Schmollers Jahrbuch 124 (2004), 355 – 369
Duncker & Humblot, Berlin

A Rights-egalitarian Pay-as-you-go Pension System

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Abstract

This paper addresses the problem of intergenerational and intragenerational distribution in a pay-as-you-go pension system. While each generation pays the pensions of the preceding generation, they also bear the burden of raising the next. The burden of child care is unevenly distributed within a generation. Demographic change affects the distribution across generations. To resolve both distributional issues this paper proposes to apply the rights-egalitarian sharing rule. Under this rule individual claims are fully respected; all gains or losses are divided equally. It can be shown that a rights-egalitarian pension system implements full compensation for human capital investments in a long-run equilibrium.

Zusammenfassung

Dieser Beitrag behandelt das Problem der inter- und intragenerationellen Verteilung einer umlagefinanzierten Rentenversicherung. Jede Generation zahlt die Renten der vorausgehenden Generation und trägt darüber hinaus die Lasten des Aufziehens der nächsten. Kindererziehungsleistungen sind innerhalb einer Generation ungleich verteilt; der demographische Wandel beeinflusst die Verteilung zwischen den Generationen. Um beide Verteilungsprobleme zu lösen, wird vorgeschlagen, eine Verteilungsregel anzuwenden, die als *rights-egalitarian distribution* bezeichnet wird. Gemäss dieser Verteilungsregel werden individuelle Rechte (Rentenansprüche) voll respektiert; die dann verbleibenden Defizite oder Überschüsse werden gleichverteilt. Es kann gezeigt werden, dass eine umlagefinanzierte Rentenversicherung, die eine rights-egalitarian distribution implementiert, im langfristigen Gleichgewicht gerade die Humankapitalinvestitionen voll kompensiert – sie kann also als eine humankapitalgedeckte Rentenversicherung bezeichnet werden.

JEL Classification: H55, D63, C71

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* This work has benefitted from stimulating discussions with Rudi Dujmovits, Georg Hirte and Clemens Puppe. Helpful comments from three anonymous referees are gratefully acknowledged.

1. Introduction

Recent public debates about pay-as-you-go pension schemes have focused on two issues. First, the change of the age structure of the population in industrialised countries puts an increasing financial burden on the working generation if current levels of pensions are to be maintained. Hence, the debate is about intergenerational distribution. Second, the burden of child care is unevenly distributed within the age-groups. Parents contribute more than others and among parents women contribute more than men. Standard pay-as-you-go public pension schemes do not take into account how the burden of child care is shared. Pensions are fully determined by one's earlier monetary contribution which is a proportion of one's wage income. Persons engaged in child care often forego a wage income and thus they also forego a corresponding pension during retirement. Since predominantly women take responsibility for raising children, they are left with insufficient pension claims and poverty is more widespread among women than among men in their old age (Stapf, 1997).

In the course of demographic change the role of families and their investment in human capital have received increasing attention. In Germany, for example, the idea of rewarding child care in the pension system was introduced into practice in 1986. Bringing up a child was rewarded with a pension entitlement equivalent to 75 % of average monetary contributions for one year. The pension reform acts of 1992 and 1996 have gradually increased the pension entitlement to an equivalent of three years of average monetary contribution for each child (Langen, 1998).

Policy makers confronted with demographic change have adjusted pension levels and contribution rates in the pay-as-you-go pension system. However, these changes seem to be *ad hoc,* reflecting the pressure of interest groups and macroeconomic policy goals rather than individual rights. There is as yet no attempt to approach the issue of intergenerational distribution in pay-as-you-go pension systems in a systematic way. The intragenerational distribution of the standard pay-as-you-go pension system is proportional to individual *monetary* contributions. Until recently, child care contributions have been largely neglected. The aim of this paper is to suggest a clear rule which is applicable to both, intergenerational and intragenerational sharing problems. We hope to open a discussion that helps to move away from *ad hoc* adjustments of pension levels and contribution rates towards a more systematic approach.

