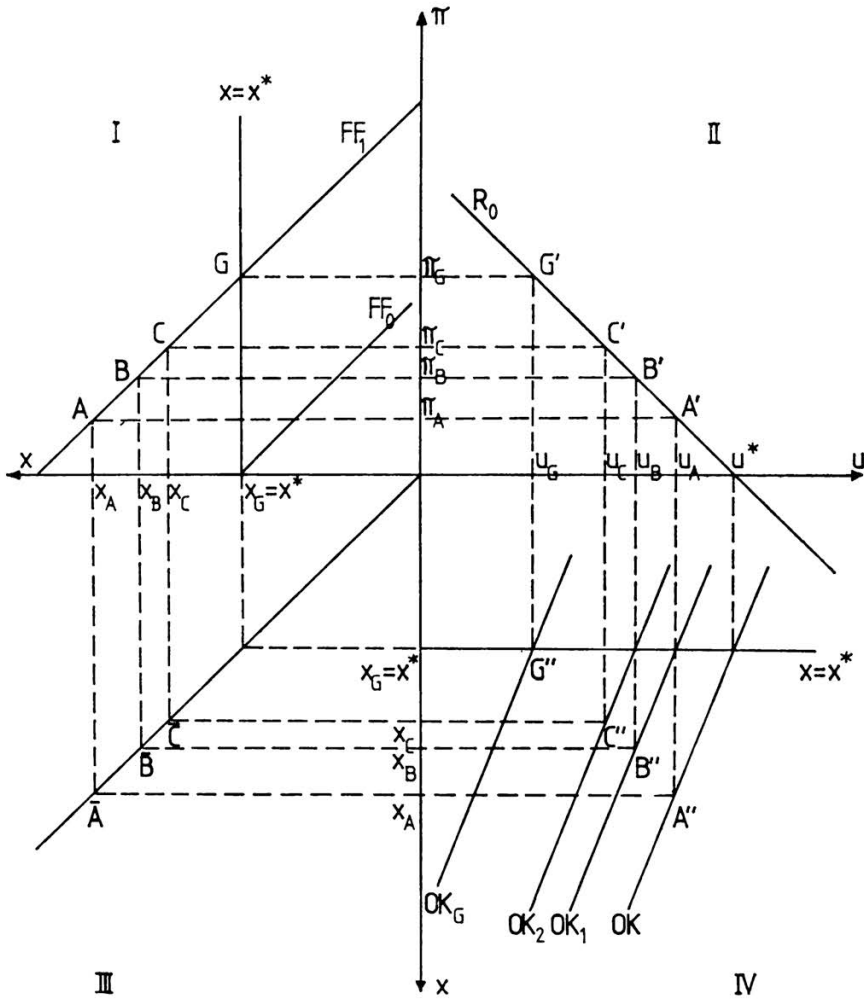
Figure 1

The starting point is the set of steady state conditions with the fol-
lowing properties:

(1) $$x_t = x^*$$

(2) $$\pi_t = \pi_t^* = 0$$

(3) $$u_t = u_{t-1} = u_t^*$$

In Fig. 1 the shift of the FF-curve to FF_1 leads to an unexpected
increase in real growth (= real expenditure) denoted by the difference

$(x_A - x^*)$ in the first quadrant. The latter is reflected onto the abscissa of the fourth quadrant and results in a point A'' along the *Okun* curve *OK* corresponding to an unemployment rate u_A. In the second quadrant above, the reduced rate of enemployment leads through the *Phillips* curve to a positive rate of inflation, namely π_A. In the first quadrant point A on the FF_1-line shows the division of the rate of growth of nominal expenditure into a real component x_A and the inflation component π_A. This concludes the "first round" effect.

A change in the rate of monetary expansion alters simultaneously the real rate of growth, the rate of unemployment, and the rate of inflation. If one examines the model in its reduced form, this becomes immediately clear.

