A Note on Activity-Level and Uncertainty

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As it is known from a number of studies which were published since 1970 the optimal activity-level of risk-averse decision-makers may differ under uncertainty from the one under certainty. After giving a short review of articles dealing with that problem this paper presents and discusses a simple model from which basic rules about the activity-level under uncertainty in comparison to the one under certainty can be derived. Two examples, the output decision of a competitive firm and the labor supply decision of an individual, are given to illustrate the results. The model presented in this paper may also be of some interest for pedagogic and didactic purposes because of its simplicity.

I. Introduction

Since the already classic work of *Agnar Sandmo* (1971), a great number of articles have appeared comparing the behavior of firms under uncertainty and certainty. *Sandmo* showed that a risk-averse price-taking single-product firm produces less under price-uncertainty than under certainty, if the certain price is equal to the mean expected price under uncertainty. *Leland* (1972) derived an analogoues result for a monopoly firm, if demand is stochastic. But not only price-uncertainty was of interest but also the effects of uncertain fixed cost, input wages etc. First analysis of this kind was delivered by *Sandmo* (1971) and *Batra* and *Ullah* (1974), and later for the case of labor-managed firms by *Paroush* and *Kahana* (1980).[1]

Further modifications and extensions were made to test whether *Sandmo's* result is still valid under different model structures:

— The multi-product firm is "of particular interest under uncertainty since the firm is able to spread its risks by output diversifications".[2]

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[1] see the survey by *Hey* (1981).

[2] *Sandmo* (1971), 72.

Neumann (1982) showed recently, that a multi-product firm might expand under price-uncertainty the output of some of its products until marginal cost exeeds expected price. The negative expected return for these products will be compensated by the benefit of a risk-pooling effect of producing these products.

— A series of articles written by *Zabel* (1971) analyses the optimal production-, inventory-, and sales-strategies under uncertainty in a dynamic multi-period framework. The same was discussed in a paper by *Hawawini* (1982) who compared the cases of price-uncertainty and price-flexibility over time.

— As mentioned above, a recent interest is the behavior of labor-managed firms under uncertainty. *Paroush* and *Kahana* (1980) as well as *Hawawini* (1982) showed that the labour-demand of labour-managed firms, focussing on expected profit per labor-unit, differ in some cases under uncertainty from the labor-demand of owner-managed firms.

— The *Sandmo* question was also asked for the labor-supply decision of individuals.[3]

Most papers in this field have the same analytical question: In which way does the optimal activity-level (e. g. output or labor-demand of a firm, labor-supply of an individual) differ under uncertainty from the one under certaintly? One might intuitively conclude from Sandmo's analysis that the activity-level of risk-averse decision-makers is lower under uncertainty. However, this is not true in every case as we will show below.

In this paper we will not add a further example of a specific uncertainty situation of a firm or an individual nor discuss its special features. Instead, we will develop a basic model of economic behavior under uncertainty which is both, as simple as possible, and able to derive some important general criterias about the optimal activity-level of decision-makers under uncertainty compared to the one under certainty. To link the results of this basic model to the cases discussed in articles mentioned above, we will illustrate them by two common examples: the output decision of a competitive firm and the labor-supply decision of an individual. Furthermore, the criterias derived from the presented model lead to interesting statements about the incentive effects of lump-sum taxes and transfers which are conditional to certain states of nature. Following these interpretations of the basic model, its limits and assumptions are discussed with reference to other approaches presented in literature.

[3] cf. *Block / Heineke* (1973).

II. The Basic Model of Economic Behavior

Most static models of economic behavior can be condensed to the problem:

$$\max \pi(x) \quad \text{s. t. } x > 0 \tag{1}$$

whereby π denotes the outcome, e. g. profit or utility, which the decisionmaker seeks to maximize, and x his or her activity level, e. g. output or labor supply. The activity level x^* which maximizes the outcome must clearly satisfy the first and second order conditions for a maximum:

$$\pi'(x^*) = 0 \tag{2}$$

$$\pi''(x^*) < 0 \tag{3}$$

To introduce uncertainty in this decision-model in the most simple way, we assume that the outcome function is different in two states of nature, so that for an activity level x the outcome will be $\pi_1(x)$ in the first and $\pi_2(x)$ in the second state. The probability that the first state occurs is q_1, that the second state occurs is q_2 $(0 < [q_2 = 1 - q_1] < 1)$. Assuming that the decisionmaker is risk-averse, and seeks to maximize expected utility, we reformulate our problem to:

$$\max \sum_{s=1}^{2} q_s u[\pi_s(x)] \quad s = 1, 2; u' > 0 > u'' \quad \text{s. t. } x > 0 \tag{4}$$

