

Dynamic Economic Strategies of Breeder Reactor Commitments

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This approach allows the user to analyze the effect of policy decisions upon the cost of electricity. The forward dynamic programming approach is adapted to an economic assessment of a nuclear reactor mix to allow many terminal conditions to be analyzed.

In particular, when the commitment history can be traced for an optimal path, the life of each unit may be computed and a retirement cost may be attributed to each terminal state to reflect the loss incurred by termination of a Light Water Reactor due to unavailability of fuel before the planned lifetime of the plant has been fulfilled.

1. Introduction

In a previous essay, *Gottinger* (1982), we have provided a cost-benefit analysis of the Light Water Breeder Reactor (LWBR) System based on a static assessment of fuel cycle costs. From the point of view of given nuclear energy choices, in this essay we look at dynamic aspects of phasing in and phasing out nuclear technologies in a way that optimize decisions for commitments in these technologies. Such commitment strategies could constitute a rational approach to intertemporal choice of technologies in the case of uncertainty, *Hammond* (1976).

A comprehensive dynamic analysis for breeder reactor commitments in the case of the Liquid Metal Fast Breeder Reactor (LMFBR) has previously been given by A. S. *Manne* (1974). In contrast to the LMFBR we must consider the LWBR as a 'conceptual reactor'. In view of the hypothetical resource use situation described by *Gottinger* (1982), the question is whether an advanced converter on a Thorium-232/U-233 fuel cycle (e.g. the prebreeder) produces U-233 at a great enough rate to make the LWBR system a more favorable technology than the Light Water Reactor (LWR). Since Thorium-232 is several times more abundant than Uranium pursuing the Th-U-233 fuel cycle has a multiplying effect on available resources.

The fuel cycle cost assumed for the breeder should be specified with some care. While the breeder has a lower fuel cycle cost than either the

LWR or the breeder, the cost of reprocessing is a significant contributor to the breeder fuel cycle cost. Under what scenarios then will the LWR possess favorable economic properties? It would appear that the main distinction between the LWR and the LWR is the lifetime constraint which applies to the LWR. In an environment of severely limited uranium availability, the penalty associated with the early termination of LWR's as generating units may be sufficient to justify the added front-end expense of additional fuel cycle cost to establish a non-terminable nuclear electric economy.

2. A Dynamic Cost Model

Consider a LWR energy system composed of three reactor types. Define the pertinent parameters as follows:

- $N_j(t)$ is the number of reactors of type j ,
- $j = 1, 2, 3$ in place at time t ,
- v_j is the cost coefficient per reactor of type j ,
- $C_j(t)$ is the cost at time t due to reactors of type j ,
- e_j is the enrichment requirement for a generating unit of type j ,
- E is the total capacity of enriching plants.

Constraint: Assume that enrichment capacity is saturated. That is, that sufficient reactors are built so that no further enriching capacity is available.

$$(1) \quad \sum_{j=1}^3 N_j(t) e_j = E$$

Type 3 is assigned to the light water breeder; thus $e_3 = 0$ and

$$(2) \quad \sum_{j=1}^3 N_j(t) e_j = \sum_{j=1}^2 N_j(t) e_j = E$$

Objective Function: The discounted total cost C_{total} , is assumed to be minimized, written

$$(3) \quad \text{Minimize } C_{\text{total}} = \int_0^T e^{-\alpha t} \sum_{j=1}^3 C_j(t) dt$$

where α is the discount rate.

This is in the tradition of formulating the intertemporal allocation problem.¹

Consider now the breeder-prebreeder relationship on the basis of the Thorium-232/U-233 fuel cycle. Assume no retirement of prebreeders.

¹ Häfele (1975).