The reduced form equations of the monetarist model are:

(4)
$$x_t = x^* + \frac{1}{1+ab}\,(\bar{m} - x^* - \pi^*) + \frac{b}{1+ab}\,(u_{t-1} - u^*)$$

(5)
$$\pi_t = \pi^* + \frac{ab}{1+ab}\,(\bar{m} - x^* - \pi^*) - \frac{b}{1+ab}\,(u_{t-1} - u^*)$$

(6)
$$u_t = u^* - \frac{a}{1+ab}\,(\bar{m} - x^* - \pi^*) + \frac{1}{1+ab}\,(u_{t-1} - u^*)$$

In the "first round" we start with $u_{t-1} = u^*$, so the last term in equations (4), (5), and (6) is zero. The monetary acceleration (i.e. the increase in the rate of money supply above the rate $x^* + \pi^*$) leads simultaneously to an increase in x_t and π_t and to a reduction in u_t. In the numerical example (Appendix I, Table 1) the coefficient a equals 0.4 and $b = 2$, so that the reduced form coefficients have the following values:

$$\frac{1}{1+ab} = 0.56 \qquad \frac{ab}{1+ab} = 0.44 \qquad \frac{a}{1+ab} = 0.22$$

A "monetary impulse" of 5 % (i.e. $\bar{m} - x^* = 5\,\%$) therefore alters the real rate of growth as well as the rate of inflation as follows:

$$\Delta\,(x - x^*) = 0.56 \cdot 5 = 2.8$$
$$\Delta\,\pi = 0.44 \cdot 5 = 2.2$$

The "impact-effect" of an increase in money supply raises the real rate of growth by 2.8 % and the rate of inflation by 2.2 %. At the same time the unemployment falls by $\Delta u = -0.22 \cdot 5 = -1.1$.

In Fig. 1 the points A on the FF_1-line, A' on the *Phillips* curve and A'' on the *Okon* curve *OK* correspond to this situation. The salient point now is that this situation is not stable. In the fourth quadrant the

11*

position of the *Okun* curve changes in the following period. The new unemployment rate is $u_{t-1} = u_A$, so that the new equation for *Okun's* Law OK_1 reads: $u_t = u_A - a(x_t - x^*)$ and thus the curve shifts to the left corresponding to new distance along the ordinate u. Further representations of the monetarist system at various times are given by the rectangles $[B, B', B'', \bar{B}]$ and $[C, C', C'', \bar{C}]$ to each of which corresponds an appropriate *Okun* curve, where the constant term of each *Okun* curve is determined by the unemployment rate of the previous period. The motion from A' to C' proceeds along the *Phillips* curve with an increasing rate of inflation and a falling rate of unemployment. The corresponding movement from A to C on the FF_1-line shows the change in the components of the nominal rate of growth in favor of the rate of inflation. No short-run equilibrium occurs as long as there exists a positive deviation $(x_t - x^*)$, which induces a shift in the *Okun* curve to the left.

The short-run adjustment process can be represented by three first order difference equations in $g_t = x_t - x^*$, π_t and u_t, derived from equations (4), (5), and (6). The solution of these difference equations are:

$$(7) \qquad g_t = \left(\frac{1}{1 + ab}\right)^t [\bar{m} - x^*]$$

$$(8) \qquad \pi_t = [\bar{m} - x^*] - \left(\frac{1}{1 + ab}\right)^t [\bar{m} - x^*] \qquad t = 1, 2, \ldots$$

$$(9) \qquad u_t = u^* - \frac{1}{b}[\bar{m} - x^*] + \left(\frac{1}{1 + ab}\right)^t \frac{1}{b}[\bar{m} - x^*]$$

The time path of $g_t = (x_t - x^*)$, π_t and u_t generated by these difference equations will converge to a short run equilibrium if $\left|\frac{1}{1 + ab}\right| < 1$. The empirical evidence suggests that $a > 0$ and $b > 0$, so we can assume that the short-run adjustment process is stable, i.e. the time path of these variables will monotonically approach to equilibrium position as $t \to \infty$.

