Inflation and Relative Price Risk*

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This paper contributes to the growing literature on the interdependence of the variances of relative prices and the general price level. The analysis concentrates on the concept of risk rather than mere variability. The relationships among sources of aggregate risk, relative price risk and inflation risk are analyzed within the frame of a multimarket model and then estimated with German data.

I. Introduction

Price level stability is the most important goal of monetary policy, according to some, apparently old-fashioned central bank laws.

The virtue of a stable general price level is obvious, provided stability is interpreted to mean constant expectation and zero variance. In such an ideal state changes in absolute market prices are identical to changes in relative prices, hence no biases are introduced into the transmission of information about relative scarcities of goods and services. This ideal state can be preserved even if we permit the general price level to change, provided the resulting rate of inflation will be held constant and provided that the policy is credible. Under those circumstances it would be easy for economic agents to decompose observed changes of absolute market prices into the economy-wide inflation component and market-specific relative price changes. In the absence of progressive taxation a regime of zero or constant inflation promotes efficient exchange and an efficient allocation of resources by avoiding the problem of errors in signal extraction.

The real world is far away from such ideal states, of course. During the fourty years since world war II we have experienced worldwide not just inflation but considerable inflation variability. There is no country where economic policies have not proven to be unsteady, moving continously from inflation to disinflation and back to reflation.

Unsteadiness of policies, most notably of monetary and fiscal policies, has been criticized by non-keynesian economists for a long time to be a potential source of harmful effects. There are two main arguments,

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both of them stressing information problems. The first argument concentrates on the apparent lack of sufficient macroeconomic information on the side of policy makers. Insufficient information may lead to policy actions that run counter to good intentions by aggravating rather than smoothing cyclical fluctuation. This has been emphasized time and again by monetarists.¹

The second argument against unsteadiness has been brought to the attention of the profession by the seminal work of *Lucas* (1973) on the signal extraction problem. A changing volatility of inflation or, equivalently, aggregative noise makes it more difficult for market participants to solve their information problem, which is to isolate the true relative price component in observed changes of market prices. To quote *Friedman* (1977) from his Nobel lecture: "The more volatile the rate of general inflation, the harder it becomes to extract the signal about relative prices from absolute prices; the broadcast about relative prices is ... being jammed by the noise coming from the inflation broadcast." The resulting confusion between aggregative and relative movements in prices fools producers, period after period, into an inefficient, transitory extension or cut-back of supply. This has extensively been discussed by the rational expectations literature of the seventies.

There is, however, still another dimension to the reduced capacity of the price system to guide economic activity. If the variabilities of relative prices and inflation are positively associated, a persistent increase in inflation variability will induce a corresponding increase in the conditional variance of relative prices, hence in relative price risk. Though an increase in relative price variability must not reduce consumer welfare for given real income² the probably more important question is whether it affects real income. In this paper we suggest that output is negatively affected through its normal component. If producers are risk-averse, they will respond to an increase in perceived relative price risk by cutting back on capacity or normal output.

The purpose of the paper is to analyse and empirically investigate the relationship among inflation, inflation risk, and relative price risk, using monthly data from West-Germany over the period 1955 to 1982. In chapter II we present a multimarkets equilibrium model of the Lucas confusion type. It is a variant of the model recently developed by *Hercowitz* (1981) and *Cukierman* (1979, 1983), the main difference being in the modelling of output supply and of the sources of aggregative noise. Chapter III serves to clarify the concept of risk and to identify the hypothesized relations among relative price risk, inflation risk, the

¹ Friedman (1968), Brunner (1970).

² Fischer (1981).

types of aggregative shocks. Corresponding estimates, covering the period 1958 to 1982 are then presented in chapter IV. We give special attention to subperiods of disinflation and, for the purpose of comparison, to subperiods of reflation, too. The final chapter V summarizes our results and draws some policy conclusions which, we believe, have been ignored by policy makers of all countries for too long a time.

II. A Multimarkets Equilibrium Model

Since the early formulations by *Lucas* (1973) and *Barro* (1976) there is a growing literature on how to model the confusion between aggregative and relative price movements. The model we are presenting below is a variant of the multimarkets equilibrium model recently formulated by *Cukierman* (1983). That model, in turn, generalized in an elegant fashion the ideas developed before by *Parks* (1978) and by *Hercowitz* (1981). A common feature of all these models is that the price elasticities of demand and supply are permitted to differ between the assumed large number of relatively small markets.

We have modified the Cukierman formulation in the following respects: First, we differentiate between the modelling of normal supply of output and of transitory supply. Second, we introduce perceived relative price risk as an explicit determinant of normal output. Third, we admit the prices of imported raw materials as an important economy-wide source of aggregate shocks and differentiate between fixed and flexible exchanges rates in the modelling of regime-induced connections between import price shocks and monetary shocks.

The model is printed in Table 1. It is based on the common assumption of log-linearity, hence all variables are measured as logs. Equations (1) to (4) describe the demand and the supply on market *i* at time *t*. The market-specific price is denoted by p_i , the general price level by P, the import price level by p^J and the money stock by *M*. Expected values are denoted by the operator *E*, and they are either conditioned on the current, market-specific information set $J_{i,t}$ or on the previous, economy-wide information set J_{t-1} .

Output demand on market *i* depends positively on perceived real cash-balances and market-specific demand shifts, w_i^d ; it depends negatively on the perceived relative price, $p_{i,t} - E(P_t \mid J_{i,t})$.³ Output supply

³ In this paper we wish to stress the conditioning role of world market prices of raw materials and of the real exchange rate via the supply side. For this reason we have not modelled the impact of the real exchange rate via export demand. In a forthcoming paper (*Neumann* and *von Hagen* 1984) we will show that an explicit modelling of the export demand does not change the solutions of the model in any substantive manner, provided the supply elasticities dominate the respective demand elasticities.

