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Welfare Implications of International Labor Migration*

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1. Introduction

In a seminal paper Berry and Soligo (1969) have demonstrated that, under quite general assumptions, international labor migration increases the income per capita of the native population in the country of immigration in the short run. If people with no or a relatively small capital endowment immigrate, then standard macroeconomic theory predicts both the wage rate in the economy to decrease and the rate of interest to rise. Berry and Soligo have shown that the decline in the natives' wage income is offset by the increase in their capital income. As a consequence the average income of the native population rises although the average income of the whole economy may well decline due to the relatively low per capita income of the immigrants. If rich people immigrate, then the rise in wages offsets the decline in capital income of the natives so that in this case the natives are again better off in terms of average income. This proposition has been confirmed and generalized by Khang (1990), Tu (1991), Borjas (1995), and others.

Kenen (1971), Rivera-Batiz (1982), Wong (1986) and Quibria (1988) have incorporated short-term effects of migration into models of international trade. It can be demonstrated that immigration never decreases the aggregate income of the natives if the terms of trade of the immigration country do not deteriorate. Again, economic theory predicts that, in general, the native population will benefit from immigration in the short run.

The results on the long-term welfare effects of migration have, however, been ambiguous so far. In their paper Berry and Soligo have conjectured that immigration raises the steady-state per capita income of the natives if the immigrants differ from the natives with respect to their "propensity to hold wealth". This conjecture has been challenged by Rodriguez (1975). In his overlapping-generations model the steady-state real income of the natives may well decline as a consequence of an initial inflow of immigrants

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because there are two effects which may, in the long run, work in opposite directions. First, per capita real income of the natives rises if immigration takes the economy closer to the Golden Rule path and falls if the capital stock moves away from its Golden Rule level. Second, income is redistributed towards the group of individuals whose savings pattern becomes relatively more scarce. Starting with Galor (1986), several authors have extended the analysis of Rodriguez by including endogenous migration in various versions of overlapping-generations models (Kemp and Kondo (1989), Kondo (1989), Galor and Stark (1991) and Meier (1994)). The welfare results in these papers are, however, very similar to those in the Rodriguez paper.

Instead of using an overlapping generations framework, we analyze a Solow-type growth model with a general neoclassical production function and constant savings rates. We will show that the conjecture of Berry and Soligo holds if all natives have the same savings rate. This result is in line with Steinmann (1994) where a CES technology is employed. However, we demonstrate that if the natives are heterogeneous in their savings behavior, immigration of individuals with a high propensity to save may lower the steady-state per capita income of the natives.

There are no welfare effects from immigration in the long run if the immigrants adopt the pattern of behavior of the native population. There are many cases of such perfect "assimilation". However, we also observe immigrants who remain a distinct sub-group with behavioral characteristics predominantly determined by their tradition. This applies obviously to the case of a perfect economic integration of two originally separate economies. While this is not immigration in a narrow sense, it is tantamount to immigration from a theoretical point of view. Suppose two countries decide to lift all barriers to the free movement of capital, labor, technology, commodities and services. In the ideal world of perfectly competitive markets all prices will be the same in the long run, and the two economies will also produce with the same technologies. However, the two populations may well remain separate and maintain their original savings rates, their rates of population growth, or other behavioral parameters. After integration the population of the united economy has two sub-populations whose members may well ask whether economic integration will be beneficial to them in the long run.

In our analysis we proceed in three steps. First we assume that the initial endowments with labor and capital are exogenously given and that these endowments change due to immigration. This is the type of framework Berry and Soligo have already analyzed. Next we consider a growing economy with an initial flow of immigration where the savings ratio of the immigrants is different from the savings ratio of the natives. Immigration is permanent, i.e. the immigrants do not return to their country of origin, and both the immigrants and the natives maintain their savings behavior over time. In the third step we allow for differences in the savings behavior among the natives. Immigration then changes the initial mix of people with respectively high and low savings rates. In all three steps we ask how immigration affects the average income of the natives as well as of the whole economy.

2. The Basic Elements of the Model and the Short-Run Effects of Migration

In our model there are only two types of individuals, "natives" and "immigrants". Their numbers respectively are $L_0 > 0$ and $L_1 \geq 0$. Total population is $L = L_0 + L_1 > 0$. We assume that every individual is also a member of the workforce so that L represents the total supply of labor. There is no unemployment and every laborer receives the same wage rate. Instead of full employment we could also assume exogenously given rates of unemployment for both population groups.

The capital stock of the economy K is partly owned by natives ($= K_0$) and partly by the immigrants ($= K_1$). We assume $K_0 > 0$ and $K_1 \geq 0$ so that $K = K_0 + K_1 > 0$. Wealth per capita is denoted by $k_0 = \frac{K_0}{L_0}$ and $k_1 = \frac{K_1}{L_1}$ respectively. No capital is owned by individuals living abroad.[1] Then the capital-labor ratio $k = \frac{K}{L}$ of the whole economy is

$$k = mk_0 + (1 - m)k_1 \tag{1}$$

where $m = \frac{L_0}{L}$ and $(1 - m) = \frac{L_1}{L}$ are the fractions of respectively the natives and the immigrants in the total population.