The short-run equilibrium is then characterized by the following equations:

$$(10) \qquad g_G = 0 \to x_G = x^*$$

$$(11) \qquad \pi_G = \bar{m}_1 - x^*$$

$$(12) \qquad u_G = u^* - \frac{1}{b}[\bar{m}_1 - x^*]$$

In Fig. 2 the short-run equilibrium is represented by the rectangle $[G, G', G'', \bar{G}]$.

The rate of inflation is exactly the difference between the rate of money supply $\bar{m}$ and the steady state real rate of growth x^* [Equation (11)]. Finally the short-run equilibrium rate of unemployment u_G is below the "natural" rate of u^*, and $\bar{m}_1 - x^* = \pi_G$ is positive. If inflationary expectations are not revised the *Phillips* curve remains stable and the short-run equilibrium prevails.

Figure 2

II. The Long-Run Equilibrium

Naturally inflationary expectations do not remain constant in the long-run, but they change according to the experience of economic agents regarding inflation. Let us assume that the model of adaptive expectations adequately captures the expectations formation process:

(13) $$\pi_t^* - \pi_{t-1}^* = \Theta\,[\pi_{t-1} - \pi_{t-1}^*]\qquad 0 < \Theta < 1$$

In the long run the two propositions $\pi = \pi_G$ (short-run equilibrium) and $\pi^* = 0$ are now clearly inconsistent. Now π_t^* starts to increase and changes until the situation $\pi^* = \pi_G$ is reached. The process of expectations adjustment implies a shift in the *Phillips* curve from R_0 to R in Fig. 2. The new steady state is characterized by the following state of affairs:

(1) $x = x^*$ and (2) $\pi_G = \bar{m}_1 - x^*$ in the first quadrant of Fig. 2, (3) $\pi_G = \pi^*$ and (4) $u = u^*$, in the second quadrant and point G_2 in the lower right hand quadrant show the equilibrium position on the original *Okun* curve *OK*.

What is the difference between the long-run equilibrium $[G, G_1, G_2, \bar{G}]$ and the short-run equilibrium $[G, G', G'', \bar{G}]$? In the short-run equilibrium the position of the *Okun* curve is stable and no adjustment in the expected rate of inflation has taken place. In the long run equilibrium the *Okun* curve returns to its original position *OK*. This implies that the unemployment rate has increased again to the level of the natural rate, i.e. the real effects on the labor market have disappeared. The rate of inflation is fully anticipated and equal to the difference between the rate of monetary expansion and the rate of real growth.

Let us now consider the long-run adjustment process. In contrast to the short-run situation where the *Phillips* curve is stable, the long-run adjustment is characterized by a shift in both the *Phillips* curve (as a result of the revision of inflationary expectations) and the *Okun* curve. Although a thorough graphical analysis is not possible, it is easy to get a feel for how the long-run adjustment process behaves considering the below developed diagram. The model is now "augmented" by the adaptive expectations equation, and consists of the following equations:

(14) $$\bar{m} = \pi_t + x_t$$

(15) $$\pi_t = \pi_t^* - b\,(u_t - u^*)$$

(16) $$\pi_t^* = \Theta \pi_{t-1} + (1 - \Theta)\, \pi_{t-1}^*$$

(17) $$u_t = u_{t-1} - a\,(x_t - x^*)$$

Let us translate this system into its reduced form:

(18) $$g_t = \frac{1}{1 + ab}\,[(\bar{m} - x^*) - (\Theta\,\pi_{t-1} + (1 - \Theta)\,\pi_{t-1}^*)] + \frac{b}{1 + ab}\,(u_{t-1} - u^*)$$

(19) $$\pi_t = \frac{1}{1 + ab}\,[ab\,(\bar{m} - x^*) + (\Theta\,\pi_{t-1} + (1 - \Theta)\,\pi_{t-1}^*)] - \frac{b}{1 + ab}\,(u_{t-1} - u^*)$$

