Improvement in Economic Position through Risk-Taking

An Attempt to Map Intertemporal Risk-Consumption Frontiers

By Karl W. Roskamp, Detroit/Mich., and
Frederick E. Tank, Toledo/Ohio

I. Introduction

A person who wishes to lift himself out the group of "low-income consumers" and enter that of "middle or higher-income consumers", could do so if he should suddenly find himself in the possession of larger economic assets, possibly the result of some fortuitous event. Another possibility is that, with given low initial assets, a person achieves the upward move through a willingness to assume larger risks. In this way he may be able to compensate for the disadvantage of insufficient economic assets. He takes the chance to succeed — or to lose even those few assets he possesses.

In this paper we investigate the possibility of overcoming a lack of sufficient economic assets through an increased willingness to take risks. We propose to study this problem with the aid of a dynamic portfolio selection model. The task is to trace out frontiers of optimal intertemporal consumption and show how these are determined by the two parameters: risk-taking and initial assets.

II. The Model and its Underlying Assumptions

We assume two persons both of whom possess human and non-human assets. For the purpose of this analysis we restrict ourselves to their non-human assets, i. e., material wealth. For both of them the latter shall be the only source of income. Additionally, we assume that the first person's initial material wealth is substantially larger than that of the second.

Incomes are derived by investing in securities. Both individuals are restricted in their choice to an opportunity set of *m* securities. For each of these securities information on rates of return is available for the

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last n investment periods. If we denote the rate of return on a security as r^i_{t-j}, where the superscript i refers to the type of security and the subscript $t - j$ to the j's backward period with reference to the present period t, we can write the historically given matrix of rates of return as:

$$(1) \qquad \begin{matrix} r^1_{t-n}, & \ldots\ldots, & r^m_{t-n} \\ \vdots & & \vdots \\ r^1_{t-n}, & \ldots\ldots, & r^m_{t-1} \end{matrix}$$

Furthermore, there shall be, unknown to both investors, a set of future rates of return on the m securities, over h forward periods. It simplifies the analysis if we assume that these periods coincide with the number of periods both investors stay in the security market. We write the matrix of future rates of returns as

$$(2) \qquad \begin{matrix} r^1_t & \ldots\ldots, & r^m_t \\ \vdots & & \vdots \\ r^1_{t+h-1}, & \ldots\ldots, & r^m_{t+h-1} \end{matrix}$$

We assume now that both investors at the beginning of period t (with the actual rate of return vector $\left[r^1_t \ldots r^m_t\right]$ still unknown) desire to maximize the expected net rate of return on their security portfolios.[1] Each investor wishes to choose only effizient combinations of securities; those combinations with the highest rate of return for a given subjectively weighted variance, an allowance for the portfolio's risk. We write the expected net rate of return as:

$$(3) \qquad N_t = G'_t X_t - \Theta X'_t C_t X_t$$

In (3) G_t is a vector of expected rates of returns in the period t. We write it as

$$(4) \qquad G_t = \left(g^1_t, \ldots, g^m_t\right)$$

We assume that each particular expected rate of return on a security is determined by a simple averaging of the known past rates of return

[1] The same assumption was made by Donald Eugene *Farrar*, in his "The Investment Decision under Uncertainty", Englewood Cliffs, New Jersey, 1962, p. 27.

on that security over n past periods.[2] Thus we have for the kth security in period t:

$$g_t^k = \frac{\sum_{j=1}^{n} r_{t=j}^k}{n} \tag{5}$$

In (3) X_t is the vector of component securities, reflecting the efficient portfolio composition. In accordance with other writers on the portfolio selection problem, we assume that the weighted variance-covariance matrix $X_t' C_t X_t$ of past rates of return is a risk indicator for the investors. C_t is the variance-covariance matrix proper and it is calculated from (1). Further, the risk shall be translated into a negative rate of return with the aid of a subjective risk aversion coefficient Θ. Both investors are supposed to be risk averters; thus Θ is positive.

