

## **Volatility Bounds for Stochastic Discount Factors on Global Stock Markets**

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### **Abstract**

In this paper we address three main issues in international asset pricing. The first question is whether it is harder to simultaneously price international assets rather than domestic assets alone. The second objective is to investigate whether investors can enhance their risk-return spectrum through international diversification. To give a complete picture, our empirical tests are not restricted to stock markets of developed countries, but also include emerging stock markets. Third, we address the question whether currency risk plays an empirically significant role in international portfolio choice. All these issues are investigated exploiting the duality between volatility bounds for stochastic discount factors and the traditional mean-variance framework to derive spanning restrictions. The empirical results depend heavily on the set of stock markets and indicate that hedging significantly increase the risk-return spectrum faced by a global investor. However, when the goal is to maximize the benefits from international diversification, exploiting conditioning information turns out by far most important. Another interesting observation is that the times of the 'diversification free lunch' in emerging markets – if they ever existed – seem to be over.

### **Zusammenfassung**

Dieser Aufsatz untersucht drei zentrale Fragestellungen des internationalen Asset Pricing. Erstens wird der Frage nachgegangen, wie sich die Bewertung internationaler Aktienanlagen von der rein nationaler Anlagen unterscheidet. Zweitens wird untersucht, ob ein Investor durch internationale Diversifikation sein Rendite-Risiko Spektrum erweitern kann. Um ein vollständiges Bild der Diversifikationsmöglichkeiten darzustellen, werden auch Emerging Markets einbezogen. Drittens wird die Bedeutung von Währungsrisiken bei der internationalen Diversifikation analysiert. Alle drei Frage-

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stellungen werden simultan durch einen ‚Spanning-Test‘ untersucht, indem die Dualität zwischen den Volatilitätsschranken für stochastische Diskontfaktoren und dem klassischen Mittelwert-Varianz Ansatz ausgenutzt wird. Die empirischen Resultate hängen stark von der Länderstichprobe ab. Generell zeigt sich aber, dass durch Währungsabsicherung die Diversifikationsmöglichkeiten eines globalen Investors signifikant erhöht werden können. Allerdings wird erst durch die Berücksichtigung von Konditionierungsinformation das internationale Risiko-Rendite Spektrum maximiert. Schliesslich wird gezeigt, dass sich das hohe Diversifikationspotential in den Emerging Markets über die Zeit stark reduziert hat.

*JEL-Classification: G11, G12, G15*

## 1. Introduction

Empirical international asset pricing has become an active research area only recently. While the theoretical foundations of international asset pricing were forcefully put forth in the pioneering models by Solnik (1974), Sercu (1980), Stulz (1981), and Adler and Dumas (1983) relatively early, the empirical tests could not keep up to the pace of these theoretical developments. Although elegant, the analytical models are notoriously hard to test empirically, because the equilibrium pricing relations depend on parameters that cannot be observed. Only recently, with the availability of sufficient international data and applying sophisticated econometric techniques, powerful tests of highly structured pricing models have become possible. However, even the most recent studies in this area require more or less restrictive assumptions. The goal of this paper is to examine some important issues of international asset pricing without directly relying on a fully-fledged asset pricing model. In their seminal paper Hansen and Jagannathan (1991) introduce a diagnostic framework that allows to extract information about the behavior of ‘valid’ stochastic discount factors directly from asset return data. They derive volatility bounds in mean-variance space for admissible stochastic discount to fall into. This framework is flexible enough to address many different issues of international asset pricing empirically.

The first obvious question in an international context is whether it is harder to simultaneously price international assets rather than domestic assets alone. Assuming that the law of one price holds, an admissible discount factor always exists, irrespective of the specific set of assets. The question is, however, whether the qualitative properties of a ‘local’ discount factor and a ‘global’ discount factor are significantly different, as judged by the minimum volatility that is necessary to price a given set of assets. Bekaert and Hodrick (1992) find that international diversification imposes stronger pricing restrictions, but they do not report any measures of statistical significance. From this point of view, the test we propose can be interpreted as a nonparametric test of market integration. A second related issue in international asset pricing is the well-known

proposition that the addition of foreign securities to a purely domestic portfolio reduces the total risk of the portfolio. This observation can be attributed to the relatively low correlations between equity returns in different markets. However, few studies have tested whether the enhancement of the risk-return spectrum is statistically significant. The relevant question is whether or not investors can mimic foreign stock returns with domestic securities. Third, we test whether a simple currency hedging strategy helps to further increase the risk-return spectrum available to a global investor. This is an important practical question for portfolio managers. Fourth, we provide new evidence for the predictability of stock returns. Using linear regression analysis, previous research has shown that returns are predictable to some extent on the basis of variables related to the business cycle. We extend these findings by testing conditional implications of modern formulations of mean-variance spanning tests. Finally, we include both developed and emerging stock markets in our analysis. Recently, emerging markets have attracted a lot of attention from both researchers and practitioners. Speidel and Sappenfield (1992) and Divecha, Drach, and Stefek (1992) argue that there is a 'diversification free lunch' available for investors in emerging markets. In contrast, Zimmermann, Drobetz, and Oertmann (2002) show that emerging markets seem very attractive *ex ante*. However, they are largely responsible for negative *ex post* performance of globally diversified portfolios. Including emerging equity markets enables us to present a fuller picture of the true benefits of global diversification.

The remainder of this paper is organized as follows. Section 2 introduces Hansen and Jagannathan (1991) volatility bounds for stochastic discount factors. We briefly show how to link this diagnostic device to its better-known counterpart, the efficient frontier for portfolio returns, and how to incorporate conditioning information by adding scaled returns. Section 3 describes a modern version of spanning tests in stochastic discount factor language. Following previous work by De Santis (1995) and Bekaert and Urias (1996), we introduce a set of orthogonality conditions, which can be used to formally test all the issues mentioned above in a unifying framework. Section 4 provides a description of our data set. The empirical results and our interpretations follow in section 5. Finally, section 6 concludes.

## 2. Stochastic discount factors and asset pricing

In their seminal paper Hansen and Jagannathan (1991) develop a very general methodology to evaluate the asset pricing implications of a given set of asset returns. This section briefly introduces their approach. First, we show how security returns can be used to derive an efficiency region for stochastic discount factors that are consistent with asset pricing data, completely di-

vanced from any parametric specification. The discussion follows closely along the original work by Hansen and Jagannathan (1991). Second, their methodology can be generalized to incorporate conditioning information, as forcefully demonstrated by Cochrane (1996). Third, we demonstrate that our empirical test design described in a later section exploits the duality between mean-standard deviation frontiers for portfolio returns and mean-standard deviation frontiers for stochastic discount factors.

## 2.1 Hansen-Jagannathan volatility bounds

Consider the  $N$ -dimensional vector  $\mathbf{R}_{t+1}$  of gross asset returns from time  $t$  to  $t + 1$  and define with  $\Phi_t$  the set of publicly available information at time  $t$ . Recall that virtually all financial asset pricing models imply that the vector of asset returns  $\mathbf{R}_{t+1}$ , multiplied by some market-wide random variable  $m_{t+1}$ , has a constant conditional expectation,

$$(1) \quad E_t(m_{t+1}\mathbf{R}_{t+1}|\Phi_t) = \mathbf{1} ,$$

where  $\mathbf{1}$  is an  $N$ -dimensional vector of ones. The gross return on the asset  $i$  at time  $t + 1$  is defined as  $R_{i,t+1} = (P_{i,t+1} + D_{i,t+1})/P_{i,t}$ , where  $P_{i,t}$  is the price of the asset at time  $t$ , and  $D_{i,t+1}$  includes dividends and other payments received at time  $t + 1$ . Econometric tests usually focus on the unconditional version of equation (1). When  $\Phi_t$  is the null information set,  $E(\cdot)$  denotes the unconditional expectation. Taking the expected value of equation (1), it follows that versions of the same equation must hold for the expectations  $E(\cdot|\Phi_t)$  and  $E(\cdot)$ . Hence, conditioning down using the law of iterated expectations gives:

$$(2) \quad E(m_{t+1}\mathbf{R}_{t+1}) = \mathbf{1} .$$

The random variable  $m_{t+1}$  is known as the intertemporal marginal rate of substitution, the stochastic discount factor, or the pricing kernel. We synonymously refer to an  $m_{t+1}$  satisfying (2) as a ‘valid’ or ‘admissible’ stochastic discount factor. The existence of an  $m_{t+1}$  guarantees that all assets with the same payoffs have the same price. Hence, the existence of a valid stochastic discount factor implies that the law of one price holds. Restricting  $m_{t+1}$  to be strictly positive allows to interpret (2) as a no-arbitrage condition.<sup>2</sup>

