Vierteljahrshefte zur Wirtschaftsforschung
70. Jahrgang, Heft 3/2001
S. 339–351

Markov-Switching Procedures for Dating the Euro-Zone Business Cycle

By Hans-Martin Krolzig*

Summary

This paper addresses the issues of identification and dating of the Euro-zone business cycle by using the Markov-switching approach innovated by Hamilton in his analysis of the US business cycle. Regime shifts in the stochastic process of economic growth in the Euro-zone are identified by fitting Markov-switching models to aggregated and single-country Euro-zone real GDP growth data of the last two decades. The models are found to be statistically congruent and economically meaningful. Based of the smoothed regime probabilities from the Markov-switching models the Euro-zone business cycle is dated and recessions from 1980Q1 to 1981Q1 and 1992Q3 to 1993Q2 are revealed. A Markov-switching vector autoregression of real GDP growth rates in eight EMU member states shows that while the business cycles in the Euro-zone have not been perfectly synchronized over the last two decades, the overall evidence for the presence of a common Euro-zone cycle is strong.

1. Introduction

The advent of Monetary Union in Europe has established a new currency, the Euro, and a new central bank, the European Central Bank (ECB). On 1 January 1999, the third and final stage of the European Monetary Union (EMU) commenced with the irrevocable fixing of the exchange rates of the currencies of the eleven member states initially participating in monetary union and with the conduct of a single monetary policy under the responsibility of the ECB. The number of participating member states increased to twelve on 1 January 2001, when Greece entered the third stage of EMU.

The constitution of the EMU has raised several interesting issues. Among them, one of paramount relevance concerns the existence of a common cycle among the member countries. A lack of business cycle synchronization could complicate the operation of monetary policy in the union and constitutes a negative indicator for joining the EMU. On the other hand it has been argued recently that the formation of a monetary union in itself creates a tendency for business cycle symmetry to emerge. If this condition holds for the EMU and the quasi-union of the Exchange Rate Mechanism of the European Monetary System, then we might expect already to be able to find an emergent "Euro-zone cycle" which will become more dominant in future years. The coincidence of business cycle turning points in the countries of the Euro-zone would provide a strong indication for the existence of a common Euro-zone cycle.

This paper deals with the existence, identification and dating of the Euro-zone business cycle. We use the approach innovated by Hamilton in his analysis of the U.S. business cycle to identify regime shifts in the stochastic process of economic growth in the Euro-zone over the last two decades. The first aim of the paper is to formulate econometric models of aggregated Euro-zone real GDP growth data, which allow the dating of the Euro-zone business cycle based of the smoothed regime probabilities implied by the model. The second aim is to investigate the degree of business cycle synchronization in the Euro zone by modelling a Markov-switching vector autoregression of real GDP growth rates in eight EMU member states.

The paper proceeds as follows: Section 2 introduces the underlying econometric methodology: the Markov-switching time series model. The data analysed in this paper are real gross domestic product (GDP) time series for the Euro-zone: we first consider aggregated GDP data, then data for eight EMU member states; Section 3 discusses the details. In Section 4 a univariate Markov-switching model in the tradition of Hamilton (1989) is used

* Department for Economics and Nuffield College, Oxford University, e-mail: hans-martin.krolzig@nuf.ox.ac.uk

for the statistical assessment of the business cycle in the Euro zone. We then move to the multivariate case and present the results for a Markov-switching vector autoregression. The investigation of real GDP growth in eight EMU member states presented in Section 5 shows the synchronization of the business cycles in the Euro-zone towards a common cycle, which supports the notion of a Euro-zone business cycle. The results are compared to the business cycle datings in the relevant literature in Section 6. Finally, Section 7 concludes.

2. Methodology

Recent theoretical and empirical business cycle research has revived interest in the co-movement of macroeconomic time series and the regime-switching nature of macroeconomic activity. The general idea behind regime-switching models of the business cycle is that the parameters of a time series model of some macroeconomic variables depend upon a stochastic, unobservable regime variable $s_t \in \{1, \ldots, M\}$ which represents the state of business cycle. The number of regimes, M, is often assumed to be two reflecting economic expansions and contractions.

The Markov-switching autoregressive time series model has emerged as a leading approach for the detection and dating of business cycle turning points. Since Hamilton's 1989 application of this technique to measure the U.S. business cycle, there has been a number of subsequent extensions and refinements (see Krolzig and Lütkepohl, 1995, and Krolzig, 1997b, for an overview).

In the original contribution of Hamilton (1989), contractions and expansions are modeled as switching regimes of the stochastic process generating the growth rate of real output Δy_t:

$$\Delta y_t - \mu(s_t) = \alpha_1(\Delta y_{t-1} - \mu(s_{t-1})) + \ldots + \alpha_4(\Delta y_{t-4} - \mu(s_{t-4})) + u_t \quad (1)$$

In (1), the two regimes are associated with different conditional distributions of the growth rate of real output, where the mean growth rate μ depends on the state or "regime", s_t. For a meaningful business cycle model, μ_1 should be negative in the first regime ("recession") and positive in the second regime ("expansion"), $\mu_2 > 0$. The variance of the disturbance term, $u_t \sim \text{NID}(0, \sigma^2)$, is assumed to be the same in both regimes.

The stochastic process generating the unobservable regimes is an ergodic Markov chain defined by the transition probabilities:

$$p_{ij} = Pr(s_{t+1} = j \mid s_t = i), \quad \sum_{j=1}^{M} p_{ij} = 1 \quad \forall i, j \in \{1, \ldots, M\} \qquad (2)$$

In the case of a two-regime business cycle model, two transition probabilities are involved

p_{12} = Pr (recession in t | expansion in $t-1$),

p_{21} = Pr (expansion in t | recession in $t-1$),

and have to be estimated together with the parameters of equation (1).

Maximum likelihood (ML) estimation of the model can be based on a version of the Expectation-Maximization (EM) algorithm discussed in Hamilton (1990). By inferring the probabilities of the unobserved regimes conditional on an available information set, it is then possible to reconstruct the regimes. Hamilton was able to show that his model is able to track the NBER reference cycle very closely. In Section 4 we will apply Hamilton's approach to an aggregated Euro-zone real GDP time series to identify the turning points of the Euro-zone business cycle.

It is important to note that, by definition, univariate Markov-switching models as proposed by Hamilton (1989) are only able to capture some of the stylized facts of the business cycle. They can represent the non-linearity or asymmetry stressed in some part of the literature but, obviously, they are unable to reflect the idea of comovement among time economic series. Since business cycle symmetry in the Euro-zone is an important indicator for the optimality of a single monetary policy, the synchronization of the business cycle in the EMU member states deserves careful screening. Our methodology for assessing the synchronization of the business cycle in the Euro-zone (Section 5) will be based on the generalization of Hamilton's model to a Markov-switching vector autoregressive model.

A Markov-switching vector autoregression (MS–VAR) is a vector autoregressive process where some of the parameters are subject to Markovian regime shifts. As we focus on modelling the business cycle we only consider shifts in the vector of mean growth rates $\mu(s_t)$:

$$\Delta y_t - \mu(s_t) = A_1(\Delta y_{t-1} - \mu(s_{t-1})) + \ldots + A_p(\Delta y_{t-p} - \mu(s_{t-p})) + u_t, \quad (3)$$

where the innovations u_t are conditionally Gaussian, $u_t \mid s_t \sim \text{NID}(0, \Sigma)$. A Markov-switching vector autoregressive model of order p and M regimes will be denoted MS(M)–VAR(p): This multivariate extension of Hamilton's original model is able to represent the non-linear, regime switching and the common factor structures of the business cycle simultaneously. It can be easily extended for the presence of cointegration (see Krolzig, 1996, for the statistical analysis of these systems and Krolzig, 2001, for applications to business cycle research).