The pay-as-you-go pension system is a redistribution mechanism. Its intergenerational distribution effects have been widely discussed in the context of pension reform towards a fully funded system on the basis of dynamic models with exogenous or endogenous fertility (see e.g. Veall, 1986; Nishimura and Zhang, 1992 and 1995; Dekkers et al., 1995; Kolmar, 1997; and Miles and

Iben, 2000). Intragenerational distribution has received less attention.[1] The approach of this paper differs in two respects from the existing literature. First, it introduces child care contributions as a factor determining pension claims into the model.[2] The individual pension claims are based on monetary contributions *and* on child care contributions. Second, the paper takes a normative approach to the distribution problem. We do *not* consider who gains and who loses under various assumptions concerning demographic change and a change of the design of the pension system, but rather we suggest a particular way of burden sharing, called the rights-egalitarian sharing rule, to determine pension entitlements and contribution rates. Under this rule individual claims are fully respected while all remaining gains or losses are divided equally. This provides a unified framework to determine the inter- and intragenerational distribution based on individual claims. With a declining population the next generation's payments are not sufficient to meet all pension claims. We will argue that the rights-egalitarian sharing rule is the only reasonable rule in this context. Herrero et al. (1999) have provided different axiomatic characterisations of rights-egalitarian sharing. The reasoning in this paper is inspired by their results.

The paper is organised as follows. Section 2 introduces the rights-egalitarian sharing rule. Section 3 discusses the motivation to use this rule. It provides three axiomatic characterisations of rights-egalitarian sharing adapted to the case of a pay-as-you-go pension system. The axioms capture desirable properties of a pension system, and rights-egalitarian sharing is shown to be the unique sharing rule satisfying the proposed axioms. Section 4 applies rights-egalitarian sharing to the pay-as-you-go pension system. The claims of all stakeholders in the pension system are defined and the pensions based on these claims are determined. Section 5 examines the dynamics of the proposed sharing rule. Section 6 concludes.

2. The rights-egalitarian sharing rule

In a standard pay-as-you-go pension system pensions are proportional to (monetary) contributions. The characteristics of proportional sharing have been analysed by Moulin (1987) who proposes several axiomatic characterisations of proportional and equal surplus sharing. Mixed rules of proportional and equal surplus sharing have been characterised by Pfingsten (1991). Chun and Thomson (1992), Bossert (1993), Marco Gil (1994) and others have examined bargaining problems with claims. Their analysis has been extended by

[1] However, Weizsäcker (1996) discusses the empirical decomposition of inequality changes into inter- and intragenerational effects.

[2] Kolmar's (1997) model also incorporates child care contributions. However the focus of his study are the demographic effects.

Herrero et al. (1999) who consider a broader class of distribution problems. Their framework is applicable to cases where losses must be shared.[3] Since this case is relevant for the analysis of the pay-as-you-go pension system with declining population we use the framework of Herrero et al. (1999). The structure of the problem is as follows. A fixed amount of money Y is to be distributed among a group N of $n \geq 2$ agents. Each agent i has a claim $c_i; c = (c_1, \ldots, c_n)$. Denote $C = \Sigma_{i \in N} c_i$.

Definition 1. *Sharing problem with claims. A sharing problem with claims is a triple $\langle N, Y, c \rangle$, where N is a finite set of agents, $|N| = n \geq 2$, $Y \in \mathbb{R}$, $c \in \mathbb{R}^n$.*

The set of all such sharing problems is denoted Ω. This explicitly allows for $C \geq Y$, $Y < 0$ and $c_i < 0$. A sharing rule determines a payoff for each agent.

Definition 2. *Sharing rule. A sharing rule is a mapping $F : \Omega \rightarrow \mathbb{R}^n$ that assigns to every sharing problem $\langle N, Y, c \rangle \in \Omega$ a payoff vector $x = (x_1, \ldots, x_n), x \in \mathbb{R}^n$, such that $\Sigma_{i \in N} x_i = Y$.*

Thus a sharing rule is always efficient in the sense that it distributes the entire budget. Agent i's payoff is $F_i(\langle N, Y, c \rangle) = x_i$. A sharing rule applied to a situation where not all the claims can be satisfied is a burden sharing rule.