The sample counterparts of the orthogonality conditions derived from (2) form the basis of many tests using Hansen’s (1982) Generalized Method of Moments (GMM). However, a specific parameterization of  $m_t$  is needed to give the equation some meaningful empirical content. For example, if  $m_{t+1}$  is

<sup>2</sup> See Harrison and Kreps (1979) and Hansen and Richard (1987). However, the no-arbitrage condition does not uniquely identify  $m_{t+1}$  unless markets are complete.

a linear function of the return on the market portfolio, the CAPM of Sharpe (1964), Lintner (1965), Mossin (1965) and Black (1972) is easily obtained.<sup>3</sup> Instead of proposing yet another parametric specification of the discount factor  $m_{t+1}$ , Hansen and Jagannathan (1991) show how to derive a set of stochastic discount factors consistent with asset pricing data, but without assuming a specific asset pricing model. They assume as little structure as possible to deduce pricing restrictions. The only two assumptions made are that (i) investors can form any portfolio of traded assets (frictionless markets) and (ii) the law of one price holds. This implies that there exists a stochastic discount factor  $m_{t+1}$  in the payoff space (which needs not be positive) such that equation (1) is satisfied. However, it must be noted that their approach is really an incomplete market model with possibly many stochastic discount factors.

The Euler-equation in (2) implies that if the stochastic discount factor is a degenerate variable, then all assets must earn identical expected returns. But given that assets exhibit different expected returns depending on their risk characteristics, the stochastic discount factor cannot be constant. This already describes an important insight: cross-sectional differences in expected returns have implications for the variance of any valid stochastic discount factor. Hansen and Jagannathan (1991) suggest lower bounds for the standard deviation of any valid stochastic discount factor, using only the returns for a given set of securities.<sup>4</sup> Specifically, they derive volatility bounds by projecting the pricing kernel unconditionally on the space of available payoffs and computing the standard deviation of this projection. It is more common in the financial literature, however, to think in returns rather than payoffs. This does not affect generality, of course, because a return is simply a payoff with a price of one. It can then be shown that the following inequality for the variance of the implied stochastic discount factor from this projection holds:<sup>5</sup>

$$(3) \quad \text{var}(m) \geq (\mathbf{1} - E(m)E(\mathbf{R}')) \text{var}(\mathbf{R})^{-1} (\mathbf{1} - E(m)E(\mathbf{R})) .$$

For the risk-free asset, denoted as  $R_f$ , it further holds that  $E(m) = R_f^{-1}$ . Therefore, as the hypothetical values of  $E(m) = R_f^{-1}$  are varied over the real line, equation (3) describes a parabola in  $[E(m), \text{var}(m)]$  space. For a given set of asset returns, which determine  $E(\mathbf{R})$  and  $\text{var}(\mathbf{R})$ , the inequality describes a lower bound for the variance of  $m_{t+1}$  as a function of its mean. Taking the square root of equation (3) delivers the Hansen / Jagannathan bound in a more familiar mean-standard deviation space. The necessary (but not sufficient)

<sup>3</sup> See Hansen and Singleton (1983) and Ferson and Jagannathan (1996), among others.

<sup>4</sup> Their approach is a generalization of earlier work by Shiller (1981) and Campbell and Shiller (1988). See Cochrane (1991) for an overview.

<sup>5</sup> Time subscripts are omitted. For the derivation see Campbell, Lo, and MacKinlay (1997), Cochrane (2001), Ferson (1995) or Ferson and Jagannathan (1996).

condition is that any valid stochastic discount factor must have a mean and a standard deviation that places it within the parabola.<sup>6</sup>

**2.2 The link to the traditional mean-variance world**

Our empirical test exploits the duality between Hansen/Jagannathan bounds and efficient frontiers for portfolio returns to derive spanning restrictions. Writing the Euler equation in (2) in terms of excess returns, pulling the expectation through, and splitting up the covariance term yields the following expression:

$$(4) \quad \sigma(m) = -E(m) \frac{E(r)}{\rho\sigma(r)},$$

where  $r$  denotes the return in excess of the risk-free rate and  $\rho$  is the correlation coefficient between  $m$  and  $r$ . By definition, a correlation coefficient must be less than one,  $|\rho| \leq 1$ , yielding the Hansen/Jagannathan bound. This implies that any  $m$  on the bound is perfectly correlated with some portfolio of excess returns. Now we can write:<sup>7</sup>

$$(5) \quad \frac{\sigma(m)}{E(m)} \geq \max\left(\frac{E(r)}{\sigma(r)}\right) = \left|\frac{E(r^*)}{\sigma(r^*)}\right| = SR,$$

defining a mean-standard deviation boundary which restricts any parametric pricing variable  $m$ . The right-hand side in equation (5) is the ‘Sharpe ratio’, denoted as  $SR$ , or the market price of risk, and depends only on sample moments of asset returns. The Sharpe ratio is the slope of a line drawn from the risk-free asset,  $R_f$ , and tangent to the efficient frontier.  $r^*$  denotes the excess return on the tangency portfolio. Both the tangency portfolio and the Sharpe ratio depend on a given value of the risk-free rate. As the latter is varied, the tangency point moves along the efficient frontier. Similarly, the Hansen/Jagannathan bound corresponds to the minimum value of  $\sigma(m)$  for each value of  $E(m)$ . Because  $E(m) = 1/R_f$ , variation of the risk-free rate implies a movement along the  $[E(m), \sigma(m)]$  boundary. This reveals a one-to-one relationship between the efficient frontier for portfolio returns and the feasible region for

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<sup>6</sup> The derivation assumes there is no linear combination of the vector of asset payoffs that is identically equal to one (i.e., there is no explicit risk-free rate). Hansen/Jagannathan regions provide lower bounds on the volatility for each possible value of  $E(m)$ . If the mean discount factor was known in advance, and so  $E(m) = 1/R_f$ , the parabola would reduce to a vertical line and the mean-variance frontier for valid discount factors to a single point. See Cochrane and Hansen (1992).

<sup>7</sup> See Ferson (1995) for a more detailed discussion.

valid stochastic discount factors. The two curves contain exactly the same information.<sup>8</sup>

### 2.3 Incorporating conditioning information

Empirical evidence shows that expected returns are to some extent predictable on the basis of instrument variables related to the business cycle. The same variables that contain explanatory power for future returns can be used to test some implications of the conditional version of the Euler-equation in (1). In fact, a simple approach to incorporate conditioning information is to augment the payoff space by scaling returns with proper instrument variables. This technique was first proposed by Cochrane (1996). Improvements in the bound can be interpreted as evidence for predictability of stock returns, where ‘improvement’ is used to denote sharper volatility bounds for stochastic discount factors.

To see this, let  $\mathbf{Z}_t$  be an  $L$ -dimensional vector of instrument variables contained in the information set  $\Phi_t$ , so that  $\mathbf{Z}_t \subset \Phi_t$ . The space of scaled returns  $\mathbf{R}_{t+1} \otimes \mathbf{Z}_t$  can in principle be infinite dimensional. Then equation (1) implies:

$$(6) \quad E_t(m_{t+1}(\mathbf{R}_{t+1} \otimes \mathbf{Z}_t)) = \mathbf{1} \otimes \mathbf{Z}_t .$$

Note that we have to introduce time subscripts again to denote the exact timing of the inflow of relevant information. Taking unconditional expectations and applying the law of iterated expectation, we get:

$$(7) \quad E(m_{t+1}\mathbf{X}_{t+1}) = \mathbf{Q}_t ,$$

where  $\mathbf{X}_{t+1} = \mathbf{R}_{t+1} \otimes \mathbf{Z}_t$  is an  $NL \times 1$  vector of payoffs obtained by scaling returns, and  $\mathbf{Q} = E(\mathbf{1} \otimes \mathbf{Z}_t)$  is an  $NL \times 1$  vector of expected prices for these payoffs. Equation (7) expresses an implication of the conditional model for its unconditional version, which is not captured by just conditioning down as in equation (2). Cochrane (2001) shows that scaled returns can be interpreted as the payoffs on actively managed portfolios. For instance, assume the investor follows a linear timing rule and uses a single instrument to determine the exposure to a single risky asset.<sup>9</sup> At the beginning of each period the investor puts  $Z_t$  money units into the risky asset, so that  $Z_t$  can be interpreted as the risky asset’s time varying investment proportion. At the end of each period, the payoff is  $Z_t R_{t+1}$ . Therefore,  $Z_t$  and  $Z_t R_{t+1}$  represent actual prices and payoffs of actively managed portfolios, respectively.