The MS–VAR in (3) characterizes business cycles as common regime shifts in the stochastic process of some macroeconomic time series. Modelling a vector of time series does not only correspond to the definition of the business cycle, but does also improve the inferences of the Markov process to extract the common "business cycle" component from the group of economic time series if the business cycle is a common feature of theses variables. The regime inference with the MS-VAR model is described in the appendix.

3. Data

In the following we apply the Markov-switching approach to the Euro-zone real GDP data. For the analysis with aggregated Euro-zone data, we use the BDH data set of Beyer, Doornik and Hendry (2001) for the historical pre-EMU period. The time series is constructed from data for Austria (AT), Belgium (BE), Germany (DE), Finland (FI), France (FR), Italy (IT), Ireland (IE), The Netherlands (NL), Portugal (PT) and Spain (ES). Data for Luxembourg and Greece (which joined in 2001) are not included. The BDH data set starts in 1979 Q4 and ends in 1998 Q1. So we extended the time series by calculating the growth rate of Euro-zone GDP in constant prices based on the reported figures in the ECB Monthly Bulletin from May 2001. We will then extend the analysis to the disaggregated level looking at a system with eight out of the ten countries mentioned before, where the data after 1998 Q1 have been drawn from the Economist Intelligence Unit (EIU) country database employing the following sources: Österreichisches Institut für Wirtschaftsforschung (AT), Banque Nationale de Belgique (BE), OECD (FI), INSEE (FR), Deutsche Bundesbank (DE), INDS (IT), Central Bureau voor de Statistiek (NL) and the Ministerio De Economia Y Hacienda (ES). The growth rates for AT and FI have been seasonally adjusted using data from 1993 Q1 onwards. Graphs of the aggregated data and of the country data can be found in Figures 1 and 5, respectively.

4. The Euro-Zone Business Cycle

We start by applying Hamilton's approach to the aggregated Euro-zone real GDP. An MS(2)–AR(1) model with shifts in the mean growth rate of real Euro-zone DGP, Δy_t is considered:

$$\begin{aligned}\Delta y_t - \mu(s_t) = \alpha(\Delta y_{t-1} - \mu(s_{t-1})) + \gamma D84q2_t \\ + \delta D87q1_t + u_t, \quad u_t \sim NID(0, \sigma^2)\end{aligned} \tag{4}$$

where *D84q2* and *D87q1* are dummy variables being 1 in the period indicated and –1 in the subsequent period. A first-order model is the preferred one among MS(2)–AR*(p)* processes with $0 \le p \le 5$ (see Table 1).

The estimated parameters of the MS(2)–AR(1) model using data from 1980 Q2 to 2000 Q4 are presented in Table 2. The estimated annualised growth rate is –1.68% in recessions and 2.48% in expansions. The dummies are found to be highly significant. The transition matrix is given by

$$P = \begin{bmatrix} 0.7722 & 0.0151 \\ 0.2278 & 0.9849 \end{bmatrix}.$$

where $P_{ji} = Pr(s_t = j \mid s_{t-1} = i) = p_{ij}$. From the estimated transition probabilities follow the following measures of the persistence of recession: the expected number of quarters a recession prevails (duration) and the unconditional (ergodic) probability of recessions. Whereas recessions (regime 1) have a duration of 4.39 quarters, expansions (regime 2) have a duration of 15 years. The last figure is surprisingly high, but it reflects the fact that within the sample periods only one transition from regime 2 to regime 1 has been identified. The unconditional probability of a recession is 0.062.

Table 2

ML estimates 1980 Q2 to 2000 Q4

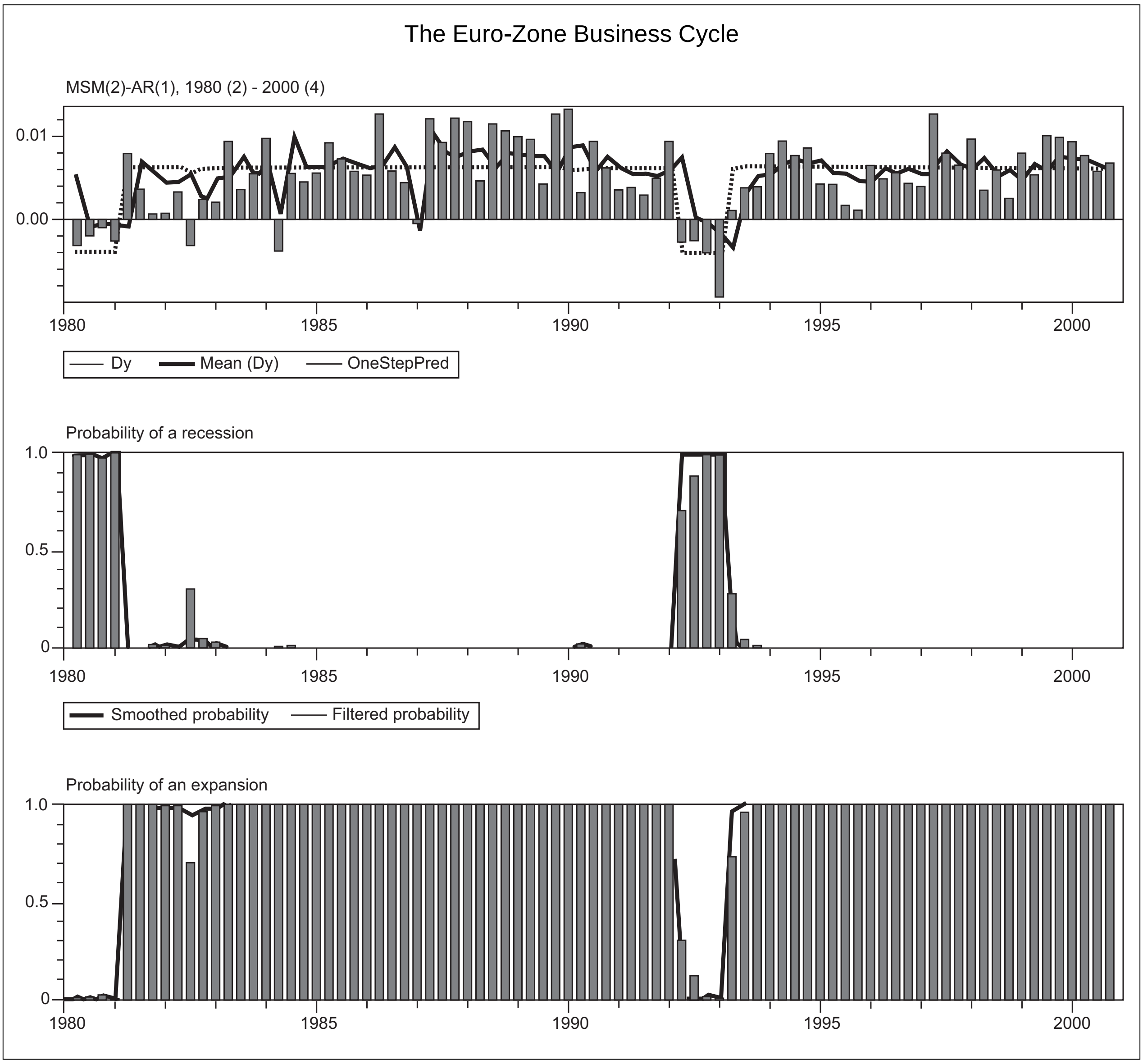
		Coefficient	Std. error	t-value
Mean growth rate				
Recession	μ_1	–0.0042	0.0017	–2.4408
Expansion	μ_2	0.0062	0.0009	6.8624
Short-run dynamics				
Δy_1	α	0.3567	0.0823	4.3329
D84q2	γ	–0.0070	0.0023	–3.0637
D87q1	δ	–0.0071	0.0022	–3.2018

