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An Empirical Law of the Stock Option Market

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The investigation of the laws governing the prices of stock options is a problem of theoretical and practical interest. The paper empirically studies the relationship between the market prices of options out of the money and the difference between stock and exercise prices, the former being fixed. We use linear regression with subsequent analysis of the residuals. The results are compared with those obtained by the Black-Scholes model. Several applications of the findings are suggested.

I. Introduction

Option prices depend on several parameters, such as stock price, striking (exercise) price, expiration time, dividends paid on stock before the expiration of the option, interest rates etc. Many attempts have been made to theoretically derive option or warrant prices under more or less restrictive model assumptions (e.g. References (1), (3), (4), (5), (7), (12), (14)).

The most famous valuation formula is undoubtedly the one proposed by *Black* and *Scholes* in 1973 for non dividend-paying stocks (3). It has prompted an extensive discussion of the model assumptions and empirical studies of the model fit. Today the formula (perhaps in an extended form allowing for dividends (16)) appears to be widely accepted.

Its greatest shortcoming is that it assumes a constant variance rate ("volatility") v^2 which is an explicit model parameter. In reality the volatility of a stock can hardly be regarded as constant over the time to maturity of the option, and, in any way, determining the variance rate poses a practical problem. Clearly, historical estimates of v (2), (8), (9) can be unreliable and dangerous if money is at stake. Implied estimates are more satisfactory as they are derived from the present market. Their rationale is as follows:

Let k options of a stock be in the market priced at O_i, $i = 1, \ldots, k$. Putting

$$O_i = \hat{O}_i(v),$$

where $\hat{O}_i(v)$ are the prices predicted by the *Black/Scholes* formula for unspecified v, estimated values $v_1, \ldots, v_k$ are obtained which can be weighted and averaged to yield a weighted implied variance rate of the stock (6), (11), (13), (15).

Various weghting schemes have been used in the literature, all more or less arbitrary or chosen on empirical grounds, so that, in principle, the application of the Black/Scholes model shares some features with empirical valuation formulas such as the one given by *Kassouf* (10).

II. An empirical law of the option market

We focus on the special problem of the relationship between the prices of options which are out of the money and the difference between the stock and exercise prices, the former being fixed. We contend that this relationships is approximately loglinear.

Fig. 1 a/b shows that the hypothesis of loglinearity is not farfetched.

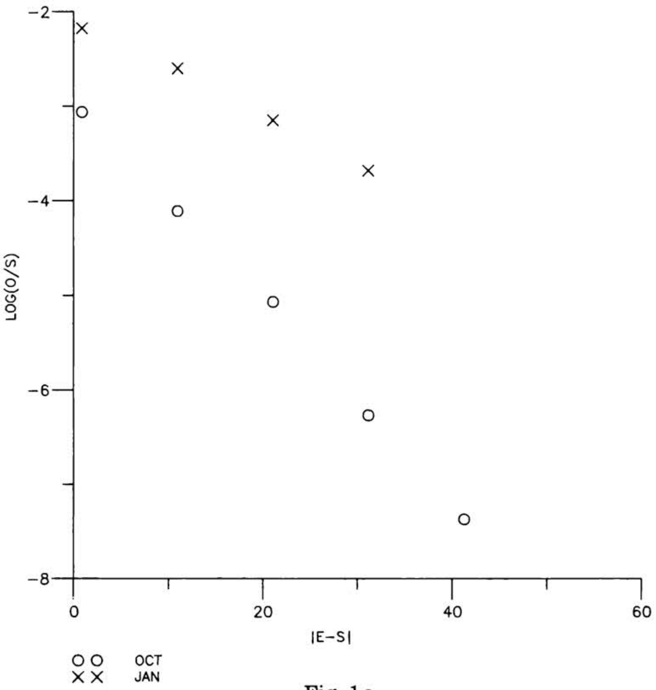

Fig. 1 a

The empirical study was based on option prices as published in the Wall Street Journal. Only informative prices were taken into consideration, that is

1. Option prices of 1/16 were excluded because at this level (the lowest possible) numerous anomalies arise. E.g., on Sept. 21st, the October $ 15, 20 and 25 puts of Homestake were all priced at 1/16.