As necessary and sufficient conditions for a maximum we obtain:

$$q_1 \frac{u_1'}{u_2'} \pi_1'(x^0) + q_2 \pi_2'(x^0) = 0 \tag{5}$$

$$\sum_{s=1}^{2} q_s [u_s' \pi_s''(x^0) + u_s'' \pi_s'(x^0)] < 0 \tag{6}$$

Inequality (6) indicates that the marginal expected utility given by (5) has to be a decreasing function in the activity level. Equation (5) requires that if the marginal outcome $\pi_1'(x^0)$ is positive, then $\pi_2'(x^0)$ must be negative, and vice versa. This follows from the assumption that marginal utility and the probabilities will always be positive. From the shape of the utility function we know that if $\pi_1(x^0)$ is greater than $\pi_2(x^0)$ the fraction of the marginal utilities in (5) must be less than one. Therefore we may distinguish nine cases listed in the matrix below. Assuming that expected outcome

$$\Sigma q_s \pi_s(x^0) = E\pi(x^0) \tag{7}$$

and the probability distribution $[q_1, q_2]$ are identical in all cases, and defining $E\,\pi'\,(x^*) = 0$ it can be easily derived from (5), — depending on $\pi_1'\,(x^0) \gtreqless \pi_2'\,(x^0)$, and $\pi_1\,(x^0) \lesseqgtr \pi_2\,(x^0)$ (implying $u_1'/u_2' \gtreqless 1$) — whether $E\,\pi'\,(x^0) \lesseqgtr E\,\pi'\,(x^*)$. Knowing, that under the standard assumption of decreasing marginal returns $(\pi'' < 0)$[4] the marginal expected outcome $E\,\pi'\,(x)$ is a decreasing function in x, it follows $x^0 \gtreqless x^*$. Therefore, x^0 may be smaller than, equal to, or even greater than x^*.

The matrix shows the nine possible combinations and the corresponding optimal activity levels: In case Aa there exists no uncertainty about the outcome. Therefore, we will take Aa as the reference case, where the optimal activity level x^0 is equal to x^*. We will denote x^* the "certainty activity level". Certainty is defined as the variable(s), which is (are) random in the corresponding uncertainty-case, being equal to the mean expected value(s) of the random one(s) under uncertainty.

Case		a	b	c
	IF:	$\pi_1'\,(x^0) = \pi_2'\,(x^0)$	$\pi_1'\,(x^0) > \pi_2'\,(x^0)$	$\pi_1'\,(x^0) < \pi_2'\,(x^0)$
A	$\pi_1\,(x^0) = \pi_2\,(x^0)$	$E\,\pi'\,(x^0) = 0$ $x^0 = x^*$	$E\,\pi'\,(x^0) = 0$ $x^0 = x^*$	$E\,\pi'\,(x^0) = 0$ $x^0 = x^*$
B	$\pi_1\,(x^0) > \pi_2\,(x^0)$	$E\,\pi'\,(x^0) = 0$ $x^0 = x^*$	$E\,\pi'\,(x^0) > 0$ $x^0 < x^*$	$E\,\pi'\,(x^0) < 0$ $x^0 > x^*$
C	$\pi_1\,(x^0) < \pi_2\,(x^0)$	$E\,\pi'\,(x^0) = 0$ $x^0 = x^*$	$E\,\pi'\,(x^0) < 0$ $x^0 > x^*$	$E\,\pi'\,(x^0) > 0$ $x^0 < x^*$

x^0 = activity-level under uncertainty
x^* = activity-level under certainty
π = outcome
π' = marginal outcome

For example, if a competitive firm is confronted with a random output price p, the corresponding certainty-case is characterized by a given output price $\bar{p}$ equal to the mean expected uncertain price ($\bar{p} = Ep$.) Because of the fact that in some cases x^0 is lower, equal or even higher than the certainty activity level, we cannot say in general that firms and individuals will have a lower activity level under uncertainty than under certainty.

