

# Is Deficit Spending Feasible in the Long Run?

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Can government in the long run spend more on goods and services than it receives by taxation? Government raises loans and levies an income tax to finance both purchases and the interest payments on public debt. The rate of interest corresponds to the marginal product of private capital. Households save a fixed share of national income and debt income, net after tax respectively. A good deal of private savings is absorbed by public borrowing, the remainder being left for private investment.

## 1. Introduction

There is a well-established body of literature on public debt and economic growth<sup>1</sup>. All these papers demonstrate that public borrowing is feasible in the long run. The present paper, on the other hand, addresses a problem which has been ignored so far: Can government in the long run spend more on goods and services than it receives by taxation? As usual, the long run is characterized by the fact that the interest payments on public debt must be paid out of the budget.

The analysis will be carried out within the following basic framework. Private firms employ private capital and labour to produce a homogeneous commodity, which serves for private consumption, private investment and public consumption. Government wishes to allocate a given fraction of national income to public consumption. Government raises loans and levies an income tax to finance both public consumption and public interest. The rate of interest corresponds to the marginal product of private capital. Households plan to save a fixed share of national income and debt income, net after tax respectively. A good deal of private savings is absorbed by public borrowing, the remainder being left for private investment. Private investment, in turn, augments private capital.

The paper is organized in the following way. In section 2, we shall discuss the long-run consequences of deficit spending. Does a steady state of growth exist? Or will public debt explode? Then, in section 3,

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<sup>1</sup> e. g. *Domar* (1944), *Modigliani* (1961), *Diamond* (1965), *Phelps and Shell* (1969), *Cavaco-Silva* (1977), *von Weizsäcker* (1979), see appendix 1.

we shall derive the golden rule of deficit spending. Finally, in section 4, deficit spending on public investment will be incorporated into the model.

## 2. Deficit Spending on Public Consumption

Private firms produce a homogeneous commodity  $Y$  by means of private capital  $K$  and labour  $L$ . For ease exposition, consider a Cobb-Douglas technology showing constant returns to scale:  $Y = K^\alpha L^\beta$  with  $\alpha > 0$ ,  $\beta > 0$  and  $\alpha + \beta = 1$ . Output  $Y$  can be devoted to private consumption  $C$ , private investment  $I$  and public consumption  $G$ :  $Y = C + I + G$ .

Government borrows and collects a tax to cover both public consumption and the interest payments on public debt. We assume that government intends to spend a given fraction  $g$  of national income  $Y$  on public consumption:  $G = gY$ . In addition, government levies a tax  $T$  at the flat rate  $t$  on national income:  $T = tY$ . If public consumption exceeds tax revenue  $G > T$ , we shall speak of deficit spending; conversely, the case  $G < T$  may be called surplus spending. Henceforth, we shall throughout assume deficit spending. Accordingly, the budget deficit equals  $G - T$ . At this stage, it is useful to define the budget deficit ratio  $h := g - t > 0$ .

Government pays the net rate of interest  $(1 - t)r$  on public debt  $D$ , so public interest amounts to  $(1 - t)Dr$ . Instead, we might suppose that government pays the gross rate of interest which then is subject to the income tax. Obviously, both approaches are equivalent. Yet to simplify matters, it is helpful to take the first approach. Let firms maximize profits under perfect competition, so the gross rate of interest corresponds to the marginal product of private capital  $r = \alpha Y/K$ . Furthermore, it is convenient to introduce the debt-capital ratio  $d := D/K$ . As a consequence, public interest equals  $(1 - t)Dr = (1 - t)\alpha dY$ .

(Net) public borrowing  $B$  and tax revenue  $T$  serve to finance both public consumption  $G$  and public interest:  $B + T = G + (1 - t)Dr$ . Solve for  $B$  and insert  $G - T = hY$  as well as  $(1 - t)Dr = (1 - t)\alpha dY$ :

$$(2.1) \quad B = hY + (1 - t)\alpha dY.$$

Public borrowing  $B$  adds to public debt:  $\dot{D} = B$  (the dot denotes the time derivative).