Table 1

Lag order selection 1981 Q2 to 2000 Q4

p	0	1	2	3	4	5
lnL	330.380	334.666	335.402	335.796	336.717	338.147
AIC criterion	–8.187	–8.270*	–8.263	–8.248	–8.246	–8.257
HQ criterion	–8.103	–8.174*	–8.155	–8.128	–8.114	–8.113
SC criterion	–7.977	–8.030*	–7.993	–7.948	–7.916	–7.897
LR test vs. *p* = 1	8.572		1.472	2.261	4.103	6.963
Marg. rejection prob.	0.0034		0.2251	0.3229	0.2505	0.1379

Figure 1



The resulting regime probabilities are plotted in the lower two panels of Figure 1. The filtered regime probabilities are shown with bars and the smooth probabilities are shown with a bold line. The filtered probability can be understood as an optimal inference on the state variable (whether the system is in a boom or recession) at time t using only the information up to time t, i. e. $Pr(s_t = m \mid Y_t)$, where m stands for a given regime. The smoothed probability stands for the optimal inference on the regime at time t using the full sample information Y_T: $Pr(s_t = m \mid Y_T)$. The time paths of the smoothed and filtered probabilities can be used to date the Euro-zone business cycle. The classification of the regimes and the dating of the business cycle amounts to assigning an observation y_t at time t to the regime m with the highest probability. The resulting dating of the business cycle turning points will be discussed in Section 6.

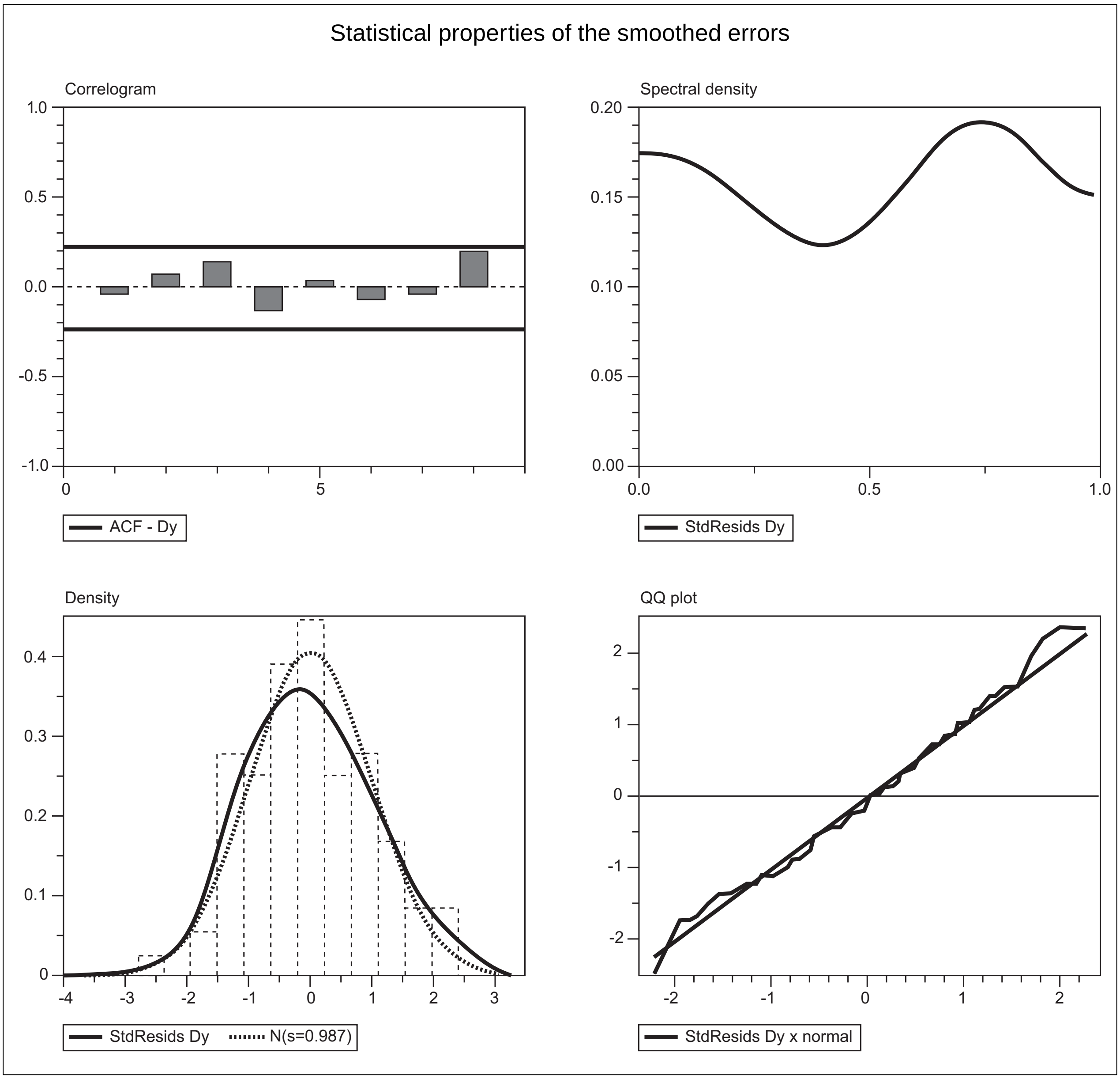
We found that the model is a congruent statistical representation of the structure of the data. The statistical properties of the residuals are visualized in Figure 2. In the smoothed and standardized Gaussian errors $\tilde{u}_t/\sigma$ associated with the MS(2)–AR(1),

$$\tilde{u}_t = \sum_{i=1}^{2} \sum_{j=1}^{2} \{(\Delta y_t - E[\Delta y_t \mid s_t = j, s_{t-1} = i, Y_{t-1}, \ldots]) \, Pr(s_t = j, s_{t-1} = i \mid Y_T)\},$$

there is no significant autocorrelation and non-normality left.

Table 3 compares the MS model to the nested linear AR(1) model. Testing for the number of regimes in an MS–VAR model is a difficult enterprise. Conventional testing approaches are not applicable due to the presence of unidentified nuisance parameters under the null of linearity.

