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The Risk Consolidation of the Insurance Industry from a Financial Perspective*

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1. Objective

In this paper we analyze the risk consolidation of insurance on the basis of the *Capital Asset Pricing Model.*[1] In principle, there are two possibilities for doing this:

- Firstly, one can assume that the risks of potential policyholders are directly traded on the capital market. In this case insurance premiums can be calculated simply by applying the so-called security market line (or the valuation formula) of the CAPM to these risks.
- Secondly, one takes the existence of financial intermediation into account, i.e. that the risks of the insureds are transferred to the capital market via insurance companies whose shares are traded on the stock exchange.

In reality the first approach is obviously not fulfilled in most cases. Few exceptions are to be found at Lloyd's, where risks are transferred directly from policyholders to private investors. In general, however, insurance is a *non-marketable good*, which means it is not traded directly on the capital market (cf. e.g. Breuer 1992, p. 621 and Gründl 1993a, p. 371). Consequently, this approach is not able to explain premiums on real insurance markets, where no price regulation exists.

Nevertheless, the first approach is the basis for most applications of the CAPM to insurance, i.e. the so-called *"Insurance CAPM"* (Cummins 1990, S. 150f.), which has to be interpreted as a "normative model" for supervi-

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[1] The standard version of the Capital Asset Pricing Model (CAPM) was developed by Sharpe (1964), Lintner (1965) and Mossin (1966).

sory authorities to calculate "appropriate insurance premiums". It proceeds on the assumption that the premiums paid by the insured have to be treated as investments in the capital market: accordingly, the claims paid represent the policyholders' yield which could be calculated using the so-called security market line of the standard CAPM (see, among many others, Quirin/Waters 1975, Kahane 1977, 1979, Biger/Kahane 1978, Fairley 1979, Hill 1979 and Kromschröder 1991). In many states of the USA the Insurance CAPM has been used for premium calculation since the 1970s (cf. Michel/Norris 1982, p. 628). This practice is coming in for more criticism because of the idealized assumption of the standard CAPM (see e.g. Müller, W. 1983, Cummins/Harrington 1985, Urrutia 1986 and Albrecht 1991).

Mayers (1972) has developed a version of the CAPM in which the existence of non-marketable assets such as human capital, pensions, government transfer payments and trust income is taken into consideration. His results were used to develop insurance models where *financial intermediation* is taken into account.[2] Turner (1987, p. 92) and Gründl (1993a, p. 379; 1993b, p. 61) derived market value functions for insurance companies. In their models one type of household exists which invests in the capital market and also concludes insurance contracts. Breuer (1994) criticized that the non-marketability of insurance was introduced ad hoc and derived the existence of insurance companies by assuming fixed transaction costs for households.[3] If there are, however, fixed transaction costs, then the capital market is segmented, i.e. households take only a limited number of assets into their portfolios and may invest in different assets (cf. Brennan 1975 and Goldsmith 1976).[4] Households then have to counterbalance the advantage of better risk diversification against the disadvantage of further transaction costs. Thus, it does not seem consistent to introduce implicitly on the one hand fixed transaction costs and, on the other hand, to use a model with one type of household which buys insurance policies and invests in a perfect capital market.

For this reason, a different approach is taken here.[5] It is taken into account that households are risk-averse to differing degrees. Therefore, two different types of households are asssumed. The first type has a relatively weak aversion to risk and invests in the capital market. The second type has a relatively strong aversion so risk and buys policies from insurance compa-

2 Mayers/Smith (1983) determined households' demand for insurance contracts simultaneously with their demand for other assets.

3 A general explanation of financial intermediation was given by Diamond (1984).

4 Levy (1978) and Mayshar (1979) provided valuation formulas for this case. They have, however, a very complex structure. Leland/Pyle (1977) showed that *adverse selection* also implies a segmentation of the capital market.

5 Cf. Kotsch (1991, 1995), where the regulation of premiums and deductibles were investigated on the basis of the CAPM.

nies. So it is possible to model financial intermediation, that is to say especially the *risk transfer via insurance companies* from the policyholder to the investors on the capital market, with the standard CAPM.

The intention of this paper is to provide a *welfare analysis of the insurance market*, i.e. to prove the existence of a competitive equilibrium and whether it guarantees a Pareto optimum. A market value function of the standard CAPM will be used to calculate the market capitalization of an insurance plc that reflects in particular the risk costs the company incurs in concluding insurance polices. Then the behaviour of an insurance company is investigated by maximizing its market value. Finally, a comparison of the premium principle obtained here with that of the Insurance CAPM is undertaken to reveal the allocation effects of the supervisory authorities on those American states using this model for price regulation.