National income and debt income, net after tax respectively, constitute disposable income  $Y_d := (1 - t)Y + (1 - t)Dr$ . We posit that households save a fixed share  $s$  of disposable income:  $S = sY_d$ . Restate private savings in terms of national income by taking account of  $(1 - t)Dr = (1 - t)\alpha dY$ :

$$(2.2) \quad S = s(1-t)(1+\alpha d)Y.$$

A good deal of private savings is absorbed by public borrowing, the remainder being left for private investment  $I = S - B$ . Substitute (2.1) and (2.2) to obtain:

$$(2.3) \quad I = s(1-t)(1+\alpha d)Y - hY - (1-t)\alpha dY.$$

Private investment, in turn, augments private capital:

$\dot{K} = I$ . Thus, the growth rate of private capital is determined by the private saving ratio  $s$ , by the budget deficit ratio  $h$ , by the tax rate  $t$ , by the debt-capital ratio  $d$ , by technology  $\alpha$ , and by the capital coefficient  $v$ :  $\dot{K} = [s(1-t)(1+\alpha d) - h - (1-t)\alpha d]/v$  with  $v := K/Y$ .

Let labour grow at the natural rate  $n$ . In the steady state, private capital accumulates at the natural rate:

$$(2.4) \quad n = \frac{s(1-t)(1+\alpha d) - h - (1-t)\alpha d}{v}$$

Here,  $h$ ,  $n$ ,  $s$ ,  $t$  and  $\alpha$  are constant;  $v$  is flexible and can adjust itself to fulfill condition (2.4). However,  $d$  is still unknown.

The debt-capital ratio  $d$  proves to be a strategic variable, as will be demonstrated now. In the steady state holds  $d = \dot{D}/\dot{K} = B/I$ . Insert (2.1) and (2.3):

$$(2.5) \quad d = \frac{h + (1-t)\alpha d}{s(1-t) - h - (1-s)(1-t)\alpha d}$$

Then solve this quadratic equation for  $d$ :

$$(2.6) \quad d = \frac{(s-\alpha)(1-t)-h}{2\alpha(1-s)(1-t)} \pm \sqrt{\frac{[(s-\alpha)(1-t)-h]^2}{[2\alpha(1-s)(1-t)]^2} - \frac{h}{\alpha(1-s)(1-t)}}.$$

A steady state of growth does only exist, if the debt-capital ratio  $d$  is real and positive. To evaluate this condition, consider the reduced discriminant of (2.6):

$A := [(s-\alpha)(1-t)-h]^2 - 4\alpha(1-s)(1-t)h$ . The discriminant vanishes at:

$$(2.7) \quad h_{1,2} = (1-t)(\alpha + s - 2\alpha s) \mp (1-t)\sqrt{(\alpha + s - 2\alpha s)^2 - (s-\alpha)^2}.$$

The analysis of (2.7) reveals that  $h_1, h_2$  are real and positive with  $h_1 < h_2$  (see appendix 2). Evidently, the reduced discriminant can be written as  $A = (h - h_1)(h - h_2)$ .

This gives rise to four distinct cases, depending on the saving ratio, the capital elasticity, the deficit ratio and the tax rate (see appendix 3).



First, if  $h_1 < h < h_2$ , then  $A < 0$ . That is to say, if the deficit ratio is moderate, the debt-capital ratio is complex. Second, if  $h \leq h_1$  and  $s \leq \alpha$ , then  $A \geq 0$  and  $d < 0$ . In other words, if the deficit ratio is low and if the saving ratio falls short of the capital elasticity, the debt-capital ratio is negative. In the first two cases, there is no steady state of growth, hence deficit spending is not feasible in the long run.

Third, if  $h \leq h_1$  and  $s > \alpha$ , then  $A \geq 0$  and  $d > 0$ . That means, if the deficit ratio is low and if the saving ratio exceeds the capital elasticity, the debt-capital ratio becomes positive. In this case, surprisingly, deficit spending is feasible in the long run. Fourth, if  $h \geq h_2$ , then  $A \geq 0$  and  $d < 0$ . That is, if the deficit ratio is high, the debt-capital ratio is negative. In this case, again, deficit spending is not feasible in the long run.

