The Role of Uncertainty in a Simple Temporary Equilibrium Model of International Trade with Quantity Rationing under Fixed Exchange Rates*

By Ulrich K. Schittko and B. Eckwert

A two-period model of temporary equilibrium with rationing and international trade under fixed exchange rates is presented, emphasizing the importance of agent's expectations of future prices and constraints. It is shown that several traditional comparative statics results are only compatible with a specific expectational structure. Especially this is the case for the reaction of the trade balance to exogeneous parameter changes.

1. Introduction

The failure of the price system to adjust immediately to its Walrasian equilibrium value gave rise to the formulation of temporary equilibrium models with quantity rationing, starting e.g. with *J. P. Benassy* (1975), *E. Malinvaud* (1977), *W.* and *K. Hildenbrand* (1978), and culminating in the work of *V. Böhm* (1980). If the planning horizon of the economic agents is not confined to one period, then the future overshadows the present in the sense that the agents have to decide now without knowing the prices and wages of tomorrow nor the quantity constraints they will have to face when the future unfolds. The following model of a small open economy tries to take this situation as a starting point in a formulation of a simple model with price expectations (depending on the current prices and wages) and uncertainty concerning future quantity constraints.

Thereby we can complement *A. Dixit's* model (1978) in several respects. First our model contains an explicit intertemporal formulation of the consumers' and producers' optimizing behaviour, especially allowing inventory decisions.

Secondly we examine the influence of a specified expectational pattern concerning future prices and wages as well as possible random

* Financial support of the Deutsche Forschungsgemeinschaft is gratefully acknowledged.

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restrictions in the labour market on the optimizing behaviour and the comparative statics properties of the whole model.

The chosen formulation concerning price and quantity constraint expectations is sufficiently general to capture the main influences of these phenomena. Our results are quite robust against more sophisticated expectational formulations. With regard to price expectations we use a one-point-distribution, meaning that future prices are expected with probability one. Individuals have no rational expectation, so that expectional errors are possible. Quantity expectations are stochastic and modelled by a discrete probability distribution. A generalization of the assumed expectational pattern is not likely to alter our results in a central way.

These enlargements produce several new insights concerning the influence of uncertainty on the properties of the possible short-run equilibria. Most of the individual decisions depend essentially on the expectational parameters. The entrepreneurial behaviour is dichotomized in the sense that the sales- and inventory-decisions are sensitive with respect to expectations, while the productions- and labour demand decisions are not. This is not a consequence of our specific expectational pattern and will be interpreted economically.

Our use of the small country assumption restricts as usual the power of the model, for it excludes some interesting disequilibrium situations, which show up in a two-country setting (compare *Schittko* and *Eckwert* (1981, 82, 83)).

To study the intrinsic dynamics would be too lengthy and is left to a subsequent paper (1982).

2. The basic model

Our economy is a small country which produces its national product by means of the single nontradable factor labour, whose price is fixed in the short run.

For the produced good the world product price is given for the small country, but there are no quantity constraints restricting the goods market decisions of the country. It can very well happen, that the domestic goods market is in disequilibrium, so that the foreign trade absorbs the excess supply or demand. We have then $p_t = \pi\, p_t^*$, $t = 1, 2$, so that prices of the outputs are translated by means of the exchange rate from foreign currency to home currency units. We assume country specific outputs to be completely substitutable in consumption, so that we have in fact a single tradable good. Our model is a two-period one,

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in which the economic agents base their behaviour in period $t = 1$ on the market dates of the present and on their subjectively certain point expectations concerning prices and wages in period $t = 2$ and their random expectations concerning the constraint levels on the labour market in the future.

The use of a two period model does not mean that the economy ends after period 2 but rather, that the agents formulate plans only one period ahead into the future.

The home country has its own money, which is the only asset serving as a store of value.

Consumption and production decisions are described by means of representative decision units, the representative consumer and the representative producer.

2.1. Consumer behaviour

Let us begin with the behaviour of the consumption side. The consumer decisions are the outcome of the maximization of a single, specified utility function which is defined on the present and future consumption possibilities comprising home and imported goods, i.e.

$$u(c_1, M_1, c_2, M_2) = u(x_1, x_2) = x_1 \cdot x_2 \ , \tag{1}$$

where $x_t := c_t + M_t$, $t = 1, 2$, denotes the consumption of the produced good at time t, which consists of consumption of the home produced good c_t and the imported good M_t.

We could have chosen another utility function, say $\psi(u(x_1, x_2)) = \log x_1 + \log x_2$, ψ a strictly monotone transformation. This utility function has the special, but important property, that the marginal utility of consumption in period 2 becomes very large, when the amount of consumption x_2 becomes smaller and smaller. This utility function has an Arrow-Pratt-measure of risk-aversion of one, so that we have risk-neutrality. Concerning the future market dates the consumer has the following point expectations

$$\begin{cases} p_2 = \psi_1(p_1) = ap_1 \\ w_2 = \psi_2(w_1) = bw_1 \ , \end{cases} \tag{2}$$

where p_t is the price level and w_t the wage rate both in period t, $t = 1, 2$. The linear functions ψ_i, $i = 1, 2$, show the way price expectations are formed. If $a > 1$, then the price expectations of the consumer are called inflationary; if $a = 1$, then they are called static expectations, and if $a < 1$, then we have deflationary expectations. Depending on the

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labour market situation in the present, the representative individual expects a rationing on the labour market in the future (period two) with different subjective probabilities. That amounts to that the representative consumer expects with a certain probability not be rationed, respectively to be rationed at a certain level $\bar{l}_2$. l_t, $t = 1, 2$, denotes the fixed labour supply of the consumer.

The situation can be summarized in the following matrix, where R^t denotes a labour market rationing in period t and N^t that the consumer is not rationed on the labour market in period t, $t = 1, 2$.