2. An insurance company's return

Let us first assume a collective of $i = 1, \ldots, n$ policyholders, who do not invest in the capital market. They have the *identically distributed and independent loss variables* C_i. The losses are realized within a given period (e.g. one year). In respect of the expected losses $\mu(C_i)$ and the variances of the losses $\sigma^2(C_i)$ for all policyholders i as well as the covariances $\mathrm{Cov}(C_i, C_h)$ of the losses of two policyholders i and h, the following apply:

$$\mu(C_i) = \bar{\mu} \,, \tag{1}$$

$$\sigma^2(C_i) = \bar{\sigma}^2 \quad \text{and} \tag{2}$$

$$\mathrm{Cov}(C_i, C_h) = 0 \,, \tag{3}$$

where $i \neq h$ and $i, h = 1, \ldots, n$. Using these equations it is possible to calculate the expected value, the variance and standard deviation of the collective's total loss:

$$\mu(\Sigma_i C_i) = n\bar{\mu} \quad \text{and} \tag{4}$$

$$\sigma^2(\Sigma_i C_i) = n\bar{\sigma}^2 \,. \tag{5}$$

If a *representative insurance company* j writes policies for the policyholders described in the equations (1) to (3) at the beginning of a period, at the end of that period it will pay its shareholders a *return*

$$Y_j = q(n_j \pi + k_j) - \Sigma_i C_i \,, \tag{6}$$

with $n_j \equiv$ number of policies issued by insurance company j, $\pi \equiv$ premium per policy, $q \equiv$ accumulation factor $1 + r$, where r is the riskless interest rate, $k_j \equiv$ equity capital of insurance company j and $C_i \equiv$ loss variable of policyholder i.

Let us suppose that at the start of the period the insurance company j can *invest in a riskless asset* both the premium income $n_j \pi$ and the equity capital paid in by the shareholders k_j.[6] At the end of the period, therefore, and taking into account the interest to be earned, insurance company j has total capital of $q(n_j \pi + k_j)$ at its disposal. This capital is used in the first place to settle claims made by the policyholders, with the remainder being channeled into returns for the shareholders.

The risk of bankruptcy is excluded from these calculations. It is assumed that insurance company j has always sufficient capital to cover possible insured losses. The exact nature of the *statutory solvency requirements* remains open in this context. We merely assume the generally prescribed ratio between equity capital and the number of policies issued $k_j = f(n_j)$, with $f' > 0$. Later, when the optimum corporate decision is analyzed, we will see that a more precise specification is not necessary in order to determine the risk costs.

For the expected value and the standard deviation of the returns of insurance company j, the following equations can be deduced from equations (4) to (6)

$$\mu(Y_j) = q\left[n_j \pi + f(n_j)\right] - n_j \bar{\mu} \quad \text{and} \tag{7}$$

$$\sigma^2(Y_j) = n_j \bar{\sigma}^2 \,. \tag{8}$$

3. The market value of an insurance company

We shall now examine the decision of an insurance company to write an additional insurance policy with the aid of a *market value formula of the Standard CAPM* (see for example Elton/Gruber 1991, p. 296):

6 The exclusion of risky assets for the insurance company does not cause the shareholders any decrease in utility since they can compensate this by reshuffling their own portfolios (cf. Müller, H.H. 1987, p. 144).

(9) $$v_j = \frac{\mu(Y_j) - \lambda\left[\mathrm{Cov}(Y_j, Y_M)/\sigma(Y_M)\right]}{q} \ ,$$

where $\mathrm{Cov}(Y_j, Y_M) \equiv$ covariance of company j's returns and the returns of the overall market portfolio M, $\sigma(Y_M) \equiv$ standard deviation of the returns of the market portfolio M and $\lambda \equiv$ risk price on the stock exchange. The market value of company j is identical with the *present value of the certainty equivalent* of the returns Y_j, which can be calculated by subtracting the risk costs from the expected returns $\mu(Y_j)$. The risk costs are the product of the risk price λ and the systematic risk $\mathrm{Cov}(Y_j, Y_M)/\sigma(Y_M)$.