<sup>8</sup> For a graphical analysis see De Santis (1995), p. 34.

<sup>9</sup> Bekaert and Liu (1999) and Ferson and Siegel (1999) show that linearity is not important.

Cochrane (1996) suggests to add managed portfolio payoffs and to proceed with unconditional moments as if conditional information did not exist. The Euler-equation in (2) must hold for these payoffs as well, and one can compute the unconditional bound for scaled returns in the usual manner. Intuitively, scaling will only improve the bound significantly if the weights that are applied contain information about future returns. We compute ‘stacked’ bounds for both scaled and unscaled returns, thereby imposing additional restrictions on the projection. Stacked bounds can never deteriorate compared to the original bound. But adding ‘noise’ (i.e., conditioning information without informational content) will not result in a significant upward shift of the bound either. In a nutshell, the space of payoffs only increases if the instrument variables are correlated with future returns.<sup>10</sup>

### 3. Tests of mean-variance spanning

Tests of mean-variance spanning can be used to measure the benefits of portfolio diversification within domestic markets or across global markets. The duality between the traditional efficient frontier and the Hansen/Jagannathan bound extends the relevance of spanning tests to the theory of asset pricing. Such tests allow to identify which assets impose the sharpest restrictions on the volatility of any valid discount factor. In the empirical framework that follows below we propose that a set of asset returns provides diversification benefits relative to some set of benchmark returns if an addition of these returns leads to a significant leftward shift of the efficient frontier. Given the volatility of stock market returns, there may be little confidence in a statistical sense that the risk-return tradeoff is truly better when new assets are added, even with reasonably long time series of data.

Let  $\mathbf{R}_{t+1}$  be an  $N \times 1$  vector of asset returns. All assets included in  $\mathbf{R}_{t+1}$  define the mean-variance efficient frontier. Mean-variance spanning tests ask whether there exists a subset of assets that span the entire mean-variance frontier. For empirical tests it is convenient to partition  $\mathbf{R}_{t+1}$  into a  $K \times 1$  vector of ‘spanning assets’,  $\mathbf{R}_{1,t+1}$ , and an  $[(N - K) \times 1]$  vector of ‘test assets’,  $\mathbf{R}_{2,t+1}$ . The null hypothesis to be tested is whether the assets in  $\mathbf{R}_{1,t+1}$  span the entire mean-variance frontier associated with  $\mathbf{R}_{t+1}$ . In other words, spanning implies that the minimum-variance frontier of  $\mathbf{R}_{1,t+1}$  is the same as the minimum-variance frontier associated with  $\mathbf{R}_{t+1}$ . We ask whether one can significantly improve the risk-return tradeoff by adding additional test assets to an already existing portfolio of spanning assets. With international data this is an important test of the benefits of international diversification. Another interpretation refers to the ‘home bias puzzle’. If the null hypothesis of spanning can-

<sup>10</sup> For the proof see Bekaert and Liu (1999), proposition 2.3.



not be rejected, any home bias in portfolio holdings could be explained by statistical uncertainty of the benefits of global diversification.

Our empirical test design is borrowed from previous work by De Santis (1995) and Bekaert and Urias (1996) for U.S. data. It is a more modern, more robust representation of the Huberman and Kandel (1987) version of spanning tests. Obviously, one can construct Hansen/Jagannathan volatility regions for any given set of returns separately. However, it is more interesting to explore whether the bound for the payoff space of spanning assets is statistically distinguishable from the lower bound associated with the combined payoff space of spanning and test assets. Taking the perspective of a Swiss investor, the question is whether the qualitative properties of a 'local' Swiss stochastic discount factor and a 'global' stochastic discount factor are significantly different, as judged by the minimum volatility that is necessary to properly price both sets of payoffs.<sup>11</sup> Finding that it is 'harder' to price international assets than Swiss assets alone (harder in the sense that an extremely volatile stochastic discount factor is required in a global context) would raise doubt about international stock market integration. However, any results can only be understood as indicative, because no particular asset pricing model has been imposed.

When a new set of test assets is added to the set of spanning assets, the key question is whether the resulting shift in the bound is significant. The projection argument put forth by Hansen and Jagannathan (1991) imposes additional restrictions on the pricing kernel as we go from the payoff space of spanning assets to the combined payoff space of spanning assets and test assets. The bound becomes tighter, i.e., it shifts upward. Again, the analogy to the mean-variance world is straightforward. Adding additional assets always shifts the efficient frontier to the left, thereby increasing the risk-return tradeoff (or Sharpe ratio) an investor faces.<sup>12</sup> A natural procedure for an econometric test is to ask whether the stochastic discount factor, which prices all assets in the universe, can be modeled as a linear function of the test assets alone. Specifically, consider a regression of  $m_{t+1}$  on a constant  $c$  and the vector of demeaned (unexpected) returns:

$$(8) \quad m_{t+1} = c + (\mathbf{R}_{t+1} - E(\mathbf{R}))' \boldsymbol{\beta} + \varepsilon_{t+1} = m_{t+1}^* + \varepsilon_{t+1},$$

with  $\mathbf{R}'_{t+1} = (\mathbf{R}_{1,t+1}, \mathbf{R}_{2,t+1})'$  and  $\boldsymbol{\beta} = (\boldsymbol{\beta}'_1 \boldsymbol{\beta}'_2)'$ . The null hypothesis of mean-variance spanning asserts that the  $N$ -dimensional vector  $\mathbf{R}_{t+1}$  is priced by a

<sup>11</sup> The question is *not* whether a common discount factor exists for the combined payoff space. If the law of one price holds, a unique stochastic discount factor that prices all payoffs simultaneously always exists in the payoff space. But the stochastic properties of the discount factors, as described by their minimum volatility required to qualify as valid stochastic discount factors, may be different.

<sup>12</sup> For a graphical analysis see De Santis (1995), p. 36.

linear combination of the  $K$  spanning assets included in  $\mathbf{R}_{1,t+1}$ . This implies that all  $N - K$  coefficients of test assets in  $\beta_2$  are equal to zero. In other words, the returns on the spanning assets,  $\mathbf{R}_{1,t+1}$ , suffice to mimic the risk-return spectrum associated with the returns on the set of test assets,  $\mathbf{R}_{2,t+1}$ . Taking the unconditional expectation of (8) and using the fact that  $E(m_{t+1}) = R_{f,t+1}^{-1}$  gives:

$$(9) \quad E(m_{t+1}) = E(m_{t+1}^*) = R_{f,t+1}^{-1} = c .$$

In mean-variance language, for a given value of  $c$  the test can be used to investigate whether the risk-return spectrum associated with  $\mathbf{R}_{1,t+1}$  is tangent to that associated with  $\mathbf{R}_{t+1}$  at the point with the highest Sharpe ratio. The idea to test whether the two bounds are tangent at the point  $E(m) = c$  captures exactly what Huberman and Kandel (1987) call intersection. There is, however, a severe complication: the econometrician usually does not observe the value of  $c$ . Assuming some arbitrary value for  $c$  and running the test is of no help. Being tangent at this arbitrary point does not imply that the two frontiers intersect for other values of  $c$  as well; in particular, it is possible that they do not intersect for the true value of  $c$ . Obviously, one could test the null hypothesis for all possible values of  $c$ , but this would be very cumbersome. To get around this problem, De Santis (1995) and Bekaert and Urias (1996) invoke the two-fund separation theorem.<sup>13</sup> Loosely speaking, this well-known theorem from portfolio theory suggests that any frontier portfolio can be obtained as a linear combination of two other frontier portfolios. Hence, if the frontiers are tangent in any two points, they must coincide at all points. Running the test for two arbitrary (but different) values of  $c$  implies testing whether the bounds coincide at all points. This is what Huberman and Kandel (1987) refer to as mean-variance spanning. Formally, defining  $c_1$  and  $c_2$  as two arbitrary values of  $E(m_{t+1})$ , such that  $c_1 \neq c_2$ , and plugging (8) into the Euler-equation in (2), the orthogonality conditions for a GMM-based test of unconditional mean-variance spanning are as follows:<sup>14</sup>

$$(10) \quad E \left\{ \begin{matrix} \mathbf{R}_{t+1}c_1 + \mathbf{R}_{t+1} [\mathbf{R}_{1,t+1} - E(\mathbf{R}_{1,t+1})]' \beta_{1,c_1} - \mathbf{1} \\ \mathbf{R}_{t+1}c_2 + \mathbf{R}_{t+1} [\mathbf{R}_{1,t+1} - E(\mathbf{R}_{1,t+1})]' \beta_{1,c_2} - \mathbf{1} \end{matrix} \right\} = \mathbf{0} .$$

As shown above, an elegant test of conditional mean-variance spanning is to add scaled returns. The system of equations in (10) can be estimated using Hansen’s (1982) Generalized Method of Moments (GMM). There are  $2 \times N$  orthogonality conditions to test and  $2 \times K$  model parameters to estimate. This leaves  $2 \times (N - K)$  overidentifying restrictions that can be used to test the

<sup>13</sup> See Huang and Litzenberger (1988) or Ingersoll (1987).