To find the optimal portfolio composition in the first period t, the investible wealth of the individual (which forms the asset constraint) is set equal to unity. Formally the optimization model can be written as a non-linear mathematical programming problem:

$$\max N_t = G_t' X_t - \Theta X_t' C_t X_t \tag{6}$$

subject to:

$$\sum_{i=1}^{m} X_t^i = 1 \qquad i = 1, \ldots\ldots, m \tag{7}$$

$$X_t^i \geq 0 \qquad i = 1, \ldots\ldots, m \tag{8}$$

The non-negativity constraints (8) are introduced to exclude negative securities (short-sales of securities). From (6), (7) and (8) an optimal portfolio composition vector X_t^{i*} is determined. Combining it with the *actual* vector of rates of return $\left[r_t^1, \ldots, r_t^m\right]$, we obtain the actual rate of return on the total portfolio for period t, denoted as $\bar{r}_t$. Once we obtain this rate of return, we compute in the next step the earnings of the total portfolio. It can be denoted

$$E_t = \bar{r}_t W_t \tag{9}$$

where W_t stands for the initial assets in currency units.

[2] Other possibilities would be to give weights to different past experiences: the more remote ones may count less than recent ones, or a disastrous experience may get a greater weight than a favorable one. See: Karl W. *Roskamp*, "Portfolio Selection and Expectations, Towards a Dynamic Simulation Model", in Heinz *Haller* and Horst Claus *Recktenwald* (eds.), Finanz- und Geldpolitik im Umbruch, Mainz 1969, p. 465 - 475.

The investor then substracts a certain number of currency units to cover his current consumption K, with the resultant net change in assets after the end of the first investment equal to

$$\Delta W_t = E_t - K \tag{10}$$

ΔW_t may be positive, zero, or negative. The wealth available for investment in the next period $t + 1$ is thus

$$W_{t+1} = W_t + \Delta W_t \tag{11}$$

The model described above lends itself in a recursive form to simulation of the asset accumulation path. The latter becomes a function of the risk aversion coefficient Θ. Through simulation of above model it will be shown what differences in the coefficient Θ are required for the poorer man to achieve the same assets accumulation (or alternatively: level of consumption, leaving initial asset intact) as the wealthier individual.

III. The Simulation Process

The described model was programmed for an I.B.M. 360 electronic computer and we briefly describe the simulation process.[3] To trace the asset accumulation of an investor over h forward investment periods, the risk aversion coefficient Θ, the initial assets W_t and the past rates of return are given. In a first step the variance-covariance matrix C_t is computed from (1). Next the expected rate of return vector G_t is determined from (1) and (5). The initial wealth is then set equal to unity and the optimal portfolio composition vector X_t^{i*} determined from (6), (7) und (8). From (9), (10) and (11) the new assets W_{t+1} available for investment in the following period, are calculated.

For the subsequent period $t + 1$, the process is repeated with the only difference that the most remote period in the historical rate of return matrix is dropped. This is the vector $\left[r_{t-n}^1, \ldots, r_{t-n}^m\right]$. Added to the historical matrix is however the latest actual rates of return vector $\left[r_t^1, \ldots, r_t^m\right]$.[4] In this way we obtain for each of the h investment periods

[3] In the simulation process the following program was used: Garth P. *McCormick*, W. Charles *Mylander III*, Anthony V. *Fiacco*, "Computer Program Implementing the Sequential Unconstrained Minimization Technique for Non-linear Programming", Research Analysis Corporation, Technical Paper RAC-TP-151, McLean, Virginia, April 1965.

[4] The chosen procedure keeps the for the formation of expectations relevant historical time period constant. It is assumed that more recent experiences count fully and very remote ones not at all.

vectors of optimal portfolio composition and, simultaneously, an asset accumulation path.

For both individuals, we then find a level of consumption, K, such that assets at the end of h investment periods are equal to initial assets. At the end of the final investment period each investor must have the same wealth he had when he selected his initial portfolio. Therefore, each investor must reckon with an intertemporal asset constraint: while asset accumulation or deaccumulation is possible in each investment period, he must egress the securities market with exactly the same amount of wealth he had when he entered the market. If we impose this additional constraint we can focus our attention on three variables only: the initial asset endowment W_t, the level of consumption K, and the risk aversion coefficient Θ. Furthermore by iteratively varying the coefficient of risk aversion, we are able to equate the consumption levels for both individuals. With this the dynamic process is finally reduced to two dimensions, risk and wealth.

