The Structure of Technology, Federal Republic of Germany, 1950 - 1973: Comment

By Peter Zweifel

In this Journal, *Conrad* and *Jorgenson* (1978, CJ henceforth) develop a translog approximation to an arbitrary production function $F(C, I, K, L, t) = 0$. They present empirical separability tests showing that a distinction between outputs (C, I) and inputs and technical change (K, L, t) is compatible with German data. Unfortunately, their work contains a couple of flaws which tend to cast doubt on the validity of their interesting results.

I.

The first point concerns the separability issue itself. Any function $F(\cdot)$ can be approximated by a second-order Taylor series in the logarithms of its arguments. In order to have logarithms throughout, let us write

$$H = \ln(F + 1) \cong h' a + \frac{1}{2} h' B h,$$

$$\text{with} \quad h' = [\ln C, \ln I, \ln K, \ln L, t]$$

$$a' = \left[\frac{\partial H}{\partial \ln C}, \frac{\partial H}{\partial \ln I}, \frac{\partial H}{\partial \ln K}, \frac{\partial H}{\partial \ln L}, \frac{\partial H}{\partial t}\right]$$

$$(1) \qquad = [\alpha_C, \alpha_I, \alpha_K, \alpha_L, \alpha_t]$$

$$B = \left[\frac{\partial^2 H}{\partial \ln C^2}, \frac{\partial^2 H}{\partial \ln C \; \partial \ln I}, \ldots, \frac{\partial^2 H}{\partial t \; \partial \ln L}, \frac{\partial^2 H}{\partial t^2}\right]$$

$$= \begin{bmatrix} \beta_{CC} & \beta_{CI} & \beta_{CK} & \beta_{CL} & \beta_{Ct} \\ & \beta_{II} & \beta_{IK} & \beta_{IL} & \beta_{It} \\ & & \beta_{KK} & \beta_{KL} & \beta_{Kt} \\ & & & \beta_{LL} & \beta_{Lt} \\ & & & & \beta_{tt} \end{bmatrix}$$

Therefore, the vector h contains the increments of the arguments around the point of expansion $(1, 1, 1, 1, 0)$; the vector a collects the first partial derivatives of the approximating function; and B is the Hessian, the symmetric matrix of second derivatives. The parameterization with α_C, β_{CC} etc. is only for convenient notation.

Eq. (1) is the entirely unrestricted translog approximation of $F(C, I, K, L, t) = 0$. In particular, it contains no separability assumptions. Since *CJ* want to test for separability, one would expect them to start from this formulation.

However, their basic equation reads

$$\ln L = H(\ln C, \ln I, \ln K, t) \tag{2}$$

and it is easily seen that this follows from (1) if and only if

$$\alpha_L = -1; \beta_{CL} = \beta_{IL} = \beta_{KL} = \beta_{LL} = \beta_{Lt} = 0 \ . \tag{3}$$

In *CJ*'s terminology, *explicit separability* of L from outputs and all other inputs is assumed here. In terms of the B matrix in (1), the entire column relating to L is set to zero. For comparison, let us consider explicit separability in terms of (C, I, t) and (K, L, t). This would call for a partitioning of B into a (C, I) and a (K, L) block, leaving the last column unaltered. Therefore,

$$\beta_{CK} = \beta_{IK} = \beta_{CL} = \beta_{IL} = 0 \ . \tag{4}$$

The last two of these restrictions have already been introduced with eq. (3). Generally speaking, *CJ* lose testable implications of the separability hypotheses because they write H in the form of eq. (2). All of their tests are *conditional upon the validity of the restrictions contained in (3)*. Unfortunately, the reader is not told whether these restrictions were tested in one of the cited forthcoming publications and what the results were.

II.

The profit maximization hypothesis is a cornerstone of *CJ*'s argument. Without it, their value share equations in terms of $q_C\, C/q_L\, L$, $q_I\, I/q_L\, L$, and $-\, q_K\, K/q_L\, L$ could not be derived. Also, all separability tests are based on its acceptance. Since the approximation of the technical relationship between inputs and outputs is given in eq. (1), profit maximization first of all entails a *change of functional form*. In the literature cited by *CJ*, this change has not been tested for; estimation has always started from value share equations. The difficulty is that one has to compare the fit of two equations with differing dependent variables; cf. *Quandt* (1974) for a possible solution. But let us suppose that the profit maximization hypothesis survived such a test. Still, two versions of it should be distinguished. In the *short run*, optimal economic profit Z^* may take on any real value, while in the *long run*, $Z^* = 0$ under the competitive conditions assumed by *CJ*, cf. *Hirshleifer* (1976, 263). Although working with annual data, *CJ* opt for the long-run version with $Z^* = 0$. Acrordingly, they have defined capital outlays $q_K K$ to include all book profits, cf. *Conrad* and *Jorgenson* (1975). This yields an equality of valued outputs and inputs,

$$\text{(5)} \qquad q_C C + q_I I + q_L L + q_K K \quad \text{or} \quad \frac{q_C C}{q_L L} + \frac{q_I I}{q_L L} = 1 + \frac{q_K K}{q_L L}.$$

Eq. (5) shows that the number of independent value share equations is reduced to two. Moreover, summing-up restrictions on translog parameters follow.

In a world of government interference leading to closed rather than open markets, it is difficult to believe that economic profits and losses of the private sector vanish within a year. In fact, several industries have been found where parts of annual book profits could be identified with economic profit Z^*, cf. *Harberger* (1954) and *Kamerschen* (1966). In view of this, one should estimate capital outlays net of Z^*, which will differ from the long-run gross concept used by *CJ*. The two versions of the profit maximization hypothesis *cannot be tested by the same set of data* on q_K and $\ln K$, and acceptance of the hypothesis with $Z^* = 0$ does not imply that the short-run version with $Z^* \neq 0$ would have been accepted. In sum, a thorough test of the profit maximization hypothesis involves three steps. Starting point is eq. (1). Then, three (and not two) share equations would have to be estimated, with data on $\ln K$ and q_K conforming to $Z^* \neq 0$. If this change of functional form is compatible with the data, then the restrictions emanating from the assumption $Z^* = 0$ may be introduced: Deletion of one of the value share equations as redundant, summing-up restrictions on parameters, and replacement of $\ln K$ and q_K by values stemming from the gross concept of capital outlays as used by *CJ*. With such a testing sequence successfully completed, there would be good reasons to join *CJ* in their acceptance of the profit maximization hypothesis and the separability results that are based upon it.

References

Conrad, K. and D. W. *Jorgenson* (1975), Measuring Performance in the Private Economy of the Federal Republic of Germany 1950 - 1973. Tübingen.

— (1978), The Structure of Technology, Federal Republic of Germany, 1950 - 1973. Zeitschrift für Sozialwissenschaften, 3, 259 - 279. (*CJ* in the text).

Harberger, A. C. (1954), Monopoly and Resource Allocation. American Economic Review 54, 77 - 87.

Hirshleifer, J. (1976), Price Theory and Applications. London.

Kamerschen, D. R. (1966), An Estimation of the Welfare Losses from Monopoly in the American Economy. Western Economic Journal 4.

Quandt, R. E. (1974), A Comparison of Methods for Testing Nonnested Hypotheses. Review of Economics and Statistics LXI, No. 1, 92 - 99.