The number of breeders which may be supported at the activation point is some function of the number of prebreeder-years which have accumulated. Assuming a constant γ kg U-233 per prebreeder unit, the number of breeders which may be supported is

$$(4) \quad N_3(t) = \frac{1}{R} \gamma \int_0^{t-1} N_2(\tau) d\tau$$

where R is the U-233 requirement, in kg, for one breeder unit. The upper limit is $t - 1$, assuming one year for processing and fabrication of U-233 fuel.

Now $N_2(t)$ may be assumed to have the form $N_2(\tau) = f [1 - e^{-a\tau}]$, which is a convenient but also realistic assumption on the saturation process up to the enrichment capacity. This function is easily integrated to yield

$$(5) \quad \begin{aligned} \int_0^{t-1} N_2(\tau) d\tau &= \int_0^{t-1} f [1 - e^{-a\tau}] d\tau = f \int_0^{t-1} d\tau - f \int_0^{t-1} e^{-a\tau} d\tau \\ \int_0^{t-1} N_2(\tau) d\tau &= f(t-1) - f \left[-\frac{1}{a} e^{-a\tau} \right]_0^{t-1} = f(t-1) + \frac{f}{a} [e^{-a(t-1)} - 1] \\ &= f \left[t-1 + \frac{1}{a} e^{-a(t-1)} - \frac{1}{a} \right] \end{aligned}$$

Now from the constraint equation

$$(6) \quad N_1(t) = \frac{1}{e_1} [E - f(1 - e^{-at}) e_2]$$

Substituting these relationships into the objective function,

$$(7) \quad C_{\text{total}} = \int_0^T e^{-at} \left\{ \frac{1}{e_1} [E - f(1 - e^{-at}) e_2] v_1 + f(1 - e^{-at}) v_2 \right. \\ \left. + \frac{1}{R} \gamma f \left(t-1 + \frac{1}{a} e^{-a(t-1)} - \frac{1}{a} \right) v_3 \right\} dt$$

$$(8) \quad C_{\text{total}} = \int_0^T \frac{E}{e_1} v_1(t) e^{-at} dt + \int_1^T \left[-f \frac{e_2}{e_1} v_1(t) (1 - e^{-at}) \right. \\ \left. + f(1 - e^{-at}) v_2(t) + f \frac{1}{R} \gamma v_3(t) \left(t-1 + \frac{1}{a} e^{-a(t-1)} - \frac{1}{a} \right) \right] e^{-at} dt$$

Now it is seen that if f is constant, the objective function may be written

$$(9) \quad C_{\text{total}} = \int_0^T \frac{E}{e_1} v_1(t) e^{-at} dt + f \int_0^T e^{-at} \left[-\frac{e_2}{e_1} v_1(t) (1 - e^{-at}) \right. \\ \left. + (1 - e^{-at}) v_2(t) + \frac{1}{R} \gamma v_3(t) \left(t-1 + \frac{1}{a} e^{-a(t-1)} - \frac{1}{a} \right) \right] dt$$

$$(10) \quad C_{\text{total}} = I_1(T) + f I_2(T)$$

When regarded as a function of f , the objective function is minimized by choosing $f = 1$ if $I_2(T)$ is negative, or by selecting $f = 0$ if $I_2(T)$ is positive.

Now in general the commitment pattern for prebreeders is not selected in advance. Therefore, it is desired that the time-dependent mix of reactors $[N_1(t), N_2(t), N_3(t)]$ be chosen to minimize the objective function independently of any particular algorithm. The relationship between the number of breeders and the prebreeder history is given, from equation (4), by

$$(11) \quad \dot{N}_3(t) = \frac{\gamma}{R} N_2(t-1)$$

where the dot denotes differentiation with respect to time.