The economy produces its output Y with the two inputs L and K. The marginal productivities of both capital and labor are positive and decreasing and there are constant returns to scale. The technology can then be represented by a per capita production function

$$y = f(k), \tag{2}$$

in which $y = \frac{Y}{L}$ is the average productivity of labor. Given our assumptions the marginal productivity of capital $f'(k)$ and the marginal productivity of labor $f(k) - kf'(k)$ are both positive and we have $f''(k) < 0$, i.e. the marginal productivity of capital is decreasing.

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1 Omitting this assumption can lead to completely different results, see Berry (1974).

The two factors are paid their respective marginal products:

$$w = w(k) = f(k) - kf'(k), \tag{3}$$

$$r = r(k) = f'(k), \tag{4}$$

where w denotes the real wage rate and r the rate of interest.

Since we assume that immigration is non-recurring, there is no further immigration after some initial immigration in the beginning of our period of analysis. The immigrants may come with or without capital. Once they have arrived they offer their services in the labor market and they invest the capital they own in the host economy. This will, in general, affect both the wage rate and the rate of interest. The newcomers may also save and accumulate capital. In this section, however, we consider only the impact of immigration on the wage-interest ratio without taking into account capital accumulation so that our analysis captures only short-run effects.

Let us now first assume that there are only natives before the newcomers arrive. Then $L_1 = 0$, $L_0 = L$, $k = k_0$, and $m = 1$ initially, and $L_1 > 0$, $m < 1$ after immigration.

The natives' income before immigration is $y_0 = f(k_0)$. After immigration the capital-labor ratio is $\bar{k} = mk_0 + (1-m)k_1$ where k_1 denotes the wealth per capita of the newcomers. The natives' new income is $\bar{y}_0 = w(\bar{k}) + r(\bar{k})k_0 = f(\bar{k}) + f'(\bar{k})(k_0 - \bar{k})$. If $k_0 = k_1$, so that the natives are as "rich" as the immigrants, then immigration does not change the capital-labor ratio and, hence, $y_0 = \bar{y}_0$. Let us, therefore, assume $k_0 \neq k_1$ so that immigration changes the initial capital-labor ratio in the economy. Then it can be shown that the income of the natives rises, i.e.

$$y_0 = f(k_0) < f(\bar{k}) + f'(\bar{k})(k_0 - \bar{k}). \tag{5}$$

A proof of this proposition can be found in Khang (1990).

The economic interpretation of this result is straightforward. If relatively rich people immigrate so that the capital-labor ratio increases ($\bar{k} > k_0$), then both the average income in the economy and the wage rate rise $(f(\bar{k}) > f(k_0), w(\bar{k}) > w(k_0))$ while the rate of interest declines $(r(\bar{k}) < r(k_0))$. The whole economy is better off in terms of average per capita income. The natives benefit from the immigration because the increase in the wage income is higher than the decline in capital income.

If the immigrants' per capita wealth is smaller than the initial capital-labor ratio, then the capital-labor ratio declines ($\bar{k} < k_0$). Average income per person in the economy and the wage rate, too, also go down

$(f(\bar{k}) < f(k_0), w(\bar{k}) < w(k_0))$. However, since f is concave, there is a relatively strong increase in the rate of interest. The natives' rise in capital income offsets the decline in the wage income they receive.

As we may assume that the immigrants' income in the host country is also larger than the income they earned in their country of origin, immigration makes no one worse off in the country of destination provided that every native owns the same per capita wealth. However, if the capital stock of the natives is not equally distributed, there may be winners and losers among the natives. Laborers, for example, with little per capita wealth will not be compensated by an increase in capital income if their real wage falls due to an immigration of relatively poor people.

Next we analyze the extent to which the natives gain from immigration. Let us define $h = k_0 - \bar{k}$ and

$$G(h) = f(k_0 - h) - (k_0 - h)f'(k_0 - h) + k_0 f'(k_0 - h) - f(k_0). \tag{6}$$

Obviously, $G(h) = \bar{y}_0 - y_0$ is the natives' gain in average per capita income from immigration while $h = (1 - m)(k_0 - k_1)$ represents the difference in per capita wealth of respectively the natives and the total population. Differentiating G with respect to h yields

$$G'(h) = -hf''(\bar{k}). \tag{7}$$

The function G is strictly decreasing for $h < 0$ and strictly increasing for $h > 0$. From this we conclude that for any arbitrary h_1 and h_0 with $sgn[h_1] = sgn[h_0]$ the inequality $|h_1| > |h_0|$ implies $G(h_1) > G(h_0)$.

Therefore, in both cases, $k_0 < k_1$ and $k_0 > k_1$, a larger difference in per capita wealth between immigrants and natives implies a larger difference in the natives' per capita income before and after immigration. However, since the function G need not be symmetric, we can in the case of $sgn[h_1] \neq sgn[h_0]$ in general not expect that $G(h_1) > G(h_0)$ if $|h_1| > |h_0|$.

An intuitive explanation of the short-term results is straightforward. The principle that the price of a factor rises if it becomes relatively more scarce obviously also applies to factor combinations. Thus if, as a consequence of immigration, the "average native" becomes relatively more scarce, then his compensation in terms of income increases.

So far we have assumed that $m = 1$ before immigration. Let us now briefly look at the situation in which $m < 1$ initially. Then there is an old immigrant population whose share in the total population is $(1 - m)$. We assume that the "new" immigrants own the same per capita wealth k_1 as the old ones. Otherwise there would be a third group after immigration, a case which we do not want to discuss here. The usage of the terms "old" and "new" does not mean that much time has elapsed between the different waves of migration.