<sup>14</sup> See De Santis (1995), equation (18), p. 10.

null hypothesis of mean-variance spanning via the chi-square test statistic for the model's goodness-of-fit. The overidentifying restrictions are obtained by assuming that the  $2 \times (N - K)$  coefficients in  $\beta_{2,c1}$  and  $\beta_{2,c2}$  are simultaneously equal to zero. The assets in  $\mathbf{R}_{1,t+1}$  span the volatility bound associated with  $\mathbf{R}_{t+1}$  if the subset of test assets in  $\mathbf{R}_{2,t+1}$  can be disregarded in a linear parameterization of the candidate stochastic discount factor. Up to a mean zero, orthogonal factor the returns on test assets are mimicked by the returns on some portfolio of spanning assets.

De Santis (1995) uses data from developed stock markets and performs spanning tests on the basis of the system of equations in (10) from the U.S. perspective. His results indicate that pricing international assets requires a more volatile discount factor than pricing U.S. assets alone. He also finds that the implied bound is usually sharper when assets are hedged against currency risk and conditional information is used. Bekaert and Urias (1996) specify a slightly different setup and examine the diversification benefits from emerging market investments using U.S.- and U.K.-traded closed-end funds. They report significant diversification benefits for the U.K. country funds, but not for the U.S. funds. Most recently, Errunza, Hogan, and Hung (1999) also apply spanning tests based on volatility bounds and report that U.S. investors can mimic foreign indices by holding domestically traded assets. They argue that it suffices for U.S. investors to invest in multinational corporations, closed-end country funds, and American Depositary Receipts (ADRs) in order to capture the benefits of international diversification.

#### 4. Characteristics of input data

*MSCI stock market indices:* Monthly data on international equity indices from 1973.06 to 1998.08 are obtained from Morgan Stanley Capital International (MSCI). The countries used in our developed markets' sample are the sixteen countries constituting the Organization for Economic Cooperation and Development (OECD), plus Hong Kong and Singapore. The OECD countries include Australia, Austria, Belgium, Canada, Denmark, France, Germany, Italy, Japan, the Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom, and the United States of America. Each national index covers around 60 percent of the respective stock market. The focus is on stocks with good liquidity and free float. Finally, there is some attempt to ensure that the index reflects the industry characteristics of the overall market. Hence, about 60 percent of each industry group are targeted for inclusion in each MSCI country index. The indices are all market capitalization weighted on a total return basis, including the dividends paid. Cumby and Glen (1990) further find that 99 percent of non-U.S. stocks covered in the MSCI-world index are readily purchasable by non-nationals. Hence, investors should have been able to implement most of the strategies tested below.

*IFC stock market indices:* Monthly data on emerging markets' equity indices from 1976.01 to 1998.08 are from the International Finance Corporation (IFC). IFC's definition of an emerging stock market is aligned only to an emerging economy criterion: if a country's GNP per capita did not exceed the World Bank's threshold for being a high income country (i.e., if a country was eligible to borrow from the World Bank), its stock market was said to be emerging. IFC indices are intended to represent the performance of the most active stocks in their respective stock markets and to be the broadest possible indicator of market movements. The target aggregate market capitalization of IFC index constituents is 60 to 75 percent of the total capitalization of all exchange-listed shares. IFC indices do not take foreign investment restrictions into account and do not attempt to replicate the composite, regional, or industry balances in overall market capitalization. The indices are all market capitalization weighted on a total return basis, including the dividends paid. The emerging markets we include in our analysis are Argentina, Brazil, Chile, Greece, India, Korea, Malaysia, Mexico, Pakistan, Taiwan, Thailand, Turkey, Venezuela, the Philippines, and Portugal.

Summary statistics for MSCI and IFC stock market returns from 1973.06 to 1998.08 and from 1976.01 (or 1985.01) to 1998.08, respectively, are shown in Table 1. The statistics include average (annualized) arithmetic and geometric returns, as well as the standard deviation. In all tests we apply continuously compounded returns measured in Swiss francs. For the MSCI markets local currency returns are translated into Swiss francs using the effective exchange rate on the last trading day of each month. For the IFC markets the returns in U.S. dollars are translated into Swiss francs by the same method. The difference between arithmetic and geometric average returns is well known. The arithmetic average assumes a rebalancing strategy, requiring equal investment in each period. Gains from one period to another are not reinvested (i.e., the total amount invested is kept constant). In contrast, the geometric average has the more intuitive interpretation of a buy-and-hold strategy. A fixed amount is invested at the beginning, and the portfolio is held until the end of the sample. It is implicitly assumed that any cash-flows occurring during the period of investigation are reinvested.

In the developed markets (arithmetic) mean returns in Swiss francs range from 16.286 percent (in Sweden) to 7.714 percent (in Singapore). The range is significantly larger across emerging markets: an average annual (arithmetic) return of 54.241 percent in Argentina is contrasted to an average annual loss of -9.342 percent in Indonesia.<sup>15</sup> There is no market among the MSCI sample that posts an arithmetic average of over 20 percent. In contrast, 9 out of the 18 IFC markets' mean returns exceed 20 percent (Argentina, Brazil, Chile,

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<sup>15</sup> It should be noted, however, that the Indonesian sample is the shortest of all emerging markets, starting only in 1990.01. It is not used in the empirical analysis below.

**Table 1: Summary statistics for stock returns**  
 Period: 1973.06 – 1998.08 (monthly data)

MSCI markets:	Starting year and month	Swiss francs returns			Hedged returns		Autocorrelations		
		Arithm. mean in % annual	Geom. mean in % annual	S.D. in % annual	Geom. mean in % annual	S.D. in % annual	$\rho_1$	$\rho_2$	$\rho_3$
Australia	1973.06	10.424	6.164	29.805	5.970	22.950	-0.022	0.010	-0.059
Austria	1973.06	8.756	6.630	20.457	5.940	19.163	0.162	-0.024	0.005
Belgium	1973.06	13.188	11.397	18.598	9.873	16.922	0.145	-0.045	0.099
Canada	1973.06	8.746	6.138	22.850	6.444	17.137	0.042	0.009	-0.038
Denmark	1973.06	10.729	8.988	18.439	7.864	17.093	0.064	0.069	-0.055
France	1973.06	12.029	9.258	23.402	8.467	21.369	0.085	-0.055	-0.052
Germany	1973.06	12.415	10.468	19.568	9.441	18.201	0.037	-0.047	0.011
Hong Kong	1973.06	15.043	8.117	38.859	10.013	36.605	0.053	-0.026	0.010
Italy	1973.06	10.908	6.034	31.107	5.135	25.335	-0.038	-0.009	0.059
Japan	1973.06	8.940	6.396	22.489	6.091	18.262	0.048	-0.001	-0.027
Netherlands	1973.06	15.591	13.814	18.432	12.958	17.169	0.049	-0.018	0.074
Norway	1973.06	9.327	5.528	27.705	4.657	25.804	0.128	-0.058	0.085
Singapore	1973.06	7.714	2.949	31.586	8.106	29.211	0.101	-0.001	0.069
Spain	1973.06	9.745	6.609	25.098	7.399	21.178	0.123	0.056	0.036
Sweden	1973.06	16.286	13.332	24.024	13.233	21.637	0.068	-0.001	0.063
Switzerland	1973.06	12.739	11.267	16.885	11.267	16.885	0.088	-0.033	0.024
United Kingdom	1973.06	13.778	10.969	25.303	10.669	22.597	0.107	-0.081	-0.024
United States	1973.06	12.303	10.137	20.672	8.520	17.029	0.033	0.049	0.030
MSCI-World	1973.06	10.475	9.070	17.442	–	–	0.082	0.044	0.016

The table continues . . .