From the constraint equation,

$$(12) \quad N_1(t) e_1 + \frac{R}{\gamma} \dot{N}_3(t+1) e_2 = E$$

$$(13) \quad N_1(t) = \frac{1}{e_1} \left[E - \frac{R}{\gamma} \dot{N}_3(t+1) e_2 \right]$$

Substituting this in the cost functional

$$(14) \quad C_{\text{total}} = \int_0^T e^{-at} \left[\frac{1}{e_1} E(t) - \frac{R}{\gamma} \dot{N}_3(t+1) e_2 \right] v_1(t) + \frac{R}{\gamma} \dot{N}_3(t+1) v_2(t) + N_3(t) v_3(t) \Big] dt$$

Writing $y(t) = \frac{R}{\gamma} N_3(t)$ and expanding $\dot{y}(t+1)$ in a Taylor series about $\dot{y}(t)$, equation (14) becomes, after truncating,

$$(15) \quad C_{\text{total}} = \int_0^T e^{-at} \left\{ \frac{1}{e_1} [E(t) - \dot{y}(t) - \ddot{y}(t)] v_1(t) + v_2(t) [\dot{y}(t) + \ddot{y}(t)] + \frac{\gamma}{R} y(t) v_3(t) \right\} dt$$

(For the sake of realism t may refer to any suitable time unit matching the actual 'fuel cycle turnover time'.)

Now the costs $v_1(t)$, $v_2(t)$ and $v_3(t)$ are dependent upon the commitment history, including capital investment, since U_3O_8 -price may be modelled as a function of cumulative consumption.

An effective method for the attack of problems where constraints limit the range of admissible functions is that of dynamic programming. In the case of prebreeder commitment, such an approach is tractable since the ability of the nuclear industry to support the U-233-Th fuel

cycle is limited by lack of reprocessing facilities. Large capital requirements for construction of reprocessing plants would normally be assumed to act as a limiting influence upon the rate of growth of reprocessing facilities.

As a means of allowing for consideration of limitations upon production capability, the Dynamic Programming formulation of the minimization problem allows the input of arbitrary limits upon the system mix. These constraints reflect assumptions upon the capacity of the nuclear service sector to meet needs arising from various proportions of light water reactors, prebreeders, and breeders. Alternatively, they could reflect policies adopted by either the Government or industry groups to support light water breeder reactor development. This method of attack yields an effective approach to the minimization problem defined by equations (1) and (3).

3. Dynamic Formulation for Reactor Plant Selection

It is desired to determine the optimal path for reactor commitment based upon time-dependent values for capital cost, operating cost, and fuel cycle cost. The stage variable in this analysis is time; the state is the vector consisting of the reactors of various types.

$$(16) \quad \underline{x} = [X_1, X_2, X_3]$$

where X_n is the commitment of reactor type n .

The control \underline{u} applied at time t is the vector of plant additions. For convenience, the elements of the \underline{x} and \underline{u} vectors shall be taken to be GW (e) of nuclear-electric additions. The analysis may be expanded to include plants of other types such as coal-fired and natural gas facilities.

We adopt constraints to help reduce the computing time of the problem at hand. In the first case, we are limited by uranium ore availability. Thus, the number of uranium-consuming reactors is limited. That is, the yearly consumption of uranium must be consistent with the capacity of the industry to provide it. A second constraint concerns availability of enriching services. The separative work required in any year must be less than or equal to available or projected available capacity. Fuel fabrication capability may also be employed as a constraint. However, it is generally taken to be a simplifying assumption that fabrication capability is not a limiting restriction. It is probably more precise to say that enriching services are a more restrictive constraint than is the availability of fuel fabrication facilities. Reprocessing availability will be a constraint for some reactor types but not for others.

The standard Light Water Reactors, the PWR and the BWR, are not entirely dependent upon a closed loop fuel cycle for their operation. On the other hand, the Light Water Breeder System is dependent upon reprocessing capability, and it should be observed that a rational decision-maker would not commit his company's resources to a breeder unless he were certain of reprocessing availability. The arguments may be extended to include various measures of social cost and adverse environmental effects as elements of cost computation or constraint formulation. The only requirement in such a case is that one has quantifiable relationships between costs and benefits, or more precisely, between the measurable impacts and the resulting component of cost. Such an undertaking is frequently impossible when dealing with matters relating to social costs, and even when such attempts are made, the results may not be accepted by the other scholars in the area. As a simplification for a dynamic programming approach, it is the case here that the social costs are taken to be equal for all reactor types.