When all other things are equal, except initial assets we can be shown how much more risk the not so wealthy man has to assume if he wishes to lift himself out of the low-income consumption group and enjoy, over h investment periods, the same level of consumption as his wealthier neighbor.

IV. Simulation Results with Hypothetical Data

For the following simulation we assumed that there is a set of four securities to choose from throughout all periods in which investments can be made. The first security has a stable 5 % rate of return in all periods. The second security has a rate of return between 6.5 % and 9 %. The third security is still more speculative and yields between 10 % to 16 %. Finally, the fourth security is a highly speculative one with rates of return varying between 5 % and 48 %. The stipulated available security set thus provides considerable opportunity for risk-taking. The hypothetical data pertaining to these securities is given in Appendix Table.

To start the simulation process the initial wealth and a risk aversion coefficient Θ is chosen. With the requirement that terminal wealth must be equal to initial wealth, we determine the consumption K which the investor can enjoy in all periods. In the process there will be changes in the portfolio composition and the earnings rate on the total portfolio.

Also, the risk associated with each portfolio (indicated by the variance of earnings) will vary from one time period to another. There may be in each period, an asset accumulation or deaccumulation but not at the end of the last period. Below we show in Table 1 and 2, data on such a process. These data were generated with the inputs given in Appendix Table 1 and with an initial wealth of 10,000 currency units.

Table 1

Intertemporally Efficient Portfolio Proportions, Earnings Rates, and Variances

Investment Period	Portfolio Composition (%)				Expected Earnings Rate	Variance*)
	X^1	X^2	X^3	X^4		
			Θ Equals 500 Consumption Equals 1114.55 Currency Units			
$t+1$	00.02	35.51	48.36	16.11	14.31	32.83
$t+2$	00.02	54.97	30.78	14.23	13.11	28.92
$t+3$	00.03	61.95	35.82	2.19	10.34	22.42
$t+4$	00.03	56.85	41.32	1.80	10.46	21.07
$t+5$	00.03	66.49	29.94	3.54	10.19	19.68
$t+6$	00.03	66.30	30.20	3.47	10.19	19.72
$t+7$	00.02	67.62	29.43	2.93	10.14	16.36
$t+8$	00.02	60.10	37.75	2.13	10.61	18.98
$t+9$	00.03	65.61	31.52	2.84	10.26	18.92
$t+10$	00.92	68.89	26.24	3.95	10.00	24.66
			Θ Equals 10000 Consumption Equals 543.91 Currency Units			
$t+1$	89.16	5.98	3.82	1.03	5.76	3.81
$t+2$	90.13	6.58	2.42	0.87	5.63	3.17
$t+3$	93.50	4.31	2.10	0.09	5.32	1.61
$t+4$	92.72	4.65	2.54	0.09	5.37	1.82
$t+5$	91.83	6.41	1.50	0.25	5.36	1.81
$t+6$	92.11	6.02	1.64	0.23	5.36	1.79
$t+7$	89.74	8.14	1.87	0.26	5.45	2.25
$t+8$	91.27	5.99	2.59	0.14	5.44	2.19
$t+9$	91.99	5.82	2.01	0.18	5.38	1.91
$t+10$	94.94	3.53	1.32	0.20	5.25	1.27

*) The variance figures in Tables 1 and 2 are the actual variances times one thousand.

In Table 1 the X^i values refer to the portfolio composition, in percent. In can be seen that with a low risk aversion coefficient of 500, the optimal portfolio in each period contains only a fractional percentage of the very low yield security. It contains however, the most risky and speculative security. With this risk aversion coefficient the overall portfolio earnings rates are between 14.3 % and 10 %. The investor can enjoy a consumption of 1114.55 currency units in each investment period without impairing his assets.

If the investor should become more conservative in his investments and chooses, everything else the same, a risk aversion coefficient of 10,000, the bulk of his portfolio consists of the very low-risk, low-yield securities. The portion of the high risk-high yield security drastically dwindles. The earnings rates are now between 5.8 % and 5.3 % and the consumption drops to a mere 543.91 currency units in each period. The risk ascribable to this portfolio has also decreased as a comparison of the variance data in Table 1 reveals.

Similar Tables were constructed for other values of the risk aversion coefficient but to conserve space, only two statistics Θ and K are shown in Table 2.