The following sharing rule is called the rights-egalitarian rule.[4]

Definition 3. *Rights-egalitarian rule. According to the rights-egalitarian rule F^* everyone receives her claim. The remaining gains or losses are distributed equally among all agents. $F_i^*(\langle N, Y, c \rangle) = c_i + \frac{1}{n}(Y - C)$, for any sharing problem $\langle N, Y, c \rangle \in \Omega$.*

The rights-egalitarian rule can be considered as a mixed rule combining proportional and equal sharing.

Before, in sections 4 and 5 of the paper, the rights-egalitarian sharing rule is applied to the problem of inter- and intragenerational distribution in a pay-as-you-go pension system, the next section provides arguments in favour of rights-egalitarian sharing.

3. Axioms for a rights-egalitarian pension system

At first sight it may seem arbitrary to use rights-egalitarian sharing instead of proportional sharing or any other sharing rule to resolve the inter- and intragenerational distribution conflicts in a pay-as-you-go pension system. To address this problem we will formulate some desirable properties (axioms) of a

3 Such bankruptcy problems are also considered by Aumann and Maschler (1985), Curiel et al. (1987) and others; see Thomson (2003) for a recent survey.

4 It is also called *divorce rule* if applied to a two-person case.

pension system and show that, if these properties are accepted, rights-egalitarian sharing is the only option.

Herrero et al. (1999) provide several axiomatic characterisations of the rights-egalitarian sharing rule for the broad class of sharing problems Ω. They also consider a restricted class of sharing problems where budget and claims are non-negative. However, the case of the pension system is an intermediate case. We restrict the claims to be non-negative, but we allow for a negative budget since there may be public borrowing to finance pensions. A sharing problem can be decomposed into two (or more) sharing problems. In the case of public borrowing, when Y is the given budget, first an amount $Y' > Y$ is distributed, while the deficit $Y - Y' < 0$ has to be covered later. Let Ω^+ be a restricted class of sharing problems such that $\Omega^+ = \{\langle N, Y, c\rangle \in \Omega \,|\, c_i \geq 0\}$. Following Herrero et al. (1999) five (refined) axioms can be stated.

The starting point is the general idea that individual rights should be respected and that this is a joint responsibility of all members of society. This idea is captured by the following *Responsibility* axiom:

Responsibility. For any sharing problem $\langle N, Y, c\rangle \in \Omega^+$ and all $i \in N$, $F(\langle N, Y, c\rangle) = (0, \ldots, 0, c_i, 0, \ldots, 0) + F(\langle N, Y - c_i, (c_1, \ldots, c_{i-1}, 0, c_{i+1}, \ldots, c_n)\rangle)$.

Responsibility says that an agent i first gets her claim; then the remaining budget is distributed assuming i has no further claim. The rule respects individual claims and makes the group responsible for obtaining the necessary budget. Other reasonable requirements are the following.

Symmetry. For any sharing problem $\langle N, Y, c\rangle \in \Omega^+$, if $c_i = c_j$ for all $i, j \in N$, then $F_i = F_j$ for all $i, j \in N$.

Compatibility. For any sharing problem $\langle N, Y, c\rangle \in \Omega^+$ and all $i \in N$, if $C = Y$, then $F_i(\langle N, Y, c\rangle) = c_i$.

Composition. For any sharing problem $\langle N, Y, c\rangle \in \Omega^+$ and any $Y_1, Y_2 \in \mathbb{R}$ such that $Y_1 + Y_2 = Y, F(\langle N, Y, c\rangle) = F(\langle N, Y_1, c\rangle) + F(\langle N, Y_2, c - F(\langle N, Y_1, c\rangle)\rangle)$.