We wish to minimize the total discounted cost to society due to the installation of nuclear-electric power. If we denote by $v(t)$ the total cost of power to society at time t , we may use Bellman's principle of optimality to establish the iterative equation of Dynamic Programming

$$(17) \quad v(t) = \text{Min}_{0 \leq \tau \leq t} [k(\tau) \Delta t + e^{-\alpha \Delta t} v(\tau + \Delta t)]$$

Reducing this to the discrete form

$$(18) \quad v(t) = \text{Min} [k(t) + a(t) \Delta t + \exp(-\alpha \Delta t) v(t + \Delta t)]$$

where

$k(t)$ is the cost of adding new units

$a(t)$ is the output rate of the current units.

The discount factor is the reciprocal of $(1 + i)$, where i is the interest rate. For quantized t and unit value of Δt , $e^{-\alpha}$ equals $(1 + i)^{-1}$. This equation says that, if one knows the ideal combination of units to generate power from year $t + 1$ to T , the terminal year in the analysis, then it is desired to determine the ideal combination of units which takes the system from the beginning of the final year. Now it is to be observed that the function $v(t + \Delta t)$ is a real-valued function of the vector \underline{x} , as we are ultimately attempting to determine the values of the state variable \underline{x} and the control variable \underline{k} as well as that of the minimum cost.

Constraints upon the problem may assume a number of forms, and in the simple analysis to be performed initially it shall be taken that the constraints be formulated in terms of state-vector quantities rather than control vector quantities. Denoting the enrichment constraint by

$e(t)$, a real quantity, and the average enrichment requirement for each reactor by the vector E , the first constraint equation may be written $E'X \leq e(t)$ where the prime denotes transpose. Similarly, if the vector u denotes U_3O_8 production required to support each reactor type, the formulation $U'X \leq u(t)$ may be used to specify the uranium production constraint. The basic equation of the reactor deployment model is

$$(19) \quad \underline{x}(t+1) = \underline{x}(t) + \underline{k}(t)$$

which states that the capacity distribution at time $t+1$ is that at time t plus any additions which might occur between t and $t+1$. There is one further constraint, an equality constraint, which states that the total capacity available in period t must be equal to the demand schedule $d(t)$. For n plant types

$$(20) \quad \sum_{i=1}^n x_i(t) = d(t)$$

Now the control vector $\underline{k}(t)$ may assume an unlimited number of representations, since the mix of reactor types may be continuously represented.

4. Exemplification of the Dynamic Programming Approach

A stage-increment technique can be developed to solve the optimization problem utilizing techniques of Dynamic Programming. The fundamental equation suggests that we may apply each control to all existing states in order to define the states at the succeeding stage. Additional assumptions are required to reduce the number of states kept during the search for an optimum in the dynamic programming algorithm.

Two approaches were combined to yield a tractable solution. The first is a tunnel constraint, the other is related to the quantization of the states themselves. Where a calculated state is identical to an existing state at some stage t , the principle of optimality requires that the state having minimum cost be retained.

A state is assumed to be a slowly-varying function of its parameters, it may be assumed the a nearby state is reflective of the same properties as a given state \underline{x} . It must be borne in mind that the state \underline{x} is an ordered n -tuple which, for the case of the LWBR system, includes as one of its components the number of prebreeder-years. Therefore, two states which are close to each other in particular have the same or nearly the same number of prebreeder-years, and hence have generated approximately the same quantity of U-233. The metric utilized to establish closeness is the sum of the absolute values of the deviations. In general,

one might use $d(x, y) = \sum_{i=1}^n |x_i - y_i|$ as the distance function for the evaluation of closeness of two states. For the system consisting of the LWBR and the LWR, the equation reduces to $d(x, y) = \sum_{i=1}^3 |x_i - y_i|$.