If $m < 1$ then y_0, the natives' income, and y_1, the immigrants' income, are both differentiable functions of m:

$$y_0(m) = f(k) + f'(k)(k_0 - k) = f(k) + f'(k)(1-m)(k_0 - k_1), \tag{8}$$

$$y_1(m) = f(k) + f'(k)(k_1 - k) = f(k) - f'(k)m(k_0 - k_1) \tag{9}$$

with $k = mk_0 + (1-m)k_1$.

Differentiation with respect to m yields

$$y_0'(m) = (1-m)(k_0 - k_1)^2 f''(k) < 0, \tag{10}$$

$$y_1'(m) = -m(k_0 - k_1)^2 f''(k) > 0. \tag{11}$$

An increase in the immigrant population, i.e. a decrease in m, therefore reduces the income of the old immigrants whereas the income of the natives rises. Whether total average income in the economy grows or declines depends again on the change in the capital-labor ratio in the economy $k(m) = mk_0 + (1-m)k_1$. Differentiation yields $k'(m) = k_0 - k_1$. Thus if m decreases as a consequence of immigration, then both the average income $y = f(k(m))$ and the capital-labor ratio $k(m)$ decrease if $k_0 > k_1$, i.e. if the immigrants are poorer in terms of per capita wealth than the natives. For $k_0 < k_1$ the opposite holds true. Thus also for $m < 1$ we have obtained the result that, on the average, the native population gains from immigration.

Emigration instead of immigration reverses all results. Furthermore, in the case of immigration the natives' gain depends on the assumption that they do not share the additional financial burden caused by the immigrants for the provision of public goods and other services.

3. The Long-Run Effects of Migration

Let us now take capital accumulation into account. In order to then study the consequences of migration we consider a strictly neoclassical growth model. We show how, in this framework, an inflow or outflow of individuals affects the steady-state income of both natives and immigrants.

Again we confine our analysis to the case in which migration takes place only in the initial period. After this period there is no further change in the ratio between the sizes of the two groups. If the two populations grew at different rates, then one group's share in total population would asymptotically become zero. Therefore, let us assume that both the immigrant and the native population grow at the same rate n. This rate is assumed to be positive.

Let time t be a continuous variable and let a dot on a variable indicate a derivative with respect to t. Then

$$\frac{\dot{L}_0(t)}{L_0(t)} = \frac{\dot{L}_1(t)}{L_1(t)} = \frac{\dot{L}(t)}{L(t)} = n \tag{12}$$

and

$$\frac{L_0(t)}{L(t)} = m, \qquad \frac{L_1(t)}{L(t)} = 1 - m, \tag{13}$$

where m is constant over time. Furthermore we now assume $0 < m < 1$.

Since all individuals in our economy are at every time t paid the same wage rate $w(t)$ and the same rate of interest $r(t)$ and since migration is nonrecurring, it can affect the economy in the long run only if immigrants and natives differ in their economic behavior. We assume that it is the rate of savings that distinguishes the members of the two groups. Let s_0 and s_1 denote the savings rate of respectively the natives and the immigrants. For our analysis we assume $s_0 > s_1$. This is not a restriction because we could rename those individuals as "natives" whose rate of savings is lower. It is of importance, however, that the two groups maintain their savings behavior over time. In other words, assimilation with respect to savings behavior does not take place.

In the following individuals with the lower savings rate s_1 will be called "low savers" while the term "high savers" denotes the individuals with a savings rate s_0. $K_0(t)$ and $K_1(t)$, the stocks of wealth of the two types of individuals, change in time according to the following two differential equations:

$$\dot{K}_0(t) = s_0[w(k(t))L_0(t) + r(k(t))K_0(t)], \tag{14}$$

$$\dot{K}_1(t) = s_1[w(k(t))L_1(t) + r(k(t))K_1(t)]. \tag{15}$$

There is no depreciation. We know from growth economics that a positive rate of depreciation does not essentially affect the results of a standard neoclassical model of capital accumulation.

From (14)-(15) and from our previous assumptions it follows that

$$\dot{k}_0(t) = s_0[w(k(t)) + r(k(t))k_0(t)] - nk_0(t), \tag{16}$$

$$\dot{k}_1(t) = s_1[w(k(t)) + r(k(t))k_1(t)] - nk_1(t) \tag{17}$$

with

$$\begin{aligned} w(k(t)) &= f(k(t)) - k(t)f'(k(t)), \\ r(k(t)) &= f'(k(t)), \\ k(t) &= mk_0(t) + (1-m)k_1(t). \end{aligned}$$

(16)-(17) is the dynamical system whose properties will be described subsequently[2].

Let k_0^* and k_1^* denote steady state values of the system. These two variables are solutions of the three equations

$$w(k^*) = \left[\frac{n}{s_0} - f'(k^*)\right]k_0^*, \tag{18}$$

$$w(k^*) = \left[\frac{n}{s_1} - f'(k^*)\right]k_1^*, \tag{19}$$

$$k^* = mk_0^* + (1-m)k_1^*. \tag{20}$$

Assuming that the two Inada conditions $\lim_{k\to 0} f'(k) = \infty$ and $\lim_{k\to\infty} f'(k) = 0$ are fulfilled, we can show that there exists a unique steady state (k_0^*, k_1^*) with $k_0^* > 0$ and $k_1^* > 0$.