[4] Note, that (5) and (6) may even hold if marginal return increases or remains constant in one state or the other $(\pi'' \geq 0)$.

To shed some light on the implications of our model we will refer to two common microeconomic problems: the output decision of a firm and the labor supply of an individual.

III. A Firm's Output Decision Under Uncertainty: An Example

Suppose, for example, a firm's profit is given by

$$\pi(x) = p(x)\,x - c(x) - c_f \tag{8}$$

where $p(x)$ denotes the output price ($p' = 0$ for a competitive firm), x the output (= activity level), $c(x)$ the variable costs ($c' > 0, c(o) = 0$), and c_f the fixed costs. We assume further the firm is risk-averse; and seeks to maximize expected utility $\Sigma\, q_s\, u(\pi_s)$; so that the optimal output policy is given by (4), (5) and (6) in a two-state context.

To generate the Sandmo case of a competitive firm facing uncertainty, we may consider the price as a random variable, so that e. g., $p_1 > p_2$. Obviously, we obtain then the matrix-case *Bb* (or *Cc* if $p_1 < p_2$), and may conclude "that under price uncertainty, output is smaller than the certainty output".[5]

If, instead, the fixed costs are random ($c_{f1} < c_{f2}$ or $c_{f1} > c_{f2}$) we receive the matrix case *Ba* or *Ca*, where the activity level or output under uncertainty does not differ from the certainty activity level.

Particularly interesting are the matrix cases *Bc* and *Cb*, where the activity-level — i. e., the firm's output — is higher under uncertainty than the certainty activity-level. We can generate these cases for example by assuming that the output price and the fixed costs are random ($p_1 > p_2$, $c_{f1} > c_{f2}$ *or* $p_1 < p_2$, $c_{f1} < c_{f2}$), and the difference in fixed cost overcompensates the revenue-difference caused by the price-difference. From this we may conclude: If a public policy intends to give an incentive to firms for higher activity this can be accomplished by announcing a flat — i. e. invariant to the output — tax (subsidy) due in more (less) profitable states of nature. But not only public policy may cause situations, in which it is rational for a risk-averse firm to produce a higher output under uncertainty than under certainty. For example, a market-garden may have higher fixed cost if the weather is cold, because it has to spend more for heating its greenhouses. However, it may receive a higher price for its products, too, if demand is determined among others by meteorological conditions: Consumers are willing to pay more for flowers and vegetables in a strong winter. Higher fixed cost can even lead to higher prices, as some suppliers may have to leave the market,

[5] *Sandmo* (1971), 66/77.

because their average cost is not covered by the market-price. We might also obtain the result that the optimal certainty activity-level is lower than the one under uncertainty, if $p(x)$ or $c(x)$ are uncertain and non-linear. Imagine, for example, a quantity-setting monopolisic firm facing uncertain demand function $p(x)$. The corresponding certainty demand function is then defined by $\bar{p}(x) = Ep(x)$. In this case the optimal output under uncertainty is higher than the one under certainty if $p_1(x^0) \gtrless p_2(x^0)$ and $p_1(1 + \eta_{p1x}) \lesseqgtr p_2(1 + \eta_{p2x})$, whereby η_{px} denotes the demand elasticity. That is, if the slope of the demand curve is lower but the price is higher in one state than in the other one, so that $\pi_1(x^0) \gtrless \pi_2(x^0)$ and $\pi_1'(x^0) \lesseqgtr \pi_2'(x^0)$ (see matrix-cases *Bc* and *Cb*). Therefore we may not conclude, as *Leland's* (1972) analysis suggests, that a risk averse monopolistic firm facing uncertain demand will always prefer to produce less under uncertainty than under certainty.

The reader should note that all results presented in this paper are derived from a very simple model of a world in which moral hazard does not exist, i. e. the probability-distribution cannot be effected by individual action.[6] On the other hand, the conclusion seems interesting enough given the recent attention focused on the disincentive rather than on incentive effects of risk-diminishing policy measures.

Although this list of examples of firms facing different forms of uncertainty can be easily expanded and explicated by refering to our basic model of economic behavior under uncertainty, we will draw our attention to another decision-making problem.