Figure 2



Since the test hypothesis is $\mu_1 = \mu_2$, the transition probabilities p_{12} and p_{21} are not identified. Formal tests of the Markov-switching model against linear alternatives employing the standardized LR test designed to deliver (asymptotically) valid inference have been proposed by Hansen (1992, 1996) and Garcia (1998): Hansen's approach delivers a bound on the asymptotic distribution of the standardized LR test. The test is conservative, tending to be under-sized in practice and of low power, and computationally demanding. The Monte Carlo experiments in Ang and Bekaert (1998) suggest that the true underlying distribution can be approximated by a $\chi^2(q)$ distribution where q is the number of linearly independent restrictions and nuisance variables under the null. The likelihood ratio statistic of 12.7373 for the test hypothesis $\mu_1 = \mu_2$ appears to support the presence of regime shifts. Using the $\chi^2(3)$ distribution, the null of linearity is rejected at a marginal significance level of 0.0052.

Table 3

Estimation statistics

	MS(2)–AR(1)	Linear AR(1)	MS(3)–AR(1)
lnL	350.9494	344.5808	353.1533
σ	0.00311	0.00390	0.00299
no. parameters	8	5	13
AIC criterion	–8.2638*	–8.1827	–8.1965
HQ criterion	–8.1702*	–8.1241	–8.0443
SC criterion	–8.0307	–8.0370*	–7.8176

DOI https://doi.org/10.3790/vjh.70.3.339

To investigate the robustness of this results further, we therefore consider an encompassing approach, and enter the filtered probability of a recession ξ_t into the linear AR(1) model. As the filtered probability $\xi_t = Pr(s_t = 1 \mid Y_t; \lambda)$ is correlated with y_t we use Instrument Variable Estimation (IVE) with ξ_{t-1} as the instrument:

$$\Delta y_t = \underset{(0.000749)}{0.0041} + \underset{(0.101)}{0.339}\Delta y_{t-1} - \underset{(0.00231)}{0.00703}\, D87q1_t \\ - \underset{(0.00239)}{0.00686}\, D84q2_t - \underset{(0.00207)}{0.00584}\xi_t . \tag{5}$$

The probability of being in a recession ξ_t is found to be highly significant stressing the importance of the found business cycle transitions.

5. Are the Business Cycles in the Euro-Zone Synchronized?

The constitution of the EMU has raised the question of a common cycle among the member countries. A lack of business cycle synchronization within the Euro area could complicate the operation of monetary policy in the union and would constitute a negative indicator for a country joining EMU. Therefore, findings of business cycle synchronism have decisive policy implications. In the following we analyse whether the business cycles in the Eurozone are synchronized.

Despite the importance of the transmission of shocks across countries, various concepts of common features and the recent appreciation of empirical business cycle research, there has been little attempt to investigate international business cycles with modern non-linear time series models. Still, most studies consider business cycle phenomena for individual countries only. First attempts at the analysis of international business cycles with Markov-switching models have been undertaken by Phillips (1991), Filardo and Gordon (1994) and Krolzig (1997a). Phillips's study of a two-country two-regime models was the very first multivariate Markov-switching analysis of all. Filardo and Gordon (1994) have extended his analysis to a trivariate two-regime model by using leading indicators for the prediction of turning points. In this paper we follow the approach proposed in Krolzig (1997a), stressing the importance of a data-driven model specification which enables us to derive new and economically meaningful results.

Table 4 gives an overview on the recent literature on MS-models of the European business cycle. Artis, Krolzig and Toro (1999) investigate the existence and identification of a European business cycle analyzing monthly industrial production data of nine EU countries from 1970 M01 to 1996 M12. Two important issues arise: (i) the convergence process of Southern Europe and (ii) the secular decline of the mean growth rates in the post-Bretton Woods era. Analysing GDP data for six EU countries from 1970 Q3 to 1995 Q4, Krolzig (2001) concludes that two-regime models representing contractions and expansions are inconsistent with these two stylised facts of the post-war economic history of Western Europe. Krolzig and Toro (2000) compare the "classical" approach proposed in Burns and Mitchell (1946) of dating and analysing the business cycle with its "modern" alternative: the Markov-switching approach. By using the model's regime probabilities as an optimal statistical inference of the turning point of the European business cycle, they demonstrate the capacity of the MS–VAR approach to generate the stylised facts of the *classical* cycle in Europe. These studies have in common the presence of a third regime, which however can not be found after 1980. It can therefore be assumed that for data beyond that period, two-regime models are an adequate description of the business cycle (see Figure 3 for the regime classification in Krolzig, 2001). Indeed, for quarter-to-quarter growth rates of detrended industrial production in seven Euro-area countries of the period 1978 Q4 to 1998 Q4, Peersman and Smets (2001) find that a two-regime MS–VAR delivers reasonable results.

Table 4

MS-studies of the European Business Cycle

Model		Data	*M*	Sample period	AT	BE	DE	ES	FI	FR	IE	IT	NL	PT	UK
MS–AR (table 2)		GDP	2	1980 Q2–2000 Q4	·	·	·	·	·	·	·	·	·	·	–
MS–VAR (table 6)		GDP	2	1980 Q2–2000 Q4	+	+	+	+	+	+	–	+	+	–	–
Peersman and Smets (2001)	(PS)	IIP	2	1978 Q4–1998 Q4	+	+	+	+	–	+	–	+	+	–	–
Artis, Krolzig and Toro (1999)	(AKT)	IIP	3	1970 M01–1996 M12	+	+	+	+	–	+	–	+	+	+	+
Krolzig and Toro (2000); Krolzig (2001)	(KT)	GDP	3	1970 Q3–1995 Q4	+	–	+	+	–	+	–	+	–	–	+

Legend:

M	number of regimes	+	country is included.
IIP	index of industrial production	·	country is included as component of an index.
GDP	gross domestic product (real)	–	country is excluded.

