Reserve Requirements and Optimal Money Balances

By Ernst Baltensperger*

The paper employs a simple general equilibrium model of the economy including a banking system to analyze the tax and efficiency effects of reserve requirements. It asks whether under certain conditions a nonzero reserve requirement against deposits can be said to be optimal from an efficiency point of view.

I. Introduction

Reserve requirements are comparable to a tax on a particular economic activity, namely, the production of deposit accounts. As such, they have efficiency effects similar to those of a tax on other types of activities. This has always been accepted in general terms. Yet, surprisingly, very little analytical attention has been paid to it as the discussions concerning reserve requirements have always been dominated by economic stability considerations.¹ In a sense, one may say that fractional reserve banking has been argued to be the source of a negative externality by increasing instability in the money supply and economic activity, and that the "reserve requirement tax" has been justified in terms of a stability argument, a view maybe best exemplified by the well-known 100 percent reserve proposal.² The underlying idea, of course, is that this would minimize the influence of private sector disturbances on the money stock by making the latter equal to the outstanding amount of government money itself.

The purpose of this paper is to examine the issue of reserve requirements from a pure efficiency (allocative) point of view,³ setting apart

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¹ For some recent studies of this kind see, e.g., Poole and Lieberman (1972), Poole (1976), Kaminow (1977), Laufenberg (1979), Sherman, Sprenkle, and Stanhouse (1979).

² See, e.g., Fisher (1935), Hart (1935), Angell (1935), Lutz (1936), Friedman (1959).

³ Of course, underlying the concern for economic stability is the notion that there are some sort of costs associated with economic shocks and instability. Thus, in the final analysis, a stability argument is an efficiency argument, too (of an entirely different kind, though, than the traditional "static" type of efficiency effects discussed below).

stability considerations for once.⁴ It is argued that, if it is true that the amount of government money held by the economy is suboptimal because of a divergence between private holding cost and social production cost, as is often asserted,⁵ and if this cannot be resolved simply by providing a return on government money,⁶ a nonzero reserve requirement against deposits may, under certain conditions, be optimal from an efficiency point of view (in the sense of a second-best solution). This requirement would, however, in all likelihood be much less than 100 percent (if positive). The analysis is conducted in the context of a simple and highly aggregated general equilibrium model of the economy. The model is analyzed first for the simplifying case where banks hold no excess reserves and for the more general case taking into account excess reserves subsequently. The payment of interest on bank reserves has often been suggested as a means to offset the tax effect of reserve requirements. Since the question of this paper is that of the optimal tax on deposits, it will obviously not be assumed that this tax effect is offset by a compensating subsidy. Some comments concerning the question of paying interest on reserves conclude the paper, however.

II. The Model

The discussion is based on the premise that stocks of money balances have the function of saving transactions and information costs and, more generally, helping the economy to find a superior pattern of transactions and allocation of activities. See, e.g., *Baumol* (1952) and all the further developments based on this approach, *Brunner* and *Meltzer* (1971), *Niehans* (1971, 1978), *Saving* (1971). Government money (currency) and bank money (demand deposits) are seen as substitutes in this function, although not perfect ones, as each type of money may have a comparative advantages with respect to certain types of transaction arrangements. The imposition of a reserve requirement against demand deposits has the effect of making deposit money more costly to produce and thus less attractive relative to government money (as well as making the use of money more expensive in a general or "average" sense).

⁵ See, e.g., Friedman (1969).

⁴ Santomero and Siegel (1981) and Baltensperger (1980) argue that it is not as easy as is often thought to prove the superiority of high reserve requirements in terms of their effects on economic stability, once the focus is shifted from exclusive consideration of short-run money stock control to output market or price level stability as the ultimate objective.

⁶ Say, because this is too costly itself, at least in the case of currency. If interest is "paid" in the form of deflation, this has its costs, too, presumably. Inflation, on the other hand, lowers the return on government money.

For the present purpose, it is sufficient to capture these notions by letting the supply of output which is available for consumption (Y) be a positive function of the real stocks of both types of money held by the public, currency (C) and deposits (D), e.g., as follows:

(1)
$$Y = \overline{Y} - T(C, D), \text{ with } T_C < 0, T_D < 0$$

(and $T_{CC} > 0, T_{DD} > 0$)

where \overline{Y} denotes the level of total output or "resources" available to the system and T the amount of output or resources used up in the form of transaction and information costs. The opportunity cost of holding currency is the yield on earning assets r ("the" market rate of interest), and that of holding deposits the difference between r and the deposit rate i (which is expressed net of service charges and conceivably negative). The two types of money thus are held up to the points where (disregarding direct holding or "storage" costs for simplicity's sake):

$$(2a) - T_C (= Y_C) = r$$

and

$$(2b) - T_D (= Y_D) = r - i .$$

To simplify the presentation, \overline{Y} is taken as exogenously given, so that problems of commodity production can be disregarded (exchange economy). However, individual consumers can shift consumption from current to future periods, or vice versa, via borrowing and lending. They are assumed to maximize the utility of their stream of consumption, subject to their budget constraints, in the usual way. This is assumed to lead to demand functions for current output or consumption (*E*), currency (*C*), and deposits (*D*) of the following form (all in real terms):

$$E = E(r), \quad E_r < 0$$

(4)
$$C = C(r, i), \quad C_r < 0, \quad C_i < 0$$

(5)
$$D = D(r, i), D_r < 0, D_i > 0$$
.

Implicit in these decisions, of course, is a demand for or supply of credit (bonds, future consumption) which is not explicitly shown here, however. The influence of r and i on T is reflected by their influence on the levels of C and D. An increase in the interest rate r influences the two demand for output components E and T in opposite ways: $E_r < 0$ indicates intertemporal substitution in favor of future consumption; $C_r < 0$ and $D_r < 0$ reflect the increased opportunity cost of holding money and lead to a positive association with T. It is assumed that the influence on E dominates. The influence of the deposit rate i on E is

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disregarded. Note also that E is treated here as an endogenous variable, jointly determined by the consumer with C and D, and thus does not appear as an independent argument in the C() and D() functions (say, as an index of the volume of transactions). Rather, the influence of transactions volume on money demands is captured indirectly by the joint dependence of all these magnitudes on r.

The behavior of banks is summarized by the following simple model, according to which they produce deposit liabilities at the expense of real costs (in terms of output not available for consumption) and hold a proportion k of their assets (= deposits) in the form of government money (reserves) and the remainder in the form of earnings assets (loans). The reserve ratio is expressed as $k = \rho + x = \rho + e(1 - \rho)$, where ρ denotes the required reserve ratio, and e the proportion of excess reserves which the bank maintains against the share of deposits not covered by required reserves. Total excess reserves thus are X = xD $= e (1 - \rho) D$. (This formulation suggests, of course, that the overall excess reserve ratio x declines, cet. par., with an increasing required reserve ratio ρ .) The cost function is expressed as F(D, e) and is charracterized by positive and increasing marginal cost of producing deposits $(F_D > 0, F_{DD} > 0)$ and positive (but decreasing) marginal productivity of excess reserves (in terms of adjustment costs of various types saved; $F_e < 0$, $F_{ee} > 0$). Thus, the bank maximizes a profit function of the following form (with D and e being the choice variables):

(6)
$$\pi = r (1 - k) D - iD - F (D, e)$$
$$= r (1 - \varrho) (1 - e) D - iD - F (D, e) ,$$

yielding the following first order optimality conditions

(7a)
$$\frac{\partial \pi}{\partial D} = r (1 - \varrho) (1 - e) - i - F_D = 0$$

(7b)
$$\frac{\partial \pi}{\partial e} = -r(1-\varrho)D - F_e = 0$$

(plus appropriate second order conditions).

This results in supply of deposit (S) and reserve demand (R) functions for the banks of the of the following type (where the banks' supply of deposits is now denoted with S, in contrast to the public's demand for deposits D):

⁷ Note that F_e is proportional to $(1 - \varrho) D$: a marginal increase in e implies an increase in excess reserves equal to $(1 - \varrho) D$ de. Its "productivity" in lowering adjustment costs thus should be proportional to this magnitude.

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(8)
$$S = S(\hat{r}, i), S_{\hat{r}} > 0, S_i < 0$$

(9)
$$R = (\varrho + e(r)(1 - \varrho)) S(\hat{r}, i), \quad e_r < 0$$

where

$$\hat{r} = r(1-k) = r(1-\varrho)(1-\varrho)$$
.

The banks' supply of credit (demand for earning assets) is implicit in these decisions and not explicitly shown. The influence of r and i on the banks' demand for output ("consumption of resources") is reflected by their influence on S and e, via the function F.⁸

The government issues non-interest-bearing government (base) money in a fixed nominal amount B. The real revenue B/p which it obtains in this way, to the extent that it exceeds the real cost H of producing and maintaining this stock of money, can be used by the government to acquire output, i.e., lowers the amount of output which is directly available for consumption by the private sector of the economy. However, since government purchases and consumption of output, in the final analysis, benefit some members of the private sector, too, they can for our purposes be treated at the same time as representing private consumption (i.e., it is assumed for simplicity, that government purchases and private demand shift in opposite ways and can be consolidated in one function E).