Table 1 (Continued)  
 Period: 1973.06 – 1998.08 (monthly data)

MSCI markets:	Starting year and month	Swiss francs			U.S. dollars			Autocorrelations		
		Arith. mean in % annual	Geom. mean in % annual	S.D. in % annual	Arith. mean in % annual	Geom. mean in % annual	S.D. in % annual	$\rho_1$	$\rho_2$	$\rho_3$
Argentina	1976.01	54.241	18.766	82.049	55.205	21.234	80.002	-0.001	0.021	-0.086
Brazil	1976.01	24.080	7.411	57.888	24.892	9.879	54.879	0.015	0.006	-0.011
Chile	1976.01	29.639	21.237	37.466	23.705	23.705	34.645	0.159	0.142	0.057
Colombia	1985.01	24.469	10.822	29.436	28.119	23.870	27.692	0.294	0.127	-0.024
Greece	1976.01	9.972	4.481	32.569	12.447	6.949	32.509	0.133	0.078	0.005
India	1976.01	13.113	8.780	29.158	15.052	11.248	27.212	0.152	0.011	-0.039
Indonesia	1990.01	-9.342	-20.271	46.194	-10.201	-19.952	45.459	0.254	-0.067	-0.010
Korea	1976.01	11.892	4.936	36.803	13.561	7.404	34.646	0.031	0.062	0.069
Malaysia	1985.01	1.646	-5.066	37.026	4.429	-1.018	33.274	0.174	0.016	0.024
Mexico	1976.01	23.448	11.417	50.681	24.055	13.885	46.541	0.225	-0.057	-0.029
Pakistan	1985.01	5.605	0.643	31.693	8.747	4.691	28.618	0.229	-0.049	-0.053
Philippines	1985.01	23.717	16.127	38.528	26.929	20.176	36.024	0.329	0.058	0.090
Portugal	1986.02	30.321	22.826	37.382	32.386	25.262	36.119	0.227	0.018	0.047
Taiwan	1985.01	24.350	11.702	50.171	26.522	15.751	45.967	0.081	0.074	0.099
Thailand	1976.01	10.694	4.399	35.822	12.083	6.867	32.585	0.113	0.138	0.087
Turkey	1987.01	40.808	20.525	62.111	41.579	21.142	62.277	0.117	0.012	-0.109
Venezuela	1985.01	17.608	5.797	49.617	21.245	9.845	48.658	0.007	0.156	-0.046
IFC-Composite	1985.01	10.083	5.902	29.080	12.526	9.951	22.662	0.181	0.115	0.035

The table provides descriptive statistics for global stock market returns. Total return indices on a monthly basis are provided by Morgan Stanley Capital International (MSCI) and the International Finance Corporation (IFC).  $\rho_i$  denotes the autocorrelation at lag  $i$ . S.D. is the standard deviation.

Colombia, Mexico, Philippines, Portugal, Taiwan, and Turkey). These higher returns do not come at zero cost. Emerging market returns are characterized by very high volatility, which explains the big differences between the arithmetic and the geometric returns for most of the IFC markets. The most dramatic example is Argentina, where the arithmetic average is 54.241 percent, but the geometric average is only 18.766 percent.<sup>16</sup> The annualized volatility in the IFC sample ranges from 82.05 percent in Argentina to 29.16 percent in India. The volatility in the developed markets is between 17 percent and 39 percent. The latter maximum is for Hong Kong, which seems to be more of an exception rather than a representative value. The reported autocorrelations in the last column of Table 1 measure persistence (i.e., the predictability of market returns on the basis of past market returns). There are six markets in the IFC sample that exhibit autocorrelations greater than 20 percent. This suggests that returns in emerging markets are easier to predict on the basis of past information.<sup>17</sup>

For the sample of MSCI markets we also report hedged returns, assuming that a Swiss investor covers all the foreign exchange risk inherent in his or her initial investment. The return in Swiss francs on foreign asset  $i$  from a unitary hedging strategy, denoted  $R_{i,t+1}^h$ , is:

$$(11) \quad R_{i,t+1}^h = R_{i,t+1} + \frac{F_{i,t} - S_{i,t+1}}{S_{i,t}},$$

where  $R_{i,t+1}$  is the (uncovered) Swiss franc return on the market index from country  $i$ ,  $F_{i,t}$  denotes the time  $t$  forward Swiss franc price for foreign currency  $i$ , and  $S_{i,t}$  is the spot price of currency  $i$  at time  $t$ . We call this a unitary hedging strategy, because for every dollar investment in, lets say, the United States the Swiss investor sells one dollar forward. Of course, this is neither a minimum-variance hedge nor a full hedge. While the initial investment is protected, the return is not. Total (full) coverage of foreign exchange risk is impossible, because the investor does not know beforehand how much foreign currency will have to be converted.

Unfortunately, direct one-month forward prices for the full sample period are only available for Canada, France, Germany, Japan, the Netherlands, the United Kingdom, and the United States from Data Resources Incorporated

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<sup>16</sup> Goetzman and Jorion (1999) argue that recent returns may not give a representative picture of the expected performance of emerging markets. In fact, emerging markets' return series suffer from a 'survivorship bias'. For example, Argentina was a very active stock market at the beginning of the last century, but ceased to exist afterwards to reemerge only in the 1970s. Their simulation results show that recently emerged markets ought to have high observed returns, and that the brevity of a market history is related to the bias in returns.

<sup>17</sup> See Harvey (1995) and Kim and Singal (2000).

(DRI).<sup>18</sup> Much shorter series are available for the remaining countries. Any missing forward prices are reconstructed using covered interest rate parity:

$$(12) \quad F_{i,t} = \frac{1 + r_t}{1 + r_t^*} S_{i,t},$$

where  $r_t$  is the 1-month Eurocurrency interest rate denominated in Swiss francs, and  $r_t^*$  is the respective foreign interest rate.<sup>19</sup> Exchange rates are taken from Datastream, the interest rates are from the Bank for International Settlement (BIS).<sup>20</sup> The motivation for the unitary hedging policy in equation (11) is based on the assumption that the investor cannot tell whether current risk premiums are positive or negative.<sup>21</sup>

Theoretically as well as empirically, that there are good reasons for the existence of currency risk premiums. Models of international portfolio choice such as Solnik (1974) and Adler and Dumas (1983) assert that investors hold foreign currencies in their portfolios for both hedging purposes and for speculative reasons. Drummen and Zimmermann (1992) show that a considerable fraction of currency risk is systematic. Empirical evidence for time variation in currency risk premiums is provided in Dumas and Solnik (1995) and De Santis and Gerard (1998). At least in theory, therefore, optimal hedge ratios should be below unity. The empirical evidence on the benefits of currency hedging is ambiguous. Glen and Jorion (1993) find that hedged strategies outperform unhedged ones, while Levy and Lim (1994) find the opposite result. The analysis of our data set in Table 1 reveals that in only four cases (Canada, Hong Kong, Singapore, and Spain) hedging leads to an increase in the average returns. In all cases, return volatility is successfully reduced compared to the unhedged returns, which is consistent with recent results by Solnik (1998).

In the conditional version of spanning tests we use two global instrument variables to construct scaled returns: (i) the world dividend yield as provided by Datastream and (ii) an aggregated G-7 term spread.<sup>22</sup> The time  $t$  dividend yield is calculated as the average value of dividends paid over the last twelve months on the Datastream world market index, divided by the value of the index at time  $t$ . The G-7 term spread is the average difference between the

<sup>18</sup> Forward prices are computed as averages of bid and ask prices.

<sup>19</sup> For the later periods, when both actual and estimated forward prices are available, the correlation between the two series is well above 0.9 for all countries. This indicates that estimated forward prices are good approximations for missing forward prices.

<sup>20</sup> Missing Eurocurrency rates for some countries during the early sample period were substituted by locally available money market instruments.

<sup>21</sup> See Perold and Schulmann (1988).

<sup>22</sup> A global dividend yield and a global term spread as instrument variables were used by Harvey (1991a,b) and Ferson and Harvey (1993), among others.



yield on long-term government bonds (with maturities of at least five years) and the 1-month Eurocurrency interest rate for the G-7 countries. The countries' relative shares of the G-7 real GDP are used as the country weights. The data for the yields on long-term government bonds and the real GDP are taken from the International Monetary Fund (IMF). The time series for Eurocurrency interest rates are from the Bank for International Settlement (BIS).

