The Structure of Technology, Federal Republic of Germany, 1950—1973

By Klaus Conrad* and Dale W. Jorgenson**

In this paper we use econometric models of production to develop tests of parametric restrictions for characterizing the structure of technology and changes in technology empirically.

I. Introduction

The purpose of this paper is to employ econometric methods introduced by Jorgensen and Lau (1978) for characterizing the structure of technology and changes in technology over time of the private domestic economy of the Federal Republic of Germany. In a previous paper[1] we have employed econometric models of production based on the translog production and price functions[2] to test the theory of production with aggregative time series data for the Federal Republic of Germany, 1950 to 1973. Using these models we have derived tests of the theory of production that do not impose restrictions on patterns of substitution implied by the assumption of additivity and homogeneity. We have accepted the hypothesis that marginal productivity functions and supply and demand functions are generated by profit maximization. Thus we can impose the restrictions implied by the theory of production on our econometric models and can proceed conditionally on the validity of the theory of production to test restrictions on the forms of the production and price functions. Our objective is to employ these econometric models to develop tests of restrictions for characterizing the structure of technology empirically.

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* Inst. für Gesellschafts- und Wirtschaftswissenschaften der Universität Bonn, Adenauerallee 24 - 42, D-5300 Bonn.

** Havard University, Dept. of Economics, 1737 Cambridge Street, Cambridge (Mass.) 02138, USA.

1 See *Conrad* and *Jorgenson* (1978).

2 Translog production and price functions were introducted by *Christensen, Jorgenson,* and *Lau* (1971, 1973). The approach to technical change presented below is due to *Jorgenson* and *Lau* (1978). An analogous approach to analyzing the structure of consumer preferences and changes in preferences over time is given by *Jorgenson* and *Lau* (1975).

A complete model of production includes a production function and necessary conditions for producer equilibrium giving relative prices as a function of net outputs and time. Our econometric model of production corresponds to a translog representation of these marginal productivity functions. The model consists of a system of equations giving the value shares and the rate of technical change as functions of the quantities and time. The system of translog marginal productivity functions can be interpreted as a first-order approximation to the underlying marginal productivity functions. Under the restrictions implied by the theory of production the system of equations can be integrated to obtain the translog representation of the production function. This representation can be interpreted as a second-order approximation to the underlying production function.[3]

We consider the case of two outputs, consumption C and investment I, and two inputs, capital K and labor L. The corresponding prices are q_C, q_I, q_K and q_L. The translog representation of the production function F,

$$L = F\,(C, I, -K, t)\ , \tag{1}$$

then takes the form:

$$\begin{aligned} L = \exp\,[\alpha_0 &+ \alpha_C \ln C + \alpha_I \ln I + \alpha_K \ln K + \alpha_t \cdot t \\ &+ \frac{1}{2}\{\beta_{CC}(\ln C)^2 + \beta_{CI}\ln C \ln I + \beta_{CK}\ln C \ln K \\ &+ \beta_{IC}\ln I \ln C + \beta_{II}(\ln I)^2 + \beta_{IK}\ln I \ln K \\ &+ \beta_{KC}\ln K \ln C + \beta_{KI}\ln K \ln I + \beta_{KK}(\ln K)^2\} \\ &+ \beta_{Ct}\ln C \cdot t + \beta_{It}\ln I \cdot t + \beta_{Kt}\ln K \cdot t + \frac{1}{2}\beta_{tt}\cdot t^2]\ . \end{aligned} \tag{2}$$

The corresponding representation of the marginal productivity functions take the form:

$$\begin{aligned} w_C &= \frac{q_C\,C}{q_L\,L} = \alpha_C + \beta_{CC}\ln C + \beta_{CI}\ln I + \beta_{CK}\ln K + \beta_{Ct}\cdot t + \varepsilon_C\ , \\ w_I &= \frac{q_I\,I}{q_L\,L} = \alpha_I + \beta_{IC}\ln C + \beta_{II}\ln I + \beta_{IK}\ln K + \beta_{It}\cdot t + \varepsilon_I\ , \\ w_K &= -\frac{q_K\,K}{q_L\,L} = \alpha_K + \beta_{KC}\ln C + \beta_{KI}\ln I + \beta_{KK}\ln K + \beta_{Kt}\cdot t + \varepsilon_K\ , \\ w_t &= \frac{\partial \ln L}{\partial t} = \alpha_t + \beta_{tC}\ln C + \beta_{tI}\ln I + \beta_{tK}\ln K + \beta_{tt}\cdot t + \varepsilon_t\ . \end{aligned} \tag{3}$$

[3] For more detailed discussion, see *Jorgenson* and *Lau* (1978).

where

$$w_t = \frac{\dot{L}}{L} - \frac{q_C C}{q_L L}\frac{\dot{C}}{C} - \frac{q_I I}{q_L L}\frac{\dot{I}}{I} + \frac{q_K K}{q_L L}\frac{\dot{K}}{K}$$

is the rate of technical change. The rate of technical change is the rate of decline of the labor input with respect to time, holding quantities C, I and K constant.

The parameters of the translog production function can be identified with the coefficients in a Taylor's series expansion to the underlying production function F. They take the values of first- and second-order partial logarithmic derivatives of the underlying production function at the point of expansion $(C, I, K, t) = (1, 1, 1, 0)$. The restrictions on the parameters implied by the theory of production are as follows:

(4)
$$\alpha_C + \alpha_I + \alpha_K = 1 \ , \qquad \beta_{CK} + \beta_{IK} + \beta_{KK} = 0 \ ,$$
$$\beta_{CC} + \beta_{IC} + \beta_{KC} = 0 \ , \qquad \beta_{Ct} + \beta_{It} + \beta_{Kt} = 0 \ ;$$
$$\beta_{CI} + \beta_{II} + \beta_{KI} = 0 \ ,$$

and

(5)
$$\beta_{CI} = \beta_{IC} \ , \qquad \beta_{CK} = \beta_{KC} \ ,$$
$$\beta_{IK} = \beta_{KI} \ , \qquad \beta_{Ct} = \beta_{tC} \ ,$$
$$\beta_{It} = \beta_{tI} \ , \qquad \beta_{Kt} = \beta_{tK} \ ,$$

Under homogeneity of degree one of the translog production function the parameters of the value shares satisfy the restrictions (4). Conversely, if the parameters of the translog marginal productivity functions satisfy (4) and (5), these functions are homogeneous of degree zero and can be generated by a translog production function with homogeneity of degree one. The identity between the value of output and input,

$$q_C C + q_I I = q_K K + q_L L \ ,$$

implies that the value shares sum to unity, so that, given the parameters of any two equations for the value shares, the parameters of the third equation can be determined from the parameter restrictions under (4). To estimate the unknown parameters we combine the first two equations with the fourth for the rate of technical change. Unrestricted, there are 15 unknown parameters to be estimated from the three equations. Given the symmetry restrictions (5), there are nine unknown parameters to be estimated.