	R^2	N^2
R^1	$1 - q_2$	q_2
N^1	$1 - q_1$	q_1

q_i denotes the corresponding subjective probabilities. Let us start to derive the optimal consumption decisions. If the consumer is not rationed on the labour market in period one, the employment in period two, L_2, is a discrete random variable whose probability distribution is given by

$$L_2 = \begin{cases} l_2 & \text{with probability } q_1 \\ \bar{l}_2 & \text{with probability } 1 - q_1 \end{cases} \tag{3}$$

Let x_{21} denote the action of the consumer, if he is not rationed on the labour marked in period two, and x_{22} the action he chooses, if he is rationed in that period. Then a random variable X_2 can be defined as

$$X_2 = \begin{cases} x_{21}\ , & \text{if} \quad L_2 = l_2 \\ x_{22}\ , & \text{if} \quad L_2 = \bar{l}_2 \end{cases} \tag{4}$$

X_2 is a discrete random variable with probability distribution q_1, $(1 - q_1)$. Let us now define a transformation by means of

$$X_2' := \frac{X_2 - x_{21}}{x_{22} - x_{21}} \tag{5}$$

which posesses a binomial distribution according to

$$X_2' \sim B\,(1, q_1) \tag{6}$$

$$X_2' = \begin{cases} 0\ , & \text{if} \quad L_2 = l_2\ . \\ 1\ , & \text{if} \quad L_2 = \bar{l}_2\ . \end{cases}$$

From (5) we obtain

$$X_2 = X_2'\,(x_{22} - x_{21}) + x_{21}\ . \tag{7}$$

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As X_2 is a random variable, the expected value of the utility function (1) is a relevant optimality criterion, i.e. our consumer (in case of non-rationing in period one) has to maximize

$$\max_{x_1, x_{21}, x_{22}} \{x_1 \cdot [x_{21} + (x_{22} - x_{21}) Eq_1 (X_2')]\} \tag{8}$$

s. t.

(i) $x_1 \geqq 0, x_{2i} \geqq 0, m_1 \geqq 0, \qquad i = 1, 2$

(ii) $p_1 x_1 + m_1 = m_0 + w_1 l_1$

(iii) $p_2 x_{21} = m_1 + w_2 l_2$

(iv) $p_2 x_{22} = m_1 + w_2 \bar{l}_2$.

To find a solution of problem (8) we use a standard method of dynamic programming, i.e. we first maximize with respect to the second period's decision variables over the constraint set of period two (given an arbitrary but fixed decision in period one). So we have to solve the following maximization problem in case of non-rationing in period one

$$\max_{x_{21}, x_{22}} E(u(x_1, X_2)) = \max_{x_{21}, x_{22}} E(u(x_1, X_2'(x_{22} - x_{21}) + x_{21}) = \tag{9}$$

$$\max_{x_{21}, x_{22}} u(x_1, x_{21} + (x_{22} - x_{21}) Eq_1 (X_2')) =$$

$$\max_{x_{21}, x_{22}} \{x_1 [x_{21} + (x_{22} - x_{21}) Eq_1 (X_2')]\}$$

s. t. (iii), (iv) and the non-negativity conditions for the decision variables.

As a solution we find

$$x_{21} = \frac{m_1 + l_2 w_2}{p_2}$$
$$x_{22} = \frac{m_1 + \bar{l}_2 w_2}{p_2} \quad . \tag{10}$$

As in our simplified set up the labour supply is fixed, the optimal solution (10) can be derived directly from (8) (iii) and (8) (iv).[1]

We have described the procedure of solving (8) in detail to prepare for the more complicated decision problem of the production sector.

We recall, that given the (x_1, m_1)-decision x_{21} denotes the optimal decision in period two, if the consumer is not rationed in this period. Otherwise the optimal decision would be x_{22}. If we substitute the

[1] This was pointed out by the referee.

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optimal solution for period two into (8), we obtain an *indirect utility function.*

$$V(x_1, m_1, p_1, w_1, l_2, \bar{l}_2, q_1, a, b) = q_1 \left(x_1 \cdot \frac{m_1 + bw_1 l_2}{ap_1}\right) + \tag{11}$$
$$+ (1 - q_1)\left(x_1 \cdot \frac{m_1 + bw_1 \bar{l}_2}{ap_1}\right)$$

The indirect utility function (11) has now to be optimized with respect to x_1, m_1, subject to the following period-one restrictions

$$x_1 \geq 0, m_1 \geq 0 \tag{12}$$

$$m_0 + w_1 l_1 = p_1 x_1 + m_1 \ . \tag{13}$$

We assume that $m_1 > 0$, which holds, if

$$q_1 bw_1 l_2 + (1 - q_1) bw_1 l_2 < m_0 + w_1 l_1 \ . \tag{14}$$

By making this assumption, which says that the wealth of period one is greater than the expected labour income of period two, we exclude boundary solutions. For the optimal decisions we then obtain

$$\begin{cases} x_1^* = \dfrac{1}{2p_1}[m_0 + w_1 l_1 + q_1 bw_1 l_2 + (1 - q_1) bw_1 \bar{l}_2] \\ m_1^* = [m_0 + w_1 l_1 - q_1 (bw_1 l_2) - (1 - q_1)(bw_1 \bar{l}_2)] \dfrac{1}{2} \ . \end{cases} \tag{15}$$

The partial derivatives of the optimal decisions with respect to the exogeneous variables are

$$\begin{aligned} &\partial x_1^* / \partial \bar{l}_2 = (1 - q_1) bw_1 / 2p_1 > 0 \ , \\ &\partial x_1^* / \partial m_0 = 1/2p_1 > 0 \ , \\ &\partial x_1^* / \partial q_1 = \frac{bw_1}{2p_1}(l_2 - \bar{l}_2) > 0 \ . \end{aligned} \tag{16}$$

The last inequality e.g. shows, that the reaction of the optimal consumption decision in period one due to a change in the subjective probability to be fully employed in period two, is proportional to the expected unemployment in that period.

Furthermore we deduce

$$\partial x_1^* / \partial a = 0$$

$$\partial x_1^* / \partial b = \frac{1}{2p_1} w_1 (q_1 l_2 + (1 - q_1) \bar{l}_2) > 0$$

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(17)
$$\partial x_1^* / \partial p_1 = -(1/p_1)\, x_1^* < 0$$

$$\frac{\partial x_1^*}{\partial w_1} = \frac{q_1 (l_1 + b l_2) + (1 - q_1)(l_1 + b \bar{l}_2)}{2 p_1} > 0 \,.$$

In (17) only the first partial derivative is surprising. Intertemporal substitution as a consequence of changes in price expectations does not occur because of our chosen utility function.

The sign reaction of m_1^* can be deduced from the budget condition of period one, given the sign reactions of x_1^*. In case of rationing in period one we calculate by a similar procedure the optimal decisions for period one as

(18)
$$x_1^* = \frac{1}{2 p_1} [m_0 + w_1 \bar{l}_1 + q_2 b w_1 l_2 + (1 - q_2) b w_1 \bar{l}_2]$$

$$m_1^* = [m_0 + w_1 \bar{l}_1 - q_2 b w_1 l_2 - (1 - q_2) b w_1 \bar{l}_2) \frac{1}{2} \,,$$

if a similar condition as before in (14) ensures the positivity of m_1^*. The sign reaction to parameter changes can be inferred from (18) like before. It is evident, that the sign reactions are qualitatively similar to the previous ones in case of non-rationing in period one, because also in that case l_1 is not a decision variable.

2.2. Producer behaviour

Let us now describe the production decision of our economy. We assume that the profits of period t are taxed fully by the government. Therefore the representative firm has no initial money balances, but has an endowment ω_0 of the consumption good, which has been stored from the last period. The firm plans to sell y_t units of the good and to buy z_t units of labour ($t = 1, 2$).