Table 1

A Modified Multimarkets Equilibrium Model

Demand on market *i*:

(1)
$$Y_{i,t}^{d} = -\psi_{i} \left[p_{i,t} - E\left(P_{t} \mid J_{i,t} \right) \right] + \alpha \left[M_{t} - E\left(P_{t} \mid J_{i,t} \right) \right] + w_{i,t}^{d}$$

(2)

$$\mathbf{Y}_{i, t}^{s} = \mathbf{Y}_{i, t}^{ns} + \mathbf{Y}_{i, t}^{ts}$$

The normal component:

(3)
$$Y_{i,t}^{ns} = -\beta^* \left[E\left(p_t^J \, \big| \, J_{t-1} \right) - E\left(p_{i,t} \, \big| \, J_{t-1} \right) \right] + \gamma_i^* \left[E\left(p_{i,t} \, \big| \, J_{t-1} \right) - E\left(P_t \, \big| \, J_{t-1} \right) \right] + E\left(w_i^s \, \big| \, J_{t-1} \right) - \delta^* R_t\left(p^R \right)$$

The transitory component:

(4)
$$Y_{i,t}^{ts} = -\beta \left[p_t^J - E(p_t^J | J_{t-1}) - p_{i,t} + E(p_{i,t} | J_{t-1}) \right] + w_{i,t}^s - E(w_{i,t}^s)$$
$$| J_{t-1} + \gamma_i \left[p_{i,t} - E(p_{i,t} | J_{t-1}) - E(P_t | J_{i,t}) + E(P_t | J_{t-1}) \right]$$

where

$$\beta^* > \beta > 0 < \gamma < \gamma_i^*$$

$$p_t^J = P_t^F + s_t$$

(6 b)
$$s_t = P_t - x_t$$

(7)
$$X_t = X_{t-1} + \varepsilon_t^x \qquad \land \qquad \varepsilon_t^x \sim N(0, \sigma_{\varepsilon_x}^2)$$

(8)
$$\Delta p_t^F = \Delta \overline{p}^F + \varepsilon_t^F \qquad \land \qquad \varepsilon_t^F \sim N(0, \sigma_{\varepsilon F}^2)$$

(9) $\Delta M_t = \Delta \overline{M} + \varepsilon_t^M \qquad \land \qquad \varepsilon_t^M \sim N(0, \sigma_{\varepsilon M}^2)$

(10)
$$\Delta w_{i, t} = \Delta \overline{w}_{i}^{d} - \Delta \overline{w}_{i}^{s} + \varepsilon_{i, t}^{wd} - \varepsilon_{i, t}^{ws} = \Delta \overline{w}_{i} + \varepsilon_{i, t}^{w}$$

$$\varepsilon_{i, t}^{w} \sim N(0, \sigma_{w}^{2})$$
 for all i

(11)
$$\sum_{i} w_{i, t} = 0$$

(12)
$$P_t = \sum_i u_i p_{i,t} \qquad \wedge \qquad \sum_i u_i = 1$$

(13)
$$p_{i,t} = \bar{P}_t + \lambda_i^* (w_{i,t-1} + \Delta \bar{w}_i) + [\lambda_i \Theta + (1 - \Theta) \lambda_m] \Delta^{-1} \\ [\alpha \varepsilon_t^M + \beta (\varepsilon_t^F + \varepsilon_t^x) + (1 - \beta \lambda_m) \varepsilon_{i,t}^w]$$

(14)
$$P_t = \bar{P}_t + \lambda_m \Delta^{-1} \left[\alpha \, \varepsilon_t^M + \beta \, (\varepsilon_t^F - \varepsilon_t^x) \right]$$

(15)
$$\bar{P}_{t} = E(P_{t} | J_{t-1}) = [\alpha(M_{t-1} + \Delta \overline{M}) + \beta^{*}(p_{t-1}^{F} + \Delta \overline{p}^{F} - X_{t-1}) + \delta^{*}R_{t}(p^{R})] \alpha^{-1}$$

(16)
$$E(P_t \mid J_{i,t}) = \bar{P}_t + (1 - \Theta) \lambda_m \Delta^{-1} [\alpha \varepsilon_t^M + \beta (\varepsilon_t^F - \varepsilon_t^x) + (1 - \beta \lambda_m) \varepsilon_{i,t}^w]$$

(17)
$$\mathbf{Y}_{t} = -\beta^{*} \left(p_{t-1}^{F} + \Delta \, \bar{p}^{F} - \mathbf{X}_{t-1} \right) - \delta^{*} \, R_{t} \left(p^{R} \right) + \Theta \, d_{m} \, \Delta^{-1} \, \alpha \varepsilon_{t}^{M} - \\ - \left(1 - \Theta \, d_{m} \, \Delta^{-1} \right) \beta \left(\varepsilon_{t}^{F} - \varepsilon_{t}^{x} \right)$$

where

(18)	$\Delta = \Theta (1 - \beta \lambda_m) + \alpha (1 - \Theta) \lambda_m > 0$
(19)	$\lambda_m = \Sigma u_i \lambda_i$
(20)	$d_m = \Sigma u_i d_i$
(21)	$\lambda_i = 1 / (\psi_i + \beta + \gamma_i)$
(22)	$d = \alpha / (\alpha + \beta + \alpha)$

(22)
$$d_{i} = \gamma_{i} / (\psi_{i} + \beta + \gamma_{i})$$
$$\lambda_{i}^{*} = 1 / (\psi_{i} + \beta^{*} + \gamma_{i}^{*}) < \lambda_{i}$$

(24)
$$\Theta = \sigma_w^2 \left(1 - \beta \lambda_m\right)^2 / \left[\sigma_A^2 + \sigma_w^2 \left(1 - \beta \lambda_m\right)^2\right]$$

(25)
$$\sigma_A^2 = \alpha^2 \sigma_{\varepsilon M}^2 + \beta^2 (\sigma_{\varepsilon F}^2 + \sigma_{\varepsilon x}^2)$$

on market *i* is considered to be the sum of a normal capacity component and a transitory component. Capacity output is the supply of goods planned for by firms in market *i* at the end of period t - 1 for period *t*. Firms reallocate the factors of production at the end of period t - 1, conditional on the information available to them at that time. They base their planning decisions upon their conditional expectations about next period's relative selling price, next period's relative price of raw materials to be imported, next period's market-specific supply shocks and, last not least, upon their perception about economy-wide relative price risk, denoted by $R(p^R)$. Perceived relative risk to be discussed in more detail below, plays an important role in determining normal output, if suppliers are risk-averse.