Under constant returns to scale this model implies the existence of a price function, defining the set of prices consistent with zero profits and the existence of conditions determining relative product and factor

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intensities as functions of prices and time.[4] The price function and these net supply functions are dual to the production function and the marginal productivity functions.[5] By exploiting the duality our second econometric model of production corresponds to the translog net supply functions which consist of a system of equations giving the value shares and the rate of technical change as functions of the prices of commodities and time. Again this system can be interpreted at a first-order approximation to the underlying net supply functions. Under the restrictions implied by the theory of production, accepted in our previous paper, this system of equations can be integrated to obtain the translog representation of the price function. This representation can be interpreted as a second-order approximation of the underlying price function P.

The translog representation of the price function P, here considered,

$$q_L = P\,(q_C, q_I, q_K, t) \;, \tag{6}$$

takes the form:

$$\begin{aligned} q_L = \exp\,[&\alpha_0 + \alpha_C + \alpha_I \ln q_I + \alpha_K \ln q_K + \alpha_t \cdot t \\ &+ \frac{1}{2}\,\{\beta_{CC}\,(\ln q_C)^2 + \beta_{CI} \ln q_C \ln q_C + \beta_{CK} \ln q_C \ln q_K \\ &+ \beta_{IC} \ln q_I \ln q_C + \beta_{II}\,(\ln q_I)^2 + \beta_{IK} \ln q_I \ln q_K \\ &+ \beta_{KC} \ln q_K \ln q_C + \beta_{KI} \ln q_K \ln q_I + \beta_{KK}\,(\ln q_K)^2\} \\ &+ \beta_{CT} \ln q_C \cdot t + \beta_{It} \ln q_I \cdot t + \beta_{Kt} \ln q_K \cdot t + \frac{1}{2}\,\beta_{tt} \cdot t^2] \;. \end{aligned} \tag{7}$$

The corresponding representation of the supply and demand functions take the form:

$$\begin{aligned} w_C &= \frac{q_C\,C}{q_L\,L} = \alpha_C + \beta_{CC} \ln q_C + \beta_{CI} \ln q_I + \beta_{CK} \ln q_K + \beta_{Ct} \cdot t \;, \\ w_I &= \frac{q_I\,I}{q_L\,L} = \alpha_I + \beta_{IC} \ln q_C + \beta_{II} \ln q_I + \beta_{IK} \ln q_K + \beta_{It} \cdot t \;, \\ w_K &= \frac{-\,q_K\,K}{q_L\,L} = \alpha_K + \beta_{KC} \ln q_C + \beta_{KI} \ln q_I + \beta_{KK} \ln q_K + \beta_{Kt} \cdot t \;, \\ -\,w_t &= \frac{\partial \ln q_L}{\partial\, t} = \alpha_t + \beta_{tC} \ln q_C + \beta_{tI} \ln q_I + \beta_{tK} \ln q_K + \beta_{tt} \cdot t \;. \end{aligned} \tag{8}$$

[4] The price function was introduced by *Samuelson* (1953) and has been discussed by *Burmeister* and *Kuga* (1970) and by *Christensen, Jorgenson,* and *Lau* (1973).

[5] A review of duality in the theory of production is given by *Diewert* (1974) and *Lau* (1974). See also: *Hotelling* (1932), *Jorgenson* and *Lau* (1974 a, 1974 b), *Samuelson* (1953), *Shephard* (1953, 1970), and *Uzawa* (1964).

We consider restrictions on patterns of substitution implied by separability. For each set of restrictions we derive the implications for the translog representation of the marginal productivity functions or supply and demand functions. A given set of restrictions on the underlying technology does not necessarily imply the corresponding set of restrictions on the translog representation, so that we distinguish two types of restrictions. First, the translog production and price functions may provide a representation of an underlying technology with a given set of restrictions. Second, the translog representation itself may be characterized by these restrictions.

II. Separability

We first consider restrictions on technology associated with groupwise separability of the production F given in (1). A production function F that is *groupwise separable* in outputs and inputs, for example, can be represented in implicit form as follows:

$$G(C, I, t) = H(L, K, t) \ .$$

We can write this production function in explicit form as follows:

$$L = F(G(C, I, t), -K, t) \ ,$$

where F is the production function and G the function independent of the inputs K and L.[6]

To derive restrictions on the parameters of the translog representation of a production function F that is groupwise separable we can differentiate the logarithm of the production function logarithmically with respect to the two outputs C and I:

[6] This definition of groupwise separability was introduced by *Jorgenson* and *Lau* (1978). The conventional definition of separability, due to *Leontief* (1947), is based on the explicit form of the production function. Under this definition the marginal rate of substitution between consumption and investment goods output is independent of capital input:

$$\frac{\partial}{\partial K} \frac{\frac{\partial F}{\partial C}}{\frac{\partial F}{\partial I}} = \frac{\partial}{\partial K} \frac{\frac{\partial G}{\partial C}}{\frac{\partial G}{\partial I}} = 0 \ .$$

This definition does not treat labor input symmetrically with the other variables in the production function. See also: *Goldman* and *Uzawa* (1964) and the references given there.

$$\frac{\partial \ln L}{\partial \ln C} = \frac{\partial \ln F}{\partial G} \frac{\partial G}{\ln C} = w_C \ , \tag{9}$$
$$\frac{\partial \ln L}{\partial \ln I} = \frac{\partial \ln F}{\partial G} \frac{\partial G}{\partial \ln I} = w_I \ .$$

