Measurement of Poverty from Grouped Observations

By Evangelos O. Simos

The elimination or amelioration of poverty has become a matter of public concern in modern societies. Based on grouped observations of income distribution, two new measures of poverty are derived and employed in this paper to study the pattern of poverty in Canada.

I. Introduction

One of the most exciting fields that has recently attracted the attention of researchers is the measurement of poverty. This interest emerged because of the interdependence of poverty with major economic variables and the complexities of government social policies. The sociological study and measurement of poverty involves two problems. First, the determination of the poverty line that should reflect the socially accepted minimal standard of living[1], and second, the construction of an index of poverty that provides a helpful tool to policy makers for balanced growth among mutually exclusive regions or groups of a society. This study deals with the second problem.

The most widely used measures of poverty despite their limitations were the "head-count ratio"[2] and the "poverty gap"[3]. The only substantial work on this subject is that of *Sen* (1973 and 1976), who has proposed an index using an axiomatic framework based on an ordinal welfare concept[4].

In this paper two new measures of poverty are introduced, the one being sensitive to the distribution of income among the poor. Our

---

[1] For valuable contributions to this problem the reader is referred to *Batchelder* (1971), *Townsend* (1954), and *Weisbrod* (1965).

[2] If Z is the poverty line and Y_i the income of the i-th individual, then the "head-count ratio" is the ratio of the number of people with income $Y_i \leqq Z$ to the total population.

[3] It is the aggregate short-fall of the income of all the poor taken together from the poverty line. See *Batchelder* (1971), p. 30.

[4] His index is derived as the weighted sum of poverty gaps, employing non-negative weights that follow a specific pattern in the sense that the larger the poverty gap the greater would be the weight attached to it.

format is as follows: Section II deals with the construction of the new poverty indices from grouped observations of income distribution. Based on data from the Twelfth Survey of Consumer Finances in Canada (1971), empirical application to the new measures is also employed. For this purpose Section II discusses the concept of the poverty line determination in Canada. Results are given in Section IV. Concluding remarks follow in the final section.

II. Two new poverty indices

Suppose there are N families, which have been grouped into k income classes.

Let n_i be the number of families in the i-th income class, $i = 1, 2, \ldots, k$, then $f_i = n_i/N$ is the relative frequency of the i-th income class.

If x_i is the sample mean for the i-th income class, the mean income of all the families (X) will be

$$X = \sum_{i=1}^{k} x_i f_i \tag{1}$$

If Z is the poverty line and k^* out of k income classes are below that line ($k^* < k$), then the mean income of poor families (X_p) will be

$$X_p = \sum_{j=1}^{k^*} x_j f_j \qquad \text{where } f_j = \frac{n_j}{N_p} \tag{2}$$

x_j = the sample mean income of the j-th income class;

n_j = the number of families into the j-th income class;

N_p = the number of families that have income below Z;

i. e.: $N_p = n_1 + n_2 + \ldots + n_{k^*}$

The first proposed poverty index is based on the approach of transfer of income from rich to poor so that every poor family's income is brought to the poverty line. This poverty index measures the percentage of the total income that has to be transferred from non-poor to poor in order to eliminate the existence of families having income less than the poverty line Z.

The amount of income that has to be transferred to poor so that no family unit would have income less than Z is given by

$$N_p Z - N_p X_p = N_p (Z - X_p) \tag{3}$$

Since the income of all families is NX, the proposed poverty index P_1 is derived as

$$P_1 = \frac{N_p}{N}\left[\frac{Z - X_p}{X}\right] = \frac{h}{X}(Z - X_p) \tag{4}$$

where h is the proportion of poor families and it is equivalent to the "head-count ratio".

In order to give sensitivity to our index P_1 with respect to the distribution of income among the poor, the second proposed poverty index P_2 takes the following form.

$$P_2 = \frac{h}{X}[Z - X_p(1 - G)] \tag{5}$$

where G is the Gini index of the income distribution of the poor.

The poverty index P_2 satisfies the conditions

$$\text{if } G = 0, \text{ then } P_2 = P_1$$

$$\text{if } G = 1, \text{ then } P_2 = hZ/X$$

The above conditions imply that if perfect inequality of income among the poor exists ($G = 1$), then every family with income below Z gets zero income. On the other hand if inequality of income among the poor does not exist then P_2 collapses to the first measure of poverty, P_1. Except in the limiting cases of $G = 0$ and $G = 1$, the structure of the poverty measure P_2 implies that the higher the inequality among the poor the larger the poverty index P_2. In practice, of course, G will never equal unity or zero. Therefore, the poverty measure P_2 satisfies Dalton's principle of transfers[5] since a transfer of income from any poor to a person with higher income (poor or non-poor), *ceteris paribus,* increases G and the poverty measure P_2 becomes larger.