Population Monotonicity. For any two sharing problems $\langle N, Y, c\rangle \in \Omega^+$ and $\langle N \cup M, Y + Y', (c, c')\rangle \in \Omega^+$, $F_i(\langle N, Y, c\rangle) - F_i(\langle N \cup M, Y + Y', (c, c')\rangle)$ is the same for all $i \in N$.

Symmetry guarantees equal treatment when all have equal claims. *Compatibility* is a natural requirement: If the budget is equal to the sum of all claims, everyone should get exactly her claim. *Composition* requires that the final outcome should be independent of the fact that the final budget may be composed of different parts. *Population Monotonicity* requires that all incumbents are affected equally when additional members (with claims and contributions) join the group.

The following propositions hold.

Proposition 1. Rights-egalitarian sharing is the unique sharing rule satisfying *Responsibility* and *Symmetry.*

The proof is obvious and will be omitted.

Proposition 2. Rights-egalitarian sharing is the unique sharing rule satisfying *Symmetry, Compatibility,* and *Composition.*

The proof is similar to Herrero et al. (1999).

Proposition 3. Rights-egalitarian sharing is the unique sharing rule satisfying *Compatibility* and *Population Monotonicity.*

The proof is given in the Appendix.

The pension system is a social institution that, in principle, provides old-age security to every member of society. In turn it requires everyone's contribution. Any established rights or claims should be respected and their fulfillment is a joint responsibility of all members of society. If we accept this and the idea of equal treatment of equals (*Symmetry*), then, according to Proposition 1, we are committed to rights-egalitarian sharing in the pension system.

Public borrowing to finance pensions is wide-spread and not an exception. The final division of the available budget should not depend on the sequencing of financial arrangements. Otherwise individual shares could be easily manipulated. The requirement of *Composition* rules out this possibility of manipulation. If we accept this and the *Symmetry* and *Compatibility* conditions, then, according to Proposition 2, we are committed to rights-egalitarian sharing in the pension system.

A third argument in favour of rights-egalitarian sharing looks at the effects of migration. Consider the immigration of a group of workers. The immigrants contribute to the budget and, of course, they have claims. Suppose their claims are equal to their contributions. If there is a deficit in the initial situation, then immigrants will help to share the burden of debt of the pension system. Equal treatment of the incumbent population requires that all should benefit equally from immigration. This is what *Population Monotonicity* requires. If we accept this and the *Compatibility* condition, then, according to Proposition 3, we are committed to rights-egalitarian sharing in the pension system.

However there is an important drawback of the rights-egalitarian solution. The final payoff F_i^* may be negative. To see this, consider, for example, the sharing problem $\langle\{i,j\},1,\{1,3\}\rangle$, where $F_i^* = -\frac{1}{2}$ and $F_j^* = \frac{3}{2}$. Hence, in order to implement the rights egalitarian solution it will be necessary that those with a negative payoff will have other means available (e.g. from savings or transfers) to meet their responsibility. We will explore the relevance of this drawback for the pension system towards the end of the next section.

4. A rights-egalitarian pension system

The viability of the pay-as-you-go pension scheme depends on the next generation's capacity to contribute to the system. Therefore, to bring up and educate children is a main factor which determines future pensions. Child care becomes even more important in an ageing society. Standard pay-as-you-go systems where pension claims are based on monetary contributions have been modified to introduce fertility-related claims.[5]

The remainder of the paper builds on the analysis of a simple overlapping generations model with a pay-as-you-go pension system. Consider a society where each generation lives for three periods: childhood, working age and old age. People receive wages during working age and pensions during old age. We explore the simplest case where each period has the same length of time. Denote subsequent generations as t and $t+1$, respectively. Furthermore, denote by

n the number of individuals,

p the pension,

m the monetary contribution.

In a pay-as-you-go pension system all pensions are paid from the working generation's contributions. The fundamental equation of a pay-as-you-go system is:

$$p_t n_t = m_{t+1} n_{t+1} \,, \tag{1}$$

where p_t and m_t are the average pension and the average monetary contribution, respectively. Unlike the standard model of a pay-as-you-go system we explicitly introduce child care contributions into the model. Denote by

k the contribution of bringing up one child,

η the reproduction rate; $\eta_t = \frac{n_{t+1}}{n_t}$.