With a production function f this input is transformed into output ω_t, $t = 1, 2$, which is instantaneously available, i.e.

(19)
$$\omega_t = f(z_t) = h z_t^{\varrho}, 0 < \varrho < 1, h > 0, t = 1, 2 \,.$$

The product is storable, and similar to the consumption sector the producers anticipate the future market dates by subjective expectations. Concerning future prices and wages we assume that their expectations are identical to those of the consumption side. If the producer is not rationed in period one on the labour market, he expects not to be rationed in period two with probability q_3, (so that his notional demand would be fulfilled) and to be rationed with probability $(1 - q_3)$ at the

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full employment level. That means Z_2 is a random variable with the following distribution:

$$Z_2 = \begin{cases} z_2(p_2, w_2) & \text{with prob. } q_3 \\ l_2 & \text{with prob. } (1 - q_3) \end{cases} \tag{20}$$

In case the producer is rationed in the first period, his expectation concerning the future labour market situation is given by

$$Z_2 = \begin{cases} z_2(p_2, w_2) & \text{with prob. } q_4 \\ l_2 & \text{with prob. } (1 - q_4) \end{cases} . \tag{21}$$

The producer maximizes his expected profit over the planning horizon. To calculate the expected value of the profit function for the case of non-rationing in period one, we transform the random variables Z_2 by means of

$$Z_2' := \frac{Z_2 - l_2}{z_2(p_2, w_2) - l_2} \tag{22}$$

which is distributed according to $B(1, q_3)$, so that we have

$$Z_2' = \begin{cases} 1 , & \text{if } Z_2 = z_2(p_2, w_2) \\ 0 , & \text{if } Z_2 = l_2 . \end{cases} \tag{23}$$

From (22) we obtain

$$Z_2 = Z_2'(z_2(p_2, w_2) - l_2) + l_2 . \tag{24}$$

We know from the production function, that

$$\Omega_2 = hZ_2^0 = \begin{cases} \omega_{21} = hz_2^0 , & \text{if } Z_2 = z_2 \\ \omega_{22} = hl_2^0 , & \text{if } Z_2 = l_2 , \end{cases}$$

so that Ω_2 is also random.

As an accounting restriction we have to consider

$$\begin{aligned} &\omega_0 + \omega_1 - y_1 = i_1 && \text{(period one)} \\ &\Omega_2 + i_1 = Y_2 && \text{(period two)} , \end{aligned} \tag{25}$$

i_1 denoting the storage activity in period one. The future sales are also random, depending on Z_2. This random variable Y_2 can be transformed by

$$Y_2' := \frac{Y_2 - (hl_2^0 + i_1)}{hz_2^0 - hl_2^0} \sim B(1, q_3) , \tag{26}$$

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whereby

$$Y_2' = \begin{cases} 1\ , & \text{if} \quad Z_2 = z_2\,(p_2, w_2) \\ 0\ , & \text{if} \quad Z_2 = l_2\ . \end{cases} \tag{27}$$

Again we have to solve (26) for Y_2.

$$Y_2 = Y_2'\,(hz_2^0 - hl_2^0) + hl_2^0 + i_1 \tag{28}$$

with

$$Y_2 = \begin{cases} y_{21} = hz_2^0 + i_1\ , & \text{if} \quad Z_2 = z_2\,(p_2, w_2) \\ y_{22} = hl_2^0 + i_1\ , & \text{if} \quad Z_2 = l_2\ . \end{cases}$$

The expected profits to be maximized are

$$E\,(\Pi) = p_1\,y_1 - w_1\,z_1 - r\,(w_1\,i_1, w_2\,i_1) + p_2\,E\,(Y_2) - w_2\,E\,(Z_2)\ . \tag{29}$$

This function is maximized with respect to y_1, z_1, i_1 and the possible realisations of the random variables under the technological constraint given by the production function and the accounting restrictions (25). In (29) we have a storage cost function $r\,(w_1\,i_1, w_2\,i_1)$ which we assume to be homogeneous of degree two. That captures the idea that storage costs grow more than proportionally with respect to storage quantity. Therefore we can rewrite the cost functions as

$$r\,(w_1\,i_1, w_2\,i_1) = (i_1)^2 r\,(w_1, w_2)\ . \tag{30}$$

$r\,(w_1, w_2)$ is a continuous and sufficiently often differentiable function, which is assumed to be strictly monotonically increasing in w_1 and w_2. As we know the distribution of Z_2 and Y_2 we can rewrite (29) as

$$\begin{aligned} E\,(\Pi) = {} & p_1\,y_1 - w_1\,z_1 - r\,(w_1, w_2)\,i_1^2 + p_2\,[q_3\,y_{21} + (1 - q_3)\,y_{22}] - \\ & - w_2\,[q_3\,z_2 + (1 - q_3)\,l_2]\ . \end{aligned} \tag{31}$$

To derive an indirect profit function we maximize (31) with respect to z_2, y_{21}, y_{22} under the constraints concerning the second period, namely

(32)

(i) $z_2 \geqq 0, y_{21} \geqq 0, y_{22} \geqq 0$

(ii) $\omega_{21} = hz_2^0$

(iii) $i_1 + \omega_{21} - y_{21} = 0$

(iv) $\omega_{22} \geqq 0, l_2 \geqq 0, y_{22} \geqq 0$

(v) $\omega_{22} = hl_2^0$

(vi) $i_1 + \omega_{22} - y_{22} = 0$.

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We obtain the following solution

$$z_2 = \left(\frac{w_2}{\varrho\, p_2\, h}\right)^{\frac{1}{\varrho - 1}} \tag{33}$$

$$\omega_{21} = h\left(\frac{w_2}{\varrho\, p_2\, h}\right)^{\frac{\varrho}{\varrho - 1}} \tag{34}$$

$$\omega_{22} = h l_2^{\varrho} \tag{35}$$

$$y_{21} = \omega_1 + h\left(\frac{w_2}{\varrho\, p_2\, h}\right)^{\frac{\varrho}{\varrho - 1}} + \omega_0 - y_1 \tag{36}$$

$$y_{22} = \omega_1 + h l_2^{\varrho} + \omega_0 - y_1 \tag{37}$$

With the help of equations (33) - (37) we can derive the *indirect profit function* $\widetilde{\Pi} := \max\limits_{(z_2,\, y_{21},\, y_{22})} E(\Pi)$ as

$$\begin{aligned} \widetilde{\Pi} = {} & p_1 y_1 - w_1 z_1 - r_1 (w_1, w_2)\, i_1^2 + p_2 \Big[q_3 \Big(\omega_1 + h\left(\frac{w_2}{\varrho\, p_2 h}\right)^{\frac{\varrho}{\varrho - 1}} + \\ & + \omega_0 - y_1\Big) + (1 - q_3)(\omega_1 + h l_2^{\varrho} + \omega_0 - y_1)\Big] - w_2 \Big[q_3 \left(\frac{w_2}{\varrho\, p_2 h}\right)^{\frac{1}{\varrho - 1}} + \\ & + (1 - q_3)\, l_2\Big] \,. \end{aligned} \tag{38}$$