Figure 3

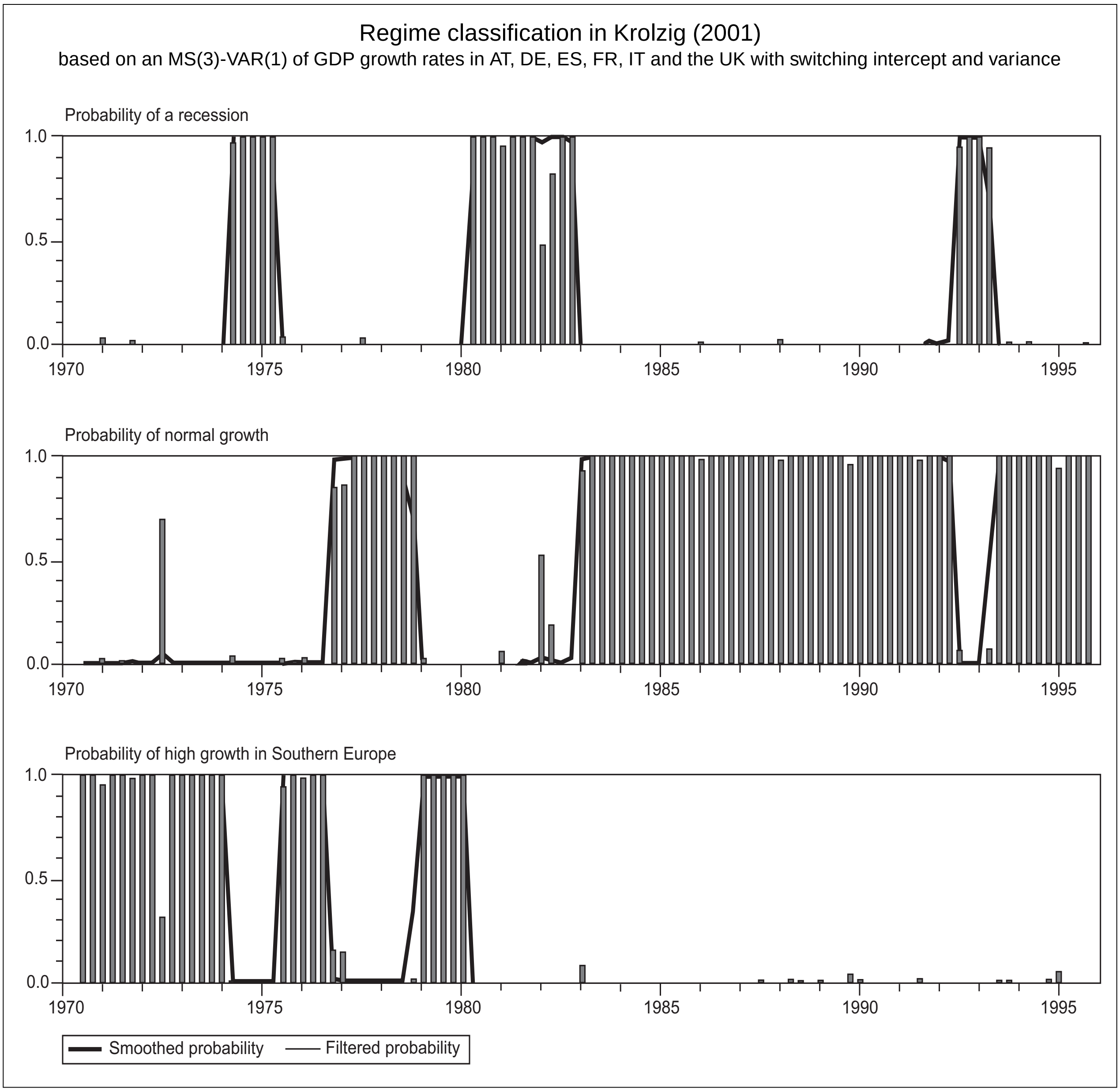
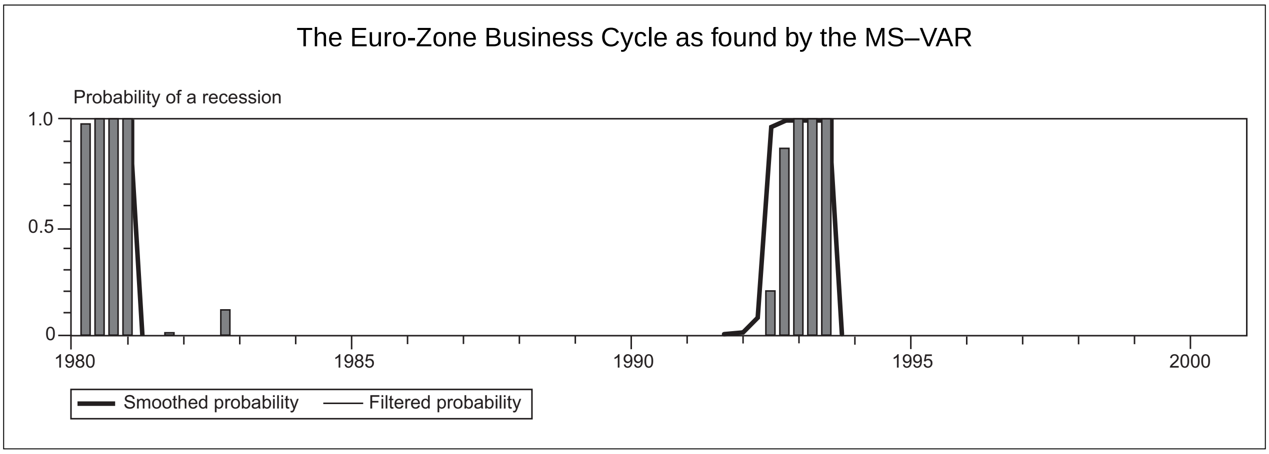


Figure 4



In spirit of the findings in the literature, we consider a multivariate extension of the MS(2)–AR(1) model analysed previously. Since a cointegration analysis gave no clear indication of the presence of a cointegrating vector, all variables are modelled in first differences: Δy_t is the vector of GDP growth rates of Austria (AT), Belgium (BE), Germany (DE), Finland (FI), France (FR), Italy (IT), The Netherlands (NL), and Spain (ES). The data for Ireland (IE) and Portugal (PT) are of poor quality and excluded from the analysis. We are again interested in shifts in the mean growth rate of real DGP representing turning points of the Euro-zone business cycle:

$$\Delta y_t - \mu(s_t) = A(\Delta y_{t-1} - \mu(s_{t-1})) + \delta D87q1_t + u_t, \quad u_t \sim NID(0, \Sigma) \tag{6}$$

where $s_t \in \{1, 2\}$ is generated by a hidden Markov chain. As in (4) D87q1 again is 1 in 1987 Q1 and –1 in 1987 Q2. The dummy D84q2 was found to be insignificant. The two regimes are distinguished by the vectors μ_1 and μ_2 of regime-conditional mean growth rates of Dy_t. Based on conventional information criteria (see Table 5), an MS(2)–VAR(0) could be selected. But the hypothesis $A = 0$ is clearly rejected in a likelihood ratio test, $\chi^2(64) = 166.914$ [0.0000]. So we work with a first-order process as in (4).

The Maximum Likelihood estimates of the MS(2)–VAR(1) model using data from 1980 Q2 to 2000 Q4 are reported in Table 6. The mean annualised growth rate in economic expansions lies between 1.72% (BE) and 2.96% (ES): The picture is more diverse for Euro-zone recessions: while BE, DE, ES, FR, IT, NL are subject to contractions of real GDP, AT and FI show only signs of growth recessions with lower, but still positive growth rates. For none of the countries $\mu_{\kappa 2}$ is significantly different from zero. The transition matrix is given by

$$P = \begin{bmatrix} 0.8015 & 0.0138 \\ 0.1985 & 0.9862 \end{bmatrix}.$$

The regimes are again persistent: the average duration of a recession is 5.04 quarters, which is not much different from the results in Section 4.

The contribution of the common business cycle to the process of economic growth in the eight Euro-zone countries is depicted in Figure 5. Major differences in the mean growth rate across regimes are evident. But there are also differences with regard to the countries. For example, some countries recover faster from the 1980/81 recession than others.