Thus the total child care contribution of generation t is $n_{t+1}k_t$. The parameter k_t is meant to capture the investment in human capital as acknowledged in the pension system. By definition k_t is commensurable with monetary contributions. We assume that the human capital investment is the same for each child in each generation, but it may differ across generations.

Since contributions will differ across persons, pensions are determined according to individual contributions. Individuals who contribute more to the system, be it monetary or child care contributions, should receive a higher pension in their old age.

[5] The case of Germany is mentioned in the introduction. Fertility related elements have also been introduced in Belgium, France, Italy, the Netherlands and in the United Kingdom (Stapf, 1997).

From (1) we obtain

$$p_t = m_{t+1} \frac{n_{t+1}}{n_t} \ . \tag{2}$$

Note that with a declining population $n_t > n_{t+1}$ the average pension p_t must fall or the average future monetary contribution m_{t+1} must rise (Aaron, 1966). Were the pensions level to be maintained, the rise in m would place the burden of change entirely on the younger generation. Keeping m constant places the burden of change on the old. There are arguments against both options. The young generation $t+1$ can claim that they did not create the change and, thus, should not be charged. Generation t will claim that they did their due to pay the pensions of the preceding generation $t-1$. Without considering child care contributions, the investment in human capital, the distribution issue can hardly be settled.

Our analysis addresses two problems: intergenerational and intragenerational distribution. We seek to answer the question whether and by how much m will have to rise when the population declines. The second problem concerns the appropriate relative weight of child care contributions as compared to monetary contributions. The total pension payments received by generation t must be distributed between individuals with different monetary and child care contributions. Both problems can be addressed using the framework of section 2. In order to do this we have to identify the relevant set of agents, the budget available for distribution, and the individual claims. In the following we define a sharing problem which covers both, intergenerational and intragenerational sharing.

We define the set of agents to be the members of two succeeding generations t and $t+1$:

$$N = N_t \cup N_{t+1} \ . \tag{3}$$

We assume that earlier generations have passed away and children (generation $t+2$) do not have any claims nor responsibilities with respect to the pension system at time t. Thus a complete pay-as-you-go pension system can be described as a succession of bi-generational sharing problems.

The resource to be distributed is the wage earned by generation $t+1$.

$$Y = y_{t+1} n_{t+1} \ , \tag{4}$$

where y_t denotes the average wage of generation t. Finally, the fixation of individual claims is straightforward. A pensioner claims a compensation for her monetary contributions m_i and for bringing up n_i children. Members of the working generation have a claim to their earnings.

$$c_i = \begin{cases} m_i + n_i k_t & \text{for} \quad i \in N_t \\ y_i & \text{for} \quad i \in N_{t+1} \end{cases} . \tag{5}$$

(3), (4), and (5) specify a sharing problem to which we apply the rights-egalitarian solution given in Definition 3. Denoting the average claim of a member of generation t by c_t, the individual payoffs are as follows:

$$x_i = c_i + \frac{1}{n_t + n_{t+1}} (Y - n_t c_t - n_{t+1} c_{t+1}) . \tag{6}$$

Using (4) and (5) this can be rewritten as

$$x_i = c_i - \frac{1}{1 + \eta_t} (\eta_t k_t + m_t) . \tag{7}$$

The individual contribution to the pension system including the child care contribution is fully acknowledged in the payoff x_i. But note that everyone has to pay an equal share of the deficit. This feature of the rights-egalitarian sharing rule causes a problem. Individuals whose claims are sufficiently small will be left with a debt. A positive payoff is received if and only if

$$c_i \geq \frac{1}{1 + \eta_t} (\eta_t k_t + m_t) , \tag{8}$$

for all i.