Table 1  
Maximum Feasible Deficit Ratio

Saving Ratio	Tax Rate				
	.1	.2	.3	.4	.5
.1	0	0	0	0	0
.2	0	0	0	0	0
.3	.012	.010	.009	.008	.007
.4	.043	.038	.034	.029	.024

To illustrate these conditions, consider a numerical example with  $\alpha = .2$ . Table 1 presents the critical values of the deficit ratio as a function of the saving ratio and the tax rate. For instance, let the tax rate be .3. If the saving ratio is .1 or .2, the critical value of the deficit ratio is 0. In this situation, deficit spending is not feasible in the long run. If the saving ratio is .3, the critical value of the deficit ratio amounts to .009. In this situation, two different cases may occur. If the deficit ratio falls short of .009, deficit spending actually is feasible in the long run. Conversely, if the deficit ratio exceeds .009, deficit spending is no more feasible in the long run.

From the empirical point of view, deficit spending seems not to be feasible in the long run. On the other hand, there may be an exception to this rule. If both the saving ratio is very high and the deficit ratio is extremely low, then deficit spending indeed is feasible in the long run. In summary, deficit spending is unlikely to be feasible in the long run. This result is in remarkable contrast to the policy recommendations presented by the German Council of Economic Advisers (see appendix 4).

Now imagine a situation where deficit spending is not feasible in the long run (that is  $s \leq \alpha$  or  $h > h_1$ ). What happens if government nevertheless makes use of deficit spending? Remember that government borrows to finance both the budget deficit and the interest payments on public debt. First of all, the share of public interest in national income rises steadily. For this reason, public borrowing and public debt expand more rapidly than national income. As a consequence, the rise in public interest accelerates. On the other hand, public borrowing crowds out private investment, thus reducing the private capital-output ratio. Accordingly, the marginal product of capital and the rate of interest increase, which reinforces the rise in public interest.

As soon as public borrowing completely absorbs private savings, private capital formation comes to a halt. This may occur either after a finite span of time or asymptotically, depending on the size of the deficit ratio. Therefore, the rate of income growth declines to  $\beta n$ . As time proceeds, the share of public interest in national income may continue to increase, thereby squeezing the deficit ratio. If the saving ratio exceeds the capital elasticity, the deficit ratio may drop until deficit spending becomes feasible. Conversely, if the saving ratio falls short of the capital elasticity, the scope for deficit spending shrinks back to zero. Ultimately, it may happen that private savings are not sufficient to cover public interest.

At this point, we come to a reinterpretation of the maximum feasible deficit ratio  $h_1$ . Suppose that the tax rate, the saving ratio and the capital elasticity are given. Then, what is the maximum public consumption ratio which is feasible in the long run? Here, two cases may occur. If  $s \leq \alpha$ , then  $g = t$ . In this situation, the maximum feasible public consumption ratio coincides with the given tax rate. On the other hand, if  $s > \alpha$ , then  $g = t + h_1$ . In this situation, the maximum feasible public consumption ratio is higher than the given tax rate.

Next have a look at the inverse problem. Let the public consumption ratio, the saving ratio and the capital elasticity be predetermined. Then, what is the minimum tax rate which is feasible in the long run? If  $s \leq \alpha$ , then  $t = g$ . In this situation, the minimum feasible tax rate corresponds to the public consumption ratio. Conversely, if  $s > \alpha$ , then  $t = (g - h_3) / (1 - h_3)$  with  $h_3 := h_1 (1 - t)^{-1}$ . In this situation, the minimum feasible tax rate is lower than the public consumption ratio.