Given $\varepsilon > 0$ and a point y , one may say that x is in a neighborhood of y if $d(x, y) < \varepsilon$. For the dynamic programming problem, all quantities are integers and therefore the neighborhood should be specified in terms of an integer. The substitution criterion therefore becomes $d(x, y) < K$ where K is an integer. For initial dynamic programming studies, K has been taken to be equal to 2. This approximation may result in propagated error, since the algorithm is that if $d(x_j, x_n) < K$, $j = 1, \dots, n-1$ then the state with minimum cost is retained and the other one is discarded. This is a sequence dependent procedure. Observe, for example, that the sequence of triples (12, 10, 11), (12, 10, 12), (12, 10, 13), (12, 10, 14) will result in the storing of the single state (12, 10, 14), while the same four states in the order (12, 10, 11), (12, 10, 14), (12, 10, 13), (12, 10, 12) will result in the retention of states (12, 10, 14) and (12, 10, 12).

Even with the imposition of constraints and simplifying approximations, the procedure of finding an optimal trajectory over a suitable planning period, say twenty years, is a formidable task. The problem may then be scaled to a manageable size in order to yield, in a reasonable amount of time, a solution which will bear some resemblance to the optimal path for the more elaborate form. For example, a stage may be taken to be two years instead of one, and power additions may be assumed to be in increments of two GW (e) rather than one. It would be desirable to investigate the effect of policy constraints upon the cost of power, or more appropriately, the cost of nuclear-electric power. It is not necessary to utilize absolute cost values; relative costs may be used to investigate the economic properties of the reactor system.

5. Computational Method

It is necessary to determine whether there will be a net benefit to society deriving from the existence of the light water breeder reactor system. It has previously been noted that the research and development cost for the LWBR system are anticipated to be small relative to those for a reactor type not in production. Thus, it is quite valid to compare the light water breeder system with the light water reactor system, since it may be assumed that production capacity and operational characteristics are similar in the two cases. It is desired that quantification be made of the discounted cost differential between light water reactors and the light water breeder system. To determine the effect of this cost differential, a Dynamic Programming approach is employed.

The objective function is the discounted cost of those components which vary with reactor type. In the comparison of the light water reactor and the light water breeder/prebreeder systems, this economic influence is primarily attributable to differences in fuel cycle cost. There is, however, a further consideration. In an optimization scheme, an objective function is minimized or maximized over a defined period. However, benefits accrue to society in the years following the defined span of the optimization period. Therefore, a terminal condition is applied to each admissible state at the end of the period to provide proper accounting for the penalties and rewards associated with each state at the final stage. Forward dynamic programming is chosen since it provides a simple and straightforward method for examining only those states for which the prebreeder/breeder combination is realizable from admissible states at previous stages. It may be observed that the familiar backward dynamic programming algorithm contains no provision for ensuring that a given state $\bar{x}(t)$ may be generated by applying admissible controls at stage $t - 1$. Therefore, backward dynamic programming could result in the evaluation of a large number of states which could possibly be generated from a given initial condition. While forward generation of states may also result in a number of states which are of no consequence, there is at least the guarantee that each state so generated is derivable from some possible physical situation at the preceding stage.

The admissibility of the state depends upon constraint conditions imposed upon the particular problem. Since it is assumed that fuel for the light water breeder reactor is produced by the prebreeder reactor, admissibility of a state is governed not only by the constraint condition but also by the system equation. The number of breeder reactors is limited to those which may be fueled with existing quantities of U-233, retaining enough in the fabrication and reprocessing loop for one reload core. Assuming a whole core inventory of 4500 kg and a reload core requirement of 1000 kg U-233, 5500 kg of U-233 is required for the base design breeder reactor. Assuming once yearly refueling of the reference design prebreeder, 310 kg of the U-233 isotope is removed each year from the prebreeder reactor, cooled and reprocessed. Thus, one prebreeder may be operated in period of $5500/310 = 17.7 \cong 18$ years in order to generate a quantity of uranium sufficient to sustain a light water breeder reactor. Assuming 18 prebreeder-years per breeder, and a delay of at least one year for cooling, reprocessing and fabrication, the number of breeders which may be sustained is computed from the number of prebreeders which have been utilized. The number of prebreeder-years may be determined by summing the contribution from the deployment of prebreeders at each stage. Prebreeder-years are computed by integrating the commitment function for prebreeders over the pre-