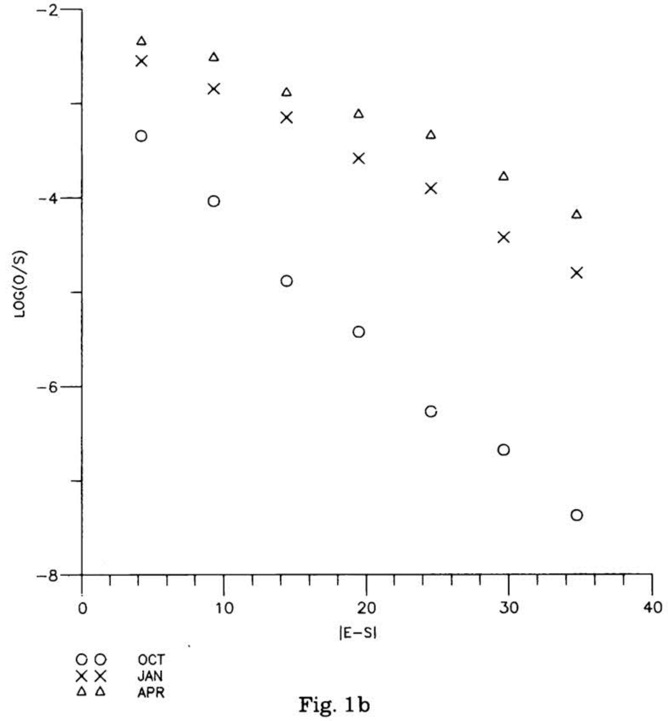

Fig. 1b

2. Stocks with less than three prices above 1/16 of options out of the money were excluded since they carried no information as to our hypothesis.

   We were slightly more restrictive in that three option prices > 1/16 for successive exercise prices had to be available in order to qualify the stock for the analysis.

For puts and calls different days had to be chosen for we failed to find one single day where the criteria of selection were met by a satisfactorily large sample of both calls and puts. The data bases for the analysis were the following:

Calls expiration in October 1982
prices of June 23rd, 1982
20 eligible stocks

Puts expiration in January 1983
prices of September 22nd, 1982
22 eligible stocks.

For each selected stock a simple linear regression of

$$\log (O/S) \quad \text{vs} \quad \Delta = 100 \cdot |S - E| / S$$

was performed (O, E, S denoting the option, exercise and stock prices respectively). Of course, log (O/S) and Δ are linearly related if and only if log (O) and $E - S$ are. The standardization was introduced in the hope that the result would prove independent of S and possibly even of the stock.

Unfortunately, no general test for linearity of a regression exists unless there are more degrees of freedoms than abscissa values, and this is not the case, here. It is, however, reasonable to assume that any alternative to linearity is either concavity or convexity. In both cases a systematic effect must show in the successive differences $d_i = r_{i+1} - r_i$ of the residuals belonging to increasing abscissa values for each stock. In case of convexity the d_i, $i = 1, 2, \ldots$ should increase, in case of concavity they should decrease.

Tables 1 a/b and 2 a/b show the results. The d_i are very small compared with the change in the log (O/S)-values as predicted by the regression. This indicates that the linear model fits well. In two cases the log (O/S) lie even on straight lines.

The Hodges-Lehmann estimators of the median differences of the d_i, $i = 1, 2, 3$ were

$$-\,0.001 \text{ and } -\,0.11 \text{ for calls}$$

and

$$0.009 \text{ and } 0.19 \text{ for puts}.$$

While there is no perceptible monotonic trend in the d_i of calls, such a trend, though very slight, can be ascertained in the samples of puts (Jonckheere test against ordered alternatives, $p < 0.01$). However, there were only 10 out of 22 stocks with a *strictly* montonic increase. On the other hand, there were 4 stocks with a strictly monotonic decrease and 2 stocks with equality of the d_i. Summarizing, deviations from linearity, if any, were small and not systematic in the majority of the stocks.

Table 1 a/b shows that the intercepts of the regression lines were densely packed, while the differences in the slopes were rather marked. Since calls and puts had about the same time to maturiy, it makes sense to compare their regression parameters, especially for those stocks appearing in both analyses. Slopes and intercepts were smaller in puts than in calls (the arithmetic means were $-\,2.39$ and $-\,6.99$ in puts versus $-\,2.27$ and $-\,6.24$ in calls). The parameters of puts of a stock were strikingly similar to those of the calls of the same stock, and, as far as were differences, there seems to be no rule for their sign.