Next, we differentiate logarithmically a second time with respect to capital K:

$$\frac{\partial^2 \ln L}{\partial \ln K \, \partial \ln C} = \frac{\partial^2 \ln F}{\partial \ln K \, \partial G} \frac{\partial G}{\partial \ln C} = \beta_{KC} \ , \tag{10}$$
$$\frac{\partial^2 \ln L}{\partial \ln K \, \partial \ln I} = \frac{\partial^2 \ln F}{\partial \ln K \, \partial G} \frac{\partial G}{\partial \ln I} = \beta_{KI} \ .$$

Given groupwise separability, equations (9) and (10) must hold everywhere; in particular they must hold at the point of approximation $(C, I, K, t) = (1, 1, 1, 0)$, where we can identify the first- and second-order partial derivatives with the parameters of the translog production function. We conclude that the parameters of the translog representation satisfy the restrictions:

$$\beta_{KC} = \varrho \, \alpha_C \ , \tag{11}$$
$$\beta_{KI} = \varrho \, \alpha_I \ ,$$

where ϱ is a constant given by

$$\varrho = \frac{\dfrac{\partial^2 \ln F}{\partial \ln K \, \partial G}}{\dfrac{\partial \ln F}{\partial G}}$$

at the point of expansion.

There are two more possible sets of groupwise separability restrictions. In a strictly analogous manner it can be shown that separability of $\{C, L\}$ from $\{I, K\}$ implies the following restrictions on the parameters of the translog production function:

$$\beta_{IC} = \varrho \, \alpha_I \ , \tag{12}$$
$$\beta_{KC} = \varrho \, \alpha_K \ ,$$

and separability of $\{C, K\}$ from $\{I, L\}$ implies:

$$\beta_{CI} = \varrho \, \alpha_C \ , \tag{13}$$
$$\beta_{KI} = \varrho \, \alpha_K \ .$$

Each set of restrictions involves two restrictions with the introduction of one new parameter. Given symmetry, the number of unknown parameters to be estimated under each set of groupwise separability restrictions is eight, one less than the number without restrictions.

Restrictions on the structure of technology do not necessarily imply the corresponding restrictions on the translog function itself. Therefore, the translog representation of a groupwise separable production function F is not necessarily groupwise separable. We distinguish between situations where the translog production function provides an approximation to the underlying production function with a certain property and situations where the translog production function also possesses that property. In the latter case we say that the translog production function possesses the property explicitly. For a translog production function to be explicitly groupwise separable in the pair of outputs $\{C, I\}$ and inputs $\{K, L\}$, it is necessary and sufficient that:

$$\varrho = 0 \ . \tag{14}$$

Under this restriction the parameters β_{KC} and β_{KI} in (11) are zero.

An example of a production function groupwise separable in outputs and inputs is a CET-CES production function:

$$(\delta_C C^{-\zeta_1} + \delta_I I^{-\zeta_1})^{-1/\zeta_1} = (\delta_K K^{-\zeta_2} + \delta_L L^{-\zeta_2})^{-1/\zeta_2} .$$

The translog approximation to this separable production function is not separable. A second example of a groupwise separable production function is a CET-Cobb-Douglas production function:

$$(\delta_C C^{-\zeta_1} + \delta_I I^{-\zeta_1})^{-1/\zeta_1} = K^{\alpha} L^{1-\alpha} .$$

The translog approximation to this separable production function is separable.

Similar restrictions must hold for $\{C, L\}$ separability from $\{I, K\}$ and for $\{K, C\}$ separability from $\{I, L\}$. Each of these restrictions is imposed, given the corresponding groupwise separability restriction, so that seven unknown parameters remain to be estimated. Similarly, a price function P is groupwise separable in outputs $\{C, I\}$ and inputs $\{K, L\}$ with prices $\{q_C, q_I\}$ and $\{q_L, q_K\}$ if and only if the price function can be represented in implicit form, as follows:

$$Q\,(q_C, q_I, t) = R\,(q_L, q_K, t) \ .$$

We can represent the price function in explicit form as follows:

$$q_L = P\,(Q\,(q_C, q_I, t), q_K, t) \ .$$

Restriction on the parameters of the translog representation of the price function P corresponding to groupwise separability in the prices $\{q_C, q_I\}$ and $\{q_K, q_L\}$ can be derived in the same way as given in (11). The translog representation of a groupwise separable price function P is not necessarily groupwise separable. The jointly necessary and sufficient conditions for groupwise separability of the translog price function are the condition (11) and the additional restriction for explicit groupwise separability, $\varrho = 0$. Similar restrictions must hold for $\{q_C, q_L\}$separability from $\{q_I, q_K\}$ and $\{q_C, q_K\}$ separability from $\{q_I, q_L\}$.

If the production function F is homogenous of degree one, groupwise separability of the production function in $\{C, I\}$ and $\{K, L\}$, for example, is equivalent to groupwise separability of the price function in the prices $\{q_C, q_I\}$ and $\{q_K, q_L\}$.[7] However, the translog representation of a groupwise separable production function is not necessarily groupwise separable; similarly, the translog representation of a groupwise separable price function, which corresponds to a groupwise separable production function, is not necessarily groupwise separable. We conclude that groupwise separability of the translog representation of the production function F does not imply groupwise separability of the translog representation of the price function P and vice versa.

III. Technical Change

We next present an approach to the characterization of changes of technology over time studied by *Jorgensen* and *Lau* (1978) and empirically implemented by them with data for the private domestic U.S. economy. In this approach time is treated symmetrically with inputs, outputs or prices in the description of the technology. To characterize changes in technology over time we employ restrictions on the production or price functions corresponding to separability in commodities and time.

We consider restrictions on technical change associated with groupwise separability of the production function F. We begin by considering a production function that is groupwise separable in the pair of outputs $\{C, I\}$ and the pair consisting of the dependent variable L and time $\{L, t\}$.[8] A production function that is separable in these two pairs of variables can be represented, implicity, in the form:

$$C\,(C, I, -K) = H\,(K, L, t) \quad .$$

[7] See *Jorgenson* and *Lau* (1978), Chapter 7.