Table 5

Lag order selection 1981 Q2 to 2000 Q4

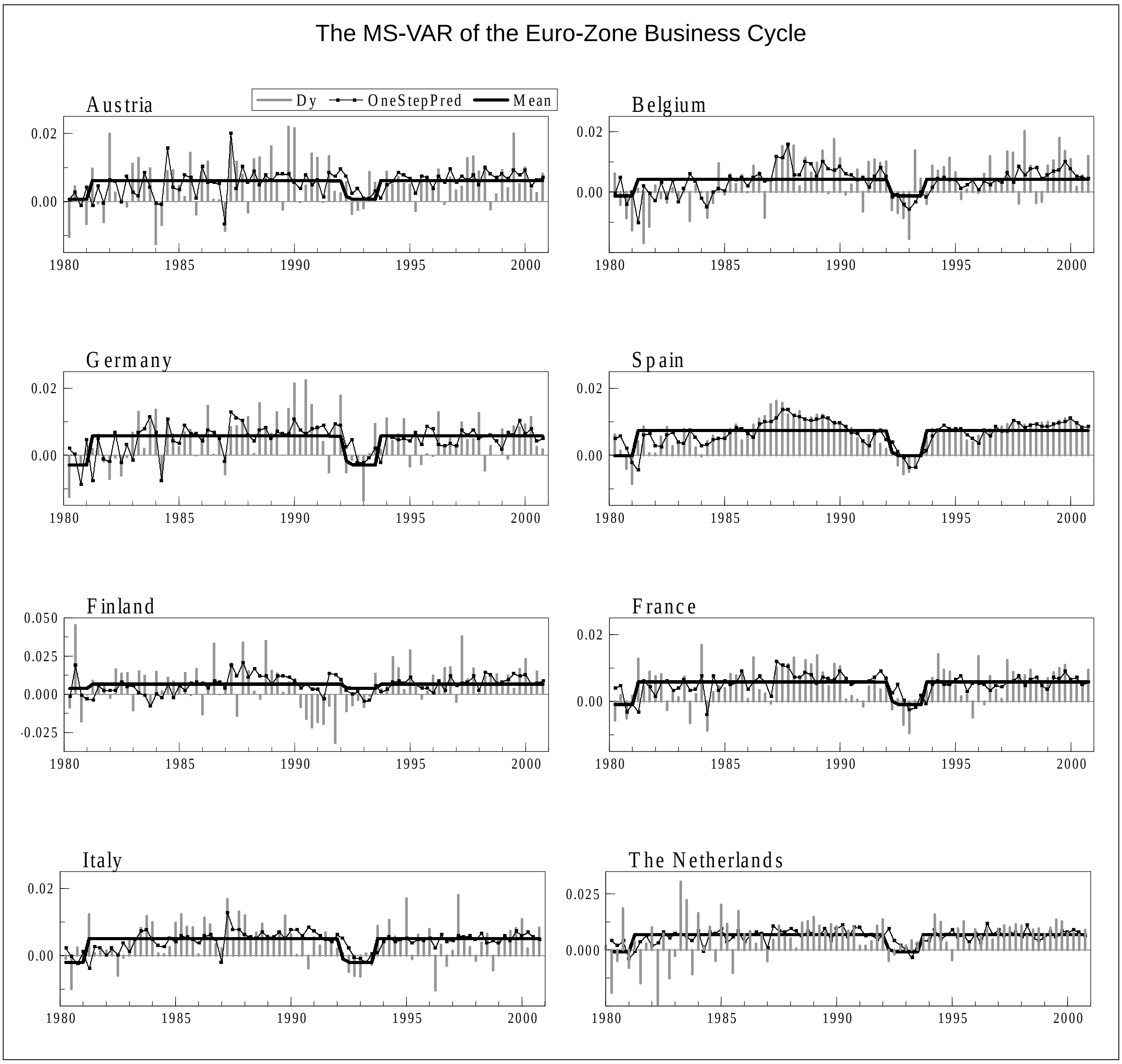
p	0	1	2	3	4	5
lnL	2353.675	2437.132	2487.432	2593.339	2653.625	2735.102
AIC criterion	–58.017	–58.51	–58.163	–59.224	–59.130	–59.572*
HQ criterion	–57.272*	–56.996	–55.880	–56.172	–55.309	–54.982
SC criterion	–56.158*	–54.731	–52.464	–51.606	–49.592	–48.115

Table 6

ML estimates 1980 Q2 to 2000 Q4

	AT	BE	DE	ES	FI	FR	IT	NL
Mean growth rate								
Recession	0.0002	–0.0019	–0.0032	–0.0005	0.0047	–0.0014	–0.0023	–0.0006
Expansion	0.0063	0.0043	0.0058	0.0074	0.0067	0.0059	0.0051	0.0068
Short-run dynamics								
AT_1	–0.1224	0.1170	0.2257	–0.0156	–0.2218	0.1225	0.0509	0.0725
BE_1	0.1599	0.0665	0.0687	0.0444	0.4109	0.0870	0.0328	0.1469
DE_1	–0.1810	–0.1848	–0.1193	–0.0679	–0.3073	–0.1076	0.0976	0.1841
ES_1	0.2199	0.8973	0.3498	0.6864	0.9093	0.3321	0.2754	0.1545
FI_1	–0.0504	–0.1221	–0.0982	–0.0008	–0.1460	–0.0687	–0.0336	0.0387
FR_1	0.0306	0.1127	–0.3911	0.1087	0.4538	–0.0615	–0.1742	–0.2040
IT_1	–0.1190	–0.0033	0.3445	0.0406	–0.1145	0.1623	0.0907	–0.1088
NL_1	0.2018	0.1242	0.1233	0.0033	–0.1020	–0.0879	0.0684	–0.1191
D87q1	–0.0115	–0.0027	–0.0074	0.0012	–0.0011	–0.0039	–0.0078	–0.0038
Standard errors								
σ	0.0063	0.0067	0.0057	0.0022	0.0131	0.0044	0.0046	0.0082

Figure 5



In equation (6) we impose the rather strong restriction which requires that all countries switch regimes simultaneously. This design of the model allows the identification and analysis of the latent Euro-zone cycle, while it restricts the propagation of country-specific shocks to the vector autoregression. In other words, the hidden Markov process is not informative about the propagation of business cycles across countries. The structure, however, allows that some countries (but not all) are unaffected by shifts in regime. A more flexible approach can be found in Phillips (1991) and Hamilton and Lin (1996) where some variables might precede others in their cycle or different variables follow completely different Markov processes. Given the relatively large number of countries considered and our special interest in the phenomenon of a *common* Euro-zone cycle, modeling separate Markov processes for each country is not very appealing. In other instances, however, it may be appropriate to allow more than one state variable, such that each variable may respond to a specific state variable. Considering two-country two-regime models of monthly growth rates of industrial production in the US, the UK, Germany and Japan, Phillips (1991) assumes separate Markov chains for each equation. Interestingly, in none of the estimated models the null hypothesis of perfectly correlated regime shifts could be rejected which supports the presumptions made here.

In the following we test the hypothesis of a common Euro-zone cycle by looking at the significance of the regime shifts in the mean growth rate of GDP in the Euro-zone countries. Under the null hypothesis, the mean growth rate of country k in recessions (regime 1) is identical to its growth rate in expansions (regime 2): $\mu_{\kappa 1} = \mu_{\kappa 2}$ for $\kappa = 1, \ldots, 8$. Since the regime-dependent means in the remaining equations of the system are unrestricted, the statistical identification of the model is ensured under the null hypothesis. The tests are nuisance parameter free. Therefore, classical likelihood theory can be invoked, and the asymptotic null distribution of the Wald test is $\chi^2(r)$ where $r = 1$ is the number of linearly independent restrictions.

Table 7

Wald tests of the regime-invariance hypothesis

$\mu_{\kappa 1} = \mu_{\kappa 2}$

Country	Test statistic	$\chi^2(1)$
AT	4.642	[0.0312] *
BE	3.365	[0.0666]
DE	10.333	[0.0013] **
ES	27.499	[0.0000] **
FI	0.116	[0.7333]
FR	13.248	[0.0003] **
IT	11.917	[0.0006] **
NL	4.375	[0.0365] *

Table 7 reports the results of the Wald specification tests. The test hypothesis $\mu_{k1} = \mu_{k2}$ is strongly rejected for Germany (DE), Spain (ES), France (FR), Italy (IT). The common cycle is less pronounced in Austria (AT) and the Netherlands (NL) where the hypothesis can still be rejected for 5% critical values, and in Belgium (BE) where the marginal significance level is 6.7%. Its weakest link is Finland where the differences in the mean in the two regimes are clearly insignificant and, hence, the common cycle appears to be irrelevant. These findings are in line the results of Peersman and Smets (2001) when analysing Euro-zone industrial production data. Overall, the notion of a common Euro-zone business cycle is justified.

