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Long-Run and Short-Run Demand Response, Discount Rate, and Pricing

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The article deals in the first instance with the relation between the traditional "Cournot-price" and the price which maximizes the present discounted value of a monopoly's profit stream. The distinction arises if different long-run and short-run demand responses are separated. Then the question is raised whether monetary policy has an impact on prices provided that a changing level of interest rates affects the proper discount rate. Finally we briefly ponder whether the results gained are restricted only to the case of monopoly or not.

I.

The orthodox theory of monopoly price has come under severe attack. In an article recently published, *Brennan, Buchanan* and *Lee* (henceforth for the sake of brevity: *B/B/L*) asserted that the price which maximizes the present value of the profit stream of a monopoly is higher than the traditional "Cournot-price".[1] The analysis is carried out under the reasonable and widely accepted assumption that the (absolute) values of demand elasticities are greater in the long-run than in the short-run. *B/B/L* detected only one exception to their rule: in the special case of long-run and short-run demand curves with an initial price that "chokes off" all demand.[2] We will see later why the old German adage "*Die Ausnahme bestätigt die Regel*" – the exception proves the rule – has no chance to be employed under the given conditions.

In this paper it is not discussed at length the reproach advanced by *B/B/L* against Cournot, namely that if he would price his mineral water according to his well known "marginal revenue equals marginal cost"-rule, he would receive less profit than possible when charging a higher price.[3] This seems somewhat abstruse and unjustified. Cournot analysed profit maximization when a monopolist is facing *one given* downward sloping demand curve, i. e. he submitted profit maximization in one period of time (which may be called

* Helpful comments by an anonymous referee are gratefully acknowledged. The usual caveats apply.

[1] See *Brennan / Buchanan / Lee* (1983).

[2] Cp. *Brennan / Buchanan / Lee* (1983), 538 - 39.

[3] Cp. *Brennan / Buchanan / Lee* (1983), 540.

"long-run") to questioning. Changing crucial assumptions always leads to different upshots (in fact, that is the criterion for a crucial assumption), the real problem is to single out which to adopt and which to discard. Once it was found sensible to separate long-run and short-run demand responses the familiar Cournot pricing was discredited because of its unaccepted premises and not because of its logical spurious derivation of the profit maximizing price.

The underlying assumption throughout the following analysis is that our monopolist is looking for one single best price prevailing in all periods of his time horizon[4]. The value of total profit (PV) comprises short-run profit (P_S), i.e. the profit in the actual period, and long-run profit (P_L) to be discounted at the proper discount rate (r) which is supposed to be constant for the first run over the field:

(1) $$PV = P_S + P_L / r \,,$$

where $r > 0$.

Point of further departure is the orthodox textbook precept of pricing, i.e. the monopolist is considering only the long-run demand curve, denoted as

(2) $$f = f(p) \,,$$

where p stands for the price charged and $f' < 0$ (a prime indicates here and henceforth the first derivative with respect to p). If we abstract from constant costs for reasons of simplicity profit per period emerges as

(3) $$P_L = f(p - k) \,.$$

The direct unit costs k are considered to be positive and constant, thus coinciding with the marginal costs. Maximum profit is reached when

(4) $$\partial P_L / \partial p = f + pf' - f'k = 0$$

holds, which implies that the Cournot-price (p^*)

(5) $$p^* = -f/f' + k$$

is charged.

After reading the $B/B/L$-article the monopolist was deeply impressed by the possibility to proliferate his profits by raising his price above the Cour-

[4] For a discussion of this strategy see *Brennan / Buchanan / Lee* (1983), 544 - 45. *Murray* (1985), 269, objects to the constant price assumption that it merely imposes a constraint on the model but does not render the model dynamic. Below we touch this issue.

not-price p^*. He confirmes the existence of several short-run demand curves intersecting the long-run demand curve at p^* reflecting different demand responses according to the length of adjustment time respectively represented. He singles out one of these as "the" relevant short-run demand curve (to this issue we will turn later), denoted as

$$g = g(p), \tag{6}$$

with $g' < 0$.