[8] *Jorgenson* and *Lau* (1978) refer to groupwise separability involving time as *groupwise neutrality*.

In explicit form, the production function F can be represented as follows:

$$L = F\,(G\,(C, I, -K), -K, t)\ .$$

Proceeding as in our analysis of groupwise separability in two pairs of commodities, we can derive two restrictions on the parameters of the translog representation:

(15)
$$\beta_{Ct} = \varrho\,\alpha_C\ ,$$
$$\beta_{It} = \varrho\,\alpha_I\ ,$$

where:

$$\varrho = \frac{\dfrac{\partial^2 \ln F}{\partial t\ \partial G}}{\dfrac{\partial \ln F}{\partial G}}$$

Groupwise separability in the variables $\{C, t\}$ and $\{K, L\}$ implies the restrictions of the form:

(16)
$$\beta_{CK} = \varrho\,\alpha_C\ ,$$
$$\beta_{tK} = \varrho\,\alpha_t\ ,$$

where:

$$\varrho = \frac{\dfrac{\partial^2 \ln F}{\partial \ln K\ \partial G}}{\dfrac{\partial \ln F}{\partial G}}$$

We next consider restrictions on the parameters of the translog representation of the production function F, where the dependent variable L is not included in either of the two pairs of variables, say $\{C, I\}$ and $\{K, t\}$. As before, we can represent the production function, implicitly, in the form:

$$G\,(C, I, L) = H\,(K, L, t)\ ,$$

by definition of groupwise separability. By differentiating implicitly two restrictions can be derived on the parameters of the translog representation:

(17)
$$\alpha_K\,\beta_{Ct} - \alpha_t\,\beta_{CK} = \varrho\,\alpha_C\ ,$$
$$\alpha_K\,\beta_{It} - \alpha_t\,\beta_{IK} = \varrho\,\alpha_I\ ,$$

where:

$$\varrho = \frac{\dfrac{\partial \ln L}{\partial \ln K} \dfrac{\partial^2 H}{\partial t \, \partial \ln L} - \dfrac{\partial \ln L}{\partial t} \dfrac{\partial^2 H}{\partial \ln K \, \partial \ln L}}{\dfrac{\partial H}{\partial \ln L} - \dfrac{\partial G}{\partial \ln L}}$$

There are two more sets of restrictions similar to the one given above where L is not included in either one of the two groups considered.

Given symmetry, there are twelve possible sets of groupwise separability restrictions, involving time. Besides the three, already given under (15), (16) and (17) we obtain the following:

$\{C, K\}$ separable from $\{I, t\}$:

$$\alpha_I \beta_{Ct} - \alpha_t \beta_{CI} = \varrho \alpha_C \; , \qquad \alpha_I \beta_{Kt} - \alpha_t \beta_{IK} = \varrho \alpha_K \; ; \tag{18}$$

$\{C, K\}$ separable from $\{L, t\}$:

$$\beta_{Ct} = \varrho \alpha_C \; , \qquad \beta_{Kt} = \varrho \alpha_K \; ; \tag{19}$$

$\{C, L\}$ separable from $\{I, t\}$:

$$\beta_{CI} = \varrho \alpha_I \; , \qquad \beta_{Ct} = \varrho \alpha_t \; ; \tag{20}$$

$\{C, L\}$ separable from $\{K, t\}$:

$$\beta_{CK} = \varrho \alpha_K \; , \qquad \beta_{Ct} = \varrho \alpha_t \; ; \tag{21}$$

$\{C, t\}$ separable from $\{I, K\}$:

$$\alpha_I \beta_{CK} - \alpha_K \beta_{CI} = \varrho \alpha_C \; , \qquad \alpha_I \beta_{Kt} - \alpha_K \beta_{It} = \varrho \alpha_t \; ; \tag{22}$$

$\{C, t\}$ separable from $\{I, L\}$:

$$\beta_{CI} = \varrho \alpha_C \; , \qquad \beta_{It} = \varrho \alpha_t \; ; \tag{23}$$

$\{I, K\}$ separable from $\{L, t\}$:

$$\beta_{It} = \varrho \alpha_I \; , \qquad \beta_{Kt} = \varrho \alpha_K \; ; \tag{24}$$

$\{I, L\}$ separable from $\{K, t\}$:

$$\beta_{IK} = \varrho \alpha_K \; , \qquad \beta_{It} = \varrho \alpha_t \; ; \tag{25}$$

$\{I, t\}$ separable from $\{K, L\}$:

(26)
$$\beta_{IK} = \varrho\, \alpha_I \ ,$$
$$\beta_{Kt} = \varrho\, \alpha_t \ .$$

Each set of restrictions involves a set of two restrictions with the introduction of one new parameter. Under symmetry, there are eight unknown parameters to be estimated.

The translog representation of a production function F that is groupwise separable in two pairs of variables involving time is not necessarily groupwise separable in these same variables. For groupwise separability of the translog representation in the pairs of variables $\{C, I\}$ and $\{L, t\}$ a necessary and sufficient set of restrictions consists of the groupwise separability restrictions given under (15) together with the explicit separability restriction:

$$\varrho = 0 \ .$$

This implies that the parameters β_{Ct} and β_{It} are zero, so that the translog function is groupwise separable.