All this can be summarized in the following general equilibrium system describing the economy:

(10)
$$E(r) = \overline{Y} - T(C, D) - F(D, e) - H(C, R)$$

$$(11) D(r,i) = S(\hat{r},i)$$

(12)
$$C(r,i) + (\varrho + e(r)(1-\varrho)) S(\hat{r},i) = \frac{S}{P}$$

The first equation describes the output market, the second the deposit market, and the third the market for government money. The levels of T, F and H appearing in he first equation are, of course, jointly determined with the levels of C, D and e, as described, and thus are themselves functions of the interest rates r and i. The description of the credit market which is implicit in the equations summarized above is eliminated from explicit consideraton with the help of Walras' law. But note that existence of a credit market implies the existence of a third asset (bonds) besides currency and deposits. The endogenous variables of the system are r, i and the price of output P and thus, via

⁸ Free entry into banking would lead, in the long run, to an equilibrium with $\pi = 0$ or $r(1 - \varrho)(1 - e) - i = F(D, e) / D$, or $F_D = F(D, e) / D$.

the appropriate demand or supply functions, the quantities C, D (= S), E and R.⁹

Obviously, the system's equilibrium does depend on the level of ρ imposed by the authorities. Our interest in this paper concerns the efficiency effects of reserve requirements. These can be measured by the influence of ρ on the expression $T + F + H \equiv W$. The most interesting question is whether, and under what circumstances, there exists an optimal reserve requirement which is non-zero. Intuitively, it appears clear that it is optimal to impose no reserve requirements against deposits, if the real quantity of government money held by the system is at its socially optimal level, which would require that the (social) cost of producing and maintaining it, at the margin, just equals its private holding cost r.¹⁰ On the other hand, if the marginal social cost of producing government money is less (e.g., zero or approximately zero, as is often assumed), and yet no interest is paid on it,¹¹ the situation is different. It can be shown that, under these circumstances, a positive tax on deposits in the form of a reserve requirement may, under certain conditions, be efficient (in the sense of a second-best solution). Note, however, that the situation is complicated by the fact that government money is used as a substitute for for deposits in the form of currency on the one hand, and as an "input" in the production of deposits on the other hand.¹²

III. The Optimal Tax on Deposits

To simplify the presentation, we will first consider the case where banks hold no excess reserves, so that all terms involving *e* vanish, and analyze the more general case subsequently.

⁹ A more explicit model would include a real resource market and deal with commodity production explicitly. However, for our purpose, the extra insights would probably not justify the extra cost in terms of analytical complication.

¹⁰ Or that government's revenue from producing money is returned *proportionately* to the holders of government money in the form of a stream of services or interest.

¹¹ See footnote 6 above.

¹² Another possible justification of a reserve requirement, which is not further pursued here, however, would be the existence of a deposit insurance scheme which subsidizes risk through improper pricing. (More generally: any situation where, for some reason, the banking system does not properly internalize risk, and thus produces too much deposits.) — Still another argument would view reserve requirements as the "price" for various services supplied by the central bank without charge to the banks. However, it would probably be more efficient to charge for these services explicitly. — Yet another argument justifying a tax on deposits (suggested by *Thomas Mayer*) would be that the "liquidity yield" on money balances escapes the income tax so that a reason for taxing them exists on equity grounds. Of course, this would apply to government money as well as deposits and would not be a reason for taxing them differentially.

The model without excess reserves (e = 0)

From equations (10) to (12) we have in this case (assuming $H_R = 0$, for simplicity¹³)

(13)
$$a_{11} dr + a_{12} di = 0$$

$$a_{21} dr + a_{22} di = -S \, ; r \, d \, \varrho$$
,

where

$$\begin{aligned} a_{11} &= E_r + (T_C + H_C) C_r + (T_D + F_D) D_r \\ a_{12} &= (T_C + H_C) C_i + (T_D + F_D) D_i \\ a_{21} &= D_r - S_r (1 - \varrho) \\ a_{22} &= D_i - S_i \end{aligned}$$

with $a_{11} < 0$, $a_{22} > 0$, and $a_{11} a_{22} - a_{12} a_{21} \equiv \Delta < 0$ (dominance of "direct" over "indirect" effects) as the usual stability conditions.

We obtain

(14)

$$\frac{dW}{d\varrho} = (T_C + H_C) dC + (T_D + F_D) dD$$

$$= (a_{11} - E_r) - \frac{dr}{d\varrho} + a_{12} - \frac{di}{d\varrho}$$

$$= - \frac{E_r S_r r a_{12}}{\Delta}$$

The sign of this expression depends on the sign of a_{12} , as $-E_r S\hat{r} r/\Delta < 0$.