Maximizing (38) with respect to the decision variables of the first period z_1, y_1, i_1 subject to

$$z_1 \geqq 0, y_1 \geqq 0, i_1 \geqq 0 \tag{39}$$

$$\omega_1 = h z_1^{\varrho} \tag{40}$$

$$y_1 \leq \omega_0 + \omega_1 \tag{41}$$

gives the optimal production, sales and storage plan for period one. By assuming inflationary expectations i.e. $a > 1$ we ensure $i_1 > 0$ and exclude by that boundary solutions of the relevant problem.

If we had assumed linear storage costs or had abstracted from them at all, our optimization problem would have led to boundary solutions or to non-uniqueness of the sales and inventory decisions.

The solutions of the maximization problem are then

$$z_1 = \left(\frac{w_1}{p_1\, \varrho\, h}\right)^{\frac{1}{\varrho - 1}} \tag{42}$$

$$\omega_1 = h\left(\frac{w_1}{p_1\, \varrho\, h}\right)^{\frac{\varrho}{\varrho - 1}} \tag{43}$$

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$$y_1 = h\left(\frac{w_1}{p_1 \varrho h}\right)^{\frac{\varrho}{\varrho - 1}} + \frac{p_1 (1 - a)}{2 r (w_1, b)} + \omega_0 \tag{44}$$

$$i_1 = - \frac{p_1 (1 - a)}{2 r (w_1, b)} \; . \tag{45}$$

One can recognize from (45) that the optimal inventory decision depends only on the difference of current and expected future prices and the storage costs.

For the partial derivatives of the optimal decisions z_1, y_1, i_1 with respect to the exogeneous variables we calculate first for the labour demand

$$\partial z_1 / \partial l_2 = \partial z_1 / \partial q_3 = \partial z_1 / \partial a = \partial z_1 / \partial b = 0 \; . \tag{46}$$

In (42) we see that the current labour demand is insensitive with respect to the price expectations. This is due to the fact that production does not take time but is nevertheless surprising at first sight, since one would expect a positive labour demand impact in reaction to an increase in the price expectation. If we define a kind of marginal storage revenue as the difference of the price expectation and the marginal storage costs, we recognize that the marginal storage revenue decreases with the storage size. Note further that marginal revenue in period one is constant and equal to p_1. The optimal behaviour of the producer implies an equality of marginal storage revenue and p_1. This can be seen by an arbitrage argument, i.e. intertemporal profits can be increased solely by changing sales at the expense of the storage level. As an optimal production plan implies the equality of marginal revenue and marginal costs and we know that marginal revenue out of sales and out of storage activity is the same in the optimal solution and equal to p_1, it is clear that the entrepreneurial decision is only affected by the price of period one.

To put it more precisely: Assume that a producer would increase his production and his storage by one unit because of a p_2-rise. Then this additional unit of production costs him more than p_1, because the marginal production costs were equal to p_1 in the producer equilibrium before the p_2-rise. Thus the producer has increased his storage by one unit at the cost of more than p_1 money units. If he alternatively would have reduced his current sales by one unit and have added this unit to his storage, then the costs of the additional storage unit would have been p_1 (the reduction of current sales revenues). Therefore it is sub-optimal for him to increase his storage by additional production.

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Furthermore we get

$$\frac{\partial z_1}{\partial w_1} = \frac{1}{p_1 \varrho h (\varrho - 1)} \left(\frac{w_1}{p_1 \varrho h} \right)^{\frac{2-\varrho}{\varrho - 1}} = \tag{47}$$

$$= \frac{1}{(\varrho - 1) w_1} \left(\frac{w_1}{p_1 \varrho h} \right)^{\frac{1}{\varrho - 1}} < 0$$

$$\frac{\partial z_1}{\partial p_1} = -\left(\frac{1}{\varrho - 1} \right) \left(\frac{w_1}{p_1 \varrho h} \right)^{\frac{2-\varrho}{\varrho - 1}} \frac{w_1}{\varrho h (p_1)^2} = \tag{48}$$

$$= \frac{1}{-(\varrho - 1) p_1} \left(\frac{w_1}{p_1 \varrho h} \right)^{\left(\frac{1}{\varrho - 1}\right)} > 0 ,$$

which show classical labour market reactions.

$$\frac{\partial y_1}{\partial l_1} = \frac{\partial y_1}{\partial q_3} = 0 , \tag{49}$$

$$\frac{\partial y_1}{\partial a} - \frac{p_1}{2 r (w_1, b)} < 0 , \quad \frac{\partial y_1}{\partial b} = - \frac{\pi p_1^* (1 - a) \partial r / \partial b}{2 r^2} > 0 . \tag{50}$$

Since the production in contrast to storage activity is not affected by a change of a, we have via $\omega_0 + \omega_1 - y_1 = i_1$ an influence on the planned sales in period one.

$$\partial y_1 / \partial w_1 = \frac{1}{p_1 (\varrho - 1)} \left(\frac{w}{p_1 \varrho h} \right)^{\left(\frac{1}{\varrho - 1}\right)} - \tag{51}$$

$$- \frac{p_1 (1 - a) \partial r / \partial w_1}{[2 r (w_1, b)]^2} = \frac{\partial \omega_1}{\partial w_1} - \frac{\partial i_1}{\partial \omega_1} \lesseqgtr 0 .$$

The sales effect of a wage change can be decomposed into a production and inventory effect. If the production effect dominates, we end up with a negative sign.

Furthermore we have

$$\frac{\partial y_1}{\partial p_1} = - \frac{w_1}{p_1^2 (\varrho - 1)} \left(\frac{w_1}{p_1 \varrho h} \right)^{\left(\frac{1}{\varrho - 1}\right)} + \tag{52}$$

$$+ \frac{1 - a}{2 r (w_1, b)} = \frac{\partial w_1}{\partial p_1} - \frac{\partial i_1}{\partial p_1} > 0 .$$

For the sign specification we used the assumption that the production effect dominates the inventory effect. In general we can state, because $\omega_0 + \omega_1 = i_1$ holds, that

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$$\frac{\partial i_1}{\partial \vartheta} = -\frac{\partial y_1}{\partial \vartheta} + \frac{\partial \omega_1}{\partial \vartheta} \tag{53}$$

for $\vartheta = l_2, q_3, b, a, w_1, p_1$.

