Zeitschrift für Wirtschafts- u. Sozialwissenschaften (ZWS) 107 (1987), S. 201 - 205
Duncker & Humblot, Berlin 41

Stock Price Distribution Versus Time to Maturity of Associated Options

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While the stock market exerts a natural influence on stock options it is less evident that the reverse is true, as well. Using samples of published option prices we examine whether the differences between stock prices and adjacent striking prices of associated options are influenced by time to options expirations. For statistical analysis the absolute differences are grouped into classes (intervals) of length one. The observed numbers in the classes are compared to the expected numbers calculated under the hypothesis that time to option maturity has no influence on stock prices.

Introduction

Since their opening starting with CBOE in 1973 the exchanges for listed stock options have been a great success in the US. Today, daily volume in options trading in terms of numbers of shares involved typically exceeds the volume in underlying stocks. Enormous economic interests are at stake both on the options writers' and buyers' side. In view of this it is hardly astonishing that not only the options market is influenced by the stock market but that the opposite is also true. Thus it is known (3) that in the expiration week of a series of options the average residual return of the underlying stocks is approximately -1.0 percent and that a partial adjustment occurs in the subsequent week with an average residual return of 0.4 percent. Also, there have been reports of manipulations of stock prices by institutional investors on options expirations.

Here we study a particular aspect of stock market prices, namely their difference to exercise prices of associated options. More precisely, we will investigate the dependence of these differences on time to option expiration. Clearly, if there is a time-dependence, this is not only in conflict with the random walk hypothesis of stock market prices but is also of interest for practical investment.

Methods and Data of Study

The method we adopt is very simple. Let S denote the stock price and E the largest (smallest) striking price $\leq S$ of associated options. The variable of interest is $D = S - E$ and its dependence on time t to expiration.

For $t = -1, 0, 1$ days as well as $t = -4, -2, 2, 4$ weeks we study and compare the empirical distribution of D. The data base consists of all stocks with options listed on one of the four exchanges (CBOE, AMEX, PHLX, PSE) and with $S \leq 100$. It is known[1] that the distribution of the fractional parts of stock prices is not uniform, lower fractions being more frequent than in view of their weight they should be. In order to avoid artificial effects introduced by this anomaly of stock prices we consider five classes $C_0, \ldots, C_4$ of stock prices defined by the inequalities $i \leq D < i + 1$, $i = 0, \ldots, 4$. Obviously, this amounts to aggregating stock prices into the five classes C_i defined by the last digits (0/5), (1/6), (2/7), (3/8), (4/9) of the nonfractional parts. The grouping is illustrated *Figure 1* with exercise prices of \$ 30, \$ 35 and \$ 40. If the stock prices are not influenced by options expirations then one would expect that, independent of t, all C_i contain approximately the same number of stock prices.

For each of the values of t specified above and each of the three expiration cycles, three consecutive instances were considered. All prices were taken as quoted by the Wall Street Journal between June and October, 1982. The exact issues used for the analysis were of June 18, July 2, 15, 16, 19, 23, 30, August 6, 19, 20, 23, September 3, 16, 17, 20, October 1, 15 comprising the expiration dates of July 16, August 20, and September 17. In all, the analysis was based on 2.398 prices[2].

Tests for equality of empirical and theoretic distributions of proportions were performed by means of appropriate chisquare statistics.

Results

Table 1 shows the numbers $n(C_i)$ of prices falling into the different classes C_i. Except for $t = -1$ day, there was a significant deviation of $n(C_i)$ from expected values. The dependence of D on t becomes more apparent when considering the ratios

$$Q_1 = n(C_2) \,/\, (n(C_0) + n(C_4))$$

and

$$Q_2 = (n(C_1) + n(C_2) + n(C_3)) \,/\, (n(C_0) + n(C_4)).$$

[1] *Abel / Boing* (1983).

[2] Stocks with options having one striking price only were excluded from analysis for in these cases $|D|$ is necessarily small.