Equation (4) describes the transitory component of output supply on market *i*. Transitory supply is decided upon during the current period *t*, when the observation of the actual, market-specific price $p_{i,t}$ induces firms to revise their relative price expectations. Hence transitory output is a function of remaining forecast errors. Applying the Le-Châte-lier-Principle⁴ we require the price elasticities of transitory supply to be smaller than the corresponding elasticities of normal output supply; see condition (5).

Our model takes four types of shocks into account; see equations (6) to (10). There are (i) aggregate domestic monetary shocks, ε^M , (ii) aggregate real shocks from foreign prices of raw materials to be imported, p^F , translated by the exchange rate, s, (iii) aggregate shocks producing deviations from purchasing power parity, x, and (iv) relative demand and supply shocks. All types of shocks are modelled as random cumulated shocks. For conveniance we assume that they are serially and

⁴ Silberberg (1978).

mutually uncorrelated. Finally note that the relative excess-demand shifts between markets are modeled as random walks with drift.

The multimarket model can be solved in the usual fashion by applying Lucas' method of undetermined coefficients. Equations (13) to (17) of Table 1 provide the rational expectations solutions for an individual market price p_i , for the general price level P, for its economy-wide expectation at the end of period t - 1, for its current expectation in market *i*, and for total output.⁵ There is no need to go through the solutions in detail. Suffice it to say, first, the general price level and total output depend both upon aggregate shocks while market-specific prices and output supplies as well as current expectations of the general price level depend in addition upon relative excess-demand shocks. Second, all individual prices and the general price level respond positively to perceived economy-wide relative price risk while output supplies, via normal components, respond negatively.

III. Inflation Risk, Relative Price Risk and the Phillips Curve

In a world of rational expectations market participants do not know the future realizations of prices and quantities but they do posess information on the probability distributions of future outcomes. Hence they base their decisions on expectations about the future realizations of economic variables using their knowledge of the stochastic properties of the forecast errors. Following Frank H. Knight we call this a situation of risk rather than of uncertainty.⁶ Under the conventional assumption of normal distributions risk is characterized by the conditional variances of white noise errors. We consider two types of risk, inflation risk and relative price risk.

It is important to note that we now introduce the potentially fruitful hypothesis that the conditional variances of monetary and real aggregate shocks may not be constant but change over time. For example, the switching between stop-and-go policies by the Bundesbank may not just change the level of monetary growth but in addition its variance. Under those circumstances economic agents are confronted in each period with the problem that the variance of monetary shocks may have changed. Consequently, they will use their updated information set in order to revise the conditional variance. Hence our hypothesis implies that agents revise each period the conditional variances for all

⁵ To be more precise: equation (17) defines the weighted average of output supplies.

⁶ Uncertainty is a situation where the probabilities of future outcomes are unknown.

types of aggregate shocks and, therefore, perceived inflation risk as well as perceived relative price risk. The underlying relationships are defined in Table 2.

There is first the risk of inflation; see expression (26). It equals the conditional variance of the general price level, as defined by equation (14). We use the information available at the end of period t - 1, J_{t-1} , as the conditioning information set.⁷ Note that we have assumed the information set J_{t-1} to be available economy-wide. This implies that our measure of perceived inflation risk is the same in all markets, though the current price level expectations differ between markets. Expression (26) indicates that inflation risk is an increasing function of the conditional variance of aggregate shocks, provided the real balance effect is dominated by substitution effects. As *Cukierman* (1983) points out this is a sufficient, but not necessary condition.

Next consider economy-wide relative price risk. Economy-wide relative price risk must neither be simply equated with the variability of relative prices nor with the variance of relative price change. Rather it is to be defined as the conditional variance of relative prices.⁸ In principle it would be of interest to differentiate between market-specific relative price risk and economy-wide relative price risk. An increase in the former would induce a shift of normal output between markets while an increase in the latter would induce a cut-back of normal output supplies in all markets. For reasons of analytical tractability, however, we do not permit market-specific relative price risk to play a role in our model; this is effected by containing Cukierman's assumption that the relative excess-demand shocks for all markets are drawn from a common distribution; see equation (8).⁹

Expression (28) is derived by subtracting equation (14) from (13), computing the conditional variance of the noise component for each relative price and summing over all markets. Economy-wide relative price risk depends upon the variance of relative excess-demand shocks and upon the conditional variance of aggregate shocks. The sign of the derivative of relative price risk with respect to the conditional variance

⁷ Alternatively one might use the current market-specific information set $J_{i,t}$. We deliberately avoid that in order to hold the empirical work within manageable proportions.

⁸ For a formal definition see expression (28) of Table 2.

⁹ In passing we note that the hypothesized cut-back of normal output supplies in all markets, in response to an incease in perceived relative price risk, feed back into asset markets as it is accompanied by a corresponding decrease in the supply of private debt to the commercial banking system. See e. g. *Mascaro* and *Meltzer* (1983).