So far, we assumed that households save a fixed share of disposable income. Now suppose, instead, that households save a fixed share of national income plus debt income as a whole, net after tax respectively:  $S = s(1 - t)Y + (1 - t)Dr$ . Under this saving function, a steady state

of growth with  $n = [s(1 - t) - h] / v$  does always exist, provided that  $h < s(1 - t)$  which imposes no severe constraint. In this situation, deficit spending generally is feasible in the long run, in contrast to the conclusions drawn above. The underlying saving behaviour, however, seems to be highly unrealistic.

### 3. The Golden Rule of Deficit Spending

In the preceding section, the focus was on the long-run feasibility of deficit spending. In the present section, on the other hand, we shall examine the long-run optimality of deficit spending. Let the public consumption ratio, the private saving ratio and the capital elasticity be given. So, which deficit ratio maximizes the sum of private and public consumption per head  $C/L + G/L$  in the long run?

The necessary condition is that the rate of interest agrees with the natural rate of growth:  $r = n$ , as is well known. This yields  $nv = \alpha$  and  $\alpha d = b$ , where  $b := B/Y$  denotes the public borrowing ratio. Further, divide  $B + T = G + (1 - t)Dr$  through by  $Y$  to obtain  $b + t = g + (1 - t)b$ , which can be restated as  $b = (g - t)/t$ . Then, insert  $nv = \alpha$ ,  $\alpha d = b$  as well as  $b = (g - t)/t$  into (2.4) and solve for the optimal tax rate:

$$(3.1) \quad t^* = \frac{g(1 - s)}{1 - \alpha - gs}.$$

Now substitute (3.1) into  $h = g - t$  to arrive at the optimal deficit ratio:

$$(3.2) \quad h^* = g \frac{(1 - g)s - \alpha}{1 - \alpha - gs}.$$

As a result, this is the golden rule of deficit spending.

Properly speaking, deficit spending prevails only if the optimal deficit ratio is positive. From the empirical point of view, it is safe to posit  $1 - \alpha - gs > 0$ . Accordingly, if  $g \leq (s - \alpha)/s$ , then  $h^* \geq 0$ . This gives rise to three distinct cases, depending on the private saving ratio and on the public consumption ratio. First, if  $s \leq \alpha$ , then  $h^* < 0$ . Under a low private saving ratio, there is no optimal deficit ratio. Second, if  $s > \alpha$  and  $g < (s - \alpha)/s$ , then  $h^* > 0$ . Under a high private saving ratio and a low public consumption ratio, an optimal deficit ratio does really exist. On the other hand, the optimal deficit ratio will be very small, since it must always stay below the maximum feasible deficit ratio:  $h^* < h_1$ . Third, if  $s > \alpha$  and  $g \geq (s - \alpha)/s$ , then  $h^* \leq 0$ . Under a high private saving ratio and a high public consumption ratio, there is again no optimal deficit ratio.



In summary, deficit spending is unlikely to be optimal in the long run. On the contrary, surplus spending will often be superior to deficit spending. Here, surplus spending is characterized by a budget surplus and public lending to stimulate private investment.

#### 4. Deficit Spending on Public Investment

So far, the analysis was confined to deficit spending on public consumption. Now the analysis will be extended to include deficit spending on public investment. Essentially, we take the same approach as in section 2. Private firms produce a homogeneous commodity  $Y$  by means of private capital  $K$ , public capital  $H$  and labour  $L$ . Consider the Cobb-Douglas technology  $Y = K^\alpha H^\beta L^\gamma$  with  $\alpha > 0, \beta > 0, \gamma > 0$  and  $\alpha + \beta + \gamma = 1$ . Output  $Y$  can be devoted to private consumption  $C$ , private investment  $I$  and public investment  $G$ .

We assume that government wishes to allocate a given fraction  $g$  of national income to public investment:  $G = gY$ . Public investment, in turn, increases public capital:  $\dot{H} = G$ . Hence, the growth rate of public capital is influenced by the public investment ratio  $g$  and by the public capital coefficient  $u$ :  $\hat{H} = g/u$  with  $u := H/Y$ . Let labour grow at the natural rate  $n$ . In the steady state, public capital develops at the natural rate:  $n = g/u$ . Here,  $g$  and  $n$  are constant;  $u$  is variable and can be adapted to meet the steady state condition. Moreover, government collects a tax  $T$  at a flat rate  $t$  on national income:  $T = tY$ . Again, we postulate deficit spending  $h := g - t > 0$ .