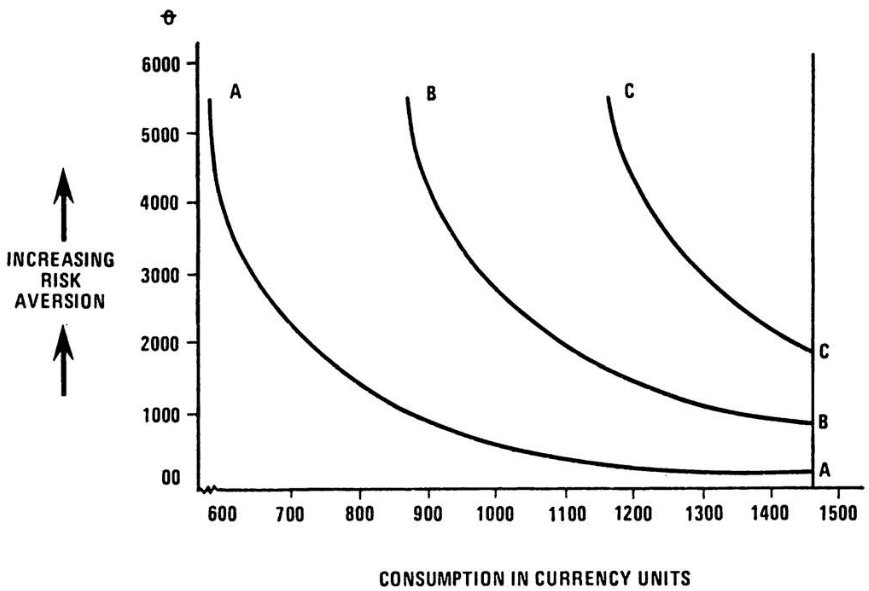
Table 2

Coefficients of Risk Aversion and Associated Levels of Consumption with Initial Wealth of 10,000 Currency Units

	Risk Aversion Coefficients	Consumption (in currency units)
Increasing Risk ↓	6500	567.67
	6000	573.32
	5500	580.02
	5000	588.07
	4500	597.90
	4000	610.22
	3500	626.08
	3000	647.30
	2500	677.08
	2000	721.95
	1500	797.30
	1000	941.34
	500	1114.55
	400	1169.19
	300	1260.83
	200	1419.06
	100	1681.68

Finally the initial asset endowment was changed for each of three beginning wealth levels, and we obtained the optimal levels of consumption for each coefficient of risk aversion. These data were mapped as risk-consumption curves in Diagram 1. The three risk-consumption curves summarize the exchange, or trade-off, possibilities between certainty and consumption for initial wealth of 10,000; 15,000 and 20,000 currency units.

DIAGRAM 1.



CURVE	INITIAL WEALTH IN CURRENCY UNITS
AA	10,000
BB	15,000
CC	20,000

Diagram 1 has on its vertical axis the risk aversion coefficient Θ. On the horizontal axis is the attainable consumption K when terminal assets are equal to initial assets. Each particular point on the curves in Diagram 1 states the optimal relationship between W, K and Θ over 10 time periods. All points on a curve have a particular initial wealth in common. An increase in initial wealth, will shift the risk-consumption curve upward and to the right. Moreover, it will be noticed that each risk-consumption curve is a downward sloping strictly convex function. This simply reflects that, with a given initial wealth, the optimal marginal-trade-off between certainty and consumption ($d\,\Theta/dK$) decreases with increasing consumption levels.

V. Results and Conclusions

A set of optmial interrelationships between initial wealth, risk aversion and consumption were computed and mapped in Diagram 1. It shows that the first person, who possesses at the beginning of the first investment period half the assets of the second person (compare Curve A with Curve C), must assume a multiple of the latter's risk, to enjoy the same intertemporal consumption. If we choose, for example, in Diagram 1 a consumption level of 1300.00 currency units for both men, the corresponding risk aversion coefficient Θ for curve $\overline{AA}$ is approximately 300. On curve $\overline{CC}$ it is however just under 3000. The 1300.00 currency unit consumption level can be attained by the wealthier person from a rather low-risk portfolio, whereas the less wealthy man is obliged to look for much more speculative investments.