(20) $$\pi_t^* = \Theta\,\pi_{t-1} + (1 - \Theta)\,\pi_{t-1}^*$$

(21) $$u_t = u^* - \frac{a}{1 + ab}\,[(\bar{m} - x^*) - (\Theta\,\pi_{t-1} + (1 - \Theta)\,\pi_{t-1}^*)] - \frac{1}{1 + ab}\,(u_{t-1} - u^*)$$

The starting point for $t = 0$ are the parameters of the short-run equilibrium:

$$\pi_0 = \pi_G, \qquad u_0 = u_G, \qquad x_0 = x^* = x_G, \qquad \text{and} \quad \pi_0^* = 0$$

In the first period (i.e. period in which expectations adjustment begins) π^* rises according to equation (16) to $\pi_1^* = \Theta\,\pi_G$. This leads through equation (19) and (21) simultaneously to an increase in π_t and u_t and to a deceleration in the real rate of growth (equation (18)). The *Phillips* curve shifts upward (from $R_0 \to R_1$) in the second quadrant of fig. 3. The position of the *Okun* curve in the first period OK_1 is given by the equation $u_t = u_G - a\,(x_t - x^*)$. The shift in the *Phillips* curve leads to an increase in the rate of unemployment from $u_0 \to u_1$ and the rate of inflation from $\pi_0 \to \pi_1$. Since the FF_1-curve in the first quadrant remains unchanged, the higher rate of inflation causes a reduction in the real rate x_1 and therefore the growth-differential $g_1 = x_1 - x^*$ becomes negative.

Since that actual rate of inflation π_1 exceeds π_1^*, the expected rate of inflation rises again ($\Delta\pi_2^* = \Theta\,[\pi_1 - \pi_1^*]$) in the second period and triggers off once more an upward shift of the *Phillips* curve from $R_1 \to R_2$.

The "second" and the "third" round are characterized by a shift in both the *Phillips* curve and the *Okun* curve; the former moves upward from $R_1 \to R_2 \to R_3$, the latter to the right ($OK_1 \to OK_2 \to OK_3$). The numerical example in Table 1 (Appendix II) and the graphical representation in fig. 4 exhibit that the fourth period differs from the former insofar as a reduction in the expected rate of inflation occurs, since the actual inflation rate π_3 falls short of the expected rate π_*. The *Phillips*

Figure 3

curve shifts downward ($R_3 \to R_4$) and the *Okun* curve moves further to the right (as a result of the increase in unemployment from u_2 to u_3).[5]

The graphical analysis may convey the impression that the *Okun* curve systematically shifts to the right in this adjustment process. The

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[5] The curves R_4 and OK_4 are not shown in fig. 3 to avoid unnecessary clutter.

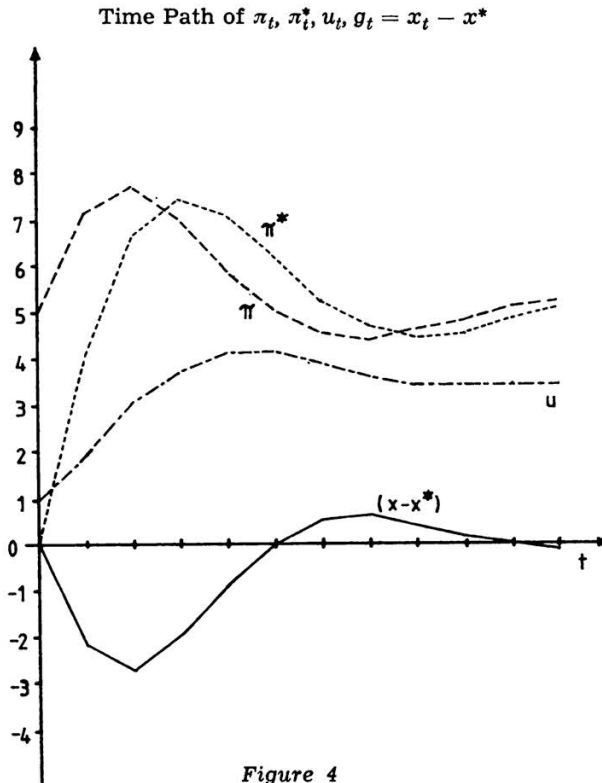Time Path of $\pi_t,\, \pi_t^*,\, u_t,\, g_t = x_t - x^*$