Table 2:

Key Variables

Notation: $\sigma_{A,t}^{*2} = E(\sigma_{A,t}^{2} | J_{t-1})$ Inflation risk: (26) $R_{t}(P) = V_{t}^{EN}(P | J_{t-1}) = a_{3}\sigma_{A,t}^{*2}$ where $a_{3} = \lambda_{m}^{2}\Delta^{-2} = 0$ (27) $\frac{\delta R(P)}{\delta \sigma_{A}^{*2}} = a_{3} [1 + 2\Theta(1 - \Theta)(1 - (\alpha + \beta)\lambda_{m})\Delta^{-1}] > a_{3}$ Relative price risk: (28) $R_{t}(p^{R}) = V_{t}^{EN}(p^{R} | J_{t-1}) = a_{1}\sigma_{A,t}^{*2} + (a_{1} + a_{3})\sigma_{w}^{2}(1 - \beta\lambda_{m})^{2}$ where $a_{1} = \Theta^{2}(\Sigma u_{t}\lambda_{t}^{2} - \lambda_{m}^{2})\Delta^{-2} > 0$ (29) $\frac{\delta R(p^{R})}{\delta \sigma_{A}^{*2}} = a_{1}(1 - 2\alpha\lambda_{m}\Delta^{-1}) + 2a_{3}\Theta^{2}[1 - (\alpha + \beta)\lambda_{m}]\Delta^{-1} \ge 0$ Ratio of inflation risk to sum of relative price risk and inflation risk:

(30)
$$RR_t = (1 - \Theta) [a_3 (a_1 + a_3)^{-2}]$$

(31)
$$\frac{\partial \Pi \Gamma}{\partial \Theta} = - [a_3^2 + a_1 a_3 (1 + 2 (1 - \Theta) \Theta^{-1}] (a_1 + a_3)^{-2} < 0$$

Variance of output

when exchange rates are flexible:

(32)
$$V_t (y - y^n) = c_1 \varepsilon_t^{M2} + c_2 (\varepsilon_t^{F2} + \varepsilon_t^{x2})$$

where $c_1 = (\alpha \Theta d_m)^2 / \Delta^2$
 $c_2 = \beta^2 (\Delta - \Theta d_m)^2 / \Delta^2 > 0$

(33)
$$\frac{\delta c_1}{\delta \sigma_A^{*2}} = -\frac{2 c_1 \alpha \lambda_m \Theta}{\Delta \sigma_w^{2} (1 - \beta \lambda_m)^2} < 0$$

(34)
$$\frac{\delta c_2}{\delta \sigma_A^{*2}} = \frac{2 c_2 \lambda_m \Theta \gamma \overline{c_1}}{(\Delta - \Theta d_m) \sigma_w^2 (1 - \beta \lambda_m)^2} > 0$$

when exchange rates are fixed:

(35)
$$V_t (y - y^n) = c_1 \varepsilon_t^{MA2} + c_3 (\varepsilon_t^{F2} + \varepsilon_t^{x2})$$
where $c_3 = [\beta \Delta - \Theta d_m (\beta + \alpha e)]^2 / \Delta^2 > 0$
$$\varepsilon_t^M = \varepsilon_t^{MA} + e (\varepsilon_t F - \varepsilon_t^x)$$

(36)
$$\frac{\delta c_3}{\delta \sigma_A^{*2}} + \frac{2 c_3 \lambda_m \Theta (\beta + \alpha e) \gamma c_1}{[\beta \Delta - \Theta d_m (\beta + de)] \sigma_w^2 (1 - \beta \lambda_m)^2} > 0$$

Rate of change of normal output:

of aggregate shocks is ambiguous; see expression (29) of Table 2. If, however, either the real balance effect or the variance of the price elasticities across markets¹⁰ is sufficiently small, relative price risk will be an increasing function of the conditional variances of monetary and real aggregate shocks. The empirical part of the paper will provide evidence in support of this conjecture.

Our measures of inflation risk and of economy-wide relative price risk contain information on the slope coefficient of the standard natural rate Phillips curve. Computing the ratio of inflation risk to the sum of relative price risk and inflation risk — se expression (30) of Table 2 we receive an indicator of the slope coefficient of the short-run Phillips curve. Like this coefficient the computed risk ratio RR is a clean, decreasing function of Lucas' Θ -coefficient which is defined by expression (24) of Table 1.¹¹ An increase in the perceived variance of aggregate shocks, for a given variance of relative shocks, reduces Θ , hence makes the Phillips curve steeper. This will be reflected by an increase in the risk ratio RR.

Another variance of interest is the observed variance of transitory output. Its definition varies with the prevailing exchange rate regime. Expression (32) of Table 2 is appropriate for a regime of flexible exchange rates while expression (35) is consistent with a regime of fixed rates. The main difference between the two definitions results from modelling the dependence of domestic monetary shocks on exogenous shocks from world market prices for the era of fixed exchange rates.

The variance of transitory output depends upon current surprises squared as well as on the perceived variance of aggregate shocks. A well-known effect of the Lucas confusion in conjunction with *monetary surprises* is as follows: An increase in the perceived variance of aggregate shocks reduces the positive impact elasticity of current monetary shocks¹² as the suppliers are induced to attribute an increased fraction of currently observed changes in market-specific prices to changes in global demand rather than changes in market-specific demands. As a result an increase in the perceived variance of aggregate shocks reduces in conjunction with monetary surprises the actual variance of transitory output.

¹⁰ The variance of the price elasticities across markets is equal to $\Sigma u_i \lambda_i^2 - \lambda_m^2$. The smaller it is, the smaller is a_1 in derivative (29).

¹¹ Note that the Θ -coefficient is constant within the context of Table 1 while it is time dependent within the context of Table 2. The time-index has been skipped in order to facilitate the notation.

¹² The impact elasticity is equal to the square root of the coefficient c_1 in expression (32).

However, this is not the complete story. There is in addition an opposite effect if the Lucas confusion occurs in conjunction with aggregate *real surprises*. An unanticipated increase in foreign prices of raw materials exerts a direct negative impact on output supply. But this negative effect is moderated by a concurrent positive impact which results from the Lucas confusion of global and relative shocks; suppliers observe an induced increase in market- specific absolute prices which they interpret to be increases in relative prices. Given that an increase in the perceived variance of aggregate shocks reduces the moderating effect of the Lucas confusion it follows that an increase in the perceived variance of aggregate shocks raises the actual variance of transitory output, provided it occurs in conjunction with aggregate real surprises; see the positive signs of the derivatives (34) and (36).

Finally consider equation (37). It portrays the rate of change of normal output as a stochastic variable. Our multimarkets equilibrium model does not contain a full-blown theory of normal output determination but concentrates on two stochastic determinants which may be very important. One of them is the real price of imported raw materials, the other one is perceived relative price risk.