The depicted relationships in Diagram 1 depend on the historical set of rates of return data and on the assumed manner of expectation formation. With another set of data and a different kind of expectation formation (i. e., error-learning process) the slopes of the traced curves and the distances between the curves would change. However, any family of curves depicting the relationship between consumption, assets and risk-taking must, in a situation in which investors are risk-averters and maximize N_t, possess the same general characteristics as those shown in Diagram 1. Each curve must be a convex function which slopes downward and to the right. Increasing initial wealth shifts them upward and to the right. Of the many possible families of curves we have mapped only one particular one.

The above curves show that an investor can substitute, quid pro quo, certainty for consumption by reducing his coefficient of risk aversion. However, the possibility to compensate for a difference in initial wealth through increased risk-taking is limited. The larger the wealth differentials the smaller the chance to catch up. Our results are obtained from a model which uses several simplifying assumptions namely: no transaction cost, transparency of the security markets, free information about investment possibilities, and an identical opportunity set of securities for both the rich and poor man. All of this works in favor of the poorer man. For him, catching up should become much more difficult if we relax some of these assumptions. If we introduce loading charges or broker' commissions, transaction cost per investment unit may be higher for the small investor than for the large one. A man with large assets and more income can afford to buy better information, investment advice, and portfolio management services. Finally, the well-to-do person may be willing to accept larger amounts of risk than the poor man can afford to take. With increasing wealth risk aversion may very well diminish. All of this would decrease the chance to overcome an initial lack in economic assets through increased risk-taking.

Appendix Table

Historical and Future Rates of Return in Percentage

	Investment Period	Security X^1	Security X^2	Security X^3	Security X^4
Historically Given Rates of Return	$t-9$	5.00	7.50	12.50	31.00
	$t-8$	5.00	8.00	14.00	28.00
	$t-7$	5.00	8.00	15.00	33.00
	$t-6$	5.00	9.00	12.00	35.00
	$t-5$	5.00	7.00	14.00	38.00
	$t-4$	5.00	7.00	13.00	30.00
	$t-3$	5.00	7.70	12.00	34.00
	$t-2$	5.00	8.50	14.00	25.00
	$t-1$	5.00	8.00	14.00	35.00
	t	5.00	8.00	12.00	28.00
New Rates of Return Inputs	$t+1$	5.00	7.70	16.00	36.00
	$t+2$	5.00	7.50	12.00	15.00
	$t+3$	5.00	8.00	14.00	10.00
	$t+4$	5.00	8.50	16.00	5.00
	$t+5$	5.00	7.00	15.00	30.00
	$t+6$	5.00	7.00	15.00	40.00
	$t+7$	5.00	8.80	12.00	45.00
	$t+8$	5.00	7.50	11.00	46.00
	$t+9$	5.00	6.50	10.00	48.00
	$t+10$	5.00	7.70	15.00	15.00

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Zusammenfassung

Verbesserung der wirtschaftlichen Position durch Risikoübernahme: Ein Versuch der Darstellung der Grenzen des Risikoaufwandes

In dem Aufsatz wird die Möglichkeit untersucht, ein anfangs zu kleines Anlagevermögen durch größere Risikobereitschaft (oder geringere Risikoabneigung) zu überspielen.

Es wird die Lage von zwei Investoren verglichen. Der eine besitzt anfänglich ein großes, der andere ein kleines Vermögen. In dem Computer-Modell wird angenommen, daß beide Anleger ihre Anlageentscheidung aufgrund des gleichen Sortiments von *m* Wertpapieren treffen können, das sowohl Wertpapiere mit geringem als auch solche mit hohem Risiko enthält. In jeder Anlageperiode werden jeweils die optimalen Portefeuilles ausgewählt. Bedingung ist, daß die Investoren nach den Anlageperioden aus dem Wertpapiermarkt mit dem Anfangs-Vermögensbestand wieder herauskommen. Da es also keinen Netto-Vermögenszuwachs gibt, haben die aus den Portefeuilles resultierenden Einkommensströme Unterschiede in den Verbrauchsniveaus zur Folge. Intertemporaler Verbrauch ist allein eine Funktion der Risikoabneigung. Es gibt demgemäß Grenzen für den intertemporalen Risikoaufwand; sie wurden in einem Simulationsprozeß errechnet und im Schaubild 1 dargestellt.