Figure 4

stability conditions and the numerical example show however that the long-run adjustment process is characterized by cyclical changes in the rate of unemployment. In periods of increasing unemployment the *Okun* curve shifts to the right, in periods with declining unemployment the *Okun* curve shifts to the left. In the long run the *Okun* converges to the equilibrium position *OK*, which prevailed before the short-run adjustment process was set in motion. The *Phillips* curve shifts according the anticipated rate of inflation ($\pi^* = \pi_G$) to the new equilibrium position *R* in fig. 3. The long-run equilibrium is characterized by two properties:

(1) $\qquad\qquad\qquad\qquad \pi = \pi^* = \bar{m}_1 - x^*$

(2) $\qquad\qquad\qquad\qquad u = u^*$

One might stress three qualifications, characterizing the long-run adjustment process:

1. For each "time period" the "relevant" *Phillips* curve is determined by the lagged inflationary expectation π_{t-1}^*, and the forecast error $\pi_{t-1} - \pi_{t-1}^*$. An unanticipated acceleration of inflation shifts the *Phillips* curve upward $(\pi_{t-1} - \pi_{t-1}^*) > 0$, the opposite $(\pi_{t-1} - \pi_{t-1}^*) < 0$ downward.

2. The position of the "relevant" *Okun* curve is determined by the lagged rate of unemployment u_{t-1}. An increase in u (i.e. $u_{t-1} > u_{t-2}$) shifts the *Okun* curve to the right, a decrease to the left.

3. Given the parameters of the *Phillips* curve and the *Okun* curve (both of them determined by the previous period) the real rate of growth together with the *Okun* curve determines the corresponding unemployment rate. The latter determines through the *Phillips* relationship the actual rate of inflation.

In Appendix II we show that the model (14), (15), (16), and (17) can be transformed in a system of four second order-difference equations. The solution to this model may be written as:

$$(22) \qquad \pi_t = \frac{2 + ab\,(1 - \Theta)}{1 + ab}\,\pi_{t-1} - \frac{1}{1 + ab}\,\pi_{t-2} + \frac{ab\,\Theta}{1 + ab}\,[\bar{m} - x^*]$$

$$(23) \qquad \pi_t^* = \frac{2 + ab\,(1 - \Theta)}{1 + ab}\,\pi_{t-1}^* - \frac{1}{1 + ab}\,\pi_{t-2}^* + \frac{ab\,\Theta}{1 + ab}\,[\bar{m} - x^*]$$

$$(24) \qquad g_t = \frac{2 + ab\,(1 - \Theta)}{1 + ab}\,g_{t-1} - \frac{1}{1 + ab}\,g_{t-2}$$

$$(25) \qquad u_t = \frac{2 + ab\,(1 - \Theta)}{1 + ab}\,u_{t-1} - \frac{1}{1 + ab}\,u_{t-2} + \frac{ab\,\Theta}{1 + ab}\,[u^*]$$

These difference equations will have complex roots, i.e. will generate a cyclical time path of the rate of inflation, the rate of inflationary expectations, the real rate of growth and the rate of unemployment if:

$$(26) \qquad\qquad ab\,(1 - \Theta)^2 < 4\,\Theta$$

Since Θ is the adjustment coefficient in equation (16), this condition says that the faster the adjustment of the inflationary expectations $(\Theta \to 1)$, the higher the probability of cyclical fluctuations of the system.

The condition for the stability of the cycles, i.e. for dampened oscillations is:

$$(27) \qquad\qquad \frac{1}{1 + ab} < 1 \,,$$

Time Path of π_t, π_t^*, u_t, $g_t = x_t - x^*$

which is (since both parameters are positive) clearly fulfilled.

One may, therefore, discern that the endogenous variables π_t, π_t^*, u_t and g_t approach their long-run equilibrium values with oscillations of decreasing amplitude.