Two implications of the formulation are of interest. First, the rates of change of total output are serially correlated because each unanticipated import price shock affects transitory output in the current period, thereafter permanent or normal output. This adds to the serial correlation resulting from the role of adjustment costs.¹³ Second, changes in perceived relative price risk affect normal output growth. An increase in relative price risk, resulting from an increase in the variances of relative or aggregate shocks, induces risk-averse suppliers to reduce the planned growth of capacities. Consequently, we have two channels for the non-neutrality of money to play a role. Channel one: unanticipated monetary shocks induce transitory changes of output. Channel two: a persistent increase in the actual and, consequently, in the perceived variability of money growth raises perceived relative price risk which in turn depresses the growth of normal output.

IV. Empirical Investigation

In the theoretical part of our paper we have shown that relative price risk and inflation risk — as defined by the second moments of the relevant conditional probability distributions — are not independent of each other but closely related. Specifically we assume that both

¹³ Sargent (1979).

types of risk move over time in response to changing conditional variances of different types of aggregate shocks.

Any testing of the relationships implied by our model requires first to construct adequate empirical counterparts to the theoretical risk variables defined in Table II. This construction work is explained in the following section 1 while the tests of our model follow in section 2. Before we turn to that a few remarks on the nature of the data used may be in order.

We use seasonally unadjusted monthly data, covering the period 1955 through 1982. The 14 most important components of the producers' price index of industrial products serve as inputs for computing relative price dispersion; they accounted for 80 percent of the total index in 1980; see Table A1 of the Appendix. The individual weights were redefined in order to compute a new index of the general price level of industrial products from these 14 price series. Note that we generally employ moving weights in order to avoid structural breaks. For each individual price series a time series of weights was derived from linearly extrapolating the trend underlying the weights published by the Federal Statistical Office for selected years. Output is approximated by the index of (net) industrial production.

There are many sources of aggregate shocks. We consider two important sources: one is monetary policy, the other one is world market prices of industrial raw materials. The time series of the narrowly defined money stock M1 is used to model monetary shocks. In principle, it would be of interest to employ a definition of the monetary base. It can be shown, however, that the base definitions at hand unavoidably lead to a biased indication of monetary policy.¹⁴ For the modelling of real aggregate shocks, finally, we employ an index of the world market prices of raw materials, measured in US-dollars, as computed by the Hamburger Weltwirtschaftsinstitut.

1. Estimation of Conditional Variances

The construction of the various conditional variances of interest involves two major steps. First, the conditional expectations are modeled for 14 relative price series, the general price level series, the foreign price series of imported raw materials, and the money stock series. Second, the estimated residuals are used to derive estimates of the conditional variances.

Consider the *first step*. Forecasts are obtained from estimating autoregressive-transferfunction models for the stationarized logarithmic

¹⁴ Neumann (1983).

¹² Zeitschrift für Wirtschafts- und Sozialwissenschaften 1985/2/3

Table 3

Variable	Differ- encing	Inputs: lags of	Box-Pierce Statistic to Lag 18 (dof)	F (dof)
M 1 P PF P 10 P 21 P 22 P 25 P 27 P 32 P 33	1,12 1,12 1 1 1 1 1 1 1 1 1 1	M 1, P, PF, B P, PF, B PF, S P 10, P, PF P 21, P, PF P 22, P, PF P 25, P, B P 32, P, PF, B P 33, P, PF, B	21.3 (15) 19.2 (12) 14.7 (15) 12.4 (14) 14.7 (14) 20.3 (17) 7.2 (16) 10.2 (17) 14.5 (15) 12.2 (15)	5.1^{**} (306; 8) 18.4^{**} (298; 12) 110.4^{**} (324; 12) 35.7^{**} (316; 8) 10.7^{**} (301; 10) 31.4^{**} (331; 3) 7.9^{**} (318; 5) 7.8^{**} (319; 4) 21.2^{**} (316; 7) 11.6^{**} (315; 8)
P 36 P 38 P 40 P 54 P 58 P 63 P 68	1 1 1 1 1 1 1	P 36, P, PF P 38, P, PF, B P 40, P, PF, B P 54, PF P 58, P, PF P 63, P, PF P 68, P, PF	17.9 (15) 16.1 (17) 14.1 (15) 10.4 (13) 26.4 (13) 19.8 (15) 21.4 (16)	8.2^{**} (317; 7) 9.5 ^{**} (318; 5) 20.8 ^{**} (317; 7) 6.9 ^{**} (316; 7) 22.7 ^{**} (314; 9) 35.0 ^{**} (317; 6) 16.2 ^{**} (317; 6)

Characteristics of Forecast Models

Note: Detailed information about the variables is given in table A1 of the data appendix.

** Means significant at the 1 % level.

series of each time series.^{15, 16} The limitation to autoregressive processes is convenient as it facilitates the construction of the Maximum Likelihood-estimators in the second step of our procedure, to be explained below. For all but one of our data series the *OLS*-forecasts are based on lags of the variable considered as well as on lags of the general prive level P, the money stock M, the world market price index of raw materials p^F , and a monetary base definition B.

An exception is the price index of raw materials. In this case it appeared advisable to add an intervention function to the set of regres-

¹⁵ Box and Jenkins (1976).

¹⁶ We found it convenient to remove the strong seasonal element in the price level, output and M1 by simple moving average or autoregressive filters at the outset of our procedure.

sors.¹⁷ This function is designed to take care of the likely fact that market participants recognized the exceptional nature of the drastic increase in world market prices in the early seventies and again in 1979. It implies the conjecture that the market participants did not assume that the basic time series behaviour was destroyed but rather that it was shifted to a higher level.

The first step of our estimation procedure provides for each variable considered a series of serially uncorrelated forecast errors with the additional property of being uncorrelated with lagged realizations of P, M, p^F , and B; see Table 3.