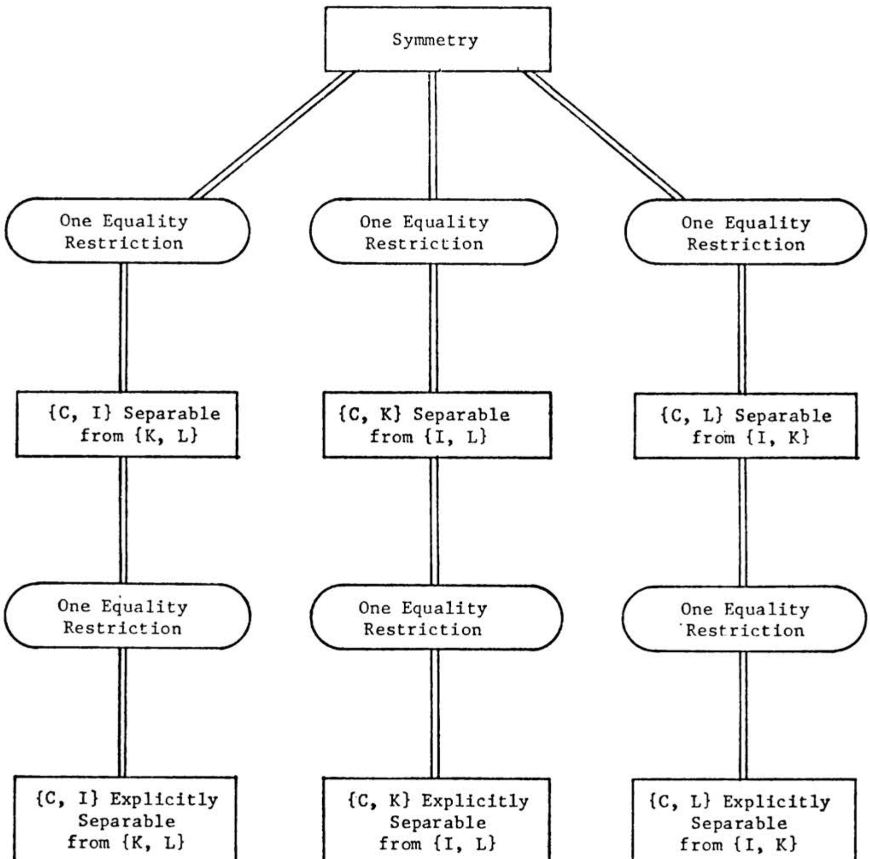
Given any set of groupwise separability restrictions involving time and the dependent variable L in the production function, there are nine possible sets of explicit groupwise separability restrictions of this type, corresponding to parts (15) - (26), excluding (17), (18) and (22) which do not involve L. Each of these restrictions is imposed given the corresponding groupwise separability restrictions, so that there are seven unknown parameters to be estimated. We will not focus attention on explicit separability in pairs of variables that exclude the dependent variable L; they can be generated by conjunction of sets of explicit groupwise separability restrictions already discussed.

Restrictions on technical change associated with groupwise separability of the price function P can be derived and tested exactly as in the model based on the translog representation of the marginal productivity functions. Groupwise separability of a price function P implies precisely analogous restrictions on the parameters of the translog price function.

IV. Tests

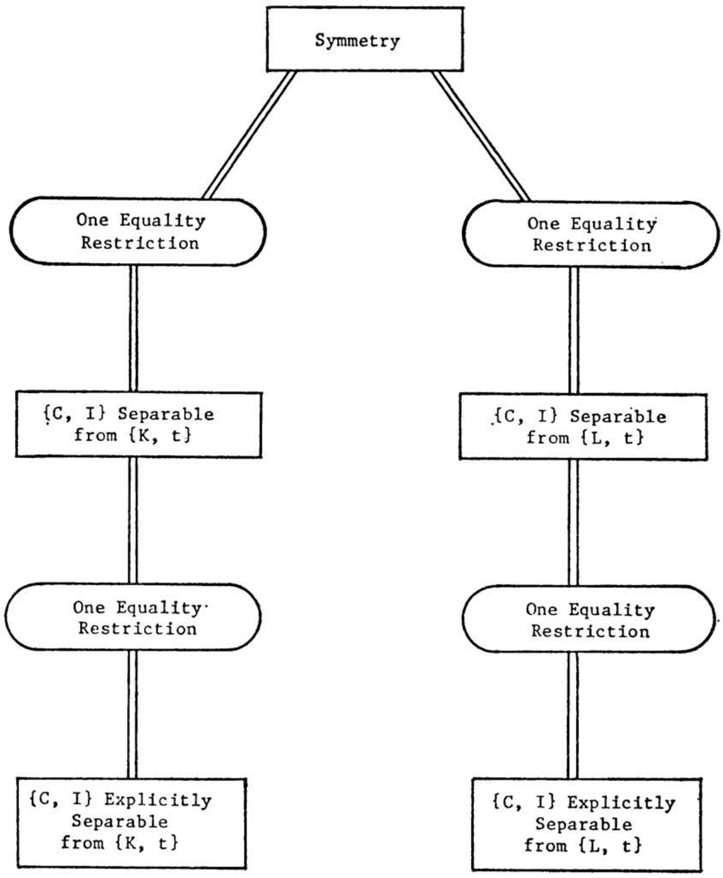
We have developed econometric models for characterizing the structure of technology and changes in technology over time. We propose to test restrictions derived from groupwise separability in commodities and time. Our proposed test procedure is presented in diagrammatic form in two figures. We first impose the symmetry restrictions implied by the theory of production. We then proceed to test the restrictions derived from groupwise separability of the production function in commodities. Given these restrictions, we proceed to test the additional restrictions implied by explicit groupwise separability. All three tests for groupwise separability are carried out in parallel. Each of the three tests for explicit groupwise separability is carried out given the corresponding

Figure 1: Tests of Groupwise Separability in Commodities



groupwise separability restrictions. The groupwise separability restrictions involve a set of two equality restrictions with the introduction of one new parameter. Given symmetry, this reduces the number of unknown parameters to be estimated by one. The explicit groupwise separability restrictions involve one additional equality restriction, leaving seven unknown parameters to be estimated.

Figure 2: Tests of Groupwise Separability in Time
(There are twelve sets of tests of this type; this diagram gives only two sets of such tests corresponding to the group $\{C, I\}$.)



Continuing with tests of groupwise separability of the production function in time, our test procedure is presented diagrammatically in Figure 2. We first test groupwise separability for each of the twelve possible groups consisting of one pair of commodities and one pair of a commodity and time. If we accept groupwise separability for any two pairs, we proceed to test explicit groupwise separability for these two pairs. All twelve tests for groupwise separability in time are carried out in parallel.

To dualize this analysis we observe that a precisely parallel test procedure can be developed for the price function with analogous tests of the restrictions on the parameters of the translog representation of the price function.