scribed interval of interest. Denoting this interval by $[0, T]$ and pre-breeder commitment by $P(t)$, thus

$$(21) \quad \text{prebreeder-years} = \int_0^T \gamma P(t) dt$$

The amount of uranium produced by each increment of capacity is assumed for this analysis to be constant. The general expression for fissile isotope production from m distinct generating units may be written

$$(22) \quad \text{U-233 produced} = \sum_{i=1}^m \int_0^T R_i(t) \gamma_i(t) dt$$

where $\gamma_i(t)$ and $R_i(t)$ are production rate of U-233 and power level of unit i .

6. Dynamic Programming Objectives

The dynamic programming approach has its objective the minimization of the cost functional subject to availability constraints. The cost functional is given by

$$(23) \quad J = \int_{t_0}^{t_f} l[\underline{x}, \underline{u}, t] dt$$

where $l[\underline{x}, \underline{u}, t]$ is the cost associated with the transition from state \underline{x} at stage t to state $\underline{x} + \underline{u}$ at stage $t + \Delta t$. The problem is simplified by the choice of a constant value for Δt . Further, the objective function J may be written

$$(24) \quad J[\underline{x}; \underline{u}] = \int_{t_0}^{t_q} l[\underline{x}, \underline{u}, t] dt + \int_{t_q}^{t_f} l[\underline{x}, \underline{u}, t] dt$$

where t_q is any quantized value for an intermediate stage, $t_0 < t_q < t_f$. The problem to be attacked is the evaluation of the costs of unit scheduling strategies over the stage interval $[0, t_f]$. The effect of a strategy enacted over the period of t_f stages may be approximated by considering no change to the system during the stage t_q to t_f . This is the case in which the control vector $\underline{u}(t)$ is constrained to be equal to the zero vector in the interval $t_q < t < t_f$. Thus, the integral

$$(25) \quad \int_{t_q}^{t_f} l[\underline{x}, \underline{u}, t] dt = \int_{t_q}^{t_f} l[\underline{x}, \underline{o}, t] dt$$

and is therefore treated as a terminal state condition. Minimization is carried out over all admissible controls applied to the integral

$$(26) \quad J_1[\underline{x}; \underline{u}] = \int_{t_0}^{t_q} l[\underline{x}, \underline{u}, t] dt .$$

The intent of the problem being the identification of optimal mixes of energy generation types under various policies, the further simplifications are made that all unit additions are brought on line at exactly the same time in each year and that each increment consists of one GW(e).

7. Cost Constraint Considerations

For the components of the fission power industry, the known possibilities for technological variation are given in Table 1. Among these the most important with respect to the introduction and maintenance of a light water breeder reactor power generation technology are the enrichment alternatives, the U-233 fabrication and reprocessing capabilities, and the thorium mining capability. Absence of U-233 handling capability would inhibit the LWBR; high costs of fabrication and reprocessing would similarly act to deny its development. Increased economy of enrichment, by the Laser Isotope Separation method, for example, would have an indeterminate effect, since such a development would benefit both the LWR and the LWBR segments.