Let us now consider the *second step* of our procedure. Assuming the conditional variances of forecast errors not to be constant within the framework of least squares estimation is equivalent to assuming heteroscedasticity. This suggests for a variable x a covariance matrix of the following form

(38)
$$E[x_t - x_t^*)(x_t - x_t^*)'] = \operatorname{diag}(\sigma_t^{*2})$$

where x_t^* denotes the conditional forecast of variable x and σ_t^{*2} the variance of the forecast error. If we combine expression (38) with an hypothesis about the behaviour over time of the variance σ_t^{*2} we can test this hypothesis against the Null of homoscedasticity. We employ simple models of the following kind

(39)
$$\sigma_t^{*2} = \delta_0 + \sum_{j=1}^n \delta_j \sigma_{t-j}^{*2}$$

where for obvious reasons all coefficients δ_j are restricted to be nonnegative. Model (39) implies that market participants are likely to assign higher probabilities to large forecast errors today if they did so in the recent past.¹⁸

Using (39), the procedure in the second step is as follows:

For each variable the forecast errors are squared and then regressed on their lags. The test statistic for rejecting the hypothesis H_0 : $\delta_j = 0$ for $j \neq 0$ is

1, 2, ..., 32 t = 1971,7 to 1973,11 and 1978,10 to 1980,1

S(t) = 100 t = 1973,120 otherwise

This helped us to avoid that expectationally high residuals create problems in the estimation of the conditional variance.

¹⁷ Following Box and Tiao (1975) we added to an autoregressive model the following step function s(t)

¹⁸ Tests of model (39) and similar models for σ_t^{*2} have recently been developed by *White* (1980) and by *Breusch* and *Pagan* (1981). These tests belong to the general class of Lagrange-multiplier test. Trivially, rejecting (39) does not imply homoscedasticity.

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(40)
$$\chi^2 = TR^2 \frac{H_0}{asy} \chi^2 (n)$$

where T is the number of observations and R^2 the multiple correlation coefficient from the estimation of (39). χ^2 converges asymptotically to a chi-square distribution under the Null.¹⁹

The test results for all variables are presented in Table 4. With the exception of three relative price series (Nos. 25, 27, and 68) we find that the Null hypothesis of constant variances is always rejected. It follows directly that the *OLS* estimators of (39) are not efficient, since they differ from maximum likelihood estimates. To obtain efficient estimates we reestimate the parameters of the transfer function together with those of (39) using a *ML* estimator that was developed following the lines of *Engle* (1982). In almost all cases a single score was found to be sufficient to arrive at efficiency. At this point we are left with estimated models of the kind of (39), where we replace σ_t^{*2} by the squared forecast errors

(39')
$$\hat{\sigma_{\iota}^2} = \hat{\delta_0} + \sum_{j=1}^n \hat{\delta_j} \hat{\varepsilon}_{t-j}^2$$

Note that $\hat{\delta}_i$ is the *ML* estimate of δ_i . Expression (39') obviously is the best estimate we can get for the conditional variance presuming that (39) holds. Finally, we construct an empirical approximation of economy-wide relative price risk, as defined by equation (28) of Table 2, by summing up the respective versions of (39') for the 14 relative price series.

(41)
$$\hat{R}_t(p^R) = \Sigma u_i \hat{\sigma}_{it}^2$$

2. Aggregate Risk as a Determinant of Relative Risk

Our multimarkets equilibrium model implies that relative price risk is positively related to the conditional variances of aggregate monetary and real shocks, provided the real balance effect is dominated by substitution effects. In this section we are going to test this implication.

Figure 1 presents our measure of relative price risk for the period of 1958 to 1982. The following observations are noteworthy: first, during a period of nine years, from 1958 through 1966, relative price risk was remarkably stable. Thereafter it started moving in large fluctuations around a gradually rising trend. Second, a first major peak appeared in the mid of 1967 when the German authorities for the first time turned

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¹⁹ Note that the test is not a test for misspecification of the underlying forecast equation because the forecast errors are serially uncorrelated.

Inflation and Relative Price Risk

Variable	Type of Model	Test-Statistic (dof)a)
<i>M</i> 1	autoregressive (AR)	10.8** (1)
PF	autoregressive (AR)	46.4** (2)
Р	autoregressive (AR)	8.8** (1)
Р	AR plus lagged variances M 1 and PF	21.2** (3)
P 10	AR	39.6** (2)
P 21	AR	32.7** (2)
P 22	AR	38.0** (1)
P 25	AR	0 (2)
P 27	AR	0,8 (2)
P 32	AR	12.5** (2)
P 33	AR	14.8** (2)
P 36	AR	1.0 (1)
P 38	AR	4.6* (2)
P 40	AR	22.0** (2)
P 54	AR	6.7* (1)
P 58	AR	57.6** (1)
P 63	AR	40.4** (1)
P 68	AR	0 (1)

Table 4

La Grange Multiplier Tests for Constant Variances

a) The test-statistic is $T \cdot R^2$ according to equation (40).

to an explicit Keynesian concept of anticyclical stabilization policies. Third, relative price risk moved heavily up and down durnig the years 1970 to 1972 when repeated waves of speculation against the dollar/DM parity signaled a possible break-down of the Bretton-Woods system. Fourth, relative price risk reached a maximum peak by mid of 1974, shortly after the oil price and the prices of other raw materials had exploded and German monetary policy had effected the most severe contraction since world war II. Fifth, a smaller peak of relative price risk appeared in 1979 at the time of the second oil price shock and a similar peak occurred at the end of 1980 when the DM/dollar rate began to fall drastically.