Government pays the net rate of interest  $(1 - t)r$  on public debt  $D$ , so public interest amounts to  $(1 - t)Dr$ . Besides, the gross rate of interest corresponds to the marginal product of private capital:  $r = \alpha Y/K$ . Owing to this, public interest equals  $(1 - t)Dr = (1 - t)\alpha dY$  with  $d := D/K$ . Public borrowing and tax revenue serve to finance both public investment and public interest:  $B + T = G + (1 - t)Dr$ . This involves  $B = hY + (1 - t)\alpha dY$ . Public borrowing adds to public debt:  $\dot{D} = B$ .

In analogy to section 2, we assume that households save a fixed share  $s$  of disposable income:  $S = s(1 - t)(1 + \alpha d)Y$ . A good deal of private savings  $S$  is absorbed by public borrowing, the remainder being left for private investment  $I = S - B = s(1 - t)(1 + \alpha d)Y - hY - (1 - t)\alpha dY$ . Private investment, in turn, augments private capital:  $\dot{K} = I$ . On account of this, the growth rate of private capital is  $\hat{K} = [s(1 - t)(1 + \alpha d) - h - (1 - t)\alpha d]/v$  with  $v := K/Y$ . In the steady state, private capital accumulates at the natural rate:  $n = [s(1 - t)(1 + \alpha d) - h -$

$(1 - t) \alpha d] / v$ . Here,  $h$ ,  $n$ ,  $s$ ,  $t$  and  $\alpha$  are given;  $v$  is flexible to accommodate itself.

Again, the debt-capital ratio  $d$  turns out to be a strategic variable.  $d$  is real and positive, if and only if  $s > \alpha$  and  $h \leq h_1$ . But this condition is identical to the feasibility condition derived in section 2. As a result, deficit spending on public investment, too, is unlikely to be feasible in the long run.

## 5. Conclusion

From the empirical point of view, deficit spending seems not to be feasible in the long run. On the other hand, there may be an exception to this rule. If both the saving ratio is very high and the deficit ratio is extremely low, then deficit spending indeed is feasible in the long run. In summary, deficit spending is unlikely to be feasible in the long run.

Now imagine a situation where deficit spending is not feasible in the long run. What happens if government nevertheless makes use of deficit spending? Public interest, public borrowing and public debt expand more rapidly than national income. In addition, public borrowing crowds out private investment, thereby increasing the rate of interest. Finally, public borrowing completely absorbs private savings, hence capital formation comes to a halt.

Of course, the emphasis was on some basic problems, and other aspects are still open to question. For example, at what speed does public debt explode? What is government's response? Does government push up the tax rate, inflate the money supply or cut back public consumption?

## 6. Appendix

### 1

Domar considers an economy suffering from secular unemployment. Government purchases of goods and services are financed by public borrowing. The interest payments on public debt are covered by a special tax. The rate of interest is given exogenously, irrespective of public borrowing. By way of contrast, Diamond considers a full-employed economy. The problem of public debt is embedded in a neoclassical growth model with overlapping generations. Government raises loans and collects a lump-sum tax to finance transfers and public interest. The rate of interest corresponds to the marginal product of capital. In the growth model by Phelps and Shell, transfers are covered by public borrowing, while public interest is covered by a lump-sum tax. In the



growth model by von Weizsäcker, government raises loans and levies a tax to finance public consumption and public interest.

## 2

Restate the discriminant of (2.7) as  $4\alpha s(1 + \alpha s - \alpha - s)$ . The term in brackets is positive since  $0 < s < 1$  and  $0 < \alpha < 1$ . Further,  $(\sqrt{\alpha} - \sqrt{s})^2 > 0$  and  $\sqrt{\alpha s} > \alpha s$  implies  $\alpha + s - 2\alpha s > 0$ . Therefore,  $h_1, h_2$  are positive.