Das Ergebnis der Untersuchung lautet, daß in einer Wettbewerbsgesellschaft der Möglichkeit, Unterschiede beim Ausgangsvermögen durch eine größere Risikobereitschaft zu kompensieren (das ist ein Freiheitsspielraum für den weniger Begüterten, sein Schicksal zu verbessern), ziemlich enge Grenzen gesetzt sind. Je größer die Wohlstandsunterschiede um so geringer die Aussicht, sie auszugleichen. Das gilt um so mehr, wenn die Kosten je Investitionseinheit (Transaktions-, Informationskosten usw.) nicht wie in der Untersuchung angenommen gleich, sondern für den kleinen Investor höher sind als für den großen. Ein Gleichziehen wird letzten Endes nahezu unmöglich, wenn der vergrößerte Wohlstand gleichzeitig mit größerer Risikobereitschaft einhergeht: Der Reiche trägt mehr Risiko, weil er es sich leisten kann.

Summary

Improvement in Economic Position through Risk-Taking: An Attempt to Map Intertemporal Risk-Consumption

In this paper the possibility of overcoming an initial lack of sufficient economic assets through increased willingness to take risk (lower risk-aversion), is investigated.

Compared are the situations of two investors, one of them possessing large, the other small economic assets. Both are risk-averters. In the computer model

it is assumed that both investors can make their asset choices from the same opportunity set of m securities, which contains low as well as high risk securities. In each investment period optimal portfolios are selected. The condition is, however, that the investors have to egress after n investment periods from the security markets with assets equal to their initial ones. Because there is no net accumulation, the income streams emanating from the two portfolios result in differences in consumption levels. Inter-temporal consumption is a function of risk-aversion only. There are thus inter-temporal risk-consumption frontiers. These were computed in a simulation process and they were mapped in diagram 1.

The conclusion of this study is that the possibility to compensate in a competitive world for a difference in initial wealth through increased risk-taking — a degree of freedom the less-wealthy has to improve his lot — is limited. The larger the wealth differential, the smaller the chance to catch up. This will be all the more so if the per unit investment cost (transaction cost, cost of obtaining information, etc.) which was assumed to be the same in this study, is higher for the small than the large investor. Finally, catching up becomes will-nigh impossible if with increased wealth goes, pari passu, a decreased risk-aversion, the rich man taking more risk because he can afford it.

Résumé

Amélioration de la position économique par l'acceptation des risques: Un essai de définition des limites de la prise des risques:

L'article examine la possibilité de multiplier un patrimoine très modeste à l'origine par la prise de risques (ou par une moindre aversion des risques).

L'on compare la situation de deux investisseurs. Au départ, l'un possède un patrimoine important, l'autre un avoir modeste. Pour le programme de l'ordinateur, l'on suppose que les deux investisseurs peuvent prendre leurs décisions sur la base du même assortiment de valeurs mobilières, lequel assortiment contient des valeurs à risque faible et des titres à risque élevé. Pour chaque période d'investissement, l'on sélectionne dans les deux cas le portefeuille optimal. L'on pose comme condition qu'à l'issue des périodes de placement, les investisseurs doivent retrouver leur patrimoine initial. Puisque l'on ne suppose aucun accroissement net du patrimoine, les revenus du portefeuille impliquent des différences dans les niveaux de consommation. La consommation intertemporelle n'est rien d'autre qu'une fonction de l'aversion des risques; il existe donc des limites à l'acceptation de risques intertemporels; ces limites furent calculées par un processus de simulation et représentées dans le graphique 1.

L'étude démontre que dans une société de compétition, il existe des limites aux possibilités de compensation des écarts de patrimoines par une plus grande audace (c. à. d. une marge de manoeuvre à la disposition des moins bien lotis

en vue d'améliorer leur sort). Plus importants sont les écarts de bien-être, moindres sont les chances de les éliminer. Et c'est encore plus vrai lorsque les coûts par opération d'investissement (frais de transaction, d'information, etc. ...) ne sont pas égaux, comme cela fut supposé dans l'étude, mais qu'ils sont supérieurs pour le petit investisseur. En fin de compte, une égalisation est quasiment irréalisable lorsque se combinent patrimoine élevé et disposition à prendre des risques: le riche prend plus de risques parce qu'il peut se le permettre.