Theorem 1: Let the production function f meet the two Inada conditions $\lim_{k\to 0} f'(k) = \infty$ *and* $\lim_{k\to\infty} f'(k) = 0$.

Then the system (16)-(17) possesses a unique equilibrium (k_0^*, k_1^*) *with* $0 < k_1^* < k^* < k_0^*$. *This equilibrium is locally asymptotically stable, i.e.*

$$\lim_{t\to\infty}[k_0(t), k_1(t)] = [k_0^*, k_1^*]$$

for every pair of initial values (k_0, k_1) *sufficiently close to* (k_0^*, k_1^*).

Proof: See Appendix A.

[2] Formally, our system bears a strong similarity to a model used by Pasinetti (1962) and Samuelson and Modigliani (1966) to study the properties of a two class society with "capitalists" and "laborers". In their analysis the two groups are also distinguished by their savings rates. However, in their model the capitalists, i.e. the ones with a higher rate of savings, do not work and do, therefore, not earn wages. With respect to the results this makes a big difference to our model.

Theorem 1 tells us that the steady state is locally asymptotically stable so that every path of capital accumulation eventually approaches the equilibrium if it starts sufficiently close to it. Hence, in the long run the steady state describes the economic outcome of our model economy.

A few results are obvious. From (18), (19) and $s_0 > s_1$ it follows that $k_0^* > k_1^*$. The immigrants' (low savers') per capita wealth k_1^* is smaller in the long run than the average capital stock k_0^* of the natives (high savers). Since the steady-state incomes of the natives and the immigrants respectively are

$$y_0^* = f(k^*) + f'(k^*)(k_0^* - k^*) \tag{21}$$

$$y_1^* = f(k^*) + f'(k^*)(k_1^* - k^*)$$

a higher savings rate implies a higher income: $y_0^* > y_1^*$. Average income in the economy is $y^* = f(k^*)$.

The variables $k_0^*, k_1^*, k^*, y_0^*, y_1^*$, and y^* are all functions of m because m is a parameter of the steady-state conditions (18)-(20) and because the steady state is unique. The question is now again the same as in the short-run analysis: How does an initial wave of immigration (i.e. a reduction in m) affect the per capita incomes of both the natives and the immigrants? In the following comparisons of incomes of the immigrant population always refer to steady-state values in the immigration country. Thus, even if losses with respect to this criterion are predicted, the immigrants will usually still be better off than if they had remained abroad.

Theorem 2: Let y_0^* and y_1^* denote the steady-state income per capita of respectively the natives and the immigrants. Then the following propositions hold true:

(a) $y_0^* > y_1^*$.

(b) If the share $(1 - m)$ of immigrants in the population increases, then the income per capita y_0^* of the natives rises while the income per capita y_1^* of the immigrants declines.

(c) An increase in the share $(1 - m)$ of immigrants in the population reduces the average income y^* of this economy in the steady state.

Proof: See Appendix B.

Theorem 2 implies

$$\frac{\partial k_0^*}{\partial m} < 0, \frac{\partial k_1^*}{\partial m} > 0, \text{and} \frac{\partial k^*}{\partial m} > 0. \tag{22}$$

We have thus obtained results similar to those in the short-run case. A reduction in the share of the native population due to an initial, non-recurring flow of immigrants increases both per capita income and per capita wealth of the natives. However, it reduces the average income of the whole economy as well as per capita income and per capita wealth of the immigrants. Again the natives are, on the average, the winners of the immigration while, in terms of income and wealth per person, the immigrant population loses. Emigration of low savers simply reverses our results. As in the short-run analysis the intuitive principle that individuals are paid according to their relative scarcity in the economy applies. The increase in the per capita income of the natives due to changing factor prices leads to higher savings. This raises their capital stock per capita and therefore also their capital income which enhances the factor price effect.

In the long run the burden on the natives from the provision of additional public services for the immigrants is probably of minor importance because these costs are likely to be paid in the first few periods after immigration and do therefore not affect steady-state values.

Our model can be given an alternative interpretation. There may be low savers and high savers among both the native and the immigrant population. If for some reason (i.e. immigration, change in savings behavior, or other events) the share of high savers in the population falls, then their per capita income rises. The low-savers' income as well as the average income in the economy both decline.

If the share of high savers in the native population equals a constant $\overline{m}$ with $0 < \overline{m} < 1$, the natives' steady-state per capita income y_N^* can be written as

$$y_N^*(m) = w(k(m)) + (\overline{m}k_0(m) + (1-\overline{m})k_1(m))r(k(m)).$$

Theorem 3:

$$\frac{dy_N^*}{dm} < 0 \quad \textit{if} \quad 0 < m < \widetilde{m} \text{ and}$$
$$\frac{dy_N^*}{dm} > 0 \quad \textit{if} \quad \widetilde{m} < m < 1$$

with

$$\widetilde{m} = \overline{m}\frac{n - f's_0 + f'(s_0 - s_1)}{n - f's_0 + \overline{m}f'(s_0 - s_1)} > \overline{m}.$$

Proof: See Appendix C.

Theorem 3 states that if there are initially only natives in the immigration country, the natives' steady-state per capita income rises when low savers arrive and falls if "some" high savers come into the country. It is possible that further immigration of high savers leads to an increase in the per capita income of the natives if there are already "many" high savers among the immigrant population.