The graphical representation in fig. 4 (based on the numerical example in Appendix II) illustrates the general solution. At $t = 1$ let inflationary expectations increase to 4 %. At that instant there will be a fall in the rate of real growth below x^*, an increase in the rate of unemployment and simultaneously an increase in the actual rate of inflation. Fig. 4 shows that the time path of the variables π_t, π_t^*, u_t and g_t follows the pattern of dampened oscillations. By reason of equation (1) $\bar{m} = \pi + x$ fluctuations in the rate of inflation lead to movements of equal magnitude but of opposite sign in the rate of real growth. If, therefore, the rate of inflation rises, the rate of real growth must correspondingly sink, since at a constant rate of money growth ($\bar{m} = 8$ % in the numerical example), equation $\bar{m} = \pi + (g + x^*)$ must always remain valid. By inspection of fig. 4 one may observe this relationship directly. The growth differential g_t is at first negative, changes sign repeatedly and converges to an equilibrium value of zero ($x - x^* = g = 0$).

Appendix I

A numerical example may illustrate the short-run adjustment process.

The following parameters are compiled from the above mentioned papers by *Dornbusch* and *König*:

$$\text{\textit{Phillips} curve:} \quad b = 2$$

$$\text{\textit{Okun} equation:} \quad a = 0.4$$

The starting point is characterized by:

$$\bar{m} = x^* = 3\,\% \qquad \pi = \pi^* = 0 \qquad u^* = 3.5\,\%$$

Now, introduce the monetary acceleration discussed in the text. The rate of monetary expansion $\bar{m}$ increases from 3 % to 8 %, so that a "monetary impulse" of $\bar{m} - x^* = 5$ % results. Table 1 shows the short-run adjustment of the variables ($x_t - x^*$), π_t and u_t. The model consists of the following equations:

(1) $\bar{m} - x^* = 5\,\%$

(2) $\pi_t = 0 - 2\,(u_t - 0.035)$

(3) $u_t = u_{t-1} - 0.4\,(x_t - 0.03)$

Table 1 (in %)

t	$x_t - x^*$	π_t	π_t^*	u_t
0	0.0	0.0	0.0	3.5
1	2.8	2.2	0.0	2.4
2	1.5	3.5	0.0	1.8
3	0.9	4.1	0.0	1.4
4	0.5	4.5	0.0	1.2
5	0.3	4.7	0.0	1.1
6	0.1	4.9	0.0	1.1
7	0.0	5.0	0.0	1.0

With the parameters chosen the system converges relatively quickly to a short-run equilibrium. The parameters of the short-run equilibrium are:

$$x_G = x^* = 3\,\% \qquad \bar{m} - x^* = \pi_G = 5\,\% \qquad u_G = 1\,\%$$

Appendix II.1

We continue the numerical example of Appendix I and illustrate the long-run adjustment process. The expectations parameter $\Theta = 0.8$.

The initial condition (the starting point) is the short-run equilibrium of Table 1 (Appendix I).

$$x_G = x^* = 3\,\% \;; \qquad \bar{m} - x^* = \pi_G = 5\,\% \;; \qquad u_G = 1\,\%$$

The model consists of the following equations:

(1) $\bar{m} = x_t + \pi_t$

(2) $\pi_t = \pi_t^* - 2.0\,(u_t - u^*)$

(3) $u_t - u_{t-1} = -0.4\,(x_t - x^*)$

(4) $\pi_t = 0.8\,\pi_{t-1} + 0.2\,\pi_{t-1}^*$

(5) $u^* = 0.035$

(6) $x^* = 0.03$

Table 2 shows the long-run adjustment of $g_t = x_t - x^*$, π_t, π_t^*, and u_t.