The dotted curve of figure 1 shows the rate of inflation, computed from our summary index of industrial prices. It is apparent that there are a few short subperiods which suggest that relative price risk may

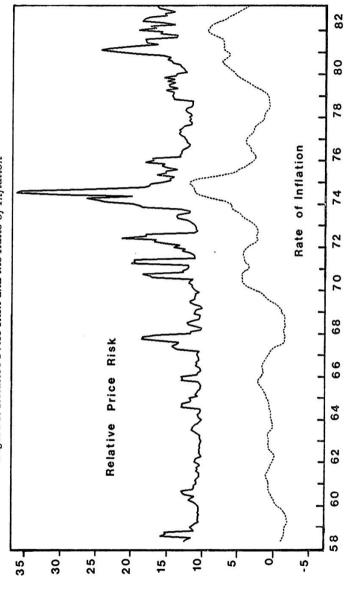


Figure 1: Relative Price Risk and the Ratio of Inflation

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Period	R (P)	$\sigma^{*2}_{\Delta M}$	$\sigma^{*2}_{\varDelta P}F$
Total Period 1958, 2-1982, 12			
Coefficient F-Statistic	.08 21.9*	.08 13.9**	.10 35.3**
Periods of Disinflation 69 observation ^{a)}			
Coefficient F-Statistic	.24 .0	.08 2.8	.004 .0
Periods of Reflation 118 observations ^{b)}			
Coefficient F-Statistic	1.12 14.9**	.14 15.1**	.11 22.5**
Period of R (p ^R)-Stability 1958, 2-1966, 12			
Coefficient F-Statistic	01 .0	05 .0	22 .0
Period of $R(p^R)$ -Instability			
Coefficient F-Statistic	.06 7.0**	.0.6 5.4**	.09 18.4**

Table 5

Dependence of Relative Price Risk on Aggregate Risk

Note that *(**) indicates the 5-(1-)percent level of significance where the number of tested parameters is.

a) 1961, 3 - 1962, 1; 1966, 4 - 1967, 5; 1974, 10 - 1975, 12; 1980, 4 - 1980, 9;

b) 1959, 1 - 1961, 2; 1964, 7 - 1965, 7; 1968, 12 - 1970, 8; 1972, 7 - 1974, 9; 1978, 9 - 1980, 3.

be positively related to the level of inflation but in general such a relationship does not seem to exist. Indeed, the class of multimarket equilibrium models to which our model belongs suggests instead that relative price variability is positively related to unanticipated inflation rather than to measured inflation.²⁰ Our paper sharpens the focus by asserting that relative price risk is not simply related to unanticipated inflation but to its conditional variance, i.e. to inflation risk.

Tests of this hypothesis are presented in Table 5. They consist of regressing our measure of relative price risk separately on three different

²⁰ Empirical work by Parks (1978), Franz (1984), and Fischer (1982) provide some support.

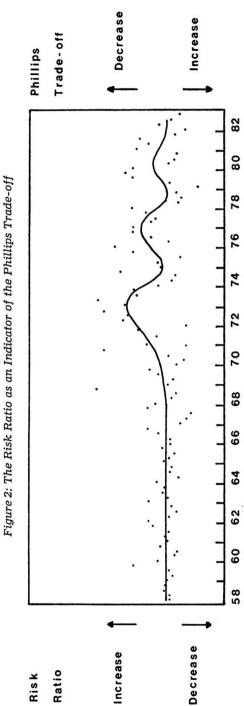
measures of aggregate risk. Consider first the three regressions for the total sample period of 1958 through 1982. Each of them is F-significant at the 1 percent level and gives the expected positive sign for the estimated regression coefficient. Thus, even though the true theoretical relations are nonlinear, our simple test provides strong evidence in support of the hypothesis that relative price risk is positively influenced by aggregate risk. We find this link not just for total aggregate risk, as summarized by inflation risk, but equally well for the two selected major components of aggregate risk, i.e. the risks arising from domestic monetary shocks and from foreign real shocks.

In honouring the general topic of this conference we have repeated the regressions for subperiods of disinflation and of reflation. Periods of disinflation have been defined to be periods where the rate of inflation falls month after month without any major interruption. Periods of reflation have similarly been collected as all periods characterized by a clear-cut rising of the rate of inflation. Table 5 indicates that for both types of periods we again find a significant positive link between relative price risk and inflation risk as well as the risk resulting from monetary shocks. It is of interest to note that the estimated coefficients are almost twice as large for periods of reflation than for periods of disinflation. But we hasten to add that these differences should not be stressed as they may purely be due to the intrinsic complications of the nonlinear relationships.²¹ Moreover, from the theoretical point of view taken in this paper there is no reason to expect that periods of disinflation are fundamentally different from periods of reflation.

In our view, a more interesting split up of the total sample period results from differentiating between periods of stable relative price risk and periods of unstable relative price risk. Inspection of figure 1 reveals that under this criterion the first nine years of our sample (1958 to 1966) constitute a period of persistent stability while the following sixteen years (1967 to 1982) provide a period of pronounced instability.

Consider the corresponding regressions of Table 5. They show what one would expect: For the stable period the existing relationships are statistically not detectable, for the very reason that during those "golden years" all conditional variances, hence risks, were impressively stable. For the later period, in contrast, where all conditional variances started moving in a troublesome fashion, the hypothesized links among them show up significantly.

²¹ For similar reasons we do not think that the estimated zero contribution of the conditional variance of world market prices during periods of disinflation does mean anything, except that by historical chance this variance did not move much during the subperiods considered.



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The differentiation between an early period of risk stability and a later period of risk instability is the more useful as it permits to draw conclusions on the likely development of the Phillips trade-off. In the previous chapter we inroduced a risk ratio RR which was defined as the ratio of inflation risk to the sum of inflation risk and relative price risk; see expression (30) of Table 2. This risk ratio is an indicator of the negative slope of the short-run Phillips curve. It has been graphed in figure 2. Each dot indicates a quarterly value of the risk ratio, defined as monthly average. An increase in the risk ratio (left hand scale of fig. 2) indicates that the short-run Phillips curve becomes steeper, hence that the sort-run trade-off (right hand scale) decreases, and vice versa.

The free-hand drawn curve in figures 2 suggests the following: During the early period of risk stability the slope coefficient of the Phillips curve remained remarkably stable, though it was not constant. From the late sixities onwards, however, when aggregate risks began to move, the slope coefficient started to increase numerically and to dance around. It appears that the Phillips trade-off worsened during the seventies, moving in large swings and generally exhibiting a high volatility; the measured variance of the risk ratio is six times larger for the unstable period than for the period of risk stability.