It is obvious from a consideration of the list in Table 1 that an extensive economic analysis could be established provided that the data were available. However, the uncertainty in many of the fuel cycle component costs will tend to render useless the conclusions which might be reached as result of such a study. Analysis of the economic dynamics of an industry, particularly in the preliminary states, is limited in scope to one or a few pertinent economic variables. Price is usually chosen to be the controlling variable. For an economic analysis of any alternative energy source, the fuel price alone is insufficient to model the energy-generating sector. It is necessary to specify the constraints on the system, and these may arise out of considerations of physical, rather than economic availability. While these constraints may generally be formulated in terms of costs, the estimates may be subjective and controversial. A case in point is the availability of uranium. The quantity of uranium, for example, within the United States is not well known. Geologists do not agree upon the amount available in a reasonably well-defined area; there is even less certainty about that which is available in the regions which have not been extensively explored.

If known reserves are taken as the governing availability criterion for the nuclear industry, the constraint upon light water reactor commitment is seen to be severe indeed. The available uranium at a forward cost of \$ 30.00 per pound or less is 640,000 short tons (ERDA, 1976). Using as a rule of thumb 200 tons U_3O_8 per GW(e) per year and assuming that each nuclear base load unit is employed for 40 years, it is seen that

the available uranium from domestic proven reserves will support approximately 80 reactors.

Table 1
Potential Sources of Technological Variation

Fuel Cycle Component	Possible Modes
Mining/Milling	<ol style="list-style-type: none">1. Surface Mining2. Drilling and Tunneling3. Thorium Mining and exploration4. Recovery of Low-Grade Deposits
Conversion	No change to current processes foreseen
Enrichment	<ol style="list-style-type: none">1. Gaseous Diffusion2. Gas Centrifuge3. Laser Isotope Separation4. Fast Breeder Economy5. Light Water Breeder Economy
Fabrication	<ol style="list-style-type: none">1. UO₂ Pellet Fabrication2. Mixed Oxide Fabrication3. Thorium Oxide Fabrication4. UO₂ Fabrication (U-233)5. UC Fabrication
Reactor Operation	<ol style="list-style-type: none">1. Optimize Plant Efficiency2. Optimize Fissile Isotope Production
Reprocessing	<ol style="list-style-type: none">1. Uranium Recovery and Recycle Only2. U-235 and Fissile Pu Recovery3. U-233 Recovery
Transportation	<ol style="list-style-type: none">1. Unlimited Transportation of Spent Fuel2. Regional Reprocessing, Distributed Energy Centers3. Regional Reprocessing, Concentrated Energy Centers

It may be noted that the commitment history for prebreeders often shows moderate commitment until the tenth year, when deployment appears to be uncharacteristiclaly heavy. This is a result of the model which divides the problem into two time periods; one in which the decision logic pertaining to system growth is employed and one in which the effect of this logic are evaluated. The computational method which provides the accounting for the evaluation of effects, or terminal constraints, is based upon the premise that a prebreeder is retired as soon as sufficient U-233 is available to allow its replacement by a breeder.

This assumption is not employed during the first phase of the analysis in which the time-dependent growth combinations are enumerated. Therefore, there is the effect of a short lifetime for some of the prebreeders deployed in the later years of the investigation, and since the

prebreeder has the highest fuel cycle cost of the three reactor types, the optimal path might be expected to correspond to that condition which minimizes the number of prebreeder-years for a given number of prebreeders. The nature of the terminal constraint formulation in this analysis yields an unusually heavy commitment during the last year of growth. It is possible that the Dynamic Programming model might be improved by an investigation of the natural boundary conditions corresponding to the variational formulation which is the basis for the LWBR study. The terminal constraint formulation might, then, be improved to include this consideration which is expected to have the effect of smoothing the year-by-year deployment of prebreeders as reflected in the prebreeder-to-LWR fraction. Since the basic economic properties of the reactor types remain the same, there is no *a priori* reason to believe that this refinement of the algorithm will result in a cumulative ten-year deployment which differs substantially from the ones given as result of the present investigation. That is, the variation in answer due to a change in the algorithm is expected to be less severe than those variations which may be induced by utilizing different values for uranium, enrichment, and reprocessing prices.