Table 2 (in %)

t	$x_t - x^*$	π_t	π_t^*	u_t
0	0.0	5.0	0.0	1.0
1	− 2.2	7.2	4.0	1.9
2	− 2.7	7.7	6.6	3.0
3	− 2.0	7.0	7.4	3.7
4	− 0.9	5.9	7.1	4.1
5	0.0	5.0	6.1	4.1
6	0.5	4.5	5.2	3.9
7	0.6	4.4	4.6	3.6
8	0.4	4.6	4.4	3.4
9	0.2	4.8	4.5	3.4
10	0.0	5.0	4.8	3.4
11	− 0.1	5.1	5.0	3.4
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
16	0.0	5.0	5.0	3.5

Appendix II.2

Consider the "expectations-augmented" model.

(1) $$\bar{m} = x_t + \pi_t$$

(2) $$\pi_t = \pi_t^* - b\,(u_t - u^*)$$

(3) $$u_t = u_{t-1} - a g_t$$

(4) $$\pi_t^* = \Theta\,\pi_{t-1} + (1 - \Theta)\,\pi_{t-1}^*$$

(5) $$g_t = x_t - x^*$$

Lag equation (4) and subtract the lagged equation from (4)

$$\pi_t^* = \Theta\,\pi_{t-1} + (1 - \Theta)\,\pi_{t-1}^*$$
$$\pi_{t-1}^* = \Theta\,\pi_{t-2} + (1 - \Theta)\,\pi_{t-2}^*$$

(6) $$\overline{\pi_t^* - \pi_{t-1}^* = \Theta\,(\pi_{t-1} - \pi_{t-2}) + (1 - \Theta)\,(\pi_{t-1}^* - \pi_{t-2}^*)}$$

Lag the *Phillips* curve and subtract from (2)

$$\pi_t = \pi_t^* - b(u_t - u^*)$$

$$\pi_{t-1} = \pi_{t-1}^* - b(u_{t-1} - u^*)$$

(7)

$$\pi_t - \pi_{t-1} = (\pi_t^* - \pi_{t-1}^*) + abg_t$$

Apply a *Koyck*-Transformation to equation (7), i.e. lag equation (7), multiply the new equation by $(1 - \Theta)$ and subtract from (7). This gives:

(8)
$$\pi_t - 2\pi_{t-1} + \pi_{t-2} = abg_t - ab(1 - \Theta)g_{t-1}$$

Transform equation (1) to $\bar{m} - x^* = g_t + \pi_t$ and rearrange, to get $g_t = \bar{m} - x^* - \pi_t$ and $g_{t-1} = \bar{m} - x^* - \pi_{t-1}$ substitute this in (8) and rearrange:

(9)
$$\pi_t = \frac{2 + ab(1 - \Theta)}{1 + ab}\pi_{t-1} - \frac{1}{1 + ab}\pi_{t-2} + \frac{ab\,\Theta}{1 + ab}[\bar{m} - x^*]$$

By analogy one can show that the solution of π_t^*, g_t and u_t is:

(10)
$$\pi_t^* = \frac{2 + ab(1 - \Theta)}{1 + ab}\pi_{t-1}^* - \frac{1}{1 + ab}\pi_{t-2}^* + \frac{ab\,\Theta}{1 + ab}[\bar{m} - x^*]$$

(11)
$$g_t = \frac{2 + ab(1 - \Theta)}{1 + ab}g_{t-1} - \frac{1}{1 + ab}g_{t-2}$$

(12)
$$u_t = \frac{2 + ab(1 - \Theta)}{1 + ab}u_{t-1} - \frac{1}{1 + ab}u_{t-2} + \frac{ab\,\Theta}{1 + ab}u^*$$

Zusammenfassung

Ein „Textbuch" zu Inflation und Arbeitslosigkeit

Dieser Beitrag untersucht die dynamische Struktur, die dem von *J. Vanderkamp* und *R. Dornbusch* vorgeschlagenen einfachen makroökonomischen Modell zugrundeliegt. Bekanntlich ermöglicht dieses Modell eine Analyse einer Änderung der Wechselwirkung zwischen der Geldmengenrate, der Rate des realen Wachstums und der Inflationsrate.