## 5. Empirical results

### 5.1 Methodological issues

In our empirical analysis Swiss stock market returns and the 90-days Eurocurrency interest rate denominated in Swiss francs serve as the benchmark (spanning) assets. The set of test assets comprises international equity indices from the MSCI and IFC databases. They are grouped according to the following geographical criteria:

- Europe 1: France, Germany, United Kingdom
- Europe 2: Austria, Belgium, Denmark, Italy, the Netherlands, Norway, Spain, Sweden
- Europe: Europe 1 plus Europe 2
- Europe plus North America: Europe, Canada, United States
- Pacific Basin: Australia, Hong Kong, Japan, Singapore
- Asia: India, Korea, Malaysia, Thailand, Pakistan, Taiwan, Philippines
- Latin America: Argentina, Brazil, Chile, Colombia, Mexico, Venezuela
- Europe 3: Greece, Portugal, Turkey
- IFC1: Argentina, Brazil, Chile, Greece, Mexico, India, Malaysia, Thailand (start 1976.01)
- IFC2: Asia Composite, Latin America Composite (start 1985.01)

'Europe 1' includes the large European equity markets, 'Europe 2' the smaller European stock markets in the MSCI set, and 'Europe 3' the European stock markets contained in the IFC database. The remaining MSCI markets are classified as 'North America' and 'Pacific Basin'. The grouping of IFC constituents is similar. In addition to 'Europe 3', there are regional subsets denoted as 'Asia' and 'Latin America'. A full set of data for emerging equity markets is only available starting 1985.01. Investigating structural breaks is particularly interesting for these markets. Taking data for all IFC countries from as late as 1985.01 onward, and dividing the sample into two periods of approximately equal length leads to a proliferation of orthogonality condi-

tions. Hence, there is a danger of unreliable results.<sup>23</sup> To preserve the power of GMM tests, 'IFC1' includes only the eight emerging stock markets with data available from as early as 1976.01, and 'IFC2' contains the two (value-weighted) composite indices for Asia and Latin America, both available starting 1985.01. This approach is somewhat pragmatic, but it helps to reduce the dimensionality of the problem. To explore whether any changes in the benefits of diversification have occurred over time (i.e., to detect a possible structural break), both 'IFC1' and 'IFC2' are divided into two subperiods of approximately equal length.

The system of equations in (10) tests the null hypothesis that the two Swiss assets suffice to span the volatility bounds associated with different international portfolio strategies. The statistical significance is measured via the chi-square test for the goodness-of-fit of the overidentifying restrictions. However, this is only hard to interpret economically. De Santis (1995) and Bekaert and Urias (1996) propose a simple method to assess the economic significance of this change. In particular, they suggest to measure the distance between the two bounds at the value of  $E(m_{t+1})$ , which corresponds to the minimum of the bound for the Swiss spanning assets. Rewriting equation (5) reveals that this measure is equal to the change in the Sharpe ratio, divided by the risk-free rate of return that is implied by the choice of  $E(m_{t+1}) = R_{f,t+1}^{-1}$ . The mean discount factor at the minimum point is very close to one and, therefore, the change in the volatility bounds at this point can be interpreted as the approximate increase in expected returns per unit of risk attainable through international diversification. In a nutshell, this measure allows an assessment of the enhanced risk-return spectrum that is available for global investors. The intuition behind this measure is empirically supported by the following observation. The minimum of the Swiss bound corresponds to a monthly risk-free rate of 0.39 percent, or 4.69 percent per year. This value is close to the observed sample mean of the one-month Eurocurrency interest rate denominated in Swiss francs, which is 0.36 percent on a monthly basis, or 4.43 percent per year. It must be noted, however, that this interpretation only applies in the unconditional framework. In the conditional case, spanning tests are expressed in terms of payoffs and returns instead of returns only. Although we also report the magnitude of the shift for all conditional versions of the model, these numbers should not be given too much importance.

To investigate the role of hedging, we estimate two versions of the model. First, we assume that investors do not hedge at all and are fully exposed to currency risk. Second, investors apply a unitary hedging strategy using currency forward contracts. Results of international portfolio theory indicate that unitary hedging can be optimal only under four stringent conditions.<sup>24</sup> First,

<sup>23</sup> For a discussion of the 'saturation ratio' see Ferson and Foerster (1994).

<sup>24</sup> See Adler and Granito (1991), Jorion (1994), and Solnik (1998).

inflation must be non-stochastic. Second, apart from inflation there are no other state-variables that influence prices and are correlated with the exchange rate. Third, the forward risk-premium must be zero at all times (i.e., uncovered interest rate parity holds). And fourth, domestic and foreign stock and bond prices must be uncorrelated with the exchange rate. All of these conditions are violated empirically, at least in the short-run. Intuitively, unitary hedging implies that investors ignore the speculative character of currencies. They constitute an independent asset class with distinctive risk-return characteristics. Unitary hedging does not optimally exploit the correlation structure between stocks and forward contracts. Therefore, if unitary hedging offers additional diversification benefits for a Swiss investor, the results rather underestimate the true importance of currency risk management.

## 5.2 Unconditional bounds for developed stock markets

Table 2 shows the results for unconditional spanning tests on the basis of the system of equations in (10), using returns from developed equity markets. We report the value of the chi-square test statistic, the *p*-value (in italics), and the magnitude of the shift of the bound. The minimum of the purely Swiss Hansen/Jagannathan bound is 0.113, and the corresponding  $E(m_{t+1})$  implies a monthly risk-free rate of 0.39 percent. Overall, our results shed doubt on the conjecture that the purely Swiss discount factor is unable to price a larger set of assets and that more volatility is required. When we use unhedged returns, in general, our point estimates are not statistically significant. The shifts of the bound are lost in sampling error, as indicated by the high *p*-values in Table 2. The smallest *p*-value is 0.194 for 'Europe 2'. From a purely statistical perspective, we therefore conclude that the null hypothesis that Swiss assets suffice to span the global Hansen/Jagannathan bound cannot be rejected. In other words, international stock returns are too noisy to detect significant gains from diversification.

The economic significance of the shifts can be evaluated on the basis of changes in the Sharpe ratio. As expected, changes in Sharpe ratios are inversely related to the *p*-values associated with mean-variance spanning tests. In line with intuition, the largest shift of the bound occurs for the 'Global' (0.135) set of test assets. It is also high when 'Europe plus North America' (0.111) and 'Europe 2' (0.096) are added, while it is considerably smaller for 'Pacific Basin' (0.028). Also intuitive, the lowest change results for the large European stock markets and the United States, 'Europe 1 plus USA' (0.003). These point estimates can be translated into ex ante gains from international diversification, using the level of Swiss stock market volatility as the benchmark (which is 4.87 percent on a monthly basis; see Table 1). The monthly risk-adjusted gain that can be expected from a global diversification strategy is equal to  $0.135 \times 0.0487 = 0.0066$  or 0.66 percent. For 'Europe plus North

America’ this number is 0.54 percent, for ‘Europe 2’ it is 0.47 percent, and for ‘Pacific Basin’ 0.14 percent. Hence, the gains from investing throughout Europe mainly stem from the different risk-return menus associated with the smaller capitalization markets. In mean-standard deviation language, the Sharpe ratio improves from a Swiss perspective, although the trading strategy implies investing in some of the smaller, more volatile markets. Nevertheless, it must be emphasized that while these changes appear large from an economic point of view, they are not statistically significant.

Table 2

**Unconditional investment strategies for MSCI stock markets**

System of non-linear equations for GMM estimation:

$$E \left\{ \begin{matrix} \mathbf{R}_{t+1}c_1 + \mathbf{R}_{t+1} [\mathbf{R}_{1,t+1} - E(\mathbf{R}_{1,t+1})]' \boldsymbol{\beta}_{1,c1} - \mathbf{1} \\ \mathbf{R}_{t+1}c_2 + \mathbf{R}_{t+1} [\mathbf{R}_{1,t+1} - E(\mathbf{R}_{1,t+1})]' \boldsymbol{\beta}_{1,c2} - \mathbf{1} \end{matrix} \right\} = 0$$

Period: 1973.06 – 1998.08 (monthly data)

Set of test assets	Investment strategy				
	d.f.	No Hedging		Full Hedging	
		Change of vol. bound	$\chi^2$ -statistic (p-value)	Change of vol. bound	$\chi^2$ -statistic (p-value)
Europe	22	0.096	22.985 (0.403)	0.103	24.928 (0.301)
Europe plus North America	26	0.111	30.363 (0.253)	0.125	38.503 (0.054)
Europe 1 plus USA	8	0.003	3.634 (0.889)	0.008	4.638 (0.796)
Europe 2	16	0.092	20.599 (0.194)	0.095	20.668 (0.191)
Pacific Basin	8	0.028	7.641 (0.469)	0.032	8.429 (0.393)
Global	34	0.135	38.485 (0.274)	0.153	51.880 (0.025)

The system of orthogonality conditions in (10) is tested by GMM for a fixed set of Swiss spanning assets and varying sets of unhedged and hedged test assets. The spanning assets are the MSCI stock market index for Switzerland and the 90-days Eurocurrency interest rate, both denominated in Swiss francs. The different sets of test assets contain international stock market indices and are defined as follows: ‘Europe 1’ (Germany, France, United Kingdom), ‘Europe 2’ (Austria, Belgium, Denmark, Italy, Netherlands, Norway, Spain, Sweden), ‘North America’ (United States, Canada), and ‘Pacific Basin’ (Australia, Japan, Hong Kong, Singapore). ‘Europe’ consists of Europe 1 and Europe 2, ‘Global’ covers all countries in the MSCI dataset. d.f. denotes the degrees of freedom.