The possibility of negative payoffs might be seen as a drawback for the implementation and political feasibility of a rights-egalitarian pay-as-you-go pension system. However, this drawback can be overcome in two ways. First, notice that for reasonable parameter values η and for a reasonable joint distribution of k and m only a minority of the population violates condition (8). Individuals who do not meet (8) have claims less than $1/(1 + \eta_t)$ of the average claim.[6] This group of the poor will be entitled to support from a basic income scheme. To guarantee implementation of the rights-egalitarian solution, we assume that the basic income scheme will also cover any debt from the pension system.[7]

Alternatively, we may introduce a restriction on the range of solutions and require

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[6] Assume, for example, $k = m$ and $\eta = 1$, then (8) is violated for pensioners with no children *and* less than average monetary contributions; (8) is violated for workers whose income is less than the average monetary contribution.

[7] The Swiss pension system may serve as example for such a policy. There is a minimum contribution for everyone beyond age 20 which applies regardless of employment status (cf. e.g. Brombacher Steiner, 1999).

$$x_i \geq x_0, \quad \text{for all} \quad i\,, \tag{9}$$

where $x_0 \geq 0$ is a minimum payoff reflecting a minimum standard of living. Requirement (9) is incompatible with any sharing rule if $Y < (n_t + n_{t+1})\,x_0$. Assuming, for simplicity, $x_0 = 0$ we require a positive budget. Hence, we consider sharing problems of a further restricted domain $\Omega^{++} = \{\langle N, Y, c\rangle \in \Omega \,|\, c_i \geq 0, Y \geq 0\}$. Since, of course, we allow for $C > Y$, an individual's rights-egalitarian payoff F_i^* may still be negative. Bossert (1993) has introduced a refinement of F_i^* for the domain Ω^{++} which meets requirement (9). This solution, which I call the truncated rights-egalitarian solution F^{**}, is defined as follows.[8]

Definition 4: *Truncated rights-egalitarian rule. According to the truncated rights-egalitarian rule F^{**} everyone receives her claim. The remaining gains or losses are distributed equally among all agents subject to the individual ability to pay. For any sharing problem $\langle N, Y, c\rangle \in \Omega^{++}$ the solution $F_i^{**}(\langle N, Y, c\rangle)$ is constructed as follows. Starting out at c, where everyone receives her claim, payoffs are reduced equally until either (i) the budget constraint Y is met or (ii) someone's payoff is reduced to the minimum payoff, whatever occurs first. In case (i) the solution is established. In case (ii) the minimum payoff is established for this player. For the remaining players the procedure is repeated until all payoffs are determined.*

The truncated rights-egalitarian sharing equalises losses subject to the minimum payoff requirement (9). It maintains the spirit of joint responsibility, but does not ask a contribution from the poor. This resolves the intragenerational sharing problem when claims differ across individuals within a generation.

To describe intergenerational sharing in more detail we look at the average pension p_t and the corresponding average monetary contribution m_{t+1}. The average rights-egalitarian pension is given by

$$p_t = m_t + \eta_t k_t - \frac{1}{1+\eta_t}(m_t + \eta_t k_t) = \frac{\eta_t}{1+\eta_t}(m_t + \eta_t k_t)\,. \tag{10}$$

From (5) and (6) (or, alternatively, from (10) and (2)) we obtain the average monetary contribution of generation $t+1$ as

$$m_{t+1} = \frac{1}{1+\eta_t}(m_t + \eta_t k_t)\,. \tag{11}$$

Before we turn to a detailed discussion of our results in the next section, let us briefly examine the rights-egalitarian pension system in the equilibrium of

[8] The rule is called *extended claim-egalitarian rule* in the literature. Bossert (1993) and Marco-Gil (1994) provide axiomatic characterisations of the rule.

the stationary state. The stationary state is characterised by $\eta_t = 1$ and $k_t = k$. We use * to denote the variables of the stationary state equilibrium. The average pension and the average monetary contribution are given by

$$p_t^* = \frac{1}{2}\left(m_t^* + k\right) \tag{10*}$$

and

$$m_{t+1}^* = \frac{1}{2}\left(m_t^* + k\right) \ . \tag{11*}$$

Of course, in the equilibrium $m_{t+1}^* = m_t^*$. Moreover, $m_t^* = k$. Thus in the equilibrium of the stationary state monetary contributions and child care contributions receive the same weight as factors determining the pensions.