Under these circumstances we can write for short-run profit

$$P_S = g(p - k). \tag{7}$$

Substituting (3) and (7) in (1) gives

$$PV = (p - k)(g + f/r). \tag{8}$$

Equation (8) is the objective function our monopolist tries to maximize. Setting the first derivative with respect to p equal to zero allows the calculation of the profit maximizing price ($\hat{p}$) for all periods taken into consideration:

$$PV' = g + f/r + (\hat{p} - k)(g' + f'/r) = 0 \tag{9}$$

and hence after some obvious manipulations

$$\hat{p} = \frac{gr + f}{-g'r - f'} + k \tag{10}$$

(second order conditions are supposed to be fulfilled). This formula is the precept of setting a profit maximizing price, superseding the traditional Cournot-price but starting with it. This rule has not been stated by $B/B/L$ in this or a comparable mode. And this fact is not only due to the omission of costs in their analysis (equation (10) holds still good in the case when k equals zero by assumption), but rather to the fact that $B/B/L$ introduced a "price history", as they named it, in the analysis. $B/B/L$ depicted the short-run demand curve by

$$g = g(p, p_0),$$

where "... p is the price in the current period and p_0 is the price to which behavior has been adjusted prior to the current price change."[5] But with this

[5] *Brennan / Buchanan / Lee* (1983), 535.

distinction the subject appears obscured. Why should *ceteris paribus* p_0 *not* coincide with p^*? The only reason is that a monopolist who has not been a profit maximizer before hence charging a "historical" price p_0 has turned now into a *homo oeconomicus* behaving in the well known manner. But it appears to be methodically incorrect to compare pricing in one setting where the pricing rule is not subject to profit maximization against another setting where the behaviour of the entrepreneur is stipulated in this way. Since the purpose of this paper is to contrast profit maximization in a Cournot-world against a framework where long-run and short-run demand curves are distinguished, it is obvious that only p^* or $\hat{p}$ and not any p_0 are the relevant prices to consider. If this assessment is shared the consequences on the traditional theory of monopoly price are not so devastating as $B/B/L$ seem to think. Equation (10) shows that the old tenet to determine the profit maximizing price, namely "marginal revenue equals marginal cost", still holds good even in the setting of $B/B/L$. However, of course, the notion "marginal revenue" has now to be interpreted in a broader sense. It no longer suffices to relate this term only to the marginal revenue of the current period, instead, the present value of future marginal revenue has to be covered too.

II.

Now the crucial question arises whether $\hat{p}$ exceeds p^* and under which conditions this will occur. Fortunately this question is easy to answer. We commence with assuming that our monopolist is charging p^* applying relation (5). He is deliberating whether a position on the relevant intersecting short-run demand curve for one period of time after raising his price can offset the discounted diminution of profit in all following periods. Since he starts from realizing the "marginal revenue equals marginal cost"-rule we can write, using the well known Amoroso-Robinson relationship,

$$p^* (1 + 1/\varepsilon_L) = k \,, \tag{11}$$

where ε_L denotes long run price elasticity.

Leaving p^* by raising his price is only worthwhile if marginal profit is positive, hence

$$p^* (1 + 1/\varepsilon_S) < k \,, \tag{12}$$

where ε_S indicates short-run elasticity, has to be fulfilled. The economic interpretation of (12) is that a reduction in quantity purchased after increasing the price has to be accompanied by a larger reduction in cost than in revenue thereby yielding marginal profit.

The combination of (11) and (12) boils down after some manipulations to

(13) $$\varepsilon_L < \varepsilon_S \text{ or } |\varepsilon_S| < |\varepsilon_L| .$$

Hence we can conclude that in every case where short-run elasticity is smaller in absolute value than long-run elasticity a higher price than p^* will emerge. And it is this assumption that has heavy theoretical and empirical support.

It has already been noted that *B/B/L* presented one exception where the orthodox "long-run marginal revenue equals marginal cost"-rule is, in their opinion, still valid. This happens in the special case of linear short- and long-run demand curves where short-run demand intersects long-run demand at the zero output level, or as *B/B/L* put it, at the "choke" price. But this case turns out to be only an exception in the setting in which *B/B/L* stated the problem. Since linear demand curves that meet the vertical axis at the same point have all the same elasticities along any horizontal line, the elasticities of long-run and short-run demand do not differ at any price. Therefore equation (13) is not fulfilled and hence p^* proves to be the profit maximizing price even in the long-run. This case emerges now not as an exception to the rule, instead, the rule cannot be applied.

III.