In case the labour demand is rationed in the first period at the level $\hat{l}_1$ the indirect profit function is

$$\begin{aligned} \tilde{\Pi} = {} & p_1 y_1 - w_1 \hat{l}_1 - r_1 (w_1, b)\, i_1^2 + p_2 \Big[q_4 \Big(\omega_1 + h \Big(\frac{w_2}{\varrho\, p_2\, h}\Big)^{\frac{\varrho}{\varrho - 1}} + \\ & + \omega_0 - y_1\Big) + (1 - q_4)(\omega_1 + h l_2^{\varrho} + \omega_0 - y_1)\Big] - w_2 \Big[q_4 \Big(\frac{w_2}{\varrho\, p_2\, h}\Big)^{\frac{1}{\varrho - 1}} \\ & + (1 - q_4)\, l_2\Big] \end{aligned} \tag{54}$$

Maximizing (54) subject to the relevant conditions leads to

$$\omega_1 = h \hat{l}_1^{\varrho} \tag{55}$$

$$y_1 = \frac{p_1 (1 - a)}{2\, r (w_1, b)} + h \hat{l}_1^{\varrho} + \omega_0 \tag{56}$$

$$i_1 = -\frac{p_1 (1 - a)}{2\, r (w_1, b)}\ . \tag{57}$$

For the partial derivatives we derive

$$\begin{cases} \partial y_1 / \partial l_2 = \partial y_1 / \partial q_4 = 0 \\ \partial y_1 / \partial a = -\, p_1 / 2\, r (w_1, b) < 0\ ; \quad \partial y_1 / \partial b = -\dfrac{\pi\, p_1^{*} (1 - a)\, \partial r / \partial b}{[r (w_1, b)]^2 \cdot 2} > 0\ , \\ \partial y_1 / \partial w_1 = -\dfrac{p_1 (1 - a)}{2} \left(\dfrac{\partial r / \partial w_1}{[r (w_1, b)]^2}\right) > 0 \\ \partial y_1 / \partial p_1 = \dfrac{1 - a}{2\, r (w_1, b)} < 0 \\ \partial y_1 / \partial \hat{l}_1 = \varrho\, h \hat{l}_1^{(\varrho - 1)} > 0\ . \end{cases} \tag{58}$$

From (58) on can see that sales do not react with respect to changes of l_2 and q_4. The sales decision is optimal, if the equality of the current price p_1 and the future price p_2 minus marginal storage costs holds. This marginal condition does not depend on l_2 and q_4. An expected price increase in period two increases the rentability of storage, so that the storage activity is stimulated at the expense of current sales plans. Current and expected future wage increases have a negative effect on the

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rentability of storage, favouring current sales. A current price increase lowers sales, because the expected future price increase dominates the current one ($a > 1$) so that storage becomes more profitable. Finally a loosening of the labour demand constraint increases production and sales plans.

Furthermore we recognize

$$\partial i_1/\partial \vartheta = -\frac{\partial y_1}{\partial \vartheta} + \frac{\partial \omega_1}{\partial \vartheta}\ ; \quad \vartheta = l_2, q_4, b, a, w_1, p_1, \hat{l}_1\ . \tag{59}$$

2.3. The government sector

The government sector is modelled quite simply. The government collects the total profits of the production sector and creates money (if necessary) to finance its parametrically given expenditures $p_1\, g$. Thereby the money supply becomes endogeneous. The special profit distribution assumption in this model has, of course, consequences for the comparative static results.

3. Equilibria of the Model

Our model consists of three active markets, the goods market, the labour market and the money market. The latter will be dropped in the following because of Walras Law. Since only the labour market can be in disequilibrium at the fixed price vector (p_1, w_1), two disequilibria are possible. The *unemployment equilibrium* is described by

$$\begin{cases} z_1\,(p_1^*\,\pi, w_1) - \bar{l}_1 = 0 \qquad (\bar{l}_1 < l_1) \\[2ex] \dfrac{p_1}{\pi}\, y_1\,(p_1^*\,\pi, w_1, \omega_0, a, b) - \dfrac{p_1}{\pi}\, x_1\,(p_1^*\,\pi, w_1, m_0, q_2, b; \bar{l}_1, \bar{l}_2, l_2) - \\[2ex] \qquad\qquad - \dfrac{p_1}{\pi}\, g - HB = 0 \end{cases} \tag{60}$$

where we have used $p_1 = \pi\, p_1^*$. HB is then the trade balance surplus in foreign currency and $\bar{l}_1$ denotes the labour supply constraint. The equation system (61) now characterizes an *overemployment equilibrium*

$$\begin{cases} -\hat{l}_1 + l_1 = 0 \\[2ex] \dfrac{p_1}{\pi}(y_1\,(p_1^*\,\pi, w_1, \omega_0, a, b; \hat{l}_1)) - \dfrac{p_1}{\pi} x_1\,(p_1^*\,\pi, w_1, m_0, q_1, b; l_1, \bar{l}_2, l_2) = \\[2ex] \qquad\qquad - \dfrac{p_1}{\pi}\, g - HB = 0\ , \end{cases} \tag{61}$$

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with $\hat{l}_1$ denoting the labour demand constraint. It is easy to show the existence and local stability of these two disequilibria.

Let us now give an effective classification of the equilibria of the model in (p_1, w_1) — space. This classification is important, because it enables us to assign certain parameter constellations to the different types of disequilibria. Furthermore the classification increases our intuitive understanding of the model. We start to derive the slope of the labour market equilibrium curve in (p_1, w_1) — space. An equilibrium in the labour market is described by

$$z_1(p_1, w_1) = \left(\frac{w_1}{p_1 \varrho h}\right)^{\frac{1}{\varrho - 1}} \overset{!}{=} l_1 \,. \tag{62}$$

The slope of this curve is given by

$$\left.\frac{\partial w_1}{\partial p_1}\right|_{z_1(p_1, w_1) = l_1} = \frac{w_1}{p_1} \tag{63}$$

Therefore we can illustrate the equilibrium locus as in fig. 1.