This result is surprising. The message from the short-run model and from Theorem 2 is that the natives always benefit from immigration if the average immigrant differs from the average native in his economic characteristics. This general rule no longer holds.

The result can be explained as follows: Migration has two effects on the income of the two groups of natives. First, the capital-labor ratio of the economy is affected which causes changes in factor prices and thus influences the incomes of the individuals. This factor price effect is similar to the one which is already known from the short-run analysis. Second, changes in income affect the capital accumulation of the two groups which in turn leads to changes in their capital income. For each group this capital accumulation effect enhances the factor price effect. However, this does not apply to the average income of the natives.

In case of infinitesimal immigration (at $m = \overline{m}$) the factor price effects for the two groups of natives just offset each other, while the capital accumulation effect is stronger for the group of high savers than for the group of low savers. If immigration is finite, the total factor price effect is always positive, while the total capital accumulation effect may still be dominated by the effect on high savers. Therefore, due to the importance of the capital accumulation effect on high savers, the per capita income of natives rises if low savers immigrate and usually falls if high savers enter the country.

We can conclude that policy-makers who are only interested in a high steady-state per capita income of the natives should promote the immigration of low savers. It should be noted, however, that this result depends crucially on the assumption that the different population groups maintain their savings behavior over time.

On the other hand politicians may also aim at reducing the income inequality among the natives. Our model tells them that immigration of low savers increases the difference between the income of low savers and the income of high savers. Thus, if it is impossible to redistribute income within the native population, then there is a conflict between different goals of immigration policy.

Finally, our analysis sheds also some light on the theory of optimal economic growth. The Golden Rule of Capital Accumulation tells us that there is an optimal rate of savings which maximizes the average consumption per capita. If the rate of savings is below its optimal level, then there is under-accumulation of capital. In this case the marginal productivity of capital is still so high that it pays off to save and invest more in order to attain a higher level of consumption. Over-accumulation as a result of excessive saving leads to a relatively low marginal productivity of capital which makes it attractive to save and invest less so that, again, consumption per capita can be increased.

In our model the rate of savings in the economy depends on the ratio between low savers and high savers. If $m = 0$, then there are only low savers and the rate of savings in the economy is s_1. For $m = 1$ the savings rate is s_0. If the optimal savings rate lies between s_0 and s_1 then it can be attained by an appropriate choice of m. Whenever immigrants and natives differ in their composition of low savers and high savers, immigration policy can be used to shift the rate of savings in the economy into the appropriate direction. This way immigration policy can be used to allow for an optimal path of capital accumulation.

4. Conclusion

We have seen that the natives nearly always benefit from immigration in terms of per capita income in the short run in a neoclassical economy with one good and two factors. Thus if the natives need not share the financial burden from the provision of additional public services, then they gain from an inflow of immigrants.

In the long run natives with a high savings rate are winners of an immigration of individuals with a low propensity to save while an immigration of individuals with a high propensity to save is beneficial to natives with a low savings rate. If the natives are heterogeneous with respect to their savings behavior, they enjoy a gain in steady-state per capita income if people with a low propensity to save come into the country and usually suffer a loss in per capita income if individuals with a high savings rate immigrate. The result indicates that the welfare propositions on the long-run effects of migration of Berry and Soligo and Steinmann are highly dependent on the assumption of a homogeneous native population.

Immigration policy may be used to implement a Golden Rule path of capital accumulation. By an appropriate mix between natives and immigrants an average savings rate in the economy can be chosen so that per capita consumption is maximized.

Our results seem to be in contrast to those derived from the standard overlapping-generations models beginning with Rodriguez and Galor. On a closer view, the differences rely on model-specific income definitions, i.e. steady-state real income in the sense of the standard overlapping-generations model is not the same as steady-state per capita income in a conventional neoclassical growth model (see Rodriguez, 1975). It is in no way clear whether one of these two or yet another income concept is appropriate for the problem. Therefore, the debate on long-run effects of migration remains open.

Appendix A: Proof of Theorem 1

a) Existence

We define the two functions

$$\varphi(k) = \frac{w(k)}{k(\frac{n}{s_0} - f'(k))},$$

$$\psi(k) = \frac{\frac{n}{s_1} - f'(k)}{(1-m)\left(\frac{n}{s_0} - f'(k)\right) + m\left(\frac{n}{s_1} - f'(k)\right)}$$

and the variable $b = \frac{k_0}{k}$. Then there exists a unique equilibrium of (16)-(17) if and only if there exists a unique solution $b^* > 0$ and $k^* > 0$ of the two equations

$$\text{(I)} \qquad b = \varphi(k),$$

$$\text{(II)} \qquad b = \psi(k).$$

This statement is true because (I) and (II) are equivalent to (18) and (19) which can be easily verified.

