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Wage and Quantity Setting with Asymmetric Quality Information: A Note

By Hellmuth Milde

In this note we discuss a labor market model with asymmetric information. A firm is considered having no possibility to identify job applicants with different abilities. Given this quality uncertainty the firm is free to set an optimal wage rate and an optimal employment level. Depending on the position of the market constraint two different solutions are analyzed. The theoretical framework of the model is based on the theory of Akerlof's 'lemon' market.

1. Stickiness in prices and wages is an essential feature of Keynesian economics. Thus, not surprisingly, the rationale for price and wage stickiness has been a subject of substantial debate. Sometimes it is argued[1] that stickiness is based on considerations of informational asymmetries. The basic argument is that in situations with imperfect and asymmetric information prices and wages perform two different economic functions: They not only serve to clear the commodity and labor market but they also affect the average quality of the products traded in these markets. As a result, the determination of equilibrium prices and wages might be independent of market clearing conditions. More specifically, the asymmetric information paradigm implies the possibility of deriving a market equilibrium in which quantities demanded do not equal quantities supplied. Changes in supply and demand conditions might have no impact on equilibrium prices and wages.

In this note we study the effects of asymmetric quality information in labor markets on the wage and quantity setting behavior of firms. In so doing we re-examine the results of a model developed by *Weiss*.[2] The purpose of this note is to show that the result derived by Weiss is only one of two possible cases. The following result will turn out to be significant: "Lemon" markets[3] are not always characterized by price stickiness and quantity rationing. Or to put it differently, we shall find that the existence of informational asymmetry is not sufficient to derive wage stickiness and job rationing. The properties of the solutions depend on the position of the market constraint.

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[1] See e.g. *Stiglitz* (1979), *Stiglitz* (1985), and *Stiglitz* (1987).
[2] *Weiss* (1980).
[3] *Akerlof* (1970).

In what follows we discuss two different regimes. In the first model the equilibrium wage is not a market clearing rate. As a consequence, there is quantity rationing in the labor market. Basically, this is the result of *Weiss*. The result of our second model asserts that there is a very traditional non-rationing equilibrium. The wage is flexible, i.e. responds to changes in market supply and/or demand, and there is no job rationing. It is still possible to derive a conventional equilibrium even under the asymmetric information paradigm.

2. Consider a two commodity world. The output quantity is x and the absolute output price is 1. The input quantity ("labor") is n and the input price ("wage rate") is w. However, labor is not a homogeneous input. Each labor unit is characterized by a specific number θ, with $\theta \varepsilon [\theta_1, \theta_2]$. θ is called "ability" or "productivity". Each member of the labor force has perfect knowledge of his θ. The firm, however, cannot observe the true value of θ for any given job applicant. This is the basic informational asymmetry. The firm has a subjective belief of the distribution of θ over the interval $[\theta_1, \theta_2]$. This prior belief is expressed by the density $f(\theta)$. By assumption, the firm cannot distinguish among different θ-types of job applicants. Therefore, the wage rate offered to applicants cannot reflect the specific abilities θ. Instead, the firm will set an average wage rate reflecting the average ability of applicants accepting the job offer. In order to simplify the structure of the model we assume that there is only one "monopoly" firm in the market offering jobs. On the other side of the market there is an infinite number of applicants seeking a job.

3. The decision problem of the applicant is: employment with the monopolist firm at the given wage w or withdrawal from the labor market. If the worker drops aut, he has the opportunity for "home production". The result of home production is a quantity of commodities according to his ability number θ. Because the worker has perfect knowledge of his θ, he will compare his θ with the wage rate w offered by the firm, thus, θ is his "reservation wage". The self-selection rule is given by:

$$\text{(1)} \qquad \begin{array}{l} \text{"firm production" (employment), if } \theta \leqslant w\,, \\ \text{"home production" (drop out), if } \theta > w\,. \end{array}$$

According to (1), only "lemons" will ask for firm employment; i.e. only low-θ-applicants stay in the market; high-θ-applicants will drop out.[4] The aggregate labor supply function can be derived from (1):

[4] It is a fairly unpalatable consequence of the model that the best workers are never employed by the firm. Note, however, that the best applicants, although not working in the firm, are not unemployed. They are "self-employed".

$$
\begin{aligned}
(2) \qquad n^s(w) &= \int_{\theta_1}^{w} f(\theta)\, d\theta = F(w), \text{ with} \\
n^s{}_w(w) &= f(w) > 0, \text{ if } \theta_1 < w < \theta_2 .
\end{aligned}
$$

In our discussion below equation (2) will perform the function of the market constraint which influences the firm's optimizing behavior.