Intuitively, rejecting the null hypothesis requires either higher average returns or lower volatility. One way to reduce volatility is to hedge the currency

component of international returns. Indeed, the entries in the right panel of Table 2 indicate that hedging helps to further improve the performance of an international portfolio strategy and to bring some of the  $p$ -values down to statistically significant levels. First, unitary hedging increases the gains from global diversification ('Global') by an additional  $(0.153 - 0.135) \times 0.0487 = 0.09$  percent. This is not obvious at all, because investors do not choose an optimal amount of forward contracts in this case. Second, unitary hedging sufficiently reduces the noise in returns. The shifts in the bounds are statistically significant for the 'Global' and 'Europe plus North America' sets of test assets, at least at the 10 percent level. Again, the results are mainly driven by the smaller European countries in 'Europe 2'.

These results allow another interesting interpretation. The advantage of the Hansen/Jagannathan apparatus is that the resulting bounds contain direct pricing implications. In the model specified in (10), coincidence of bounds implies that a stochastic discount factor that prices a portfolio of Swiss assets equally qualifies as a valid stochastic discount factor for an international set of stocks. Therefore, the results in Table 2 can be interpreted as a failure to reject the null hypothesis of stock market integration, at least when unhedged returns are used. On the other hand, after eliminating the greater part of foreign exchange risk, the Swiss discount factor is at least unable to price the 'Global' and 'Europe plus North America' sets. However, such interpretations must be made with utmost care, because we test a nonparametric setup (i.e., a framework that does not assume a specific asset pricing model to hold).<sup>25</sup>

### 5.3 Conditional bounds for developed stock markets

Scaling returns by proper instrument variables allows to recover some implications of the conditional model in equation (1) within the unconditional version of the Euler-equation in (2). Tests of conditional mean-variance spanning are easily derived by using scaled payoffs and continuing as if conditioning information did not exist (i.e., estimating the system in (10) in the usual way). As described above, we use two global instrument variables to condition the model: (i) the world dividend yield as provided by Datastream and (ii) an aggregated G-7 term spread. Unfortunately, a severe practical problem is the rapid proliferation in the number of orthogonality conditions to be tested. For this reason, the 'Global' dataset has to be excluded from the analysis. As shown above, adding conditioning information expands the space of assets to be priced by actively managed portfolios. Accordingly, the conditional volatility bound must be tighter than the associated unconditional bound. The disadvantage is that the elegant Sharpe ratio analogy cannot be applied, because the

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<sup>25</sup> See Chen and Knez (1996) for market integration tests on the basis of volatility bounds.

test is now expressed in terms of prices and payoffs rather than returns anone. Therefore, little confidence should be put in the reported magnitudes of the shifts.

Table 3

**Conditional investment strategies for MSCI stock markets**

System of non-linear equations for GMM estimation:

$$E \left\{ \begin{matrix} \mathbf{R}_{t+1}c_1 + \mathbf{R}_{t+1} [\mathbf{R}_{1,t+1} - E(\mathbf{R}_{1,t+1})]' \boldsymbol{\beta}_{1,c1} - \mathbf{1} \\ \mathbf{R}_{t+1}c_2 + \mathbf{R}_{t+1} [\mathbf{R}_{1,t+1} - E(\mathbf{R}_{1,t+1})]' \boldsymbol{\beta}_{1,c2} - \mathbf{1} \end{matrix} \right\} = 0$$

Period: 1973.06–1998.08 (monthly data)

Set of test assets	Investment strategy				
	d.f.	No Hedging		Full Hedging	
		Change of vol. bound	$\chi^2$ -statistic ( <i>p</i> -value)	Change of vol. bound	$\chi^2$ -statistic ( <i>p</i> -value)
Europe	66	0.181	100.897 (0.004)	0.213	100.008 (0.004)
Europe plus North America	78	0.196	100.312 (0.045)	0.233	104.483 (0.024)
Europe 1 plus USA	24	0.021	29.194 (0.194)	0.044	20.399 (0.674)
Europe 2	48	0.160	101.817 (0.000)	0.192	142.599 (0.000)
Pacific Basin	24	0.061	74.681 (0.000)	0.126	97.882 (0.000)

The system of orthogonality conditions in (10) is tested by GMM for a fixed set of Swiss spanning assets and varying sets of unhedged and hedged test assets. The spanning assets are the MSCI stock market index for Switzerland and the 90-days Eurocurrency interest rate, both denominated in Swiss francs. The different sets of test assets contain international stock market indices and are defined as follows: ‘Europe 1’ (Germany, France, United Kingdom), ‘Europe 2’ (Austria, Belgium, Denmark, Italy, Netherlands, Norway, Spain, Sweden), ‘North America’ (United States, Canada), and ‘Pacific Basin’ (Australia, Japan, Hong Kong, Singapore). ‘Europe’ consists of Europe 1 and Europe 2. The global information variables used to scale returns are the world dividend yield and a G-7 term spread of interest rates. d.f. denotes the degrees of freedom.

Results are shown in Table 3. As expected, the point estimates for the conditional bounds are higher than the corresponding unconditional ones. More important, the *p*-values drop dramatically for all sets of test assets, except for ‘Europe 1 plus United States’. This is true for hedged as well as unhedged portfolio positions. The shifts of the Hansen/Jagannathan bound become significant at the 1 percent level for three sets: ‘Europe 2’, ‘Europe’, and ‘Pacific Basin’. The sharp drop of the *p*-value for unhedged returns in ‘Europe 1 plus United States’ and its immediate increase when hedged returns are used are only hard to interpret. However, both shifts are statistically insignificant.

The results so far have three major implications. First, the dramatic improvement in the volatility bound for scaled returns provides another strong evidence for the predictability of stock returns. This supports previous studies using simple linear regression analysis. Second, active asset allocation with periodic adjustments of portfolios based on new information helps to exploit the correlation structure between international stock returns. The risk-return spectrum of a global investor expands both economically and statistically. In light of the recent findings in Bekaert and Liu (1999) and Ferson and Siegel (1999) that optimal scaling functions are nonlinear, the simple linear scaling rule might even underestimate the true benefits of incorporating economic instrument variables. Finally, once conditioning information has been taken into account, hedging becomes an issue of minor importance for a Swiss investor.

#### 5.4 Bounds for emerging stock markets

Using emerging market data in tests of mean-variance spanning is of great practical interest. Alternative investment strategies have become very popular in the last few years, and emerging market investments are usually classified under this category. The attraction emerging markets have received can be explained by their distinct risk-return profiles, coupled with their low correlations with other stock markets.<sup>26</sup> Divecha, Drach, and Stefek (1992) and Speidel and Sappenfiel (1992) even argue that a ‘diversification free lunch’ awaits investors in emerging stock markets. In fact, IFC markets exhibited higher average returns than developed markets in the past. The average correlations between IFC and MSCI stock markets have also been lower than those between the developed stock markets themselves. Intuitively, this should make it easier to reject the null hypothesis of mean-variance spanning.

Table 4 reports the results for mean-variance spanning tests using various subsets of IFC stock market indices. Panel A displays the results for the period from 1985.01 to 1998.08. For ‘Asia’ (India, Taiwan, Thailand, Malaysia, Korea, Pakistan, and the Philippines) the null hypothesis of spanning can be rejected at the 5 percent level of significance. For a Swiss investor portfolio diversification over these seven markets would have led to a monthly gain of  $0.12 \times 0.0487 = 0.58$  percent. This is a substantial change, recalling that global diversification over all 18 stock markets in the MSCI database has been associated with an only slightly higher monthly gain of 0.64 percent. The results are different for ‘Latin America’. This set contains some of the more established markets in the IFC database (such as Argentina, Brazil, Chile, and Mexico). Although the monthly risk-adjusted gain remains economically significant (0.40 percent monthly), the shift is only moderate from a statistical

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<sup>26</sup> See Harvey (1995).

point of view with a  $p$ -value of 0.173. Unfortunately, the results for 'Europe 3' are only hard to interpret. The shift of the bound is neither economically nor statistically significant for this subset of IFC markets.