We now look into the properties of the functions φ and ψ. For this purpose we define $\tilde{k}$ as a solution of the equation $\frac{n}{s_0} = f'(\tilde{k})$. Since $f''(k) < 0$, $\lim_{k \to \infty} f'(k) = 0$, and $\lim_{k \to 0} f'(k) = \infty$, the solution $\tilde{k}$ exists. Furthermore, it is unique and positive. From $f''(k) < 0$ and $\frac{n}{s_1} > \frac{n}{s_0}$ it follows that

$$\frac{n}{s_1} - f'(k) > \frac{n}{s_0} - f'(k) > 0 \text{ for } k > \tilde{k}$$

which implies

$$\varphi(k) > 0 \text{ and } \psi(k) > 0 \text{ for } k > \tilde{k}.$$

Further properties of φ and ψ are:

(aa) $$\operatorname{rlim}_{k\to\tilde{k}}\varphi(k)=\infty,$$

(bb) $$\lim_{k\to\infty}\varphi(k)=\lim_{k\to\infty}\frac{\frac{f(k)}{k}-f'(k)}{\frac{n}{s_0}-f'(k)}=0 \text{ (by L'Hospital's Rule),}$$

(cc) $$\varphi'(k)=\frac{(b-1)kf''(k)-b\left(\frac{n}{s_0}-f'(k)\right)}{k\left(\frac{n}{s_0}-f'(k)\right)}$$

so that $\varphi'(k)<0$ for $k>\tilde{k}$ and $b>1$.

(dd) $$\psi(\tilde{k})=\frac{1}{m}>1,$$

(ee) $$\lim_{k\to\infty}\psi(k)=\frac{\frac{n}{s_1}}{(1-m)\frac{n}{s_0}+m\frac{n}{s_1}}$$

so that $1<\lim_{k\to\infty}\psi(k)<\frac{1}{m}$.

(ff) $$\psi'(k)=\frac{(b-1)f''(k)}{(1-m)\left(\frac{n}{s_0}-f'(k)\right)+m\left(\frac{n}{s_1}-f'(k)\right)}$$

so that $\psi'(k)<0$ for $k>\tilde{k}$ and $b>1$.

Finally,

$$\varphi'(k)<\psi'(k) \text{ for } k>\tilde{k}.$$

This can be seen from the inequality

$$\frac{(b-1)f''(k)k-b\left(\frac{n}{s_0}-f'(k)\right)}{k\left(\frac{n}{s_0}-f'(k)\right)}<\frac{f''(k)(b-1)}{(1-m)\left(\frac{n}{s_0}-f'(k)\right)+m\left(\frac{n}{s_1}-f'(k)\right)}$$

which is equivalent to

$$(b-1)kf''(k)m\left(\frac{n}{s_1}-\frac{n}{s_0}\right)<b\left[(1-m)\left(\frac{n}{s_0}-f'(k)\right)+m\left(\frac{n}{s_1}-f'(k)\right)\right]\left(\frac{n}{s_0}-f'(k)\right).$$

This inequality, however, must be true because the left-hand side is negative and the right-hand side is positive.

The following Figure shows φ and ψ for $k \geq \tilde{k}$:

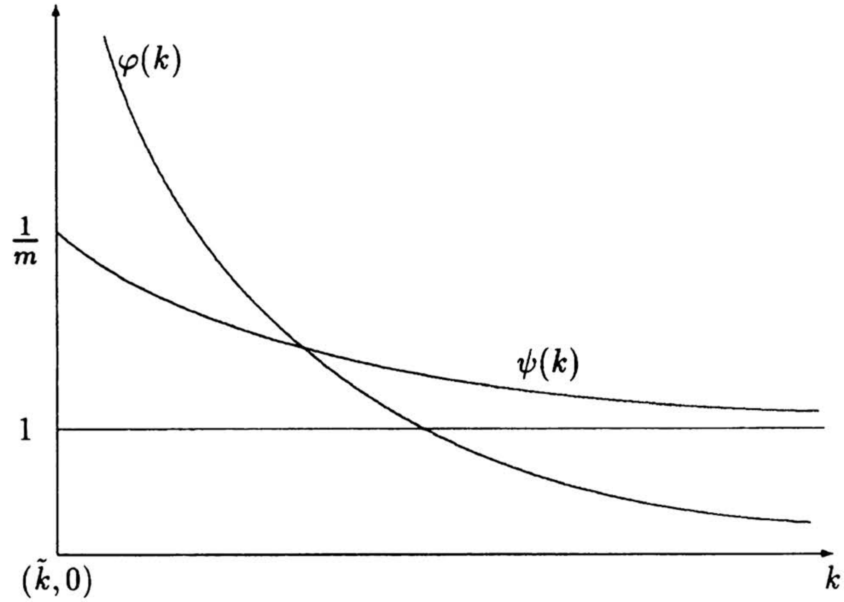

Figure: Existence and uniqueness of a steady state

From this the existence of a unique pair $(b^{\bullet}, k^{\bullet})$ with $b^{\bullet} > 0$, $k^{\bullet} > 0$ which solves (I) and (II) is obvious. $\psi(k^{\bullet}) > 1$ implies $b^{\bullet} > 1$ and $0 < k_1^{\bullet} < k^{\bullet} < k_0^{\bullet}$ because $k^{\bullet} = mk_0^{\bullet} + (1-m)k_1^{\bullet}$ and $k_1^{\bullet} < k_0^{\bullet}$. $k_1^{\bullet} > 0$ follows from $k^{\bullet} > \tilde{k}$ in combination with (19).

b) Stability

Let M denote the Jacobian matrix of (16)-(17), i.e.

$$M = \begin{pmatrix} \dfrac{\partial \dot{k}_0}{\partial k_0} & \dfrac{\partial \dot{k}_0}{\partial k_1} \\ \dfrac{\partial \dot{k}_1}{\partial k_0} & \dfrac{\partial \dot{k}_1}{\partial k_1} \end{pmatrix} = \begin{pmatrix} g_1 & g_2 \\ h_1 & h_2 \end{pmatrix}$$

Given the uniqueness of the equilibrium it suffices to show that trace $M = g_1 + h_2 < 0$ and $\det M = g_1 h_2 - h_1 g_2 > 0$ in equilibrium.