5. The dynamics of a rights-egalitarian pension system

We now consider a rights-egalitarian pay-as-you-go system in a long-run perspective. We start the analysis with the assumption that the child care contribution is the same for each child across all generations; $k_t = k$ for all t. Any arbitrary contribution m_t would approach the equilibrium $m^* = k$ in the long run. This can be seen when rewriting the difference equation (11) as

$$\Delta m \equiv m_{t+1} - m_t = \frac{\eta_t}{1+\eta_t}\left(k - m_t\right) \ . \tag{12}$$

Δm is positive if and only if $m_t < k$. Thus we have the following two propositions.

Proposition 4. Under rights-egalitarian sharing, if $m_t = k$ for some t, then the average monetary contribution m remains constant after t independent of the reproduction rate.

This result is intuitively appealing. If $m = k$ holds, the pay-as-you-go pension system can be seen as a succession of fair intergenerational exchanges. Generation t receives a full compensation for their investment in the human capital of generation $t+1$. Note that the average pension of generation t is $p_t = \eta_t m_{t+1}$. With population decline the level of pensions is lower than in the stationary state.

Proposition 5. Under rights-egalitarian sharing, if the reproduction rate η is constant, the average monetary contribution m will approach the long-run equilibrium level $m^* = k$.

This follows from (12) and the observation that Δm has the sign of $k - m_t$.

It is obvious from our results that under rights-egalitarian sharing the pension system is not viable if child care contributions are ignored. In this case $k = 0$, and according to proposition 5 the contribution will decline and approach zero in the long run.

Following an argument by Becker and Lewis (1973) it is reasonable to assume that, as the number of children declines, the investment in the human capital of each child will increase. Denote $\kappa_t = \frac{k_{t+1}}{k_t}$. Suppose that $\kappa_t = \kappa$, $\eta_t = \eta$, and $\kappa\eta = 1$, so that overall human capital investments are constant over time. To examine the features of a balanced path, where $\frac{m_{t+1}}{m_t} = \kappa$, we obtain from (11):

$$\frac{m_{t+1}}{m_t} = \frac{1}{1+\eta_t}\left(1 + \eta_t \frac{k_t}{m_t}\right) . \tag{13}$$

Using $\kappa = \frac{m_{t+1}}{m_t}$ and $\kappa = \frac{1}{\eta}$ it follows that $\frac{k_t}{m_t} = \kappa^2$. The result is summarised in the following proposition.

Proposition 6. Under rights-egalitarian sharing and for any constant reproduction rate η, if human capital investments are constant over time, $\kappa\eta = 1$, then, on a balanced path, the ratio of child care and monetary contributions is $\frac{k_t}{m_t} = \kappa^2$. Hence, with a declining population ($\eta < 1$), as human capital investments are more in terms of quality than quantity of children, child care contributions receive a larger weight as compared to monetary contributions.

The main conclusion to be drawn from this section is as follows: If child care contributions are properly accounted for and reflect the investment into the human capital of the next generation, then the long-run equilibrium reflects the "true" property rights. The older generation will receive a pension payment from the young which is equivalent to their human capital investments. This conclusion is in line with findings of Eckstein and Wolpin (1985) and Bental (1989) who study an optimal population problem. Under rights-egalitarian sharing the pay-as-you-go system approaches a fully-funded system where the accumulation of capital is entirely in terms of human capital.