## 3

First, if  $h_1 < h < h_2$ , then  $A = (h - h_1)(h - h_2) < 0$ . Second, if  $h \leq h_1$ , then  $A = (h - h_1)(h - h_2) \geq 0$ . From  $s \leq \alpha$  follows  $[(s - \alpha)(1 - t) - h] / [2\alpha(1 - s)(1 - t)] < 0$ . This together with  $h / [\alpha(1 - s)(1 - t)] > 0$  gives  $d < 0$ . Third, if  $s > \alpha$ , then  $h_1 < (s - \alpha)(1 - t)$ . Take account of  $h \leq h_1$  to arrive at  $h < (s - \alpha)(1 - t)$ . Accordingly,  $[(s - \alpha)(1 - t) - h] / [2\alpha(1 - s)(1 - t)] > 0$  and  $d > 0$ . Fourth, if  $h \geq h_2$  and  $s \leq \alpha$ , then  $A \geq 0$  and  $d < 0$ , as in the second case. Fifth, if  $h \geq h_2$  and  $s > \alpha$ , then  $A \geq 0$  and  $d < 0$ . Proof:  $h \geq h_2$  provides  $h > (1 - t)(\alpha + s - 2\alpha s)$ . Combine this with  $(\alpha + s - 2\alpha s) > (s - \alpha)$  to reach  $h > (1 - t)(s - \alpha)$ . Thus,  $[(s - \alpha)(1 - t) - h] / [2\alpha(1 - s)(1 - t)] < 0$  and  $d < 0$ .

## 4

The concept of full-employment borrowing (potentialorientierte Kreditaufnahme) advanced by the German Council of Economic Advisers CEA (Sachverständigenrat 1975/76) draws heavily on the Domar model (1944). It rests on the crucial hypothesis that the rate of interest is given exogenously.  $B + T = G + (1 - t)Dr$  together with  $B = Dn$  can be restated as  $G - T = Dn - (1 - t)Dr$ . If  $n \geq (1 - t)r$ , then  $G - T \geq 0$ . That is, deficit spending is feasible in the long run, provided the natural rate of growth exceeds the net rate of interest (Lerner's rule).

The crucial hypothesis, however, seems to be inappropriate. In the long run, the rate of interest tends to the marginal product of private capital. In this case, the rate of interest depends on the deficit ratio, the tax rate, the saving ratio, the natural rate of growth, and on technology. Suppose, e. g., that government increases the deficit ratio. This crowds out private investment, thereby raising the rate of interest. Implicitly, it has been shown in the present paper that as a rule  $n < (1 - t)r$ . As a consequence, the feasibility condition set up by the CEA generally is not fulfilled.

### Summary

Government raises loans and levies an income tax to finance both purchases of goods and services and interest payments on public debt. Government purchases exceed tax revenue. The rate of interest corresponds to the marginal product of private capital. Households save a fixed share of national income and debt income, net after tax respectively. A good deal of private savings is absorbed by public borrowing, the remainder being left for private investment. Does a steady state of growth exist? Or will public debt explode? Theoretical analysis shows that as a rule deficit spending is not feasible in the long run.

### Zusammenfassung

Der Staat nimmt Kredite auf und erhebt eine Einkommensteuer, um den Kauf von Gütern und Dienstleistungen und die öffentlichen Schuldzinsen zu finanzieren. Die öffentlichen Käufe übersteigen das Steueraufkommen. Der Zinssatz entspricht dem Grenzertrag des privaten Kapitals. Die privaten Haushalte sparen einen festen Anteil des Volkseinkommens und der öffentlichen Schuldzinsen, jeweils nach Steuer. Ein guter Teil der privaten Ersparnis wird von der öffentlichen Kreditaufnahme absorbiert; die verbleibende Ersparnis bestimmt die Höhe der privaten Investitionen. Existiert ein langfristiges Gleichgewicht? Oder explodieren die öffentlichen Schuldzinsen? Die theoretische Analyse zeigt, daß Defizitfinanzierung in der Regel langfristig nicht möglich ist.

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