In order to simplify the notation we write $w' = w'(k)$, $f' = f'(k)$, and $f'' = f''(k)$ in the following calculations:

$$\begin{aligned}
g_1 &= s_0[w'm + f' + k_0 f''m] - n = (s_0 f' - n) + s_0 m f''(k_0 - k) \\
&= (s_0 f' - n) + m(1-m)s_0(k_0 - k_1)f''. \\
g_2 &= s_0[w'(1-m) + k_0 f''(1-m)] = s_0(1-m)^2(k_0 - k_1)f''. \\
h_1 &= s_1[w'm + f''mk_1] = s_1 m^2(k_1 - k_0)f''. \\
h_2 &= s_1[w'(1-m) + f' + (1-m)k_1 f''] - n = (s_1 f' - n) + s_1 m(1-m)(k_1 - k_0)f''.
\end{aligned}$$

From this we get

$$\text{trace } M = g_1 + h_2 = (s_0 f' - n) + (s_1 f' - n) + m(1-m)(s_0 - s_1)(k_0 - k_1)f'' < 0$$

and

$$\begin{aligned}
\det M = g_1 h_2 - g_2 h_1 &= (s_0 f' - n)(s_1 f' - n) \\
&+ (s_1 f' - n)m(1-m)s_0 f''(k_0 - k_1) \\
&+ (s_0 f' - n)m(1-m)s_1 f''(k_1 - k_0) \\
&= (s_0 f' - n)(s_1 f' - n) - m(1-m)f''n(s_0 - s_1)(k_0 - k_1) > 0
\end{aligned}$$

where the inequalities for trace M and det M hold for $k_0 = k_0^*$, $k_1 = k_1^*$, and $k = k^*$.

Appendix B: Proof of Theorem 2

In this proof we consider only steady-state values (i.e. solutions of the equations (18), (19), and (20)). These three equations define k_0, k_1, and k as functions $k_0 = k_0(m)$, $k_1 = k_1(m)$, and $k = k(m)$ of the share m of the natives in the population. This is due to the uniqueness of the steady state. By definition

$$\begin{aligned}
y_0 &= w(k(m)) + f'(k(m))k_0(m) = y_0(m), \\
y_1 &= w(k(m)) + f'(k(m))k_1(m) = y_1(m), \\
y &= f(k(m)) = my_0(m) + (1-m)y_1(m) = y(m).
\end{aligned}$$

In order to prove $y_0 > y_1$ it suffices to show that $k_0 > k_1$. This, on the other hand, follows from equations (18) and (19) and from the assumption $s_0 > s_1$.

Let us now calculate $k_0'(m)$ and $k_1'(m)$. In order to prove assertion (b) we have to show that $k_0'(m) < 0$ and $k_1'(m) > 0$. Subsequently we write w' for $w'(k)$, f' for $f'(k)$, k_0' for $k_0'(m)$, and k_1' for $k_1'(m)$.

Differentiating the two equations

$$\begin{aligned}
s_0(w(k) + f'(k)k_0) - nk_0 &= 0, \\
s_1(w(k) + f'(k)k_1) - nk_1 &= 0
\end{aligned}$$

we get

$$s_0(w'k' + f'k'_0 + k_0 f''k') - nk'_0 = 0,$$
$$s_1(w'k' + f'k'_1 + k_1 f''k') - nk'_1 = 0.$$

Since $k' = mk'_0 + (1-m)k'_1 + (k_0 - k_1)$, we finally obtain

$$g_1 k'_0 + g_2 k'_1 = b_1,$$
$$h_1 k'_0 + h_2 k'_1 = b_2$$

where

$$b_1 = -s_0(1-m)(k_0 - k_1)^2 f'',$$
$$b_2 = s_1 m(k_0 - k_1)^2 f''$$

and where g_1, g_2, h_1, and h_2 are defined as in the proof of Theorem 1.

Solving this system of two linear equations yields

$$k'_0 = (-f'')\frac{(k_0 - k_1)^2}{\det M} s_0(1-m)(s_1 f' - n) < 0,$$
$$k'_1 = f''\frac{(k_0 - k_1)^2}{\det M} s_1 m(s_0 f' - n) > 0,$$

with $\det M = g_1 h_2 - g_2 h_1 > 0$.

The inequality signs for k'_0 and k'_1 then follow from $(s_0 f' - n) < 0$ and $(s_1 f' - n) < 0$.

Inserting equations (18) and (19) into the definitions of y_0 and y_1 we get $y_0 = \frac{n}{s_0} k_0$ and $y_1 = \frac{n}{s_1} k_1$. Thus we get $y'_0 = \frac{n}{s_0} k'_0 < 0$ and $y'_1 = \frac{n}{s_1} k'_1 > 0$. Therefore, a decrease in m (i.e. an increase in $(1-m)$) increases y_0 and decreases y_1.