Let us now assume that besides insurance company j a firm p exists. It is a *manufacturing company*, standing for the remaining companies traded on the stock exchange, which distributes the exogenously determined returns Y_p at the end of the period. Moreover, it is now assumed that the loss variables of the insureds C_i are uncorrelated with manufacturing company's returns Y_p, i.e. $\rho(C_i, Y_p) = 0$ for all i, which in turn means there is no correlation between the two companies' returns:

(10) $$\mathrm{Cov}(Y_j, Y_p) = -\,\mathrm{Cov}(\Sigma_i C_i, Y_p) = -\,\Sigma_i \mathrm{Cov}(C_i, Y_p) = -\,\Sigma_i \rho(C_i, Y_p)\sigma(C_i)\sigma(Y_p) = 0 \ .$$

According to (9) *insurance company j's market value* is, taking (7), (8) and (10) into account:[7]

(11) $$v_j = \frac{q\left[n_j\pi + f(n_j)\right] - n_j\bar{\mu} - \lambda n_j\bar{\sigma}^2/(\ldots)^{0.5}}{q} \ ,$$

with $(\ldots) \equiv \left[n_j\bar{\sigma}^2 + \sigma^2(Y_p)\right]$. Insurance company j is assumed to *maximize its net market value*. Thus it will increase the number of its policies n_j if the achieved increase in market value equals or – better still – exceeds the amount of equity capital which has to be injected as a result of solvency requirements:

(12) $$dv_j/dn_j \geq f' \ .$$

[7] The derivation of the CAPM market value formula is based on the so-called μ-σ-approach, the application of which is admissible if the utility functions of the investors on the capital market are quadratic or the distribution classes of the investors' incomes linear (cf. Sinn 1989, Chap. 2D). Owing to the independence of the risks involved, the incomes of the investors in this case are distributed such that, if the number of risks is sufficiently large, a normal distribution may be adequately approximated because the central limit law of convergence applies. It is thus admissible to use the market value function.

If the share and insurance markets are characterized by *perfect competition*, i.e. company j does not exert market power in either and therefore cannot influence the riskless interest rate r, the risk price λ and the insurance premium π, the following implication can be drawn from condition (12) and market value formula (11):

$$q\pi \geq \bar{\mu} + \lambda \frac{\bar{\sigma}^2 - \dfrac{\bar{\sigma}^4}{2(\bar{\sigma}^2 + \sigma^2(Y_p)/n_j)}}{(\ldots)^{0.5}} \, . \tag{13}$$

Generally, the accumulated premium earned through the marginal insurance policy must thus be equal to at least expected claims plus a *risk loading*. (For a general review of the various premium principles see e.g. Bühlmann 1970, pp. 86ff., Goovaerts/Vylder/Haezendonck 1984, p. 35f. or Heilmann 1987). Here, the risk loading is equal to the marginal risk costs incurred by an additional insurance policy, which can be calculated as the product of the risk price λ and the enhancement of company j's systematic risk with the issue of an additional policy.

As the number of insureds n_j rises, the denominator of the marginal risk costs $(\ldots)^{0.5}$ increases, while the denominator of the small fraction after the minus sign on the right-hand side $2(\bar{\sigma}^2 + \sigma^2(Y_p)/n_j)$ decreases. Both effects cause the risk loading contained in the premium to fall, so that we find *falling marginal risk costs*, which disappear as $n_j \to \infty$:

$$\lim_{n_j \to \infty} \lambda \frac{\bar{\sigma}^2 - \dfrac{\bar{\sigma}^4}{2(\bar{\sigma}^2 + \sigma^2(Y_p)/n_j)}}{(\ldots)^{0.5}} = 0 \, .$$

Marginal condition (13) shows further that the exact nature of the solvency requirements is irrelevant for determining a company's optimum market value, since the variable f' can be eliminated from both sides of the inequality. The *economic argument* for this lies in the fact that company j's equity capital and its outside capital from underwriting are invested in an interest-bearing form, thus incurring no opportunity costs for the investors in the form of lost interest on their equity capital.

4. Is there a competitive equilibrium in the insurance market?

If a competitive equilibrium is to exist in the insurance market, the insurance companies' marginal costs must either be increasing or at least constant. Otherwise – in the case of falling marginal costs – the firms would enjoy economies of scale. *Cut-throat competition* would ensue and finally lead to a monopolization (or at least oligopolization) of the insurance market. In that case, marginal condition (13) would be invalid and would have to be replaced by the monopolist's rule of "marginal revenue equals marginal costs".

4.1 Other operating costs of insurance companies

In accordance with the previous result, namely that risk consolidation leads to economies of scale, the insurance market ought really to be characterized by a trend towards a natural monopoly, with marginal conditions being exchanged. Conditions in actual insurance markets, however, where a large number of companies are active, do not confirm the existence of or any trend towards a monopoly. To explain this, it is often said that the positive economies of scale to be gained from risk consolidation are more than outweighed by the *diseconomies of scale* stemming from *other operating costs*[8] - at least from a particular company size upwards – so that, on the whole, marginal costs do not decrease. In that case the insurance market is characterized by a competitive equilibrium.

For the purposes of an analysis of risk consolidation, the exact development of the overall marginal costs can remain open since the above proof that economies of scale are to be gained from risk consolidation *does not depend on the type of equilibrium* present in the market. What this in effect means is that the marginal risk costs on the right-hand side of marginal condition (13) are in any case valid. Only the left-hand side of the equation would have to be altered in the case of a monopoly.