We have seen that the regression lines for different stocks were not equal. It might still, however, be true that, for a given stock, they do not depend on the stock price S. An empirical check of this hypothesis is difficult and must rely on few data because large changes of stock prices require some time to

Table 1 a

STOCK	INTERC.	SLOPE
FEDEXP	− 1.944107	− 7.94420
FLUOR	− 2.531504	− 2.95640
HALBTN	− 2.366208	− 4.91840
HOMESTK	− 2.027928	− 2.89740
MERCK	− 2.868246	− 8.36761
MONSAN	− 2.497658	− 10.60607
PENNZ	− 2.221898	− 3.03217
STORTEC	− 2.240172	− 3.81818
TELDYN	− 2.052711	− 8.53185
AMEXP	− 2.368962	− 9.64208
DIGEQ	− 2.251492	− 9.15036
HUTTON	− 1.815444	− 3.78057
LILLY	− 2.540796	− 11.78133
MERRIL	− 2.077980	− 5.36468
COMSAT	− 2.177308	− 9.29057
OAK	− 2.164429	− 2.73604
WESTUN	− 2.208561	− 6.73506
BALDUN	− 2.388497	− 4.81473
DSHAM	− 2.546658	− 3.36863
WANGB	− 2.023090	− 5.05841

Table 1 b

STOCK	INTERC.	SLOPE
BURLN	− 2.493704	− 6.51496
EASTKD	− 2.626853	− 12.17279
FEDEXP	− 2.203536	− 7.34584
HOMESTK	− 2.031629	− 4.37037
IBM	− 2.900605	− 6.53806
JOHNJ	− 2.761150	− 9.10545
MMM	− 2.575750	− 8.86155
MONSAN	− 2.894036	− 9.44603
PEPSI	− 3.237197	− 8.24209
TELDYN	− 2.019670	− 6.77224
TEXIN	− 2.251436	− 5.65140
AMEXP	− 1.981937	− 10.68347
DIGEQ	− 2.248021	− 8.94600
DUPONT	− 2.821536	− 8.84224
HUTTON	− 1.941320	− 4.62676
MERRIL	− 1.840691	− 5.77109
MOTORLA	− 2.365029	− 9.87831
PROCG	− 2.953498	− 8.93818
WESTNG	− 2.585875	− 7.68488
WESTUN	− 2.081289	− 10.57049
BALDUN	− 1.844625	− 6.32225
WANGB	− 2.111259	− 6.38418

Table 2 a

Successive Differences of Residuals

NO	D1	D2	D3	D4
FEDEXP	− .235175	− .006917	.244398	–
FLUOR	.202733	− .202733	–	–
HALBTN	.020637	.017681	.002938	–
HOMESTK	.160299	.077412	.263515	–
MERCK	.026622	.026622	–	–
MONSAN	.095310	.095310	–	–
PENNZ	.071381	.053032	− .120991	.0305587
STORTEC	.127967	.127967	–	–
TELDYN	− .062600	.065234	− .024378	–
AMEXP	− .052680	.052680	–	–
DIGEQ	− .113763	.048756	.048756	–
HUTTON	.220916	.220916	–	–
LILLY	.309520	.309520	–	–
MERRIL	.314304	− .314304	–	–
COMSAT	.038481	− .038481	–	–
OAK	.111572	− .111572	–	–
WESTUN	− .235002	.235002	–	–
BALDUN	.009475	.343481	− .467449	–
DSHAM	.412088	− .412088	–	–
WANGB	− 235002	.235002	–	–

Table 2 b

Successive Differences of Residuals

NO	D1	D2	D3	D4	D5
BURLN	− .018870	.018870	–	–	–
EASTKD	− .105532	.042684	.048619	–	–
FEDEXP	− .078327	.078327	–	–	–
HOMESTK	− .022607	− .059962	.102557	–	–
IBM	− .155301	.020430	.128061	–	–
JOHNJ	− .111572	.111572	–	–	–
MMM	− .008687	.072049	− .087379	–	–
MONSAN	.426039	− .380436	.377249	− .421258	–
PEPSI	− .020411	.020411	–	–	–
TELDYN	− .142430	− .291465	.253466	.194043	− .157934
TEXIN	− .221645	− .012083	− .237756	–	–
AMEXP	− .011236	− .011236	–	–	–
DIGEQ	.033605	− .257272	.161438	.110145	–
DUPONT	− .053815	.053815	–	–	–
HUTTON	− .000000	.000000	–	–	–
MERRIL	− .031143	.006598	.022346	–	–
MOTORLA	− .020469	− .168685	− .184189	.549780	–
PROCG	.159227	− .159227	–	–	–
WESTNG	.235002	.235002	–	–	–
WESTUN	.000000	.000000	–	–	–
BALDUN	− .154809	.154809	–	–	–
WANGB	.018184	− .018184	–	–	–

occur so that the change in time to maturity exerts influence on the option prices. The examples given in Table 3 show that the parameters of the linear regression remain constant even after pronounced short-term changes of the stock price. This stability is remarkable in view of the extreme sensitivity of the parameters to small variations of the option prices.