IV. An Individual's Labor Supply Decision Under Uncertainty: An Example

A well-known question handled by general microeconomic theory deals with the optimal labor supply of an individual. A common approach is:

$$\begin{aligned} &\max \pi(x) = U(y, l) \\ &\text{with} \quad y = (wx + Y)/p \\ &l = T - x \\ &\text{s.t.} \quad x > 0 \end{aligned} \tag{9}$$

where y denotes consumption, l leisure, w and p the prices for labor and consumption, x the labor supply, Y non-labor income (including flat —

[6] see *Schulenburg* (1978).

i. e. labor invariant — taxes and transfer payments), and T the total available time. As first order condition for a maximum we obtain:

$$\pi'(x) = \frac{w}{p} U_y - U_l = 0, \text{ or} \tag{10}$$

$$\frac{w}{p} - \frac{U_l}{U_y} = 0$$

The parallel of this problem to the example for a firm discussed in the previous section is obvious: the real wage rate w/p may be interpreted as the price or marginal revenue of labor supply, the fraction of the partial derivatives of the utility function can be considered as the marginal costs of labor. The difference is, that in this case the marginal costs U_l/U_y are not only dependent on the activity level but also on the available time T and the nonlabor income Y. Although this causes some analytical problems (see Appendix), we may refer normally to our basic model described in (1) and draw some conclusions for the labor supply of a risk-averse individual under uncertainty:

— If the consumption price is random, individuals will tend to have a lower labor supply than the certainty labor supply (matrix case *Bb, Cc*).

— If the wage rate is random, individuals will tend to have a lower labor supply than the certainty labor supply (matrix case *Bb, Cc*).

— If the available time (and therefore the marginal cost of labor) is random, individuals will tend to have a lower labor supply than the certainty labor supply (matrix case *Bb, Cc*).

— If individuals are uncertain about the real wage rate they will receive for their labor, appropriate flat taxes and transfers (which randomize the non-labor-income Y) may give an incentive to supply more labor than the certainty labor supply (matrix case *Bc, Cb*).

Particularly the first and the third form of uncertainty seem widespread: Normally employees know in advance the nominal wage rate but are uncertain about the inflation rate and therefore about the real wage. Also, the time available for leisure and work is uncertain because it is very dependent of the individual's health which cannot be predicted with certainty.

V. Generalization of the Results and the Inherent Problems of the Presented Model

The model presented in this paper showed in the most simple framework that even for a risk-averse decision-maker, a general judgement about the activity-level under uncertainty in comparison to the one under certainty cannot be given without further assumptions and specifications of the type of uncertainty. Furthermore, appropriate flat taxes, subsidies and transfer-payments conditional to certain states of nature may give incentives to increase the activity-level of firms and individuals.

However, the simplicity of the presented model is due to two assumptions which will now be discussed. Firstly, we considered only a two-point distribution. Secondly, our basic model contains only one decision variable; or, if there are several, the objective-function has to be specified (e. g. assuming separability of the objective-function as it is done in the Appendix).

Certainly, the limitation of the model on a two-state distribution is restrictive for a generalization of the results derived from the model, because the number of cases in the presented matrix will increase rapidly with the number of states. Three approaches are known to analyse multi-state cases:

a. A nearly forgotten one was developed in an early paper by *Krelle*[7] who showed in a graphic representation that every multi-state distribution can be transformed into a two-point distribution problem. While the application of *Krelle's* approach is limited, because the transformation-process is not independent from the individual's preferences, it has some appeal from a pedagogical point of view.

b. The mean-variance approach has a long tradition, whereby the probability distribution is characterized by its first two moments, its mean $E\,\pi$ and its variance $V\,\pi$. Using this approach we may write our maximization problem stated in (4) as follows:[8]

$$\text{(4')} \qquad \max U(x) = U(E\,\pi(x), V\,\pi(x)) \ ; \quad U_E > 0 > U_{EE}, U_V < 0 = U_{VV} \qquad \text{s.t.} \quad x > 0$$

The first and second order condition for a maximum are given by

$$\text{(5')} \qquad U_x(x^0) = U_E\,E\,\pi_x + U_V\,V\,\pi_x = 0$$

$$\text{(6')} \qquad U_{xx}(x^0) = U_{EE}\,E\,\pi_x^2 + U_E\,E\,\pi_{xx} + U_V\,V\,\pi_{xx} < 0 \ .$$

If $V\,\pi_x$ is dependent on a shift parameter z which increases the variance(s) of the random variable(s) — i.e. output price, fixed cost, wage rate — but leaving the mean expected value(s) of the random (variable(s) unchanged we receive via partial differentiating of (5')

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[7] see *Krelle* (1957), 640 - 645.