6. Dating Business Cycle Turning Points in the Euro-zone

In Markov-switching models, the classification of the regimes and the dating of the business cycle amounts to assigning every observation y_t to a given regime $m = 1, \ldots, M$. The rule that is applied here is to assign the observation at time t according to the highest smoothed probability, i. e.:

$$m^* = \arg\max_m Pr(s_t = m \mid Y_T)$$

At every point in time, a smoothed probability of being in a given regime is calculated (the inference is made using the whole set of data points), and we will assign that observation to a given regime according to the highest smoothed probability. For the simplest case of two regimes, the rule reduces to assigning the observation to the first regime if $Pr(s_t = 1 \mid Y_T) > 0.5$ and assigning it to the second regime if $Pr(s_t = 1 \mid Y_T) < 0.5$. The latter procedure allows a corresponding dating of the Euro-zone business cycle which is given in Table 8. The peak date denotes the period t just before the beginning of a recession, i. e. $Pr(s_t = 1 \mid Y_T) < 0.5$ and $Pr(s_{t+1} = 1 \mid Y_T) > 0.5$; the trough is the last period of the recession.

The resulting regime classification is independent of the weight of any country in the index. Scaling one of the countries would result in the same regime classification. Note, however, that larger countries tend to be associated with less noisy time series than smaller countries. Fluctuations in the former countries will deliver clearer signals and, therefore, will have a stronger impact on the regime probabilities. It is important to stress the fact that the models considered here are not addressing the issue of which countries drive the Euro-zone cycle but whether that cycle can be extracted and dated.

In Table 8 we also compare the datings of the Euro-zone business cycle identified in this paper with European business cycle datings in the literature (see Table 4) as far as they are also based on the Markov-switching approach. The most striking result is that the business cycle classifications based on our aggregated and multivariate analysis are almost identical which might indicated the quality

Table 8

Datings of the Euro-Zone Business Cycle

MS–AR			MS–VAR			PS			AKT			KT		
Peak	Trough	h	Peak	Trough	h	Peak	Trough	h	Peak	Trough	h	Peak	Trough	h
									1974 M7	1975 M7	1.00	1974 Q1	1975 Q2	1.25
[1980 Q1]	1981 Q1	1.00	[1980 Q1]	1981 Q1	1.00	1979 Q4	1983 Q1	4.25	1979 M10	1982 M8	2.83	1980 Q1	1982 Q4	2.75
						1985 Q4	1987 Q1	1.25						
1992 Q1	1993 Q1	1.00	1992 Q2	1993 Q3	1.25	1990 Q1	1992 Q3	2.50	1990 M9	1992 M9	2.00	1992 Q2	1993 Q2	1.00
						1995 Q2	1996 Q1	0.75						

Notes: [1980 Q1] is the period before the beginning of the sample; h is the duration of recessions.

of the BDH data. The major difference with regard to the results of Artis, Krolzig and Toro (1999), Krolzig and Toro (2000) and Krolzig (2001) is the relative short duration of recessions. As these papers find some indication for the missing business cycle synchronization the prolonged duration of recessions may reflect the need to subsume the UK and continental cycle. The outlier in Table 8 is marked by the study of Peersman and Smets (2001). As they use detrended data, the common cycle they identify has all the characteristics of a *growth* cycle. As highlighted by Table 8, their growth recessions have much a longer duration and are found much more frequently. The interest shown by academics in the *growth* cycle at the expense of the *classical* cycle, has been questioned in a number of recent papers (Pagan, 1997a, 1997b; Harding and Pagan, 2001), which stress the dominating relevance of the classical cycle to policy makers and the business community.

7. Conclusions

In this paper we used the approach innovated by Hamilton in his analysis of the US business cycle to identify a common business cycle in the Euro-zone. The analysis consisted in fitting a Markov-switching model to (i) aggregated Euro-zone real GDP growth data and (ii) the system of real GDP growth rates of the EMU member states. The models are found to be statistically congruent and economically meaningful.

The regime identification distinguishes between recessions and expansion in the Euro-area. For the sample period, 1980 Q2 to 2000 Q4, both models identify two recessions: The peaks of the cycle are identified as 1980 Q1 which is just the period before the start of the sample and 1992 Q2. The troughs are 1981 Q1 and 1993 Q2/1993 Q3. While the business cycles in the Euro-zone have not been perfectly synchronized over the last two decades (especially the Finish economy appears to be driven by a cycle of its own), the overall evidence for the presence of a common Euro-zone cycle is strong. This suggests that the conception of a common Euro-zone business cycle is an intelligible one.

In view of the criticisms that can be directed to conventional methods of business cycle identification, and more especially, in view of the policy significance of the type of results obtained, it is important to supplement those methods by others. In particular, findings of business cycle synchronism are an important indicator of the optimality of monetary union and deserve careful screening. The findings in this paper contribute to that end. Further research is required towards a rigorous analysis of the similarities in business cycle features among EMU member states in order to provide a deeper understanding of the interdependence in macroeconomic activity in the Euro-zone necessary to devise economic policies.

8. Appendix: Regime Inference in Markov-switching VAR Models

In the following, we give a brief introduction to the calculation of the filtered and smoothed regime probabilities essential for the dating of the business cycle with Markov-switching VAR models. Recall equation (3) as the VAR of y_t conditional on the regime vector $(s_t, \ldots, s_{t-p})$ and the definition of the hidden Markov chain in equation (2) as the transition equation of the regime vector.

By invoking the law of Bayes, the (posterior) probability $Pr(s_t, \ldots, s_{t-p} | Y_t)$ of the regime vector $(s_t, \ldots, s_{t-p})$ conditional on all available information at time t is given by

$$Pr(s_t, \ldots, s_{t-p} | y_t, Y_{t-1}) = \frac{p(y_t | s_t, \ldots, s_{t-p}, Y_{t-1})\, Pr(s_t, \ldots, s_{t-p} | Y_{t-1})}{p(y_t | Y_{t-1})} \tag{7}$$

where $p(y_t | s_t, \ldots, s_{t-p}, Y_{t-1})$ is the probability density of observing y_t conditional on the regime vector $(s_t, \ldots, s_{t-p})$ as defined in (3),

$$Pr(s_t, \ldots, s_{t-p} | Y_{t-1}) = \sum_{s_{t-p-1}=1}^{M} Pr(s_t | s_{t-1})\, Pr(s_{t-1}, \ldots, s_{t-p\,1} | Y_{t-1})$$

is the (prior) probability of the regimes $(s_t, \ldots, s_{t-p})$ given the information set of the previous period and $p(y_t | Y_{t-1})$ is the marginal density of y_t given that information set Y_{t-1}:

$$\begin{aligned} p(y_t | Y_{t-1}) &= \textstyle\sum_{s_t} \cdots \sum_{s_{t-p}} p(y_t, s_t, \ldots, s_{t-p} | Y_{t-1}) \\ &= \textstyle\sum_{s_t} \cdots \sum_{s_{t-p}} p(y_t | s_t, \ldots, s_{t-p}, Y_{t-1})\, Pr(s_t, \ldots, s_{t-p} | Y_{t-1}). \end{aligned}$$