Now, note that individual optimization on the part of consumers implies $(-T_c) = r$ and $(-T_D) = r - i$, and individual optimization on the part of banks $(-F_D) = r(1-\varrho) - i$.

Thus, we can write

 $a_{12} = -(r - H_C) C_i - r \varrho D_i$

or, for the case with $H_C = 0$,

$$a_{12} = -r \left(C_i + \varrho D_i\right) \ .$$

In this case, this is clearly positive at $\rho = 0$, and thus $\frac{dW}{d\rho}$ $(\rho = 0) < 0$, i.e., a marginal increase of ρ from zero to a positive level will lower W and hence be efficient.¹⁴ The rate of change of W with respect to ρ becomes zero if we are at the point where

¹³ This, of course, also implies disregarding the central bank's cost of controlling and policing reserve requirements.

¹⁴ More generally, this holds as long as $(r - H_C) > 0$.

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$$C_i + \varrho D_i = 0$$

which defines an "optimal" reserve requirement

(15)
$$\varrho^* = -\frac{C_i}{D_i} > 0 .15$$

If $D_i + C_i > 0$, i.e., if an increase in the deposit rate, cet. par., raises the total demand for money (of both types together, although it lowers the component *C*) i.e., if the "direct" effect D_i dominates, we will have $0 < \varrho^* < 1$. The value for ϱ^* can, of course, also be expressed in terms of the demand elasticities $\varepsilon_{Di} = D_i(i/D)$ and $\varepsilon_{Ci} = C_i(i/C)$ and the currency-deposit ratio:

(15')
$$\varrho^* = -\frac{\varepsilon_{C_i}}{\varepsilon_{D_i}} \cdot \frac{C}{D} \; .$$

Verbally, this can be explained as follows: Social optimization (minimization of W) requires that deposit production is extended to the point where $-T_D = F_D$, and currency is held up to the point where $-T_C = H_C$. With a zero reserve requirement (a zero "tax" on deposits), private optimization satisfies the first of these conditions since individuals hold deposits up to the point where $-T_D = r - i$, and banks expand deposit production until $r - i = F_D$. However, if $H_C < r$, and individuals hold currency up to the point where $-T_{c} = r$, the quantity of currency held by the system is suboptimal. Under these conditions (and assuming a divergence between r and H_c as given), a marginal increase in ρ (= a tax on deposits) will, via a marginal increase in C combined with a marginal reduction in D, lead to a marginal reduction in W, since in the neighbourhood of the initial situation, i.e., at $\rho = 0$, $(T_D + F_D) = 0$, but $(T_C + H_C) < 0$. The higher D_i relative to C_i , the faster will the point be reached where the divergence between these two expressions disappears, i.e., the lower is ρ^* .

The model with excess reserves (e > 0)

Consider next the situation where banks also hold excess reserves. In this case, government money is used not only as a substitute for bank deposits (in the form of currency), but also as an input in the production of deposits (in the form of reserves). This tends to lower ϱ^* as compared to the situation where the second of these uses of government money is disregarded. This can be best visualized by considering the extreme case where government money is only used in the form of bank

¹⁵ Expressed more generally,
$$\varrho^* = - \frac{(r - H_C)}{r} \frac{C_i}{D_i}$$
.

reserves, with C = 0. In that case, if is clear that the imposition of a reserve requirement could not lead to a reduction in W, but would only lead to a reduction in D,¹⁶ and thus to a loss with no corresponding gain.

In the general case with both e > 0 and C > 0, we have, from equations (10) to (12)

$$\begin{aligned} a_{11} dr + a_{12} di &= 0 \\ a_{21} dr + a_{22} di &= -S_r^2 r (1-e) d\varrho \\ \end{aligned}$$
with $a_{11} &= E_r + (T_C + H_C) C_r + (T_D + F_D) D_r + F_e e_r \\ \end{aligned}$
and $a_{21} &= D_r - S_r^2 (1-\varrho) (1-e) ,$

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(16)

and everything else as before.

This yields

$$\frac{dW}{d\varrho} = (T_C + H_C) \frac{dC}{d\varrho} + (T_D + F_D) \frac{dD}{d\varrho} + F_e \frac{de}{d\varrho}$$
$$= (a_{11} - E_r) \frac{dr}{d\varrho} + a_{12} \frac{di}{d\varrho}$$
$$(17) = -\frac{E_r S_r r (1 - e) a_{12}}{\Delta}$$

which is again negative if a_{12} is positive and vice versa, as $-E_r S_r (1-e)/\Delta < 0$.