Ein „monetärer Impuls" beeinflußt sowohl die Rate des realen Wachstums als auch die Inflationsrate. Es wird aufgezeigt, daß der Anpassungsvorgang auf kurze Sicht (d. h. bevor die Inflationserwartungen revidiert werden) ausschließlich durch einen Anstieg in der *Okun-Kurve* erklärt wird, was zu einem kurzfristigen Gleichgewicht bei verringerter Arbeitslosenzahl und höherer Inflationsrate führt.

Auf lange Sicht passen sich Inflationserwartungen entsprechend dem Prozeß der adaptiven Erwartungen an.

Der langfristige Anpassungsvorgang wird durch ein Ansteigen sowohl der *Phillips*- als auch der *Okun*-Kurve gekennzeichnet. Dieser Vorgang kann durch ein System von Differenz-Gleichungen zweiter Ordnung nachvollzogen werden. In der Arbeit wird nachgewiesen, daß dieses System zwar zyklische Bewegungen hervorruft, daß jedoch unter den sehr plausiblen ökonomischen Voraussetzungen der zeitlichen Entwicklung der Inflationsrate, der Rate der zu erwartenden Inflation und der Arbeitslosenzahl auf lange Sicht es sich einem Gleichgewicht mit gedämpften Schwingen nähert.

Während des langfristigen Anpassungsvorganges können Stagflations-phänomene mit einer steigenden Arbeitslosenziffer und hohen Inflations-raten auftreten.

Summary

A "Textbook"-Model of Inflation and Unemployment

This paper investigates the dynamic structure underlying the simple macro-economic model suggested by *J. Vanderkamp* and *R. Dornbusch*. This model allows an analysis of the interaction of a change in the rate of money supply with the rate of real growth and the rate of inflation. A "monetary impulse" affects both the rate of real growth and the rate of inflation. It is shown that the adjustment process in the short-run (i. e. before inflationary expectations are revised) is exclusively explained by a shift in the *Okun* curve, leading to a short-run equilibrium, with a reduced rate of unemploy-ment and a higher rate of inflation.

In the long-run inflationary expectations adjust according to an adaptive expectations formation process. The long-run adjustment process is character-ized by a shift in both the *Phillips* curve and the *Okun* curve. This process can be modeled by a system of difference-equations of the second order. It has been proven in the paper that this system generates cycles, but that under most plausible economic assumptions. The time-path of the rate of inflation, the rate of "expected" inflation and the rate of unemployment approaches the long-run equilibrium position with dampened oscillations. During the long-run adjustment process stagflation phenomena with a rising rate of unemployment and high rates of inflation may occur.

Résumé

Un livret sur l'inflation et sur le chômage

Cet article étudie la structure dynamique qui est à la base du modèle macroéconomique simple proposé par *J. Vanderkamp* et *R. Dornbusch*. On n'ignore pas que ce modèle permet l'analyse d'une variation de l'interaction des taux de la masse monétaire, de la croissance réelle et de l'inflation.

Une „impulsion monétaire" influence tant le taux de l'expansion réelle que celui de l'inflation. L'on met en évidence qu'à court terme (c. à d. avant toute révision des anticipations inflationnistes) le processus d'adaptation ne s'ex-

plique que par une ascension de la Courbe *d'Okun,* ce qui conduit à un équilibre précaire comportant une réduction du chômage et une hausse du taux d'inflation.

A long terme, les expectatives inflationnistes s'ajustent au processus des anticipations adaptives.

Le processus d'adaptation à long terme se caractérise par une montée de la courbe de *Phillips* et de celle d'*Okun.* Ce processus peut se réaliser complémentairement par un système d'équations de second degré des écarts. L'étude établit que si ce système engendre des mouvements cycliques, il tend néanmoins à long terme dans les très plausibles conditions économiques de l'évolution dans le temps du taux d'inflation, du taux de l'anticipation inflationniste et du nombre de demandeurs d'emplois à réaliser un équilibre aux oscillations amorties.

Au cours du long processus d'ajustement peuvent apparaître de phénomènes de stagflation avec une croissance du chômage et une élévation des taux d'inflation.