In view of the aggregate risk situation of the seventies the observed instability of the Phillips slope is not surprising. Changes in aggregate risk and, therefore, in relative price risk do, however, not only impinge on the slope of the short-run Phillips curve but — according to our formulation of the output supply by risk-averse suppliers — simultaneously on the shift parameter of the long-run Phillips curve.

We hope to provide reliable evidence on the latter relationship in future work.

Finally, we like to point out an important implication of the evidence presented in this paper. The considerable instability of the slope of the standard natural rate Phillips curve, observed over the seventies, implies that any estimate of the Phillips curve for that period will be heavily biased if it relies on the assumption of constant coefficients.

V. Summary and Conclusion

In this paper we have examined the economic relevance of aggregate risk. Following Knight we define risk as the conditional variance of forecast errors. In the theoretical part of the paper we have used a multimarkets equilibrium model of the Lucas confusion type, with different price elasticities across markets, in order to show that the variances of monetary and real aggregate shocks do not only produce

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volatility of transitory output but, more importantly, affect positively economy-wide relative price risk. Thus, an increase in the variances of aggregate shocks raises perceived relative price risk. And this in turn induces risk-averse suppliers to reduce normal output growth.

The empirical evidence presented was based on monthly data for West-Germany, covering the period 1958 to 1982. It is noteworthy that West-Germany experienced a period of "golden years", from 1958 to 1966, where relative price risk and the short-run tradeoff of the standard natural rate Phillips curve remained remarkably stable. Whether mere coincidence or not, with the advent of the Keynesian concept of stabilization policy, in 1967, the West-German economy moved into an era of pronounced instability, with widely moving aggregate risks from monetary and real shocks.

Our major empirical results are as follows: *First*, relative price risk and total aggregate risk, as summarized by inflation risk, are positively related. *Second*, and more specifically, selected types of aggregate risk, created by monetary shocks as well as by shocks from the worldmarket prices of raw materials, have explanatory power for relative price risk.

These observations lead us to conclude that it is time for policy makers to give more consideration to the problem of macroeconomic risk. Policy makers have learned that it is more important, at least with respect to real effects of stabilization policies, to look at the rates of change of economic variables rather than at levels. Most of them are used, meanwhile, to formulate monetary targets in terms of monetary rates of change and to derive the target rate from expectations about future real growth as well as a "tolerable" rate of change of the general price level.

What policy makers have not learned yet and, indeed, is not yet widely understood among economists is that the public's perception of the variances of economic variables is a matter of equal importance. Policies, whether fiscal or monetary, that leave much room for shortrun manoeuvring reduce predictability, hence raise aggregate risk. They are detrimental to normal output growth and employment in spite of any transitory gains.

Price level stability is the most important goal of monetary policy, according to some central bank laws. And we have no reason to belittle it. But we think that it is time for the policy makers to adopt as an equally important goal of stabilization policies the goal of keeping the aggregate risk, induced by their own policies, at a minimum. This requires not just to set clear targets for a sufficiently long time ahead but, moreover, to stick to them irrevocably, never mind what the "needs of the day" may call for. In short, it requires to be credible.

Summary

This paper studies the interconnection of aggregate and relative price risk. A multimarkets equilibrium model is developed which extends the Lucas supply function by giving special attention to the role of perceived, economywide relative price risk. The model implies that an increase in aggregate risk simultaneously shifts the long-run Phillips curve to the right and steepens the short-run curves. Using data from Germany for the sample period 1958 through 1982 it is shown that the movement over time of relative price risk is positively related to concurrent movements of inflation risk as well as two major sources of aggregate risk.

Zusammenfassung

Dieser Aufsatz untersucht die Beziehungen zwischen aggregativem Risiko und dem Risiko der relativen Preisentwicklung (*RPR*). In einem Multimarkt-Gleichgewichtsmodell des Lucas-Typs zeigen wir die simultane Abhängigkeit des *RPR* und des Inflationsrisikos von aggregativen Risikovariablen. Veränderungen aggregativen Risikos liefern eine Erklärung der Instabilität der Phillipskurve, deren Steigungs- und Lageparameter sich mit Bewegungen aggregativen Risikos ändern.

Wir entwickeln Maße für zwei Arten aggregativen Risikos, das Inflationsrisiko und *RPR*. Sie werden als Prozesse der bedingten Varianzen einer verallgemeinerten Formulierung von Transferfunktionsmodellen konstruiert. Mit Daten der Bundesrepublik der Jahre 1958 – 1982 zeigen wir, daß Bewegungen des *RPR* positiv mit denen des Inflationsrisikos und der aggregativen Risikofaktoren korrelieren.

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Appendix	A.1:	Data	Used
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Variable		Explanation			
Money stock	М	Currency holdings and d of domestic nonbanks	Currency holdings and demand desposits of domestic nonbanks		
Extended Monetary base	В	Currency in circulation demand deposits of commercial banks with the Bundesbank, and cumulated liberated reserves			
Foreign prices	p^F		Index of worldmarket prices of raw materials (measured in US dollars)		
General price level	p =	$\Sigma u_i p_i$			
Market-specific prices p _i :			Relative (perc		
			in 1958	in 1980	
	P 10	Electric power industry	5.8	14.3	
	P 21	Mining industry	6.1	2.8	
	P 22	Mineral-oil products Petrol-chemical products	3.6	8.1	
	P 25	Quarries	4.2	3.7	
	P 27	Iron and steel	8.6	3.9	
	P 32	Machines and equipment	10.3	9.0	
	P 33	Road vehicles	5.7	9.2	
	P 36	Electric-technical products	6.0	10.0	
	P 38	Metal ware	5.2	3.8	
	P 40	Chemical industry	10.2	9.4	
	P 54	Timber trade	3.1	3.5	
	P 58	Plastics	1.3	3.2	
	P 63	Textile industry	9.9	3.4	
	P 68	Foodprocessing industry	19.9	15.4	

Note: Source for time series M and B: Deutsche Bundesbank, Frankfurt. Source for time series p; HWWA-Institut für Wirtschaftsforschung, Hamburg. Source for domestic price series: Statistisches Bundesamt, Wiesbaden.