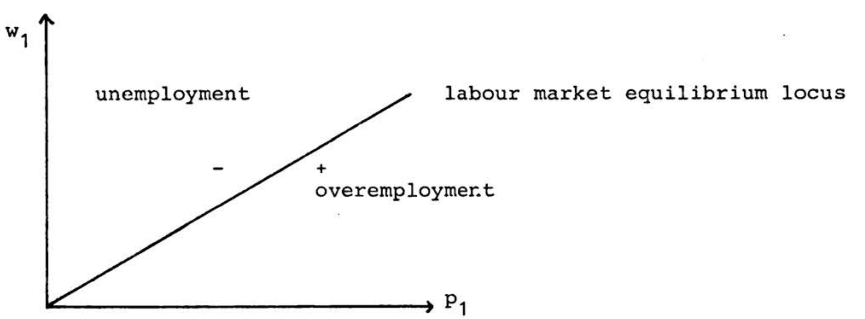

fig. 1

The trade balance in case of unemployment (the effective trade balance) in period one is defined as

$$\begin{aligned} HB &= \frac{p_1}{\pi} y_1^* (p_1 \pi_1, w_1, \omega_0, a, b) - \frac{p_1}{\pi} x_1 (p_1^* \pi, w_1, m_0, q_2, b; \bar{l}_1, \bar{l}_2, l_2) - \\ &- \frac{p_1}{\pi} g = \frac{p_1}{\pi} \left[\frac{p_1 (1 - a)}{2 r (w_1, b)} + h \left(\frac{w_1}{p_1 \varrho h} \right)^{\left(\frac{\varrho}{\varrho - 1}\right)} + \omega_0 - \frac{1}{2 p_1} \cdot \right. \\ &\left. \cdot \left[m_0 + w_1 \left(\frac{w_1}{p_1 \varrho h} \right)^{\left(\frac{1}{\varrho - 1}\right)} + q_2 (w_1 b l_2) + (1 - q_2)(w_1 b \bar{l}_2) \right] - g \right] . \end{aligned} \tag{64}$$

From (64) we calculate

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$$\frac{\partial HB}{\partial p_1} = \frac{1}{\pi}\left[\underbrace{\frac{p_1(1-a)}{r(w_1, b)}}_{(-)} + \underbrace{\left(\frac{w_1}{p_1 \varrho h}\right)^{\left(\frac{\varrho}{\varrho - 1}\right)}\left(\frac{h(\varrho - 2)}{2(\varrho - 1)}\right)}_{(+)}\right] \tag{65}$$

and

$$\frac{\partial HB}{\partial w_1} = \frac{1}{\pi}\left[\underbrace{-\frac{(p_1)^2(1-a)\frac{\partial r}{\partial w_1}}{(2r(w_1, b))^2}}_{(-)} + \underbrace{\left(\frac{w_1}{p_1 \varrho h}\right)^{\left(\frac{1}{\varrho - 1}\right)}\left(\frac{2-\varrho}{2p_1(\varrho - 1)}\right)}_{} - \underbrace{\frac{b}{2p_1}(q_2 l_2 + (1-q_2)\bar{l}_2)}_{(+)}\right] \tag{66}$$

The first term in (65) and (66) represents the storage effects of the parameter variation, while the second term in (65), respectively, the second and third term in (66) represent the effects on the excess goods supply in our economy.

Notice first, that the storage effects $\frac{p_1(1-a)}{r(w_1, b)}$ and $-\frac{(p_1)^2(1-a)\frac{\partial r}{\partial w_1}}{(2r(w_1, b))^2}$ of (65) and (66) are always of opposite signs. The slope of the $HB = 0$-curve in (w_1, p_1)-space, given by $\left.\frac{dw_1}{dp_1}\right|_{HB=0} = -\frac{\partial HB/\partial p_1}{\partial HB/\partial w_1}$, is therefore positive, if we either assume *both* storage effects to be dominant or weak.

So we can specify

$$\left.\frac{\partial p_1}{\partial w_1}\right|_{HB=0}^{U} > 0 \tag{67}$$

in the unemployment region normally.

Note that for special values of the expectational parameters the slope of the $HB = 0$-curve in the unemployment region could also be negative. This can be summarized in the following picture (fig. 2).

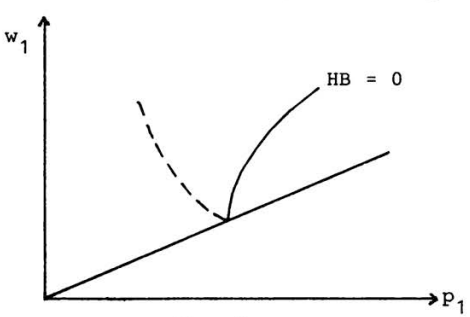

fig. 2

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When the economy faces overemployment the trade balance is given by

$$(68)\quad HB = \frac{p_1}{\pi}\,(y_1\,(p_1^*\,\pi, w_1, \omega_0, a, b; l_1)) - \frac{p_1}{\pi}\,x_1\,(p_1^*\,\pi, w_1, m_0, q_1, b; l_1, \bar{l}_2, l_2) - \frac{p_1}{\pi}\,g\,.$$

$$= \frac{p_1}{\pi}\left[\frac{p_1\,(1-a)}{2\,r\,(w_1, b)} + hl_1^o + \omega_0 - \frac{1}{2\,p_1}\,(m_0 + w_1\,l_1 + q_1\,(bw_1\,l_2) + (1-q_1)\,(bw_1\,\bar{l}_2)) - g\right].$$

For the partial derivatives we derive

$$(69)\qquad \frac{\partial\,HB}{\partial\,p_1} = \frac{1}{\pi}\left[\underbrace{\frac{p_1\,(1-a)}{r\,(w_1, b)}}_{(-)} + \underbrace{hl_1^o + \omega_0 - g}_{(+)}\right] \lesseqgtr 0$$

$$(70)\qquad \frac{\partial\,HB}{\partial\,w_1} = \frac{p_1}{\pi}\left[\underbrace{-\frac{p_1\,(1-a)\,\partial\,r/\partial\,w_1}{2\,(r\,(w_1, b))^2}}_{(+)} - \underbrace{\frac{1}{2\,p_1}\,(l_1 + q_1\,bl_2 + (1-q_1)\,b\bar{l}_2)}_{(-)}\right] \lesseqgtr 0$$

If we assume the storage effects in (69) and (70) both to be either dominant or nondominant, we get opposite signs for (69) and for (70). We can conclude, that

$$(71)\qquad \left.\frac{\partial\,w_1}{\partial\,p_1}\right|_{HB\,=\,0} = -\,\frac{\partial\,HB/\partial\,p_1}{\partial\,HB/\partial\,w_1} > 0\,,$$

which is illustrated in fig. 3.

Note that for special values of the expectational parameters the slope of the trade balance equilibrium curve in the region of overemployment can be negative as illustrated by the broken line in fig. 3.