6. Discussion and conclusion

Demographic change causes problems for the financing of pay-as-you-go pension systems. This raises the question of burden sharing between generations. Moreover, raising children and investments in their human capital are important contributions to the functioning of the pay-as-you-go system. Accordingly, such investments should be rewarded. Pension entitlements should result from both, monetary and child care contributions. This raises the question of intragenerational sharing of pension entitlements. There are many possible solutions to these sharing problems. This paper argues that the rights-egalitarian sharing rule has much to recommend itself. It is the unique rule that satisfies three different sets of axioms: (i) *Symmetry* and *Responsibility,* (ii) *Symmetry, Compatibility,* and *Composition,* and (iii) *Compatibility* and *Population Monotonicity.* We have identified these axioms as reasonable requirements for a pension system.

It is shown that under rights-egalitarian sharing in the long-run equilibrium the monetary contribution of the young equals the investment into human capital of the old. Thus the pension is a fair compensation for human capital investments.

Rights-egalitarian sharing does not solve all distributional issues. It captures the idea of fair compensation for contributions to a task which is seen as a joint responsibility. Put into practice, monetary contributions would be levied as a lump sum tax. Hence, rights-egalitarian sharing might not be politically feasible unless it is implemented in a broader social security framework which guarantees a basic (after tax) income. Still, the distributional consequences of rights egalitarian sharing are regressive. However, a proportional tax on wage would violate *Responsibility, Composition* and *Population Monotonicity.* Under a proportional tax people with higher wages pay more to cover a remaining debt. This violates *Responsibility* which asks equal sharing of joint responsibilities. Similarly, people with higher wages (and a higher tax bill) would gain more from immigrants contributing to the budget; thus, *Population Monotonicity* is violated. Moreover, under a proportional sharing rule outcomes can be manipulated and, thus, *Composition* will be violated. To see this consider a sharing problem and its decomposition into two steps: First all claims are satified by public borrowing. Then the debt is collected. As no claims remain after the first step, the debt will shared equally. This results in an outcome different from the proportional solution of the original sharing problem.

Throughout the paper I have assumed exogenous fertility. If this assumption is dropped, rights-egalitarian sharing can be criticised on grounds of inefficiency. It does not provide the right incentives to invest in human capital, since the returns to investments cannot be fully appropriated by those who contribute to child care. Although being justified, this critique applies to other sharing rules as well. The only way to avoid the problem is a fully child care based pension which gives zero weight to monetary contributions.[9] The rights-egalitarian sharing scheme proposed in this paper is a compromise between a traditional pay-as-you-go pension system based on monetary contributions and a fully efficient scheme where pensions are the full return on human capital investments.

9 Sinn (1998) discusses the pros and cons of fertility related claims.

Appendix

Proof of Proposition 3: First, rights-egalitarian sharing F^* satisfies *Compatibility* since $F^*(\langle N, C, c\rangle) = c$. Also F^* satisfies *Population Monotonicity.* Consider two sharing problems $\omega = \langle N, Y, c\rangle \in \Omega^+$ and $\omega' = \langle N \cup M, Y + Y', (c, c')\rangle \in \Omega^+$. We have $F^*(\omega) = c_i + \frac{1}{n}(Y - C)$ and $F^*(\omega') = c_i + \frac{1}{n+m}(Y + Y' - C - C')$. Thus $F^*(\omega) - F^*(\omega') = \frac{1}{n}(Y - C) - \frac{1}{n+m}(Y + Y' - C - C')$ does not depend on individual claims and is the same for all $i \in N$.

To show uniqueness we show that if F satisfies *Compatibility and Population Monotonicity,* then $F = F^*$. Consider the sharing problems ω and ω'. Suppose $Y - C = -(Y' - C')$. Then, $Y + Y' = C + C'$. By *Compatibility* $F_i(\omega') = c_i$ for all $i \in N \cup M$. By *Population Monotonicity* $F_i(\omega) - F_i(\omega') = F_j(\omega) - F_j(\omega')$ for all $i, j \in N$. Hence, $F_i(\omega) - c_i = F_j(\omega) - c_j = \lambda$, where $\lambda \in R$. It follows that $F_i(\omega) = c_i + \lambda$. Each individual receives her own share plus a fixed amount which is the same for all.

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