The results could be driven by the most recent years, when correlations have increased notably. To explore this possibility, subset 'IFC 1' (Argentina, Brazil, Chile, Greece, Mexico, Thailand, India, Korea) and 'IFC 2' (Asia Composite and Latin America Composite) are divided into two subperiods of approximately equal length. Estimation results are reported in Panel B of Table 4. Indeed, for the more advanced emerging markets in 'IFC 1' the gains from international diversification during the early period from 1976.01 to 1986.12 were huge. A Swiss investor could have gained  $0.260 \times 0.0487 = 1.27$  percent on a monthly basis, and this shift was significant at the 1 percent level. It must have been this impressive increase in risk-adjusted returns that established the myth of a 'diversification free-lunch' in emerging stock markets. Unfortunately, these times seem over. A look at the results for the more recent period from 1987.01 to 1998.08 shows that the magnitude of the shift in the bound has dropped significantly. The Hansen/Jagannathan bound still becomes tighter, but monthly gains from diversification have come back to a level comparable to the smaller European markets (0.46 percent per month) and are no longer significant at the 10 percent level. Results are similar for 'IFC 2', the set containing the (value-weighted) composite indices for Asia and Latin America. Over the early period from 1985.01 to 1991.06 the gains of 0.54 percent per month are significant at the 5 percent level. In the most recent period from 1991.07 to 1998.08, however, the magnitude of the shift drops to a monthly 0.41 percent and is lost in sampling error.<sup>27</sup> These results are consistent with Goetzman and Jorion (1999). Their simulation evidence shows that high returns and low covariances are temporary phenomena that can be attributed to the recent emergence of Asian and Latin American stock markets. In particular, the history of emerging stock markets immediately after re-emergence provides an overly optimistic picture of future investment performance.

Finally, the columns on the right side of Table 4 contain the results for the conditional model. The point estimates rise to almost unbelievable levels, but again, one must be careful with interpretations. Nevertheless, the results can be interpreted as evidence that IFC returns are better predictable than MSCI returns.<sup>28</sup> On the other hand, given that the conditional version of the model must be understood as an actively managed portfolio strategy, the gains from diversification might only be hard to realize. The strategy requires periodic adjustments of portfolios based on new available information, which might be difficult due to thin trading and a variety of direct and indirect barriers for foreign investors. Finally, from an asset pricing perspective, the findings in

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<sup>27</sup> In a recent paper, Errunza, Hogan, and Hung (1999) find similar results.

<sup>28</sup> See Harvey (1995) and Hargis and Maloney (1996).



Table 4 indicate that emerging stock markets have become increasingly integrated with the world stock market.

Table 4

**Investment strategies for IFC stock markets**

System of non-linear equations for GMM estimation:

$$E \left\{ \begin{array}{l} \mathbf{R}_{t+1}c_1 + \mathbf{R}_{t+1} [\mathbf{R}_{1,t+1} - E(\mathbf{R}_{1,t+1})]' \boldsymbol{\beta}_{1,c1} - \mathbf{1} \\ \mathbf{R}_{t+1}c_2 + \mathbf{R}_{t+1} [\mathbf{R}_{1,t+1} - E(\mathbf{R}_{1,t+1})]' \boldsymbol{\beta}_{1,c2} - \mathbf{1} \end{array} \right\} = 0$$

Period: 1985.01 – 1998.08 (monthly data)

Set of test assets	Investment strategy					
	Unconditional			Conditional		
	d.f.	Change of vol. bound	$\chi^2$ -statistic ( <i>p</i> -value)	d.f.	Change of vol. bound	$\chi^2$ -statistic ( <i>p</i> -value)
<b>Panel A:</b>						
Asia	14	0.120	24.267 (0.043)	42	0.450	109.042 (0.000)
LatinAmerica	12	0.082	16.422 (0.173)	36	0.231	101.765 (0.000)
Europe 3	6	0.012	8.272 (0.219)	18	0.127	30.493 (0.033)
<b>Panel B:</b>						
IFC1 (1976.01 – 1986.12)	16	0.260	55.916 (0.000)	48	0.342	77.776 (0.004)
IFC1 (1987.01 – 1998.08)	16	0.094	23.040 (0.113)	46	0.734	85.415 (0.000)
IFC2 (1985.01 – 1991.06)	4	0.111	15.617 (0.029)	12	0.297	30.773 (0.002)
IFC2 (1991.07. 1998.08)	4	0.084	5.439 (0.245)	12	0.345	30.119 (0.003)

The system of orthogonality conditions in (10) is tested by GMM for a fixed set of Swiss spanning assets and varying sets of unhedged and hedged test assets. The spanning assets are the MSCI stock market index for Switzerland and the 90-days Eurocurrency interest rate denominated in Swiss francs. The different sets of test assets contain international stock market indices and are defined as follows: 'Asia' (India, Taiwan, Thailand, Malaysia, Korea, Pakistan, Philippines), 'Latin America' (Argentina, Brazil, Chile, Columbia, Mexico, Venezuela), and 'Europe 3' (Greece, Portugal, Turkey). 'IFC1' contains the indices for Argentina, Brazil, Chile, Greece, Mexico, Thailand, India, Korea (all starting 1976.01), and 'IFC2' consists of the (value-weighted) composite indices for Asia and Latin America (both starting 1985.01). The global information variables used to scale returns are the world dividend yield and a G-7 term spread of interest rates. d.f. denotes the degrees of freedom.

Overall, our empirical results largely confirm previous results for U.S. data from a Swiss perspective. De Santis (1995) and Bekaert and Urias (1996) also

report that it is harder to price international assets than domestic assets. The changes in Sharpe ratios reported from our estimation are similar in magnitude to those in their analysis. Perhaps most important, they also find that using conditioning information helps to exploit the benefits of international diversification. Finally, our results for emerging markets over two distinct subperiods confirm the findings in Errunza, Hogan, and Hung (1999). They report that the gains from international diversification beyond domestic diversification portfolios have diminished recently, as many emerging countries have liberalized their stock markets.

## 6. Conclusion

The aims of this paper have been manifold. First, we ask whether it is harder to price international assets rather than domestic ones. This is a nonparametric test of market integration. Second, we shed light on the benefits of international diversification from a Swiss perspective for both developed and emerging stock markets. Third, we reexamine the predictability of stock returns on the basis of well-known instrument variables. Finally, we explore the importance of currency hedging for the risk-return menu faced by a global investor. All these issues are analyzed in a unifying framework, using the Hansen/Jagannathan volatility bounds for stochastic discount factors. Their approach shifts the interest from familiar mean-variance representations for portfolio returns to mean-variance efficiency regions for admissible stochastic discount factors. In particular, given any set of asset returns, they demonstrate how to calculate a feasible region in mean-variance space for valid stochastic discount factors to fall into.

We perform modern versions of spanning tests in the spirit of De Santis (1995) and Bekaert and Urias (1996) to examine whether a Swiss investor can mimic foreign returns by holding only domestically traded assets. The point estimates of the bounds indicate that it is harder to price international assets than Swiss assets. This is especially true for the sample of emerging equity markets. The spanning tests indicate the required increase in the minimum volatility for valid stochastic discount factors when going from purely Swiss portfolio holdings to a portfolio that is diversified across different sets of global stock markets. When currency hedging is incorporated into the analysis, volatility bounds become even tighter. In other words, hedging helps to actually realize the benefits of international diversification. The magnitude of the effect might even be underestimated, because we only implement a simple unitary hedge. To give a notion of the actual magnitude of the improvement, we find that the annual expected gain was roughly 6 percent per year for a portfolio of all European and North American stock markets in the MSCI dataset over the sample period. Unitary hedging would have led to an additional 1 percent increase.

While economically large, we find that the shifts of the volatility bound are often lost in sampling error, at least for the developed markets. This provides evidence for global stock market integration. The insignificance of our estimates could also be interpreted as a possible explanation for the home bias in portfolio holdings. Most interesting, emerging stock markets have become increasingly integrated from a Swiss investor's point of view. This interpretation requires due care, because our test design does not rely on any specific form of an asset pricing model. Yet, the times – if they ever existed – when 'diversification free lunches' were readily available in these markets seem over.

Adding scaled returns and continuing as if conditioning information did not exist allows to recover some implications of conditional asset pricing for unconditional models. Using a linear rule to scale returns with the world dividend yield and an aggregated G-7 term spread, significant shifts (both economically and statistically) in the bounds can be interpreted as additional, more sophisticated evidence for the predictability of stock returns. We conclude that the dominant strategy for Swiss investors is to actively manage their portfolios on the basis of new available information. Although this strategy involves dynamic adjustment of portfolios across different geographical regions and, hence, is harder to implement on a day-to-day basis, it allows global investors to significantly enhance their Sharpe ratio.

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