Table 3: **Changes of the parameters of the linear regression of log (O/S) vs. $|E - S|$ after sharp movements of stock prices.**

		Motorola maturity Apr. 1984		St. Oil Ohio maturity March 1984		Coleco maturity Apr. 1984	
date[(a)]		Dec. 2nd	Dec. 14th	Dec. 2nd	Dec. 14th	Dec. 16th	Dec. 23rd
stock price		142¼	135¾	45¾	41¾	25⅜	20⅝
calls:	intercept	(b)	− 2.56	− 2.61	− 2.86	− 1.86	− 1.88
	slope		− 0.055	− 0.154	− 0.165	− 0.115	− 0.1
puts:	intercept	− 2.88	− 2.74	− 2.94	(b)	− 1.92	− 1.64
	slope	− 0.065	− 0.065	− 0.254		− 0.159	− 0.16

(a) Year 1983. – (b) No calculation possible.

Without presenting a detailed analysis we finally note that for options *in the money* a loglinear relation between the premia and $\triangle$ holds, too, though outliers are more frequent.

III. Discussion

The empirical investigation has shown that for fixed stock prices S a loglinear relationship between the prices O of options out of the money and the difference between S and the exercise price E very approximately holds. Observe that this is not a consequence of the interval $E - S$ being small (so that it necessarily follows by Taylor expansion approximation). For a stock priced at about \$ 100, a \$ 40-interval (such as in Fig. 1) is not small as any investor knows, and, moreover, linearity between $S - E$ and some other function of O (say O or O^2) definitely does *not* hold.

The relationship put forward in this article is not incompatible with *Black/Scholes*'s formula as Fig. 2 shows. However, we have been unable, so far, to deduce approximate loglinearity from this formula.

Our finding gives a strong support to a special estimator for implied volatility of a stock, viz. the value v which minimizes

$$\sum_i (\log (O_i) - \log (\hat{O}_i))^2 .$$

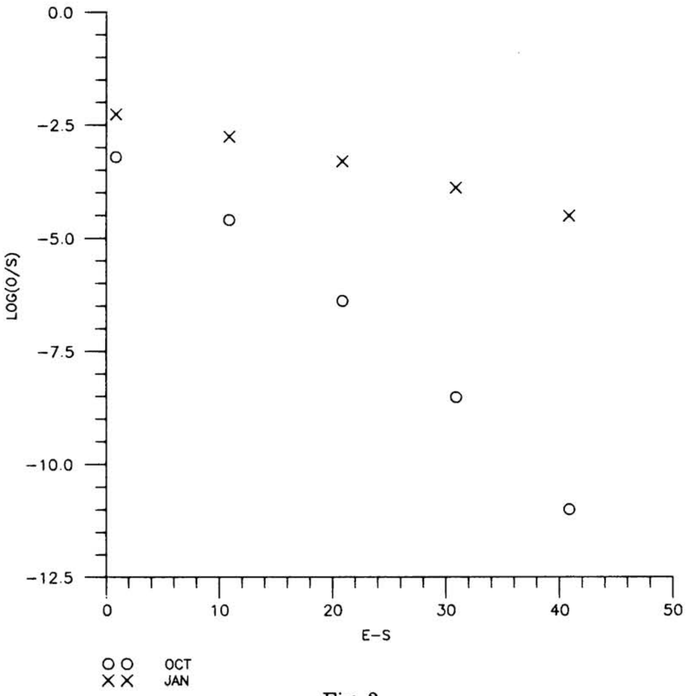

Fig. 2

Apart from this, it can be applied in several ways.

First, it can be used for predictions of option prices for future stock prices. If changes in stock prices do not occur rapidly, prognosis will require interpolation between the prices predicted for the available times to maturity.

Second, it allows the easy detection of options which are under- or overvalued with respect to other options of the same stock.

Third, it can be exploited for practical investment. We suggest the following (winning?) strategy:

Determine the intercept and slope for the puts and calls of the same stock. Calculate the value $S_{\min}$ such that a given spraddle, i.e. a combination of one put and one call of the stock with

$$E_{\text{call}} > S > E_{\text{put}} ,$$

assumes its minimum[1]. Invest when $S = S_{\min}$. Take profit on any movement of the stock price.

[1] Given the regression parameters, $S_{\min}$ can be determined analytically by standard calculus.

Summary

It is shown that the relationship between the prices of options out of the money and the difference between stock and exercise prices, the former being fixed, is approximately loglinear. This finding can be applied in various ways.

Zusammenfassung

Es wird gezeigt, daß eine approximativ loglineare Beziehung besteht zwischen den Preisen von Optionen, die nicht im Geld sind, und der Differenz zwischen Basispreis und Aktienkurs, sofern letzterer konstant gehalten wird. Dieses Resultat besitzt eine Reihe von Anwendungen.

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