4.2 Does a competitive equilibrium in the insurance market lead to a welfare optimum?

If we assume that insurance companies' overall marginal costs will rise owing to diseconomies of scale with the other operating costs and that thus a competitive equilibrium exists in the insurance market, the question is raised whether a welfare optimum is achieved in this equilibrium, i.e. whether the conditions of *Pareto optimality* are fulfilled. In order to answer this question we shall compare below the individual marginal costs incurred by insurance company j in writing an additional policy with the social marginal costs and so discover whether or not external effects occur in the stock market.

[8] Cf. e.g. Houston/Simons (1970), Colenutt (1977), Doherty (1981) and Johnston/Flanigan/Weisbart (1981).

Insurance company j's individual marginal costs can be calculated as the product of the risk price and the *enhancement of the company's systematic risk* as a result of the addition policy:

$$(14) \qquad \lambda \frac{d\left[\mathrm{Cov}(Y_j, Y_M)/\sigma(Y_M)\right]}{dn_j} = \lambda \frac{\bar{\sigma}^2(\ldots)^{0.5} - n_j \bar{\sigma}^4/2(\ldots)^{0.5}}{(\ldots)} .$$

The social marginal costs, on the other hand, are equal to the risk price multiplied by the *increase in the overall risk* in the stock market:

$$(15) \qquad \lambda\left[d\sigma(Y_M)/dn_j\right] = \lambda\bar{\sigma}^2/2(\ldots)^{0.5} .$$

By comparing equations (14) and (15), it can be proved that insurance company j's individual marginal costs are greater than the social marginal costs:

$$\lambda \frac{\bar{\sigma}^2(\ldots)^{0.5} - n_j \bar{\sigma}^4/2(\ldots)^{0.5}}{(\ldots)} > \lambda\bar{\sigma}^2/2(\ldots)^{0.5} ,$$

$$2(\ldots) - n_j\bar{\sigma}^2 > (\ldots) \quad \text{or}$$

$$\sigma^2(Y_p) > 0 .$$

Since, by definition, the variance of manufacturing company p is greater than zero, the inequality is true.

Thus, insurance company j has *positive external effects* on the other firms in the stock market.[9] From a welfare-economic point of view, therefore, the number of policies chosen by the company in accordance with marginal condition (13) is too low. The issuing of an insurance policy can enhance risk consolidation in the stock market. But it is not only the issuer of the policy, insurance company j, but all companies in the stock market that profit from this. This is why insurance company j calculates higher than necessary marginal risk costs, thus triggering a loss in welfare.

[9] Turner (1987, p. 92) and Gründl (1993a, p. 379; 1993b, p. 61) derived a somewhat different valuation formula for an insurance company (stock corporation), because they based their approaches on the modified CAPM of Mayers (1972), who takes the existence of non-marketable assets into account. As mentioned in the introduction their models might not reflect realistic cases. Thus they arrive at the result that the underwriting of insurance companies has no effect on the risk consolidation of the capital market, i.e. the market values of other firms traded in the capital market are affected by all risks of the households, both those that are insured and those that are not. Casual empiricism, however, indicates that this is not the case.

5. The insurance of perfectly positively correlated risks

Hitherto we have examined the insurance of uncorrelated risks. We will now take a look at the case of perfectly positively correlated risks.

5.1 No internal risk consolidation

Let the distributions of the insured risks continue to be characterized by equations (1) and (2). Owing to the perfectly positive correlation the following, and not equation (3), applies

$$\mathrm{Cov}(C_i, C_h) \equiv \rho(C_i, C_h)\sigma(C_i)\sigma(C_h) = \bar{\sigma}^2 \ ,$$

since $\rho(C_i, C_h) \equiv 1$ and $\sigma(C_i) = \sigma(C_h) \equiv \bar{\sigma}$ for all $i, h = 1, \ldots, n_j$. For the variance of the returns of insurance company j we now arrive at the equation

$$\sigma^2(Y_j) = \sigma^2(\Sigma_i C_i) = \Sigma_i \sigma^2(C_i) + \Sigma_i \Sigma_h \mathrm{Cov}(C_i, C_h) = n_j \bar{\sigma}^2 + n_j(n_j - 1)\bar{\sigma}^2 = (n_j \bar{\sigma})^2 \ ,$$

where $i \neq h$. If this is substituted in the market value formula (9), insurance company j's stock exchange price will be[10]

$$(11') \qquad v_j = \frac{q\left[n_j \pi + f(n_j)\right] - n_j \bar{\mu} - \lambda (n_j \bar{\sigma})^2 / (\ldots)^{0.5}}{q} \ ,$$

where $(\ldots) \equiv \left[(n_j \bar{\sigma})^2 + \sigma^2(Y_p)\right]$.