The firm's decision problem is setting the optimal wage rate and the optimal employment level, i.e. the optimal number of applicants hired. Note, that with incomplete information the firm is in a position to set both input price and input quantity independently. The output quantity produced by the firm depends on the number of workers and also on the average ability of the labor force. Because we assume that information is distributed asymmetrically, the firm has no choice but to use the *average quality* $\bar{\theta}$ as the appropriate quality index. The firm's production function is given by:

$$
\begin{aligned}
(3) \qquad x = x(n^d, \bar{\theta}),\ & x_n, x_{\bar{\theta}} > 0 , \\
& x_{nn}, x_{\bar{\theta}\bar{\theta}} < 0 , \\
& x_{n\bar{\theta}} \lesseqgtr 0 .
\end{aligned}
$$

The firm's calculation of $\bar{\theta}$ is important for the working of the model. One possibility is take the (unconditional) expectation of θ, $E[\theta]$, which is a constant parameter of the density $f(\theta)$. Using $E[\theta]$, implies that *all* workers with θ, taken form the basic interval $[\theta_1, \theta_2]$ are relevant for the calculation of the average. As the firm knows, however, this is definitely not the case. According to (1) the relevant interval for *employed* workers is $[\theta_1, w]$; see also supply function (2). Therefore, the sophisticated manager of the firm will calculate a *conditional* expectation:

$$
\begin{aligned}
(4) \qquad \bar{\theta}(w) &= \int_{\theta_1}^{w} \theta f(\theta)\, d\theta \Big/ \int_{\theta_1}^{w} f(\theta)\, d\theta, \text{ with}^5 \\
(4a) \qquad \bar{\theta}_w &= [w - \bar{\theta}(w)] \quad h(w) > 0, \\
(4b) \qquad \bar{\theta}_{ww} &= (w - \bar{\theta})\ (h_w - h^2) + h \gtreqless 0, \text{ where} \\
(4c) \qquad h(w) &= f(w)/F(w) > 0, \\
(4d) \qquad h_w &= h(f'/f - f/F) \gtreqless 0 .
\end{aligned}
$$

[5] Note that this analysis could usefully exploit the analogy to a Lorenz curve. The numerator and demominator in (4) are the ordinate and abscissa of the Lorenz curve of f(.). So the average quality at wage w is the slope of the ray from the origin to the appropriate point of the Lorenz curve (not quite, but up to a normalization).

Taking the mostly used density functions (normal, exponential, uniform) we find:

$$h_w < 0 \text{ and } \bar{\theta}_{ww} < 0.$$

Thus, $\bar{\theta}$ is a rising and concave function of w; see figure 1:

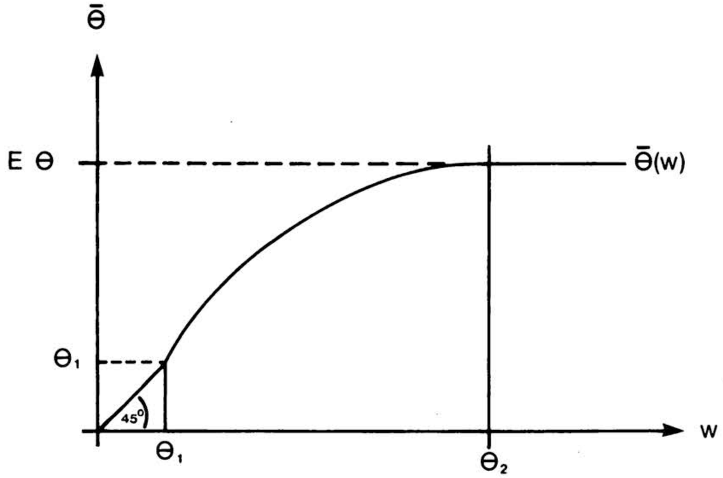

Figure 1: Conditional Expectation

4. Assuming a risk-neutral firm the expected profit is given by:

(5) $$E[\Pi] = x(n^d, \bar{\theta}(w)) - w\,n^d, \text{ with}$$

(5a) $$E[\Pi]_n = x_n - w \gtreqless 0,$$

(5b) $$E[\Pi]_{nn} = x_{nn} < 0,$$

(5c) $$E[\Pi]_w = x_{\bar{\theta}}\,\bar{\theta}_w - n^d \gtreqless 0,$$

(5d) $$E[\Pi]_{ww} = x_{\bar{\theta}}\,\bar{\theta}_{ww} + \bar{\theta}_w^2\,x_{\bar{\theta}\bar{\theta}} < 0,$$

(5e) $$E[\Pi]_{wn} = x_{n\bar{\theta}}\,\bar{\theta}_w - 1 \gtreqless 0.$$

All partial derivatives listed in (5a) to (5e) characterize the objektive function (5) completely with respect to the decision variables n^d and w. Equation (5a) is known from the literature dealing with firm behavior under certainty. Equation (5c), however, is due to uncertainty and information asymmetry. From the interaction of (5a) and (5c), combined with the concavity properties (5b), (5d), and (5e) we can derive a special shape of the iso-$E[\Pi]$-function. In the n^d-w-space the iso-$E[\Pi]$-function is a *closed* contour (circle, ellipse). Increasing levels of $E[\Pi]$ ar represented by smaller contours located inside the larger contours. The best possible point (comparable to the graphical representation of "bliss"-point or satiation point) is denoted S in figure 2.