Let us now consider a numerical example. In this way it is possible to derive a formula of the profit maximizing price that seems to be more relevant for practical purposes than equation (10). We start from the recognition that the price setting practitioner, as well as the man of empirical economic research, has good reasons to assume iso-elastic demand curves over a relevant range.[6] The general form of iso-elastic demand curves is

(14) $$q = A^{-\varepsilon} p^{\varepsilon} ,$$

where q denotes the quantity purchased, A a positive constant and ε the constant elasticity. Differentiating (14) with respect to p leads to

(15) $$q' = \varepsilon q / p .$$

Supposing that the short-run demand curve $g(p)$ and the long-run demand curve $f(p)$ are both iso-elastic we are entitled to write

[6] Cp. *Helmedag / Leitzinger* (1984), 36.

(16) $$g' = \bar{\varepsilon}_S g/p$$

and

(17) $$f' = \bar{\varepsilon}_L f/p\ ,$$

where $\bar{\varepsilon}_S$ and $\bar{\varepsilon}_L$ indicate the constant short- and long-run elasticities respectively.

Substituting (16) and (17) in (10) yields

$$\hat{p} = \frac{-gr - f}{\bar{\varepsilon}_S gr/\hat{p} + \bar{\varepsilon}_L f/\hat{p}} + k$$

whence

(18) $$\hat{p} = \frac{k\,(\bar{\varepsilon}_S gr + \bar{\varepsilon}_L f)}{gr\,(1 + \bar{\varepsilon}_S) + f\,(1 + \bar{\varepsilon}_L)}\ .$$

Equation (18) can be further simplified when a special kind of short-run demand curves is taken into account. And good reasons exist for assuming a rectangular hyperbola as short-run demand curve representing a constant price elasticity of unity. This implies that the consumers have set aside a certain total budget for the commodity of the monopolist considered, therefore *Marshall* called this demand function "constant outlay curve".[7] And it is this feature that seems to provide an *economic* criterion to differentiate between the long-run and the short-run. *B/B/L* themselves noted that several short-run demand curves can be assumed, an entire familiy issuing from any initial price-quantity equilibrium, all reflecting different consumer adjustments.[8] But which of these is the one to be chosen out to enter in (10) or (18)? To this important question *B/B/L* do not give a proper answer. They "solve" the problem by defining short-run to last one half of the long-run, i.e. the time in which full consumers' adjustments have taken place.[9] Clearly, this is a possibility that cannot be criticized purely on logical grounds. But the arbitrary element (why not one third or one quarter of long-run?) thereby entering the analysis would be smaller when defining the short period to have the length of time the consumers' budget *in toto* is not affected by a price impulse. However, if the foregoing is accepted, the actual length of one period of time is depending on the specific good traded. It will probably be longer for cigarettes than for bananas. It must be added that a constant outlay curve has the lowest elasticity one could, for economic reasons, insert in

[7] Cp. *Marshall* (1952), 691.

[8] Cp. *Brennan / Buchanan / Lee* (1983), 534.

[9] Cp. *Brennan / Buchanan / Lee* (1983), 534.

a short-run iso-elastic demand curve. For if the constant elasticity were smaller than unity in absolute value, marginal revenue would be negative, i.e. price and revenue would be moving in the same direction. Hence the monopolist could theoretically proliferate his short-run profit beyond all limits but losing presumably all future revenue when the single price strategy prevails. If a short run demand curve intersecting the long-run demand at the initial price is exhibiting an elasticity smaller than unity (which is, for instance, always true in the linear and costless case of pricing mineral water) and supposing that the monopolist only knows the elasticity at that point, he could be led astray when he exaggerates the range where marginal revenue is negative or smaller than marginal cost. He might then raise his price too much. In the light of this, the constant outlay curve gains and deserves relevance as a fair approximation of a short-run demand curve. Equation (18) reduces under conditions of a rectangular hyperbola as short-run demand function to

$$\hat{p} = \frac{k\,(-\,gr/f + \varepsilon_L)}{1 + \varepsilon_L}, \tag{19}$$

which gives for $r = 0$ also the traditional Cournot-price.

Let us finally turn to the already announced numerical example. We assume two iso-elastic demand functions with $\varepsilon_S = -1$ and $\varepsilon_L = -3$. Long-run demand may be depicted by

$$f = A_f^3 p^{-3}.$$

Supposing $A_f = 2$ we get

$$f = 8/p^3$$

and

$$f' = -\,24/p^4.$$

Let short-run demand be represented by

$$g = A_g p^{-1}$$

with

$$g' = -\,A_g/p^2.$$

For the magnitude A_g one has to insert the revenue at that price-quantity combination where short-run and long-run demand curves were intersecting. The constant marginal costs (k) may have the value of unity.