Individual optimization on the part of consumers and banks now implies $(-T_C) = r$, $(-T_C) = r - i$, $r(1 - \varrho)(1 - e) - i = F_D$, and $r(1 - \varrho)D = -F_e$.

Therefore,

$$a_{12} = -(r - H_C) C_i - r (\varrho + e (1 - \varrho)) D_i$$

or, with $H_C = 0$,

$$a_{12} = -r \{C_i + (\varrho + e(1 - \varrho)) D_i\}.$$

At $\rho = 0$, this reduces to

$$-r(C_i+eD_i)$$
.

This expression would have to be positive in order for $\varrho^* > 0$ to exist; i.e., $C_i + eD_i$ would have to be negative, or $e < -C_i/D_i$ (at $\varrho = 0$). Otherwise, an increase in ϱ would raise, not lower W. If a $\varrho^* > 0$ exists, it has to satisfy

¹⁶ Which is already suboptimal in this case, even with $\varrho = 0$. That is, if anything, a subsidy would be appropriate here, rather than a tax.

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$$\varrho^* = -\left(\frac{C_i}{D_i} + e\right) \frac{1}{1-e} \ .$$

 $C_i + (o^* + e(1 - o^*)) D_i = 0$, or

Note that this approaches $-C_i/D_i$ as *e* approaches zero, and zero as *e* approaches $-C_i/D_i$.¹⁷ Loosely speaking: The larger the use of government money as an input in the production of deposits in the form of reserves, relative to its use as a substitute for deposits in the form of currency, the less appropriate it would be to impose a tax on deposits in the form of a reserve requirement (always assuming for given, of course, that the private cost of holding government money exceeds the social cost of producing it).

Applying this approach to the question of the structure of reserve requirements against different types of deposit liabilities would suggest low (or zero) requirements for liabilities which are weak (or no) substitutes for currency (i.e., for which $\partial C/\partial i_j$ is relatively small or zero).

Payment of interest on reserves

Consider, finally, the question of paying interest on bank reserves in this context. This has often been suggested as a means to offset the tax effect of reserve requirements. It is clear that if, for reasons as discussed above, a positive reserve requirement is socially optimal, it should not be offset by payment of interest on reserves to this extent. On the other hand, paying interest on reserves would ensure that deposits are produced where $F_D = -T_D$, even if banks have a need for excess reserves (e > 0). However, it will then not be possible anymore to correct the relative use of currency and deposits via imposing a reserve requirement as discussed above, as the latter looses its character as a tax. Such a correction would require then that interest is paid on excess reserves only, but not on required reserves. More generally: interest may be paid on all reserves, and the banks pay a tax of $r \rho^* = (-C_i/D_i) r$ per dollar of deposits. In any case: it is not clear, generally speaking, that it would be advisable to pay interest on bank reserves (unless interest is also paid on currency, which I assume to be infeasible for cost reasons).

¹⁷ Of course, $H_C > 0$ (and/or $H_R > 0$) would require appropriate modifications in the expression for ϱ^* .

Summary

A simple general equilibrium model of the economy has been used to analyze the tax effects of reserve requirements. It is argued that, if it is true that the amount of government money held by the economy is suboptimal because of a divergence between private holding cost and social production cost, as is often asserted, and if this cannot be resolved simply by providing a return to government money,¹⁸ a nonzero reserve requirement against deposits may, under certain conditions, be optimal from an efficiency point of view (in the sense of a second-best solution). This requirement would, however, in all likelihood be much less than 100 percent (if positive).

Zusammenfassung

Der Beitrag verwendet ein einfaches Gleichgewichtsmodell eines Wirtschaftssystems zur Analyse der Steuer- und Effizienzeffekte von Mindestreserven. Er geht aus von der wohlbekannten Vorstellung, wonach als Resultat einer Divergenz zwischen den privaten Kosten der Geldhaltung und den gesellschaftlichen Produktionskosten die vom System gehaltene Menge an staatlichem Geld suboptimal ist. Unter dieser Voraussetzung und unter der Annahme, daß dieses Problem nicht einfach durch eine Verzinsung des staatlichen Geldes gelöst werden kann¹⁸, kann ein positiver Mindestreservesatz auf Depositenverbindlichkeiten unter bestimmten Bedingungen effizienzmäßig optimal sein (im Sinne einer Second-best-Lösung). Dieser Reservesatz würde aber normalerweise wesentlich geringer sein als 100 Prozent.

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¹⁸ See footnote 6 above.

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