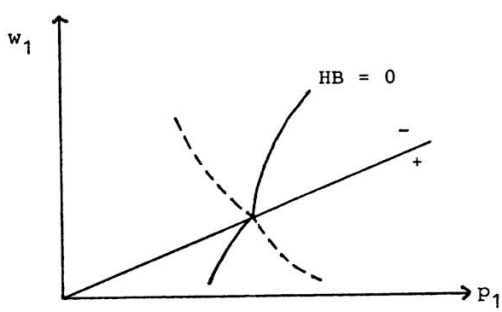

fig. 3

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4. Comparative Statics

4.1. Unemployment equilibrium

First we study the influence of parameter changes on the endogeneous variables $\bar{l}_1$ and HB, when the economy is not fully employed. The equation system is given by

$$(72)\qquad \begin{cases} K_1 := \bar{l}_1 - z_1(\pi p_1^*, w_1) = 0 \\ K_2 := p_1^*[y_1(p_1^*\pi, w_1, \omega_0, a, b) - x_1(p_1^*\pi, w_1, m_0, q_2, b; l_1, \bar{l}_2, l_2) - \\ \qquad\qquad - g] - HB = 0 \end{cases}$$

The implicit function theorem gives us the effects of a parameter change on the endogeneous variables $\bar{l}_1$, HB in a neighbourhood of the equilibrium solution. μ denotes a special parameter of interest:

$$(73)\qquad \begin{bmatrix} \dfrac{\partial \bar{l}_1}{\partial \mu} \\ \\ \dfrac{\partial HB}{\partial \mu} \end{bmatrix} = -\frac{1}{D} \begin{bmatrix} \dfrac{\partial K_2}{\partial HB}\dfrac{\partial K_1}{\partial \mu} - \dfrac{\partial K_1}{\partial HB}\dfrac{\partial K_2}{\partial \mu} \\ \\ -\dfrac{\partial K_2}{\partial \bar{l}_1}\dfrac{\partial K_1}{\partial \mu} + \dfrac{\partial K_1}{\partial l_1}\dfrac{\partial K_2}{\partial \mu} \end{bmatrix},$$

whereby

$$D := \frac{\partial K_1}{\partial \bar{l}_1}\frac{\partial K_2}{\partial HB} - \frac{\partial K_1}{\partial HB}\frac{\partial K_2}{\partial \bar{l}_1} = -1 < 0\ .$$

For a change in government expenditures we derive by means of (73)

$$(74)\qquad \partial \bar{l}_1/\partial g = 0\ ;\quad \partial HB/\partial g = -p_1^* < 0\ .$$

In a system of fixed exchange rates a change of government expenditures has no influence on the employment level. Since the goods market is always equilibrated, there exists no transmission mechanism from the goods market to the labour market. The negative reaction of the trade balance to an increase of g is obvious. Wage rate policy results in

$$(75)\qquad \partial \bar{l}_1/\partial w_1 = \partial z_1/\partial w_1 < 0$$

and

$$(76)\qquad \begin{aligned} \partial HB/\partial w_1 &= \left[-\frac{\partial x_1}{\partial \bar{l}_1}\frac{\partial z_1}{\partial w_1} + \frac{p_1^*}{\pi}\left(\frac{\partial y_1}{\partial w_1} - \frac{\partial x_1}{\partial w_1}\right)\right] \\ &= \left[\underbrace{-\frac{w_1}{2p_1}\underset{(-)}{\frac{\partial z_1}{\partial w_1}}}_{(+)} + \frac{p_1^*}{\pi}\left(\frac{\partial y_1}{\partial w_1} - \underset{(+)}{\frac{\partial x_1}{\partial w_1}}\right)\right] \gtrless 0 \end{aligned}$$

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In (76) we have $\partial y_1/\partial w_1 < 0$ for high price expectations. If the goods market effects are dominant, we end up with a negative sign. A variation of the fixed labour supply leads to

$$\partial \bar{l}_1/\partial l_1 = 0 \ , \quad \partial HB/\partial l_1 = -\frac{q_2 w_1 b}{2} < 0 \ . \tag{77}$$

The employment level $\bar{l}_1$ is independent of the aggregate labour supply l_1. Since unemployment can be measured by

$$U := l_1 - \bar{l}_1 \ , \tag{78}$$

we can derive the effect of labour supply variations on the excess labour supply as $\partial U/\partial l_1 = 1$. As is intuitively clear, a reduction of the aggregate labour supply causes a reduction of unemployment. The trade balance reaction is at first sight surprising, since the employment level in period one is determined by the demand side. But note that a reduction of the labour supply reduces the expected income of period two, because the consumer expects with probability q_2 not to be rationed in that period. So the expected loss of income in the future has a consumption demand effect in period one. We further deduce the effects of an exchange rate policy

$$\partial \bar{l}_1/\partial \pi = \partial z_1/\partial \pi > 0 \ . \tag{79}$$

$$\frac{\partial HB}{\partial \pi} = \left(-\frac{w_1}{2\pi}\frac{\partial z_1}{\partial \pi} + p_1^* \frac{\partial y_1}{\partial \pi} - \frac{1}{\pi^2} p_1 x_1\right). \tag{80}$$

A devaluation has positive labour market effects. The trade balance reaction depends on two real and a kind of monetary effect.

1. The real consumption demand increases (labour market effect). 2. The real goods supply increases. 3. The consumption demand in foreign currency decreases nominally. The trade balance reacts positively if the labour market induced consumption effect is dominated by the two remaining effects. Depending on the price expectations an opposite sign specification of (80) could be reasonable. Finally we examine the influence of changes of the expectational parameters a, b, q_2, q_3 on the unemployment equilibrium of our model.

$$\partial \bar{l}_1/\partial a = 0 \ , \quad \partial \bar{l}_1/\partial b = 0 \ , \quad \partial \bar{l}_1/\partial q_2 = \partial l_1/\partial q_3 = 0 \tag{81}$$

$$\frac{\partial HB}{\partial a} = p_1^* \frac{\partial y_1}{\partial a} < 0 \tag{82}$$

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$$\frac{\partial HB}{\partial b} = \frac{p_1}{\pi}\frac{\partial y_1}{\partial b} - \frac{1}{2\pi}(q_2 w_1 l_2 + (1-q_2) w_1 \bar{l}_2) \tag{83}$$

$$= \underbrace{-\frac{1}{2\pi}\frac{p_1^2 (1-a)\,\partial r/\partial b}{r^2}}_{(+)} - \underbrace{-\frac{1}{2\pi}(q_2 w_2 l_2 + (1-q_2) w_1 \bar{l}_2)}_{(-)} \lessgtr 0$$

(81) tells us that expectational variations do not affect the employment level of our economy. In our regarded disequilibrium the employment level is according to (72) demand determined. The labour demand however does not depend on expectations. Increases in price expectations lower current sales plans und worsen thereby the trade balance (compare (82).

Higher wage expectations increase storage costs and thereby current sales plans as well as total current consumption.

If the positive storage effect is dominated by the consumption effect, then we have a negative sign in (83).

$$\partial HB/\partial q_2 = -\frac{1}{2\pi} b w_1 (l_2 - \bar{l}_2) < 0\ , \quad \partial HB/\partial q_3 = 0\ . \tag{84}$$

The expectational parameters have no influence on the entrepreneurial labour demand decisions, so that the level of employment does not vary when expectations change.