Together with the condition for maximization of net market value (12) we can deduce from (11′):

$$q\pi \geq \bar{\mu} + \lambda \frac{n_j^3 \bar{\sigma}^4 + 2n_j \bar{\sigma}^2 \sigma^2(Y_p)}{(\ldots)^{1.5}} \ .$$

The exact development of the *marginal risk costs* (MRC) is shown by the first derivation:

$$MRC' = \lambda\Big[\left[3n_j^2 \bar{\sigma}^4 + 2\bar{\sigma}^2 \sigma^2(Y_p)\right](\ldots)^{1.5} - \left[n_j^3 \bar{\sigma}^4 + 2n_j \bar{\sigma}^2 \sigma^2(Y_p)\right] 1.5 (\ldots)^{0.5} 2n_j \bar{\sigma}^2\Big] \Big/ (\ldots)^3 \ .$$

10 Owing to the assumed perfectly positive correlation between the insured risks the central limit law is no longer fulfilled and the distribution of the investors' incomes does not converge towards a normal distribution. In order to use the market value formula, which is derived on the basis of the so-called μ-σ-approach, we must now assume a quadratic utility function for the shareholders.

$MRC' > 0$ applies, since $(...) > 0$ and

$$\left[3n_j^2\bar{\sigma}^4 + 2\bar{\sigma}^2\sigma^2(Y_p)\right](\ldots) - \left[n_j^3\bar{\sigma}^4 + 2n_j\bar{\sigma}^2\sigma^2(Y_p)\right]3n_j\bar{\sigma}^2 > 0 \ ,$$

$$3n_j^4\bar{\sigma}^6 + 2n_j^2\bar{\sigma}^4\sigma^2(Y_p) + 3n_j^2\bar{\sigma}^4\sigma^2(Y_p) + 2\bar{\sigma}^2\sigma^4(Y_p) - 3n_j^4\bar{\sigma}^6 - 6n_j^2\bar{\sigma}^4\sigma2(Y_p) > 0 \ ,$$

$$2\bar{\sigma}^2\sigma^4(Y_p) > n_j^2\bar{\sigma}^4\sigma^2(Y_p) \quad \text{or}$$

$$2\sigma^2(Y_p) > \sigma^2(Y_j) \ .$$

Insurance company j is a representative firm in the insurance market, while company p stands for all other firms whose shares are traded in the stock market. This is why the greater-than condition will usually be fulfilled and there are rising marginal risk costs. The market value v_j is then concave in n_j and an equals sign can be substituted for the greater-than-or-equal-to sign from condition (12):

$$q\pi = \bar{\mu} + \lambda \frac{n_j^3\bar{\sigma}^4 + 2n_j\bar{\sigma}^2\sigma^2(Y_p)}{(\ldots)^{1.5}} \ . \tag{16}$$

This shows that no internal risk consolidation takes place in the case of perfect positive correlation.

5.2 External risk consolidation in the shareholders‘ portfolios

It must not be concluded from the above result that no risk consolidation at all is triggered by the additional policy. As the risks accepted by insurance company j are perfectly positively correlated, no balancing between them is possible. However, consolidation is achieved with manufacturing company p's risk. This fact becomes clear when one realizes that via the stock market the insured risks finally end up in the *shareholders' portfolios*, and it is there that a balancing occurs with company p's producing risk, with which there is no correlation. In such a case insurance company j functions merely as a *risk intermediary*, i.e. it keeps a necessary capital stock for the payment of policyholders' claims and arranges for the risks to be passed on to the shareholders. However, no internal risk consolidation is carried out in the company itself.

This risk consolidation in the shareholders' portfolios can be proven if we consider the *enhancement of the overall risk* in the stock market brought about by the sale of an additional insurance policy. If no risk consolidation took place, the overall risk $\sigma(Y_M)$ would have to increase in line with the marginal risk $\bar{\sigma}$. But in fact the overall risk increases less than $\bar{\sigma}$:

$$d\sigma(Y_M)/dn_j < \bar{\sigma} \quad \text{since}$$

$$2n_j\bar{\sigma}^2/2(\ldots)^{0.5} < \bar{\sigma} \quad \text{or, after transformation,}$$

$$0 < \sigma^2(Y_p)\,.$$

The variance of manufacturing company p is, by definition, positive, thus fulfilling the inequality.