In the first round our monopolist acts like a traditional Cournot monopolist, i.e. he charges the profit maximizing price

$$p^* = -f/f' + k = 3/2$$

selling the quantity

$$f(p^*) = 8 / (3/2)^3 = 2.37037 .$$

Short-run profit turns out to be

$$P_S = p^* f(p^*) - kf(p^*) = 3.55556 - 2.37037 = \\ = 1.18519 .$$

Let the proper discount rate (r) equal 0.1. Calculating the value of total profit gives

$$PV(p^*) = P_S + P_S/r = 1.18519 + 11.8519 = \\ = 13.0371 .$$

Suppose now a long time has passed, at least long enough that consumers do not interpret a change in price as a price strategy and that the long-run and short-run demand responses as described above would not be affected. So equation (10) or (19) can enter the stage. Inserting the relevant magnitudes and functions in (19) leads to the quadratic equation

$$A_g \hat{p}^2 + 16\hat{p} - 24 = 0 .$$

For A_g the periodical revenue of the monopolist when charging p^*, i.e. 3.55556, has to be taken. Then the economically relevant solution

$$\hat{p} = 1.55364$$

emerges.

We see that $\hat{p}$ exceeds p^*. The same is true for the value of total discounted profit the monopolist gains:

$$PV(\hat{p}) = 13.0775 > 13.0371 = PV(p^*) .$$

We substantiate thereby the previous analysis, namely that there is always an incentive to raise the profit maximizing price above the Cournot-price – given the sole proviso that short-run demand elasticity is smaller than long-run elasticity in absolute value.

One might ask now if there is a further incentive to raise the price once more above the reached $\hat{p}$-level when the time passed has been long enough to settle short-run demand response at the higher price. But this is not so. Charging $\hat{p}$ we have the long-run revenue 3.3142, which could be inserted as A_g in a further short-run constant outlay curve. Employing (19) again we calculate $p^{\#} = 1.54975$ which is smaller than $\hat{p}$. The total value of profit $PV(p^{\#}) = 12.9914$ is smaller than $PV(\hat{p})$ *and* even smaller than $PV(p^*)$. Hence we can infer that no third round of pricing will be opened.

IV.

Hitherto we treated the proper rate of discount as constant. But, in fact, this rate will probably not be constant. On the one hand it is influenced by the individual assessment which rate is regarded as "proper", corresponding, if one likes to put it in the language of the theory of property rights, to the minimum expected rate of return the enforcible property right of the monopoly considered should yield. On the other hand the proper rate of discount is certainly influenced by the level of interest rates which indicates the opportunity costs to exert the monopoly right and to stay in the market. Since it is a well known empirical phenomenon that the various interest rates are more or less closely bound up with one another, we might, consequently, suppose that the proper discount rate will move in the same direction as the level of interest rates does. This level, in return, is at least co-determined by money supply. Thus it seems worthwhile to inquire into the impact on prices if monetary policy alters the level of interest rates, hence, presumably, affecting the proper discount rates. Differentiating (10) with respect to r we get

$$\frac{\partial \hat{p}}{\partial r} = \frac{g\,(-g'\,r - f') + g'\,(g r + f)}{(-g'\,r - f')^2} \tag{20}$$

The signum of (20) depends on the signum of the nominator which has to be greater than zero for (20) to be positive:

$$g' f > g f' .$$

After some manipulations we obtain

$$-f/f' < -g/g' ,$$

or

$$p_f^* < p_g^* . \tag{21}$$

Equation (21) states that the Cournot-price calculated for the long-run demand curve (p_f^*) has to be lower than that calculated for short-run demand (p_g^*). This condition is always fulfilled in the given framework. We therefore can conclude that a rise in the proper discount rate – caused perhaps by a policy of dear money – will *always* induce a rise in the profit maximizing price $\hat{p}$. Note that this influence only occurs if short-run and long-run demand responses are distinguished. It doesn't happen if the firm tries to maximize the present discounted value of its profit stream without, however, separating different demand responses. Suppose a monopolist considers only the long-run demand function but discounts future profits. His profit function in this case emerges as

$$PV = (p - k)\,(f + f/r)\,, \tag{22}$$

which obviously depends on r. However, this is not valid for the price which maximizes (22). Instead, the traditional Cournot-price p^* proves to be optimal. Hence the proper discount rate doesn't matter for pricing under this conditions.