This is due to the fact that in our model production does not take time. The trade balance however depends sensitively on the price-, wage-, and constraint expectations.

4.2. Overemployment equilibrium

In this type of equilibrium which is described by (85) the employment level is given by the fixed labour supply l_1.

$$\begin{aligned} K_3 &:= -\hat{l}_1 + l_1 = 0 \\ K_4 &:= p_1^* [y_1 (p_1^* \pi, w_1, \omega_0, a, b; \hat{l}_1) - x_1 (p_1^* \pi, w_1, m_0, q_1, b; l_1, \bar{l}_2, l_2) - \\ &\qquad - g] - HB = 0\ . \end{aligned} \tag{85}$$

As endogeneous variables we have $\hat{l}_1$ and HB. The comparative statics properties can be obtained like before and are summarized as follows.

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For a change in government demand we have

$$\partial \hat{l}_1/\partial g = 0 \ , \quad \partial HB/\partial g = -p_1^* < 0 \ . \tag{86}$$

Wage rate policy results in

$$\begin{aligned} &\partial \hat{l}_1/\partial w_1 = 0 \\ &\partial HB/\partial w_1 = p_1^* \frac{\partial y_1}{\partial w_1} - \left(\frac{1}{2\pi}\,[l_1 + q_1 bl_2 + (1 - q_1)\, b\bar{l}_2]\right) \\ &= \underbrace{-\frac{p_1^2}{\pi}\,\frac{(1-a)\,\partial r/\partial w_1}{2r^2}}_{(+)} \underbrace{-\left(\frac{1}{2\pi}\,[l_1 + q_1 bl_2 + (1 - q_1)\, b\bar{l}_2]\right)}_{(-)} \gtrless 0 \end{aligned} \tag{87}$$

$$\frac{dD_L}{dw_1} = \left(\frac{w_1}{\pi\, p_1^* \varrho\, h}\right)^{\left(\frac{2-\varrho}{\varrho-1}\right)} \frac{1}{\pi\, p_1^* \varrho\, h\,(\varrho - 1)} < 0 \ , \tag{88}$$

D_L denoting excess labour demand.

These multipliers lend themselves to a completely analogous interpretation as the preceeding ones.

A change in l_1 gives

$$\begin{aligned} &\partial \hat{l}_1/\partial l_1 = 1 \ ; \quad \partial HB/\partial l_1 = p_1^* \frac{\partial y_1}{\partial \hat{l}_1} - \\ &\qquad - \frac{1}{2\pi}\,[w_1 (1 + q_1 bw_1)] \gtrless 0 \ . \end{aligned} \tag{89}$$

An exogenous increase in the labour supply stimulates sales plans as well as consumption demand. The net effect depends on the relative strength as shown in (89).

Exchange rate policy leads to

$$\partial \hat{l}_1/\partial \pi = 0 \ ; \quad \partial HB/\partial \pi = p_1^* \frac{\partial y_1}{\partial \pi} + \frac{p_1^*}{\pi} x_1 \gtrless 0 \tag{90}$$

where the sign of the trade balance reaction depends on the price expectation.

The higher the price expectation, the more likely a negative trade balance reaction will follow. This atypical result underlines the importance of expectations.

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$$\partial D_L/\partial \pi = \frac{\partial}{\partial \pi}\left[\left(\frac{w_1}{\pi p_1^* \varrho h}\right)^{\frac{1}{\varrho - 1}} - l_1\right] > 0 \ . \tag{91}$$

For the influence of a change in the expectational parameters we calculate

$$\partial \hat{l}_1/\partial a = \partial \hat{l}_1/\partial b = \partial \hat{l}_1/\partial q_1 = 0 \tag{92}$$

$$\partial HB/\partial a = p_1^* \frac{\partial y_1}{\partial a} < 0 \tag{93}$$

$$\frac{\partial HB}{\partial b} = \left[p_1^*\left(\frac{\partial y_1}{\partial b} - \frac{\partial x_1}{\partial b}\right)\right] = p_1^*\left[\underbrace{\frac{-\pi p_1^*(1-a)\,\partial r/\partial b}{2r^2}}_{(+)} - \frac{\partial x_1}{\partial b}\right] \gtrless 0 \tag{94}$$

$$\frac{\partial HB}{\partial q_1} = -\frac{1}{2\pi} w_1 b (l_2 - \bar{l}_2) < 0 \ . \tag{95}$$

We recognize that the reaction of the trade balance essentially depends on the expectational parameters, while the labour market situation is not influenced by expectations. In a model with an explicit temporal production structure, which we will present in the future, of course, this conclusion does not hold.

5. Concluding Remarks

Though our model has a dramatically simplified structure, it became evident that expectations play a significant role in classifying the effective equilibria and for the results of comparative statics. As a consequence of our specification of the production process mainly price expectations are responsible for the qualitatively different results. Since the production and labour demand decisions do not depend on expectational parameters in our model, the influence of these parameters shows up in the trade balance only via the goods demand decisions. Even very traditional results concerning the effectiveness of a devaluation can be upset by our simple expectational structure. Wage expectations would become more decisive, if the production process is specified differently. We saw that contrary to Dixits' results even in the case of fixed exchange rates the trade balance shows different reactions depending on the kind of expectations. The expectations concerning the labour market constraints would also play a more distinctive role, if we would admit goods market rationing. On this question work is in progress.

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Summary

In a two period model of temporary equilibrium with quantity rationing and international trade under fixed exchange rates expectations concerning future prices and constraints play a significant role in classifying the effective equilibria and for the results of comparative statics. If the production sector can hold inventories (as in our model), the expectational structure influences significantly the sales but not the production and labor demand decisions. This surprising result depends on the way the production process is modelled, revealing the role of an atemporally formulated production structure.

Zusammenfassung

In einem temporären Gleichgewichtsmodell einer offenen Volkswirtschaft mit Mengenrationierung (bei festen Wechselkursen) spielen Erwartungen bezüglich zukünftiger Preise und Mengenschranken bei der Effektivklassifikation der Gleichgewichte und für die Resultate der komparativen Statik eine wichtige Rolle. Wenn für den Produktionssektor Lagerhaltung zugelassen wird (wie in unserem Modell), beeinflußt die Erwartungsstruktur signifikant die Verkaufs- aber nicht die Produktions- und Arbeitsnachfrageentscheidung. Dieses überraschende Resultat hängt von der Art der Modellierung des Produktionsprozesses ab und offenbart die Rolle einer atemporal formulierten Produktionsstruktur.

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DOI https://doi.org/10.3790/schm.103.5.461 | Generated on 2025-11-17 05:30:59
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