5.3 Positive external effects between the companies

To prove mathematically that risk consolidation also takes place in the case in point, we deliberately examined the enhancement of the overall risk in the stock market and not that of company j's systematic risk. Here, too, assuming the policyholders' risks are perfectly positively correlated, insurance company j will profit only partially from the enhanced risk consolidation, because of the *positive cost externalities* between the companies in the stock market. The social marginal costs associated with issuing an additional policy are thus less than company j's individual marginal costs, i.e. the rise in the overall risk caused by an additional policy is less than the rise in insurance company j's systematic risk:

$$d\sigma(Y_M)/dn_j < d\big[\mathrm{Cov}(Y_j, Y_M)/\sigma(Y_M)\big]/dn_j\,, \quad \text{since}$$

$$2n_j\bar{\sigma}^2/2(\ldots)^{0.5} < \big[2n_j\bar{\sigma}^2(\ldots)^{0.5} - (n_j\bar{\sigma})^2 2n_j\bar{\sigma}^2/2(\ldots)^{0.5}\big]/(\ldots)\,,$$

$$(\ldots) < 2(\ldots) - (n_j\bar{\sigma})^2 \quad \text{or}$$

$$0 < \sigma^2(Y_p)\,.$$

By definition, this inequality is fulfilled, which means that, also in the case examined here, welfare losses will occur if there is a general competitive equilibrium in the insurance and stock markets.

6. Perfect positive or perfect negative correlation between the insured risks and the remaining risks in the stock market

Extending the above analysis, we will now deal with a further two extreme cases: in the one, all the risks traded in the stock market are perfectly positively correlated; in the other, there is a perfect positive correlation between the insured risks but a perfect negative one with the manufacturing risk of company p.

6.1 No internal risk consolidation

In the case of a perfectly positive correlation between all risks, i.e.: $\rho(C_i, C_h) = 1\ \forall i \neq h$ and $\rho(C_i, Y_p) = 1\ \forall i$, equation (10) is to be replaced by $\operatorname{Cov}(Y_j, Y_p) = -\, n_j\bar{\sigma}\sigma(Y_p)$. The standard deviation of company p remains exogenous. The market value of insurance company j is no longer (11′) but:

$$v_j = \frac{q\left[n_j\pi + f(n_j)\right] - n_j\bar{\mu} - \lambda \dfrac{(n_j\bar{\sigma})^2 - n_j\bar{\sigma}\sigma(Y_p)}{\left[(n_j\bar{\sigma})^2 - 2n_j\bar{\sigma}\sigma(Y_p) + \sigma^2(Y_p)\right]^{0.5}}}{q}$$

or, simplified after applying the binomial theorem to the denominator of the risk cost expression:

$$\text{(17)} \qquad v_j = \frac{q\left[n_j\pi + f(n_j)\right] - n_j\bar{\mu} + \lambda n_j\bar{\sigma}}{q} \,.$$

Owing to the perfect positive correlation between the policyholders' claims and the returns of manufacturing company p, insurance company j's uncertain returns serve to lessen the overall level of risk in the share market. If company p distributes low returns, fewer claims will be made by the insureds and company j will pay higher returns. If company p distributes higher returns, there will be many insured claims to be paid, thus causing insurance company j to pay lower returns. All in all, this relationship leads to a reduction in the total distribution $\sigma(Y_M)$, for which insurance company j receives a bonus in the form of the *negative risk costs* $-\lambda n_j\bar{\sigma}$.

If the insured risks are perfectly negatively correlated with the other risks in the share market, i.e. $\rho(C_i, Y_p) = -\,1\ \forall i$, the following is true: $\operatorname{Cov}(Y_j, Y_p) = n_j\bar{\sigma}\sigma(Y_p)$. By analogy, the market value of insurance company j may be calculated as:

$$\text{(17}'\text{)} \qquad v_j = \frac{q\left[n_j\pi + f(n_j)\right] - n_j\bar{\mu} - \lambda n_j\bar{\sigma}}{q} \,.$$

Here the insurer's activity increases the stock market risk and we have the *standard case of positive risk costs*.

Together with condition (12) for the maximization of net market value the marginal conditions

(18) $$q\pi = \bar{\mu} - \lambda\bar{\sigma} \quad \text{or}$$

(18′) $$q\pi = \bar{\mu} + \lambda\bar{\sigma}$$

follow from the two market value formulae (17) and (17'). As the right hand side of each equation is independent of n_j, in both cases insurance company j has *constant marginal costs*, i.e. no internal risk consolidation takes place. It is remarkable that the compounded insurance premium, too, may contain a risk markdown, as can be seen in equation (18). This markdown can be explained by the above-mentioned reduction in the overall level of risk achieved by selling an additional insurance policy in the case of a perfect positive correlation between insured and other risks.