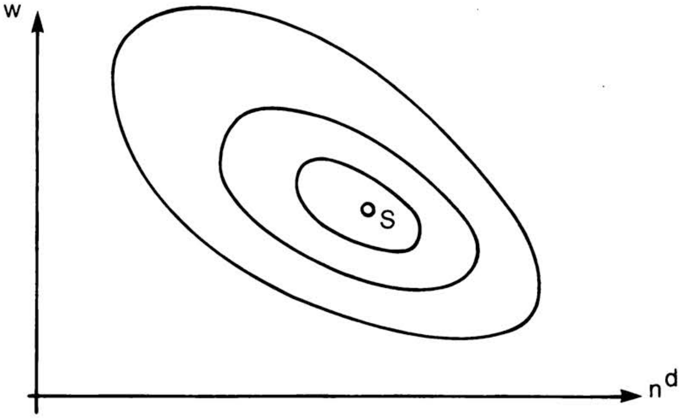

Figure 2: Closed Iso-Profit Contours

Without the self-selection mechanism we find $\bar{\theta}_w = 0$ which implies $E[\Pi]_w < 0$. As a result, the loops are no longer closed. Isoprofit-curves with well known shape are depicted in figure 3. Note that in this case the connecting line of all zero slope points on different iso-$E[\Pi]$-curves results in the conventional labor demand function.

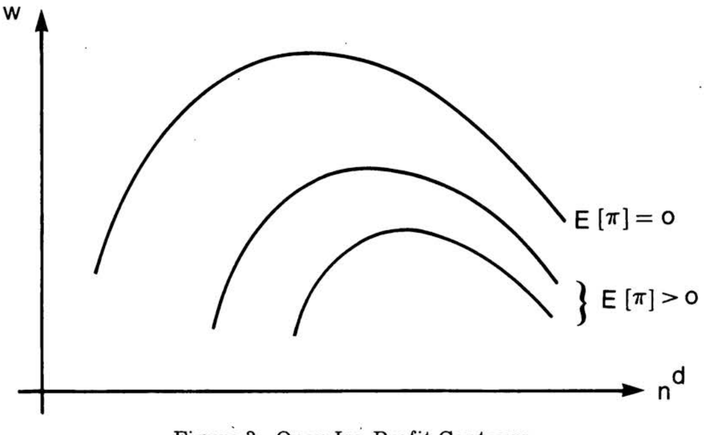

Figure 3: Open Iso-Profit Contours

The properties of the equilibrium solution depend exclusively on the position of the market constraint (2) relative to the point S of the map of iso-$E[\Pi]$-contours. We distinguish two different "regimes". In the first case the constraint is not binding. This is the quantity rationing case. In the second case the constraint is in fact binding. It is the traditional tangency solution. The formal analysis is Kuhn-Tucker maximization:

$$\underset{w,\, n^d > 0}{\text{Max}}\, E[\Pi], \text{ subject to } n^d \leq n^s(w).$$

The Lagrangean expression is:

$$L = E[\Pi] + \lambda\,[n^s(w) - n^d], \tag{6}$$

where λ is the endogenously determined Lagrange multiplier. From (6) we obtain the first order conditions:

$$L_w = E[\Pi]_w + \lambda n^s{}_w = 0, \tag{7a}$$

$$L_n = E[\Pi]_n - \lambda = 0, \tag{7b}$$

$$\lambda\,(n^s - n^d) = 0, \tag{7c}$$

$$n^s - n^d \geq 0, \tag{7d}$$

$$\lambda \geq 0. \tag{7e}$$

Two different cases can be distinguished:

Case (i) : $n^s - n^d > 0, \lambda = 0, E[\Pi]_w = 0, E[\Pi]_n = 0.$

Case (ii) : $n^s - n^d = 0, \lambda > 0, -E[\Pi]_n/E[\Pi]_w = 1/n^s{}_w.$

5. The first case (i) is illustrated in figure 4.

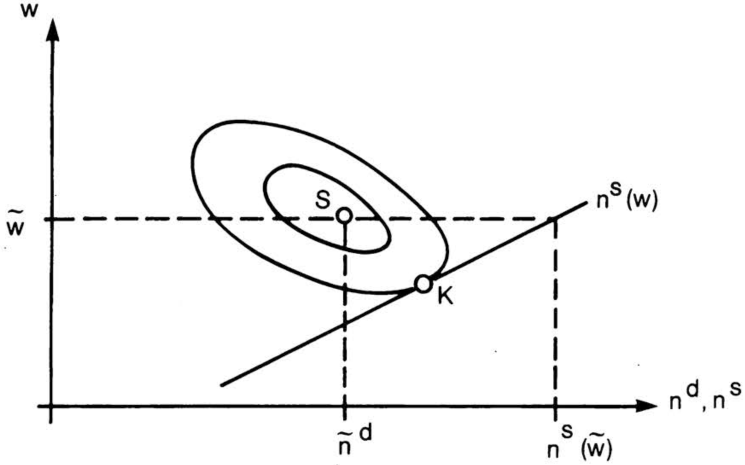

Figure 4: Equilibrium with Non-binding Constraint

The tangency point K is clearly not an optimal solution. The optimum is S with $\tilde{w}$ and $\tilde{n}^d$. The market supply function n^s is not a binding constraint: λ is zero. First order conditions are given by:

(8a) $$E[\Pi]_n = 0,$$

(8b) $$E[\Pi]_w = 0.$$

The solution in figure 4 implies an optimal volume of excess supply, $n^s(\tilde{w}) - \tilde{n}^d > 0$. There is no incentive to eliminate the excess supply by reducing the wage rate so that some workers are unable to find employment. The unemployed workers are not related systematically to their levels of θ. The firm cannot observe differences in θ by assumption.

In figure 5 the second case (ii) is depicted.

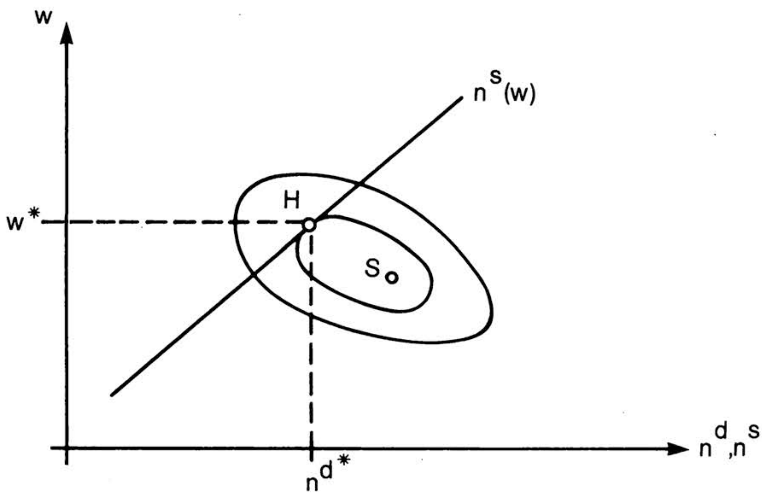

Figure 5: Equilibrium with Binding Constraint

The "bliss"-point S is not attainable anymore. The optimal solution is H with w^* and n^{d*}. There is no longer quantity rationing: $n^s(w^*) - n^{d*} = 0$. The first order condition is characterized by a well-known tangency solution:

(9) $$-\frac{E[\Pi]_n}{E[\Pi]_w} = \frac{1}{n^s_w}.$$

That is, the optimum H is characterized by a tangency between an iso-$E[\Pi]$-contour and the supply constraint.

In figure 4, the wage rate is not used to solve the allocation problem, thus, we can derive equilibrium quantity rationing. The wage rate has a purely informational function. In figure 5, on the other hand, even under uncertainty and information asymmetry, the wage rate performs both an informational and a traditional allocative function. In this case there is no longer quantity rationing.

The interesting implication of the model is that it provides an explanation of different patterns of wage and quantity adjustment. Sometimes these adjustments result in full employment, sometimes they do not. There might occur a labor market failure. Thus, at the equilibrium wage rate there are sometimes more jobs demanded than offered. The shortage of the firm's information, in conjunction with the self-selection of workers might prevent mutually advantageous labor market transactions from taking place. On the other hand, the uninformed firm obtains correct but very limited information from observing the self-selection process. If the number of job applicants is very small this additional information might be sufficient to absorb all applicants, thus achieving a non-rationing solution.

Summary

In this note we have presented an analysis of the labor market equilibrium with asymmetric quality information. We found that each of the two cases we examined hat very specific properties. In the first case there was a non-binding, in the second case a binding market constraint. We demonstrated that the position of the market constraint was of crucial importance for the properties of the market equilibrium.

Zusammenfassung

Gegenstand des Beitrages war ein Arbeitsmarkt mit asymmetrisch verteilten Qualitätsinformationen. Zwei Fälle wurden diskutiert. Im ersten Fall war die Marktrestriktion bindend, im zweiten Fall nicht. Entsprechend wurde eine Rationierungslösung oder eine Markträumungslösung abgeleitet. Es zeigte sich, daß die Lösungen davon abhängen, ob die Marktrestriktion dominiert oder nicht.

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