Using (7) the filtered regime probabilities for a sample $Y_T = (y_T, \ldots, y_1)$ can be calculated by a forward recursion for $t = 1, \ldots, T$ initialized by $(s_0, \ldots, s_{1-p})$. Suppose, for example, $p = 0$ and $M = 2$ where one regime represents "recessions" and the other "expansions", then the filter recursion can be simplified to the following odds ratio of the regimes:

$$\frac{\text{Pr}(\text{‘recession at time } t\text{’} | Y_t)}{\text{Pr}(\text{‘expansion at time } t\text{’} | Y_t)} =$$

$$\frac{\text{p}(y_t | \text{‘recession’})}{\text{p}(y_t | \text{‘expansion’})} \frac{\text{Pr}(\text{‘recession at time } t\text{’} | Y_{t-1})}{\text{Pr}(\text{‘expansion at time } t\text{’} | Y_{t-1})}.$$

The probability of a recession given all available information at time t depends (i) on the likelihood of observing y_t in a recession relatively to an expansion and (ii) on the predicted probability of a recession based on the information set Y_{t-1} of the previous period. The regime inference can improved by using future observations in which case the resulting regime regime probabilities, $Pr(s_t | Y_\tau)$ with $\tau > t$, are called "smoothed" (see Kim, 1994, for an algorithm).

Acknowledgements

Financial support from the UK Economic and Social Research Council under the grant L138251009 is gratefully acknowledged. I am also grateful to Andreas Beyer and Jurgen Doornik for providing me with their BDH data. All the computations reported in this paper were carried with the MSVAR class for Ox (see Krolzig, 1998). Helpful comments were received from the seminar audience at the DIW Workshop "Measurement Problems of Business Cycles in the EMU", Berlin, 2001, and an unknown referee.

References

Ang, A., and G. *Bekaert* (1998): Regime switches in interest rates. Research paper 1486. Stanford University.

Artis, M., H.-M. *Krolzig* and J. *Toro* (1999): The European Business Cycle. Discussion paper 2242, CEPR. London.

Beyer, A., J. A. *Doornik* and D. F. *Hendry* (2001): Constructing historical Euro-zone data. In: Economic Journal, 111, 308–327.

Burns, A. F., and W. C. *Mitchell* (1946): Measuring Business Cycles. New York: NBER.

Filardo, A. J., and S. F. *Gordon* (1994): International Co-Movements of Business Cycles. Federal Reserve Bank of Kansas, RWP94-11.

Garcia, R. (1998): Asymptotic null distribution of the likelihood ratio test in Markov switching models. In: International Economic Review, 39.

Hamilton, J. D. (1989): A new approach to the economic analysis of nonstationary time series and the business cycle. In: Econometrica, 57, 357–384.

Hamilton, J. D. (1990): Analysis of time series subject to changes in regime. In: Journal of Econometrics, 45, 39–70.

Hamilton, J. D., and G. *Lin* (1996): Stock market volatility and the business cycle. In: Journal of Applied Econometrics, 11, 573–593.

Hansen, B. E. (1992): The likelihood ratio test under non-standard conditions: Testing the Markov switching model of GNP. In: Journal of Applied Econometrics, 7, 61–82.

Hansen, B. E. (1996): Erratum: the likelihood ratio test under non-standard conditions: Testing the Markov switching model of GNP. In: Journal of Applied Econometrics, 11, 195–199.

Harding, D., and A. *Pagan* (2001): Dissecting the cycle. In: Journal of Monetary Economics (forthcoming).

Kim, C.-J. (1994): Dynamic linear models with Markov-switching. In: Journal of Econometrics, 60, 1–22.

Krolzig, H.-M. (1996): Statistical analysis of cointegrated VAR processes with Markovian regime shifts. SFB 373 Discussion Paper 25/1996, Humboldt Universität zu Berlin.

Krolzig, H.-M. (1997a): International business cycles: Regime shifts in the stochastic process of economic growth. Applied Economics Discussion Paper 194, Department of Economics, University of Oxford.

Krolzig, H.-M. (1997b): Markov-Switching Vector Autoregressions. Modelling, Statistical Inference and Application to Business Cycle Analysis. Berlin: Springer.

Krolzig, H.-M. (1998): Econometric modelling of Markov-switching vector autoregressions using MSVAR for Ox. Discussion Paper, Department of Economics, University of Oxford.
http://www.econ.ox.ac.uk/research/hendry/krolzig

Krolzig, H.-M. (2001): Business cycle measurement in the presence of structural change: International evidence. In: International Journal of Forecasting, 17 (3), 349–368.

Krolzig, H.-M., and H. Lütkepohl (1995): Konjunkturanalyse mit Markov-Regimewechselmodellen. In: K. H. Oppenländer (ed.): Konjunkturindikatoren. Fakten, Analysen, Verwendung. München: Oldenbourg, 177–196.

Krolzig, H.-M., and J. *Toro* (2000): Classical and modern business cycle measurement: The European case. Discussion Paper in Economics 60, University of Oxford.

Pagan, A. (1997a): Policy, theory and the cycle. In: Oxford Review of Economic Policy, 13, 19–33.

Pagan, A. (1997b): Towards an understanding of some Business Cycle characteristics. In: Australian Economic Review, 30, 1–15.

Peersman, G., and F. *Smets* (2001): Are the effects of monetary policy in the Euro area greater than in booms. Working paper 52, European Central Bank. Frankfurt.

Phillips, K. (1991): A two-country model of stochastic output with changes in regime. In: Journal of International Economics, 31, 121–142.

Zusammenfassung

Markov-Regimewechselmodelle zur Datierung des Konjunkturzyklus in der Eurozone

Zur Identifikation und Datierung des Konjunkturzyklus in der Eurozone wird der von Hamilton zur Analyse des US-Konjunkturzyklus vorgeschlagene Markov-Regimewechselansatz auf vierteljährliche aggregierte und länderspezifische Zeitreihen des realen Bruttoinlandsproduktwachstums der zwei letzten Jahrzehnte angewandt. Mit den statistisch kongruenten und ökonomisch sinnvollen Modellen werden Regimewechsel im stochastischen Wachstumsprozess der Wirtschaft in der Eurozone identifiziert. Basierend auf den implizierten geglätteten Regimewahrscheinlichkeiten kann eine Datierung des Konjunkturzyklus in der Eurozone vorgenommen werden: Rezessionen werden für die Perioden erstes Quartal 1980 bis erstes Quartal 1981 und drittes Quartal 1992 bis zweites Quartal 1993 notiert. Für die multivariate Analyse realer BIP-Daten von acht EMU-Mitgliedstaaten wird das zuvor betrachtete univariate stochastische Konjunkturzyklusmodell zu einem Mehrländermodell generalisiert. In Analogie zu den Ergebnissen der aggregierten Analyse belegen die geschätzten vektorautoregressiven Prozesse mit Markov-Regimewechseln die Bedeutung gemeinsamer Schocks: Obgleich die Synchronisation der Konjunkturzyklen in der Eurozone nicht perfekt ist, können mit der Ausnahme von Finnland für jedes Land simultane Regimewechsel in der mittleren Wachstumsrate identifiziert werden.