V. Estimation and Test Statistics

Our empirical results are based on the same annual time series data for the private domestic economy of the Federal Republic of Germany for the period 1950 - 1973 as those employed in our paper on tests of the theory of production.[9] As the value shares sum to unity, only two of their random variables are distributed independently. We have fitted the two equations for the value shares of consumption and investment and one equation for the rate of technical change generated by translog representation of production and price functions.[10] As we impose the symmetry restrictions our estimates of the unknown parameters satisfy these restrictions. In Table 1 we present estimates of the unknown parameters associated with restrictions implied by groupwise separability. Parameter estimates for the translog representation of the price function are given in Table 2. We give the estimates only for those specifications which we will discuss in the last section.

To test the validity of restrictions implied by groupwise separability of production and price functions in commodities and in time, we employ test statistics based on the likelihood ratio λ, where

$$\lambda = \frac{\max\limits_{\omega} L}{\max\limits_{\Omega} L}$$

The likelihood ratio is the ratio of the maximum value of the likelihood function L for the econometric model of production ω, subject to restriction to be tested, to the maximum value of the likelihood function for the model Ω without restriction.

There are 24 observations for the period 1950 - 1973 for each behavioral equation so that the number of degrees of freedom available for statistical tests of restrictions on the structure of technology and changes in

[9] See *Conrad* and *Jorgensen* (1975), p. 80, Table 20, Column 4 (L) and Column 5 (q_L); p. 70, Table 14, Column 4 (q_C), Column 5 (C), Column 7 (q_I) and Column 8 (I); p. 51, Table 9, Column 3 (K) and Column 4 (q_K) (q_K is normalized to one in 1962 and real capital input is scaled to real property income). These data were employed by *Conrad* (1975).

[10] Our estimator is based on the method of maximum likelihood presented by *Malinvaud* (1970).

Table 1: **Parameter estimates, translog production function**

Parameter	1. Symmetry	2. $\{C, I\}$ explicitly separable from $\{K, L\}$	3. $\{C, I\}$ explicitly separable from $\{K, t\}$	4. $\{C, I\}$ separable from $\{L, t\}$	5. $\{C, L\}$ separable from $\{K, t\}$	6. $\{C, t\}$ separable from $\{K, L\}$	7. $\{I, L\}$ explicitly separable from $\{K, t\}$
α_C	.999 (.005)	.995 (.004)	.993 (.005)	.999 (.005)	.997 (.005)	.996 (.005)	.996 (.005)
β_{CC}	.507 (.123)	.401 (.029)	.362 (.018)	.366 (.047)	.30 (.039)	.241 (.052)	.65 (.091)
β_{CI}	− .385 (.051)	− .402 (.029)	− .418 (.021)	− .329 (.025)	− .343 (.047)	− .339 (.051)	− .478 (.015)
β_{CK}	− .122 (.094)	—	.056 (.007)	− .036 (.064)	.043 (.010)	.098 (.011)	− .173 (.088)
β_{Ct}	.008 (.002)	.006 (.0008)	.005 (.0007)	.005 (.0008)	.004 (.0009)	.0028 (.001)	.011 (.002)
α_I	.64 (.003)	.639 (.002)	.64 (.002)	.64 (.003)	.64 (.003)	.64 (.003)	.642 (.002)
β_{II}	.408 (.028)	.402 (.027)	.401 (.027)	.387 (.024)	.396 (.028)	.396 (.03)	.478 (.015)
β_{IK}	− .023 (.036)	—	.016 (.005)	− .058 (.026)	− .053 (.034)	− .057 (.035)	—
β_{It}	.002 (.0009)	.0018 (.0006)	.0015 (.0005)	.003 (.0005)	.003 (.001)	.003 (.001)	—
α_K	− .639 (.005)	− .635 (.005)	− .632 (.0053)	− .639 (.005)	− .637 (.005)	− .636 (.005)	− .639 (.005)
β_{KK}	.145 (.083)	—	.176 (.069)	.942 (.073)	.01 (.043)	− .04 (.039)	.173 (.084)
β_{Kt}	− .01 (.0017)	− .008 (.0007)	− .0065 (.0006)	− .008 (.001)	− .007 (.0006)	− .006 (.0006)	− .011 (.002)
α_t	− .058 (.005)	− .058 (.005)	− .058 (.005)	− .058 (.005)	− .058 (.005)	− .059 (.005)	− .058 (.005)
β_{tt}	.0019 (.0007)	.0019 (.0007)	.0019 (.0007)	.002 (.0007)	.0018 (.0007)	.0018 (.0007)	.002 (.0007)
ϱ		—	—	.005 (.0007)	− .067 (.015)	.098 (.01)	

Table 2: **Parameter estimates, translog price function**

Parameter	1. Symmetry	2. $\{C, K\}$ separable from $\{I, L\}$	3. $\{C, L\}$ explicitly separable from $\{K, t\}$	4. $\{C, t\}$ separable from $\{K, L\}$
α_C	.993 (.007)	.994 (.006)	1.003 (.008)	.992 (.006)
β_{CC}	.651 (.18)	.592 (.118)	.345 (.169)	.677 (.149)
β_{CI}	− .538 (.17)	− .470 (.070)	− .345 (.169)	− .541 (.149)
β_{CK}	− .112 (.07)	− .122 (.058)	—	− .136 (.013)
β_{Ct}	− .003 (.0008)	− .003 (.004)	—	− .003 (.006)
α_I	.638 (.006)	.636 (.004)	.632 (.006)	.638 (.006)
β_{II}	.25 (.164)	.172 (.025)	.125 (.171)	.24 (.152)
β_{IK}	.287 (.052)	− .298 (.045)	.221 (.034)	.301 (.036)
β_{It}	.011 (.0006)	.011 (.0006)	.009 (.170)	.011 (.0006)
α_K	− .63 (.004)	− .631 (.004)	− .635 (.004)	− .631 (.004)
β_{KK}	− .175 (.046)	− .176 (.043)	− .221 (.178)	− .165 (.037)
β_{Kt}	− .008 (.0005)	− .008 (.0005)	− .009 (.0004)	− .008 (.0005)
α_t	.058 (.005)	.058 (.005)	.058 (.005)	.058 (.005)
β_{tt}	− .0018 (.0007)	− .0018 (.0007)	− .0016 (.0007)	− .0018 (.0007)
ϱ		− .473 (.072)	—	−. 137 (.013)

technology is 72 for either model. For normally distributed disturbances, the likelihood ratio is equal to the ratio of the determinant of the restricted estimator of the variance-covariance matrix of the disturbances to the determinant of the unrestricted estimator, each raised to the power — $(n/2)$. Our test statistic for each set of restrictions is based on minus twice the logarithm of the likelihood ratio, or:

$$-2 \ln \lambda = n \left(\ln |\hat{\Sigma}_\omega| - \ln |\hat{\Sigma}_\Omega|\right) ,$$

where $\hat{\Sigma}_\omega$ is the restricted estimator of the variance-covariance matrix and $\hat{\Sigma}_\Omega$ is the unrestricted estimator. Under the null hypothesis this test statistic is distributed, asymptotically, as chi-squared with number of degrees of freedom equal to the number of restrictions to be tested.