The results of both the static and dynamic analysis do suggest, however, that close attention be given those concepts which hold promise for extending the usefulness of the nation's uranium resources. Thus, assuming the LWBR system to be among the economically more attractive alternatives to existing light water reactors, it is seen that research and investment in an enriching technology such as *Laser Isotope Separation* can yield substantial benefits to society. Because the energy requirements for Laser Isotope Separation are significantly lower than those of either the gaseous diffusion or the centrifuge process, the prospect exists of lowering enrichment price while expanding the utility of the nation's resource base.

With respect to the consumption of U_3O_8 , commitment of an LWBR system would have the effect of increasing demand for uranium resources for the period in which the presently-known resource base of approximately 640,000 metric tons U_3O_8 will support burner reactors. Hence, the importation of uranium would have little effect upon the desirability of an LWBR system. In fact, commercial implementation of a light water breeder system would increase requirements for uranium in all but the long term. Hence, national policy should permit and encourage a uranium import program if the light water breeder becomes a commercial reality.

Reprocessing is essential for the existence of any breeder reactor program. If plutonium recycle is denied the operators of light water re-

actors, it is possible that the retrieval of U-233 may be allowed even though the recovery of PU is not permitted, basically for the reason of denaturization of the Th-U-233 cycle². However, it is unlikely that capital investment would be made in a commercial reprocessing facility for separation of uranium and thorium before a market is guaranteed. Hence, reprocessing of plutonium and U-233 is likely a requisite for commercial introduction of the prebreeder.

It is concluded, therefore, that policy implementations which would affect the light water reactor system will also have similar effect upon the economic properties of the LWBR system. The policy toward development of the LWBR is, with the exception of reprocessing, independent of those policies which may govern the rest of the fuel cycle. Any effort to improve the economic characteristics of nuclear-electric power would center upon improvement of the LWR fuel cycle or capital cost, and would thus concentrate most heavily upon enriching and U₃O₈ costs.

8. Energy Policy Analysis

The dynamic programming techniques developed and employed in this analysis of the LWBR system may be used to evaluate other combinations of plant types. Further studies should include the Liquid Metal Fast Breeder Reactor, the High Temperature Gas Cooled Reactor, and the Gas Cooled Fast Reactor. For these designs, development expenditures must be included in the capital cost formulation.

The various fissile fuel loading patterns possible for the prebreeder suggest that detailed studies of core physics be performed before commitment to any new algorithm for reactor use. Removal of the design constraints associated with current handling of the enriching function make possible a wide range of fuel loadings and thys may permit a more economical use of fuel in a reactor. When analyzing the in-core economics of fuel loadings, it is necessary to ensure that costs of fuel preparation are properly handled. Utilization of oxide fuel pellets makes the fabrication of fuel rods with enrichment of composition gradations a relatively straightforward procedure.

Summary

In extending a previous static cost-benefit assessment to the case of intertemporal, dynamic allocation of resources within a nuclear energy regime an approach is suggested which allows the simultaneous determination of prices, and the optimal reactor mix is based upon the formulation of the

² See *Conaes* (1980), 218.

energy-generating cost calculation as a variational problem. The implementation of these concepts in computational form is done via *dynamic programming*. An algorithm is developed in which the number of the reactor types and power requirements are assumed and the optimal plant commitment schedules are generated for any set of hypothesized economic conditions. The application of this algorithm to the system containing Light Water Reactor, Prebreeders and Breeders is made and costs are generated.

Zusammenfassung

In Erweiterung einer bisher durchgeführten statischen Kosten-Nutzen-Analyse auf den Fall einer dynamischen Ressourcenallokation für ein nukleares Energiesystem wird in diesem Beitrag ein Ansatz vorgeschlagen, der die Berechnung einer optimalen Reaktorkombination als die Lösung eines Variationsproblems mit Hilfe der dynamischen Optimierung ermöglicht.

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