6.2 Perfect or no external risk consolidation in the shareholders' portfolios

If the insured risks and company p's manufacturing risk are perfectly positively correlated, *perfect risk consolidation* is achieved in the shareholders' portfolios. Both the overall risk and company j's systematic risk are reduced by the marginal risk $\bar{\sigma}$ (see next paragraph).

In the alternative case of perfect negative correlation between the insured risks and the manufacturing risk, however, both the overall risk and insurance company j's systematic risk increase by the marginal risk $\bar{\sigma}$ of the additional insurance policy. The writing of insurance policies involves *no risk consolidation* here, neither within insurance company j nor in the investors' portfolios. Even under such circumstances, however, it is possible to write insurance policies.

6.3 No external effects between the companies

To check for the existence of external effects, we must once again compare the increase in the overall risk caused by an additional policy with the increase in insurance company j's systematic risk. With $\rho(C_i, Y_p) = 1\ \forall i$, the following is true:

$$\sigma(Y_M) = \left[(n_j\bar{\sigma})^2 - 2n_j\bar{\sigma}\sigma(Y_p) + \sigma^2(Y_p)\right]^{0.5} = \sigma(Y_p) - n_j\bar{\sigma} > 0\ ,$$

$$\frac{\mathrm{Cov}(Y_j, Y_M)}{\sigma(Y_M)} = -\ n_j\bar{\sigma}$$

and where $\rho(C_i, Y_p) = -\ 1\ \forall i$

$$\sigma(Y_M) = \left[(n_j\bar{\sigma})^2 + 2n_j\bar{\sigma}\sigma(Y_p) + \sigma^2(Y_p)\right]^{0.5} = n_j\bar{\sigma} + \sigma(Y_p) \ ,$$

$$\frac{\mathrm{Cov}(Y_j, Y_M)}{\sigma(Y_M)} = n_j\bar{\sigma} \ .$$

After differentiating the four equations with regard to n_j we arrive at:

$$\frac{d\sigma(Y_M)}{dn_j} = \frac{d\left[\mathrm{Cov}(Y_j, Y_M)/\sigma(Y_M)\right]}{dn_j} = +/- \ \bar{\sigma} \ .$$

Thus there are no external effects between the companies and no allocation distortions occur.

7. Comparison with the Insurance CAPM

Finally, we will compare the results of the present analysis with the *Insurance CAPM* which has been used in many states of the USA since the 1970s to regulate insurance premiums on the basis of the Standard CAPM. Whereas this practice of regulation has usually been criticized because of the idealized assumptions of the Standard CAPM (see e.g. Müller, W. 1983, Cummins/Harrington 1985, Urrutia 1986 or Albrecht 1991), here it will be demonstrated that the approach is in general misleading.

7.1 Average vs. marginal costing

The Insurance CAPM is based on the following approach: the policyholders pay the insurance companies premiums in return for cover for losses which they may or may not suffer. From this point of view they receive a return: $R_z \equiv S/(n_j\pi) - 1$. According to the model, R_z should be calculated using the *security market line* of the Standard CAPM:

$$\mu(R_g) = r + \left[\mu(R_M) - r\right]\beta_g \ ,$$

where $R_g \equiv$ return per money unit invested in a security g traded in the capital market, $R_M \equiv$ return per money unit invested in the market portfolio. The so-called *beta factor* may be defined as follows:

$$\beta_g \equiv \frac{\mathrm{Cov}(R_g, R_M)}{\sigma^2(R_M)} \ .$$

If we now apply the security market line equation to the underwriting return R_z – assuming the collective defined in equations (1) to (3) – we arrive at a formula for calculating the total premiums paid by the policyholders to insurance company j:

$$n_j \pi = \frac{n_j \bar{\mu} - \lambda \mathrm{Cov}(S, R_M)/\sigma(R_M)}{q} ,$$

with $\lambda = \left[\mu(R_M) - r\right]/\sigma(R_M)$.

By solving the equation for $q\pi$, we arrive at the *premium principle per policy* as implied in the Insurance CAPM:

$$q\pi = \bar{\mu} + \lambda \frac{\mathrm{Cov}(Y_j, Y_M)/\sigma(Y_M)}{n_j} , \tag{19}$$

since $S \equiv \Sigma_i C_i, i = 1, \ldots, n_j$, and $\mathrm{Cov}(Y_j, Y_M) = -\ \mathrm{Cov}(\Sigma_i C_i, Y_M)$. The compounded premium per policy should be, according to the Insurance CAPM, equal to expected claims plus a risk loading/markdown, the latter being the product of the risk price and the average systematic risk of insurance company j. The marginal conditions (13), (16), (18) and (18') reveal, however, rational behaviour of the economic agents in the insurance market requires *marginal costing*, i.e.

$$q\pi = \bar{\mu} + \lambda \frac{d\left[\mathrm{Cov}(Y_j, Y_M)/\sigma(Y_M)\right]}{dn_j} ,$$

and *not average costing*, as premium principle (19) suggests. It thus becomes clear that application of the Insurance CAPM leads to *further misallocations* if the insured risks are correlated with the other risks traded on the stock exchange, i.e. if $\mathrm{Cov}(S, Y_M) \neq 0$. These misallocations occur over and above the allocation distortions brought about by the external effects.