To control the overall level of significance for each series of tests of the production and price representation, we set the level of significance for each series at .05. We assign a level of significance of .01 to tests of groupwise separability in commodities. To tests of groupwise separability in time we assign a level of significance of .04. Within our set of tests for groupwise separability in commodities, we can distinguish two stages: groupwise separability for each of the three possible groups and explicit groupwise separability for each of these groups. We assign a level of significance of .00167 to each of these six tests. Similarly, within our two stages of tests for groupwise separability in time we assign a level of significance of .02 to groupwise separability and .00167 to each of the twelve tests at that stage. Similarly, we assign a level of significance of .02 to groupwise explicit separability and .00167 to each of the twelve tests at that stage.

In our complete series of tests for econometric models of production based on translog production and price functions, only tests of groupwise separability and groupwise explicit separability are "nested", none of the other tests are "nested" so that the sum of levels of significance for all tests provides an upper bound to the overall level of significance for all of these tests considered simultaneously.

VI. Conclusion

Our objective has been to develop tests of restrictions for characterizing the structure of technology and changes in technology over time. For econometric models of production based on translog representations of the marginal productivity and supply and demand functions, we have assigned levels of significance to each of our tests of hypotheses about the structure of technology and changes in technology over time so as to control the overall level of significance for all tests at .95. The probability of a false rejection for one test among the collection of tests is less than or equal to .05. With the aid of critical values for our test statistics given in Table 3, we can evaluate the results of our tests given in Table 4. If the test statistic for one of the hypotheses summarized in Table 4 is larger than its corresponding critical value, given in Table 3, we reject the hypothesis at the assigned level of significance.

Table 3

Critical values of χ^2

Degrees of freedom	Level of significance				
	.10	.05	.01	.005	.001
1	2.71	3.84	6.64	7.88	10.83

We first test groupwise separability in commodities under the translog representation of a production function. The results of our tests of groupwise separability in commodities for each possible group are presented in Table 4. Our first conclusion is that the production func-

Table 4

Test statistics for translog production and price functions

Hypothesis	Degrees of freedom	Production	Price
Given symmetry			
Groupwise separability in commodities			
$\{C, I\}$, $\{K, L\}$,	1	.56	15.33
$\{C, K\}$, $\{I, L\}$,	1	49.59	.26
$\{C, L\}$, $\{I, K\}$,	1	15.29	12.13
Groupwise separability in time			
$\{C, I\}$, $\{K, t\}$,	1	.84	24.05
$\{C, I\}$, $\{L, t\}$,	1	1.86	52.67
$\{C, K\}$, $\{I, t\}$,	1	37.06	26.99
$\{C, K\}$, $\{L, t\}$,	1	31.37	76.40
$\{C, L\}$, $\{I, t\}$,	1	41.37	11.47
$\{C, L\}$, $\{K, t\}$,	1	3.67	4.79
$\{C, t\}$, $\{I, K\}$,	1	14.32	13.20
$\{C, t\}$, $\{I, L\}$,	1	36.34	21.27
$\{C, t\}$, $\{K, L\}$,	1	6.17	.15
$\{I, K\}$, $\{L, t\}$,	1	10.96	14.09
$\{I, L\}$, $\{K, t\}$,	1	1.17	34.73
$\{I, t\}$, $\{K, L\}$,	1	12.7	32.37
Given separability in commodities			
Explicit groupwise separability in commodities			
$\{C, I\}$, $\{K, L\}$,	1	2.95	10.07
$\{C, K\}$, $\{I, L\}$,	1	7.82	27.27
$\{C, L\}$, $\{I, K\}$,	1	23.39	.06
Given separability in time			
Explicit groupwise separability in time			
$\{C, I\}$, $\{K, t\}$,	1	6.4	13.26
$\{C, I\}$, $\{L, t\}$,	1	29.6	23.92
$\{C, K\}$, $\{I, t\}$,	1	16.85	10.33
$\{C, K\}$, $\{L, t\}$,	1	.11	0.46
$\{C, L\}$, $\{I, t\}$,	1	.68	7.92
$\{C, L\}$, $\{K, t\}$,	1	17.50	9.29
$\{C, t\}$, $\{I, K\}$,	1	6.86	2.98
$\{C, t\}$, $\{I, L\}$,	1	79.90	53.15
$\{C, t\}$, $\{K, L\}$,	1	43.86	61.27
$\{I, K\}$, $\{L, t\}$,	1	20.52	62.50
$\{I, L\}$, $\{K, t\}$,	1	6.09	35.20
$\{I, t\}$, $\{K, L\}$,		13.57	34.92

tion is groupwise separable in outputs $\{C, I\}$ and inputs $\{K, L\}$. Our test reveals that the usual presentation of a production function with separability in inputs and outputs is a valid specification which we can not reject with German data. After having accepted groupwise separability we can proceed to test explicit groupwise separability of outputs $\{C, I\}$ from inputs $\{K, L\}$; we accept this hypothesis.

To obtain further simplifications in our representation of technology we next test groupwise separability in time. From the results in Table 4 we observe that we accept groupwise separability in time for the pair $\{C, I\}$ from $\{L, t\}$ and explicit groupwise separability in time for the pair $\{C, I\}$ from $\{K, t\}$. Our results are consistent with groupwise separability of the two outputs $\{C, I\}$ from the group consisting of the two inputs and time $\{K, L, t\}$. This implies that we can construct an index of real output from price and quantity data on consumption and investment goods output. Output can be represented as a function of the two inputs and time.