[8] see e. g. *Borch* (1974), 38 - 52, 130 - 137, who discusses implications of the utility index function $U(E, V) = E(b - E) - V; b > 0, 2E < b$.

(10) $$dx^0/dz = -U_V V\pi_{xz}/U_{xx} .$$

Since the denominator U_{xx} and U_V are negative, dx^0/dz has the opposite sign as $V\pi_{xz}$. For example, dx^0/dz is obviously negative for a competitive firm facing price uncertainty and zero if fixed cost is random. Thus, the optimal activity-level is lower under increased price uncertainty and remains unchanged under increased fixed cost uncertainty. Let $Vp(z)$ be the variance of the price and $Vc(z)$ the one of the fixed cost. Then, the variance of the outcome is given by $V\pi = x^2 Vp(z)$ for price uncertainty and $V\pi = Vc(z)$ for fixed cost uncertainty. In the first case we obtain $V\pi_x = 2x\,Vp(z)$ and $V\pi_{xz} = 2x\,Vp_z > 0$. In the second case $V\pi_x = V\pi_{xz} = 0$. These results are consistent with the findings in II. However, it seems rather difficult to use the mean-variance approach if both are uncertain, price and fixed cost. The variance of the profit is then given by

(11) $$V\pi = x^2\,Vp + Vc - 2x\,\mathrm{Cov}(p, c_f) ,$$

so that we obtain

(12) $$V\pi_x = 2x\,Vp - 2\,\mathrm{Cov}(p, c_f) , \quad \text{and}$$

(13) $$V\pi_{xz} = 2x\,Vp_z - 2\,\mathrm{Cov}(p, c_f)_z .$$

If there is a positive association — that is, if small values of p tend to be associated with small values of c_f and large values of p with large values of c_f — the covariance in (11) will be positive. In this case $V\pi_x$ and also $V\pi_{xz}$ might be negative (see (12) and (13)), so that it is optimal for a risk-averse firm to produce more if uncertainty increases. If, on the other hand, there is a negative association of p and c_f — that is, if small values of p tend to be associated with large values of c_f and large values of p with small values of c_f — the covariance will be negative. $V\pi_{xz}$ and therefore dx^0/dz will then be positive.

Although it could be shown that the mean-variance approach leads to the same results like our basic model presented in II, we should note that a von-Neumann-Morgenstern expected utility function can only be transformed to a mean-variance objective-function, if and only if the underlying utility function is quadratic and/or the distribution of the random variable is normal.[9]

c. In a series of articles *Diamond, Rothschild* and *Stiglitz* (1971, 1974) discussed definitions of increased risk and its applications. They developed the concept of 'mean preserving in risk'. Let x be the activity-level and k the random variable. The optimal activity-level is then given by

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[9] see e. g. *Levy / Markowitz* (1979), 308.

$$\max U(x) = \int u(x, k) f(x)\, dx \quad \text{s.t.} \quad x > 0 \tag{4''}$$

cr

$$\int u(x^0, k)_x f(x)\, dx = 0 \tag{5''}$$

$$\int u(x^0, k)_{xx} f(x)\, dx < 0\ . \tag{6''}$$

Stiglitz and his co-authors showed that $\int u_{xkk} f(x)\, dx < 0\ (> 0)$ implies a decrease (increase) of x^0 if the risk increases leaving the mean of the random variable unchanged. If u_{xkk} is uniformly singed for all k, and this is the crucial step in this approach, we can easily derive for specified examples if the activity-level will increase, decrease or will be constant when the 'mean-preserving' risk increases. If u_{xkk}, however, is not uniformly signed, this approach will lead us to no conclusions about the activity-level under uncertainty in comparison to the one under certainty.

To sum up, although the limitation of the model on a two-state distribution lessens the applicability of the results, there is some evidence that the results derived in a two-state of nature world will normally also hold in a multi-state context. Furthermore, one may find comfort with the experience that in most cases in reality decision-makers consider only two states of nature, rain and sunshine.