The decisive question is whether the welfare losses due to average costing are compensated by those caused by the positive external effects or whether a sort of additive eclipsing occurs. In practice one would expect both cases to appear across the various lines of business. From taxation theory we know that the "excess burden" in a market rises progressively with the deviation from the optimum price. Thus, if in reality the two opposing distortions cancel each other out in some lines and are compounded in others, the negative effects will predominate and the application of the Insurance CAPM will, *on balance*, bring an *additional welfare loss*.

7.2 Security market line and optimum corporate conduct

The reason for the additional misallocations caused by average costing lies in the fact that the CAPM security market line specifies only how company shares are evaluated on a stock exchange made up of rational investors. However, this equation does not also imply rational behaviour on the part of the companies; it is valid *in the general sense* given. Should a company not behave optimally, its shares cannot be sold at the highest price. Nevertheless, the security market line and the equivalent market value formula are valid, too, for this company's shares.

Optimum behaviour on the part of a company require not only the validity of these two formulae, however, but also the maximization of that company's market value, which implies marginal costing, as the marginal conditions calculated above show. This result is made possible by splitting up the uncertain premium income into a quantity and a price component as then it is possible to explicitly calculate the dependence of an insurance company's risk on the number of policies written. The purely schematic application of the security market line to the policyholders' total premium payments, as is customary in the Insurance CAPM, leads in contrast to average costing and thus, as we have seen, to welfare losses.

8. Summary

The free play of market forces in the insurance market leads to a *competitive equilibrium* provided the insurance companies do not have decreasing marginal costs. There are two grounds for assuming this. Firstly, empirical studies show that, from a particular size upwards, insurers are affected by diseconomies of scale as regards customary operating costs, which can cancel out economies of scale in the area of risk. Secondly, three of the four (extreme) cases treated in this paper revealed rising or constant marginal risk costs.

The existence of a competitive equilibrium on the insurance market, however, is no guarantee of a *welfare optimum*. It was shown that the companies traded on the stock exchange have positive external effects on each other, which is why the transaction volume in the insurance market is too low. From a social perspective the insurance companies write too few policies because the positive cost externalities related to risk consolidation lead them to calculate higher than necessary marginal costs. This is why a Pareto optimum is not achieved in a competitive equilibrium.

If we compare the premium principles described above – which arise from the rational behaviour of insurance companies and policyholders and the free play of market forces – with the *Insurance CAPM premium formula*, a considerable flaw in the latter models becomes apparent. Rational action on the part of insurance companies is ensured by marginal costing and not by premiums calculated on average costs. But it is precisely this which the application of these models entails, with the result that the supervisory authorities of those American states using the Insurance CAPM are probably producing welfare losses.

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Zusammenfassung

In der vorliegenden Arbeit wird die Risikokonsolidierung von Versicherungsaktiengesellschaften auf der Grundlage des Capital Asset Pricing Models untersucht. Im Fall unkorrelierter Risiken besitzen Versicherer Größenvorteile, weil der in der Versicherungsprämie enthaltene Risikoaufschlag mit zunehmender Anzahl der Versicherungsverträge gesenkt werden kann. Dies gilt allerdings nicht bei korrelierten Risiken. Die erzielten Ergebnisse zeigen des weiteren, daß rationales Verhalten der Versicherungsunternehmen eine Grenzkostenkalkulation erfordert, während das sogenannte "Insurance-CAPM" eine Durchschnittskostenkalkulation impliziert. Es wird außerdem eine wohlfahrtsökonomische Analyse des Konkurrenzgleichgewichts auf dem Versicherungsmarkt vorgenommen.

Abstract

This paper investigates the risk consolidation of insurance companies (stock corporations) on the basis of the Capital Asset Pricing Model. In the case of uncorrelated risks insurers enjoy economies of scale as the risk markup contained in the premium can be reduced by increasing the number of policies. However, this is not true of correlated risks. The results show furthermore that rational behaviour on the part of insurers involves their carrying out marginal-cost calculations, whereas the so-called "Insurance CAPM" has been implicitly based on average-cost calculations. The equilibrium on the insurance market under the conditions of perfect competition is analyzed from a welfare-economic viewpoint.

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