Moreover, another aspect must be mentioned. Not only the price *level* will be increased with a rising discount rate, in addition the *structure* of prices is affected too. Since a change in the level of interest rates, in so far as the proper discount rates are caused to vary, will yield different individual profit maximizing prices according to the conditions of demand respectively, relative prices before and after the adjustment has taken place do not coincide. It must be stressed that the increasing price level and the change in price structure after raising the proper rate of discount does not result by raising the individual marginal costs, but this effect, when it emerges, will amplify price variations.

These results deserve a short deliberation whether they appear merely under conditions of monopoly or whether they can be attached to other market forms too. The reason why the problem of pricing has been approached, in the first instance, by an analysis of the monopoly case is easy to grasp: here we face stable long-run and short-run demand curves as long as the total "market" demand is stable, whereas in other market forms the individual demand curves a firm is facing will shift when one or more other firms of the group or industry alter their prices. And indeed, in the same way as Cournot's rule was generalized to what *Stackelberg* dubbed *„Gesetz des erwerbswirtschaftlichen Angebots"*[10] this could be done with the precept of pricing stated in equation (10) or (18). In like manner as the traditional "marginal revenue equals marginal cost"-tenet could always be employed, if the producers only face negatively inclined demand functions, the "en-

[10] Cp. *Stackelberg* (1951), 186.

hanced" version holds good, too, under these circumstances. That means that (10) or (18) can be used in any cases where the firm is able to play an active role in the process of competition.

Over and above this one must not forget the possibility of autonomous price intervals, i.e. though a price variation of the firm under consideration leads to a shift of the demand curves which the other firms are facing their quantity sold at their prices respectively is *not* affected. Hence the simpler treatment of monopoly pricing is always feasible provided that the firm operates in its individual "monopolistic" range since then no reactions of competitors must be allowed for. The sketched demand conditions are connected with the name of *Gutenberg* and his so called doubly kinked demand function[11]; a theory which comes much closer to the conception of monopolistic competition as developed by *Robinson* than to that advanced by *Chamberlin*.

Profit maximizers in capitalism do not need instructions on how to maximize profit although it may happen that they do not exactly know how to do it technically, but they do it – at least by a procedure of trial and error. Therefore our previous example of a monopolist who does not raise his price until having read the *B/B/L*-article was presumably false: the monopolist in real life (if one ever exists in the rigorous sense of the term) has charged a higher price than the Cournot-price long before the paper of *B/B/L* was published. What seems to be more striking is the fact that monetary policy, in so far as it affects the proper discount rates, *has* an impact directly on the level and structure of prices. However, the direction of this impact differs crucially from that frequently submitted. Any monetary policy performed to control inflation through restriction of high powered money leading to a rise in the proper discount rates will trigger off an avalanche of impulses to raise prices too. Inspecting equation (10) we see that this can only be compensated in a second round by shifting the demand curves to the left, i.e. particularly after a reduction in *income*. Hence an old and commonly shared mode of thinking has also come under severe attack, namely the relationship between a restrictive monetary policy and inflation appears scathed if not reversed.

Summary

The paper starts with focusing on a new development in the theory of monopoly price. Given the widely accepted assumption that short-run elasticities are lower than long-run elasticities, the orthodox "marginal revenue equals marginal cost"-rule must be broader interpreted. In fact, the profit maximizing price exceeds the traditional Cournot-price. Furthermore it is shown that a restrictive monetary policy

[11] Cp. *Helmedag* (1982).

accompanied by a rising level of interest rates will, in so far as the proper rate to discount future profits is affected, lead *ceteris paribus* to a higher price level and a changed price structure. It is argued that the recognitions gained are not merely valid in the monopoly case but can be applied in general.

Zusammenfassung

Anfangs befaßt sich der Artikel mit einer neuen Entwicklung der Theorie der Monopolpreisbildung. Unter der weit verbreiteten Annahme, daß kurzfristige Preiselastizitäten kleiner sind als langfristige, muß die traditionelle „Grenzerlös = Grenzkosten"-Regel umfassender interpretiert werden. Tatsächlich ist der gewinnmaximale Preis höher als der Cournotsche. Darüber hinaus wird gezeigt, daß eine restriktive Geldpolitik, insoweit wie der Kalkulationszinsfuß, der zur Diskontierung künftiger Gewinne dient, mit dem Niveau der Zinssätze steigt, ceteris paribus höhere Preise und eine geänderte Preisstruktur zur Folge hat. Es wird die These vertreten, daß die erlangten Ergebnisse nicht nur für den Monopolfall gelten, sondern allgemein anwendbar sind.

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