If options expirations exert no influence on stock prices then, on expectation,

$$Q_1 = 0.5,\ Q_2 = 1.5.$$

Figure 2 shows the actual time-dependence of Q_1, Q_2 as obtained from our analysis. While, for $t = -1$ day, there is no clear deviation of Q_1, Q_2 from expected values, there is distinct peak both of Q_1 and Q_2 at $t = 0$. Though leveling off, the values remain elevated for $t \leq 4$ weeks. Four weeks before the maturity of the options the phenomenon found at $t = 0$ seems to be reversed, Q_1, Q_2 now being smaller than expected.

Discussion

We have shown that the differences of stock prices to adjacent options exercise prices depend in a characteristic way on the time t to options expirations. This statement is of a different nature than the finding of *Klemkowsky*[3] that the return of stocks is influenced by the expiration of associated options. While temporal variations in price distributions imply variations in returns the opposite is not necessarily true. Moreover, we have found that the main effect takes place not during the whole of the final week before expiration (as suggested in[3]) but on the expiration date itself.

A few remarks on the methodology employed in this study are in order. Though not the only possible measure of distance between stock and exercise prices, the difference is the simplest and most reasonable one. Thus, e.g., the ratio S/E would have been less appropriate in our study because it tends to one as $E \to \infty$ so that the analysis of the time-dependence of S/E would have had to take the absolute level of stock prices into account. In the investigation, stock prices above \$ 100 were excluded because the exercise price pattern below \$ 100 is different from the pattern above \$ 100 so that the ranges of D for $S \leq \$ 100$ and $S > \$ 100$ do not coincide. The small loss of information entailed by the grouping of prices (produced by omission of fractional parts) is compensated for by a great simplification of the calculation and interpretation of the results as well as by the avoidance of bias coming from the "rounding" of option prices mentioned above. Moreover, since the grouping we chose appears natural and had been fixed prior to the study, all statistical inferences drawn for the aggregated data are strictly valid.

We put forward two possible explanations for our findings. The first consists in the observation that option writers have a natural interest that

[3] *Klemkowsky* (1978).

options expire worthless. Options writers being more frequently among the institutional investors than option buyers they may have the financial potential for attempting to push the stock prices in a direction that serves their purposes. Now calls and puts af a stock with identical conditions simultaneously expire worthless only if S is equal to their common striking price.

Second, and more naturally, exercising of options may play an essential role. The exercice of options prior to maturity mostly occurs with puts. The deeper puts are in the money the earlier they may be exercised in a profitable way (see[4] for a thorough discussion of the situations where options should be exercised). On expiration dates the puts which remain to be exercised are mainly those having striking prices close to the stock price. Puts with striking price $E' = E + 5$, however, will be exercised only if $D' = E' - S$ is large enough, taking transaction costs into account, to render the exercise profitable. We think it likely that the writers, when the puts are assigned to them, more frequently (must) sell the stock they receive than option buyers are obliged to purchase them, puts often serving as a hedge. Thus the effect coming from the exercise of puts will be a drop of S towards E except for very small D', an event which will produce a price distribution of the form seen at $t = 0$. Similary, the exercise of calls with striking price E is profitable unless D is too small and again is likely to produce a drop of S towards E because owners of calls mostly do not wish or are unable to keep the stock in their portfolios. The selling will extend over a certain period whence the persistance of the distibutional form of stock prices found at $t = 0$.

Summary

An empirial analysis shows that the differences between stock prices and adjacent exercise prices of associated options are heavily influenced by time to options expirations. On expiration dates and, to a lesser degree, during the two following weeks, these differences have a significant bias towards zero whereas the opposite holds four weeks before expiration.

Zusammenfassung

Anhand von publizierten Optionspreisen wird untersucht, wie die Differenz zwischen Aktienkursen und den benachbarten Basispreisen zugehöriger Optionen von der Restlaufzeit der Optionen abhängt. Dabei stellt sich heraus, daß zu Verfallterminen diese Differenz gegenüber einer zufälligen Verteilung signifikant zu geringeren Werten hin verschoben ist, während vier Wochen vor dem Verfall das Gegenteil zutrifft.

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[4] See *Cox / Rubinstein* (1985), *Merton* (1973).

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