Continuing with our analysis of groupwise separability in time, we accept groupwise separability for the pair $\{K, L\}$ from $\{C, t\}$. However, we reject groupwise separability of the pair $\{K, L\}$ from $\{I, t\}$, so that our results are not consistent with groupwise separability of the two inputs $\{K, L\}$ from the groups consisting of the two outputs and time $\{C, I, t\}$. We conclude that we cannot construct an index of real input from price and quantity data on capital and labor input. Equivalently, we conclude that technical change is not Hicks-neutral.

Finally, we accept groupwise separability for the pair $\{C, L\}$ from $\{K, t\}$ and explicit groupwise separability for the pair $\{I, L\}$ from $\{K, t\}$, so that our results are consistent with groupwise separability of the group $\{K, t\}$ from the group $\{C, I, L\}$. This implies that we can construct an index of the two outputs and labor input from price and quantity data on these commodity groups. Under this restriction technical change is Solow neutral. We conclude that our results are consistent with either of two simplifications of our representation of technology, namely, groupwise separability of the group of two outputs $\{C, I\}$ from the group of two inputs and time $\{K, L, t\}$ or groupwise separability of the group $\{C, I, L\}$ from the group $\{K, t\}$.

The results of our tests of separability in goods and time for the translog price function are consistent with groupwise separability of the group $\{C, K\}$ from $\{I, L\}$, explicit groupwise separability of the group $\{C, L\}$ from $\{K, t\}$, and groupwise separability of the group $\{K, L\}$ from $\{C, t\}$. No combination of these restrictions implies simplifications of our representation of technology similar to those we obtained for the translog production function.

The result that the price function is not groupwise separable in the input prices from the output prices is no contradiction to the result we have obtained under the translog representation of the production function. Even if the underlying price function is separable, the test results obtained under the translog production function can differ from the test results obtained under the translog price function because the translog price function is not dual to the translog production function. If we know the true specification of the underlying separable production function and the true specification of the underlying price function then both functions must be separable in the same partitioning.

Summary

The purpose of this paper is to employ econometric models of production to develop tests of parametric restrictions for characterizing the structure of technology and changes in technology empirically. Our models are based on the translog production function in two outputs and two inputs and the translog price function in the corresponding prices. We consider restrictions on patterns of substitution and technical change implied by separability. We present empirical tests of each set of restriction for time series data of the Federal Republic of Germany for the period 1950 - 1973.

Zusammenfassung

Das Ziel dieses Beitrages ist die Verwendung eines ökonometrischen Produktionsmodells zur Konstruktion von Tests von Parameterrestriktionen für die Charakterisierung der Struktur der Technologie und ihrer zeitlichen Änderung. Die Modelle basieren auf der Translog Produktionsfunktion in zwei Gütern und zwei Faktoren und der Translog Preisfunktion in den entsprechenden Preisen. Es werden Parameterrestriktionen unter der Hypothese der Separierbarkeit betrachtet. Mit Zeitreihen für die Bundesrepublik Deutschland führen wir empirische Tests der einzelnen auferlegten Restriktionen durch.

References

Burmeister, E. and K. *Kuga* (1970), The Factor-Price Frontier, Duality, and Joint Production, The Review of Economic Studies 37 (1970), S. 11 - 19.

Christensen, L. R., D. W. *Jorgenson* and L. J. *Lau* (1971), Conjugate Duality and the Transcendental Logarithmic Production Function, Econometrica 39 (1971), S. 255 - 256.

— (1973), Transcendental Logarithmic Production Frontiers, Review of Economics and Statistics 55 (1973), S. 28 - 45.

Conrad, K. (1975), Optimierung und Simulation mit einem Nichtlinearen Makromodell für die Bundesrepublik Deutschland, Meisenheim am Glan 1975 (Verlag Anton Hain).

Conrad, K. and D. W. *Jorgenson* (1975), Measuring Performance in the Private Economy of the Federal Republic of Germany, 1950 - 1975, Tübingen 1975 (Mohr (Siebeck)).

Conrad, K. and D. W. *Jorgenson* (1978), Tests of a Model of Production for the Federal Republic of Germany, 1950 - 1973, European Economic Review, forthcoming.

Diewert, W. E. (1974), Applications of Duality Theory, in: M. D. *Intriligator* and D. A. *Kendrick* (Hrsg.), Frontiers of Quantitative Economics, Amsterdam 1974 (North-Holland Bd. 2).

Goldman, S. M. and H. *Uzawa* (1964), A Note on Separability in Demand Analysis, Econometrica 32 (1964), S. 387 - 399.

Hotelling, H. (1932), Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions, Journal of Political Economy 40 (1932), S. 577 - 616.

Jorgenson, D. W. and L. J. *Lau* (1974 a), Duality and Differentiability in Production, Journal of Economic Theory 9 (1974), S. 23 - 42.

— (1974 b), The Duality of Technology and Economic Behaviour, Review of Economic Studies 41 (1974), S. 181 - 200.

— (1975), The Structure of Consumer Preferences, Annals of Economic and Social Measurement 4/1 (1975), S. 49 - 101.

— (1978), Duality and Technology, Amsterdam 1978 (North-Holland) forthcoming.

Lau, L. J. (1974), Applications of Duality Theory: Comment, in: M. D. Intriligator and D. A. Kendrick (Hrsg.), Frontiers of Quantitative Economics, Amsterdam 1974 (North-Holland Bd. 2).

Leontief, W. W. (1947), Introduction to a Theory of Internal Structure of Functional Relationships, Econometrica 15 (1947), S. 361 - 373.

Malinvaud, E. (1970), Statistical Methods of Econometrics, Amsterdam 1970 (North-Holland).

Samuelson, P. A. (1953), Prices of Factors and Goods in General Equilibrium, Review of Economic Studies 21 (1953 - 1954), S. 1 - 20.

Shephard, R. W. (1953), Cost and Production Functions, Princeton 1953 (Princeton University Press).

— (1970), Theory of Cost and Production Functions, Princeton 1970 (Princeton University Press).

Uzawa, H. (1964), Duality Principles in the Theory of Cost and Production, International Economic Review 5 (1964), S. 216 - 220.