If there is more than one decision variable (e.g. income and leisure) the case-matrix may only be applied to the considered problem, if further specifications of the second derivative of the utility function are assumed. Otherwise, in a deterministic as well as in a stochastic model framework the income-effect may overcompensate the substitution effect of a price-change. Therefore, we assumed in the application of our model to the optimal labor-supply decision of an individual separability of the utility-function (see Appendix). *Block* and *Heineke* (1973) discussed the same problem by assuming inferiority of labor and decreasing absolute risk aversion. In addition, they considered the case of a quadratic utility-function. Only empirical investigations concerning the form of individuals' utility-functions can decide if the conclusions presented in the case-matrix are also valid in a multi-decision-variable case or if we have to modify them.

Summary

The paper examines the impact of uncertainty on the activity-level of decision-makers. A basic model presented and discussed in this paper leads to some general rules concerning the optimal activity-level under uncertainty in comparison to the one under certainty. The study shows in contrast

to a general belief that given certain conditions the activity-level of risk-averse decision-makers might even be greater than the one under certainty. Flat taxes, subsidies and transfer payments conditional to the occurrence of certain states of nature may therefore be suited as incentives to increase the activity-level of firms and individuals.

Zusammenfassung

In diesem Aufsatz werden die Auswirkungen verschiedener Formen der Unsicherheit auf das Aktivitätsniveau von Wirtschaftseinheiten untersucht — z. B. auf die optimale Produktionsmenge von Unternehmen oder auf das Arbeitsangebot von Individuen. Ziel der Analyse ist es, zunächst in einem möglichst einfachen und generellen Modell einige Grundregeln abzuleiten, die — wie anhand von zwei Beispielen gezeigt — auf die meisten Fälle anwendbar sind. Außerdem wird gezeigt, daß im Gegensatz zu einer weit verbreiteten Meinung, das optimale Aktivitätsniveau eines risikoaversen Akteurs bei bestimmten Unsicherheitsformen sogar größer unter Unsicherheit als unter Sicherheit sein kann. Subventionen, Steuern und Transferzahlungen, deren Gewährung an den Eintritt bestimmter Zustände gebunden ist, können eine Aktivitätsniveau-erhöhende Anreizwirkung haben.

Appendix

In this Appendix the comparative model of labour supply under uncertainty and certainty will be presented in more detail. Taking the problem described by (9), we can ask the question: what are the differences in labor supply under certainty and under uncertainty if w, p, Y and/or T are random?

The individual is maximizing expected utility

$$\text{(A1)} \qquad \max \Sigma\, q_s\, u_s\, ((w_s\, x + Y_s)/p_s, T_s - x) \ , \quad \text{s. t.} \quad x > 0 \ .$$

Assuming $u_{yl} = u_{ly} = 0$ we obtain as conditions for a maximum

$$\text{(A2)} \qquad \Sigma\, q_s\, u_{sy}\, (x^0)\, w_s/p_s - \Sigma\, q_s\, u_{sl}\, (x^0) = 0 \ ,$$

$$\text{(A3)} \qquad \Sigma\, q_s\, u_{syy}\, (x^0)\, w_s^2/p_s^2 - \Sigma\, q_s\, u_{sll}\, (x^0) < 0 \ .$$

1. If only T is random $(T_1 \neq T_2)$ A2 becomes

$$\text{(A4)} \qquad u_y\, (x^0)\, w/p - \Sigma\, q_s\, u_{sl}\, (x^0) = 0 \ .$$

Because the assumption of risk-aversions requires

$$\text{(A5)} \qquad \Sigma\, q_s\, u_{sl}\, (T_s - x^0) > u_l\, (\Sigma\, q_s\, T_s - x^0)$$

it follows

$$\text{(A6)} \qquad 0 < u_y\, (x^0)\, w/p - u_l\, (\Sigma\, q_s\, T_s - x^0)$$

and therefore $x^0 < x^*$.

2. If only Y is random ($Y_1 \neq Y_2$) and considering risk-aversion

$$\Sigma\, q_s\, u_{sy}\, ((wx + Y_s)/p) > u_y\, ((wx + \Sigma\, q_s\, Y_s)/p) \tag{A7}$$

we obtain $x^0 < x^*$.

3. Analogous to 1. and 2. it is easy to show that if the wage rate or the price level are random $x^0 < x^*$.

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