Assertion (c) of this theorem can be shown by using the definition $k = mk_0 + (1-m)k_1$. Differentiating with respect to m yields

$$k' = mk'_0 + (1-m)k'_1 + (k_0 - k_1).$$

Hence

$$k' = \frac{f''m(1-m)(k_0 - k_1)n(s_0 - s_1)}{(s_0 f' - n)(s_1 f' - n) - f''m(1-m)(k_0 - k_1)n(s_0 - s_1)}(k_0 - k_1) + (k_0 - k_1)$$
$$= \frac{(s_0 f' - n)(s_1 f' - n)(k_0 - k_1)}{\det M} > 0.$$

From this we get $y' = f'k' > 0$, i.e. immigration reduces both the capital-labor ratio and the average income per capita.

Appendix C: Proof of Theorem 3

Differentiating y_N^* with respect to m yields

$$\frac{dy_N^*}{dm} = w'k' + k_N f''k' + r(\overline{m}k_0' + (1-\overline{m})k_1')$$

with $k_N = \overline{m}k_0(m) + (1-\overline{m})k_1(m)$.

Inserting our results for k', k_0' and k_1' from the proof of Theorem 2 we get

$$\begin{aligned}\frac{dy_N^*}{dm} = &\frac{-f''(k_0-k_1)}{\det M}[(k-k_N)(s_0f'-n)(s_1f'-n)\\ &+f'(k_0-k_1)[\overline{m}(1-m)s_0(s_1f'-n)-(1-\overline{m})ms_1(s_0f'-n)]].\end{aligned}$$

To determine the sign of $\frac{dy_N^*}{dm}$, we can omit positive factors, so

$$\begin{aligned}sgn\left(\frac{dy_N^*}{dm}\right) &= sgn\left[(m-\overline{m})(k_0-k_1)+(k_0-k_1)\left(\overline{m}\frac{(1-m)f'}{f'-\frac{n}{s_0}}-m\frac{(1-\overline{m})f'}{f'-\frac{n}{s_1}}\right)\right]\\ &= sgn\left[\overline{m}\frac{f'-mf'-f'+\frac{n}{s_0}}{f'-\frac{n}{s_0}}-m\frac{f'-\overline{m}f'-f'+\frac{n}{s_1}}{f'-\frac{n}{s_1}}\right]\\ &= sgn\left[\overline{m}\left(f'-\frac{n}{s_1}\right)\left(\frac{n}{s_0}-mf'\right)-m\left(\frac{n}{s_1}-\overline{m}f'\right)\left(f'-\frac{n}{s_0}\right)\right]\\ &= sgn[m\overline{m}f'(s_0-s_1)+n(m-\overline{m})+f'(s_1\overline{m}-s_0m)]\\ &= sgn[f'\overline{m}(m-1)(s_0-s_1)+(n-f's_0)(m-\overline{m})].\end{aligned}$$

From the last expression it is obvious that

$$\begin{aligned}&\frac{dy_N^*}{dm}<0 \quad \text{if} \quad 0<m\le\overline{m} \text{ and}\\ &\frac{dy_N^*}{dm}>0 \quad \text{if} \quad m\to 1.\end{aligned}$$

Furthermore we see that

$$\frac{dy_N^*}{dm}=0 \quad \text{if} \quad m=\tilde{m}=\overline{m}\frac{n-f's_0+f'(s_0-s_1)}{n-f's_0+\overline{m}f'(s_0-s_1)}>m,$$

and if we differentiate

$$C \equiv f'\overline{m}(m-1)(s_0-s_1)+(n-f's_0)(m-\overline{m})$$

with respect to m, we get

$$\frac{dC}{dm}=f'\overline{m}(s_0-s_1)+n-f's_0+[\overline{m}(m-1)(s_0-s_1)-s_0(m-\overline{m})]f''k',$$

which is clearly positive for $m \geq \overline{m}$, so we can conclude that

$$\frac{dy_N^*}{dm} < 0 \quad \text{if} \quad 0 < m < \widetilde{m} \text{ and}$$
$$\frac{dy_N^*}{dm} > 0 \quad \text{if} \quad \widetilde{m} < m < 1.$$

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Zusammenfassung

In diesem Beitrag werden kurz- und langfristige Wirkungen einer Zuwanderung auf das Einkommen der einheimischen Bevölkerung untersucht. In der kurzen Frist führt Immigration gewöhnlich zu einer Änderung des Lohn-Zins-Verhältnisses, was sich in einer Erhöhung des Einkommens der Einheimischen niederschlägt. Langfristig kann eine Zuwanderung die gesamtwirtschaftliche Sparquote ändern. Wenn die Einheimischen ein heterogenes Sparverhalten aufweisen, erhöht (senkt) die Immigration von Individuen mit niedriger (hoher) Sparneigung ihr Pro-Kopf-Einkommen. Einheimische mit einer hohen (niedrigen) Sparquote profitieren immer von einer Verringerung (Erhöhung) der gesamtwirtschaftlichen Sparquote.

Abstract

In this paper both short-term and long-term effects of immigration on the income of the native population are analyzed. In the short run immigration usually changes the wage-interest ratio which increases the income of the natives. In the long run immigration may also have an impact on the rate of savings. If the natives are heterogeneous in their savings behavior, then immigration raises (lowers) their average income if the immigrants have a low (high) propensity to save. Natives with a high (low) rate of savings always benefit from a decline (rise) in the aggregate savings rate.

JEL-Klassifikation: F22