If we divide the population into m mutually exclusive regions or groups based on (4) or (5) we may derive the poverty index of the s-th group or region, where $s = 1, 2, \ldots m$. In that case, the poverty index in the whole population, P^*, is equal to the weighted average of the indices in each region or group, the weights being proportional to the income share of each region or group.

$$P_q^* = \frac{1}{X^*}\sum_{s=1}^{m} X_s F_s P_{qs} \tag{6}$$

$q = 1, 2$; where

X_s = the mean income of all families (X) into the s-th group or region;

[5] See *Atkinson* (1970), pp. 247 - 249.

F_s = the proportion of population in the s-th group or region

($F_s = \frac{N_s}{\overline{N}}$, where $\overline{N}$ = all families in the society)

X^* = the mean income of all families in the society

P_{qs} = the q-th poverty index in the s-th group or region

Finally, the contribution of each region or group to total poverty can be easily derived by

$$C_{qs} = \frac{X_s F_s P_{qs}}{P_q^* X^*} \tag{7}$$

where C_{qs} is the contribution of the s-th region or group to the total poverty if the measure of poverty is P_q ($q = 1, 2$).

III. Determination of poverty line in Canada, 1971

One way to solve the problem of measuring the poverty line is the calculation of the income that would buy the minimum nutritional requirement of a family. Canada has no official "poverty lines" but in 1961 low income cut-offs were developed as a result of the demand for data on the low income population. These were only "statistical lines" applicable on the average and not in specific instances. Recently, a research study of the low income cut-offs (*Statistics Canada* 1973 and *Love and Oja* 1975) was intended to revise the 1961 measures.

The rationale used in that development was that families of different sizes are at equivalent levels of incomes when they spend the same proportion of their income on "necessities" (food, shelter, and clothing). For the determination of the low income cut-offs, the low income level of living was defined to be that income where a certain percentage (70 % in 1959) of income was spent on "necessities". The 70 per cent criterion was chosen in 1959 in light of the overall expenditure-income ratio of 50 per cent. Since the 1969 data indicated a decline by 8 percentage points in the overall expenditure-income ratio to 42 per cent the criterion was lowered to 62 per cent in order to maintain the original 20 percentage point difference between the criterion and the overall expenditure-income ratio. The basis for the determination of the low income cut-offs was an Engel curve regression of consumption expenditure (on necessities) by the family unit on income. Linear and logarithmic specifications of the Engel curve were run which yielded the same results. The resulting low income cut-offs for 1969 and the updating for 1971 employing as the updater the consumer price index are as follows:

Family Size	Low income cut-offs based on linear regression	
	1969	1971
1	2363	2512
2	3426	3642
3	4372	4647
4	5199	5526
5	5812	6178
6	6380	6782
7 +	6995	7435

In order to determine a poverty line for 1971 for a "family unit" (families and unattached individuals) we have to define the average size of family units. For 1971 the average size of family units was 3.06[6], but the data indicated a positive relation between income and family size. The higher the income group, the greater the average size of the family unit. Since we deal only with low income families of which income is less than $ 7,435 according to revised cut-offs we consider only the first six income groups to compute the average size of family units. We derived a weighted average size of family units using as weights the number of family units in the corresponding group. The resulting "average size of poor family units" is 2.434, which gives a poverty line in the amount of $ 4,078 based on the 1971 low income cut-offs. Therefore, we decided that $ 4,000 would be the poverty line which is very consistent with the revised low income cut-offs. According to our determination of the poverty line, 25.8 per cent of the population were poor during 1971 in Canada.

IV. Empirical results

The source of data used for this purpose is the Income Distribution by Size in Canada (1971), which is based on data collected during the Twelfth Survey of Consumer Finances from approximately 35,000 households across the country. The basic data, from which the poverty indices were computed was in grouped form consisting of fourteen income classes. Estimation of mean or total income of each income class is not available. One possibility is the use of the mid-point of each income class as an approximation. We do not follow that approach since the estimates of the mean income of each income class for the whole

[6] See *Statistics Canada* (1974), Table 40.

population is available and is different from the corresponding mid-point. We decided to use the mean income of each income class of the population as the mean income of the corresponding income class of different subgroups of the population, based on the approach that each income class into every subgroup follows the same distribution as the corresponding income class of the whole population. The poverty line was defined as $ 4,000 [see Section III].

Table I gives the poverty indices within two groups of family units classified according to the sex of the head of the family unit. Family units with male heads having an 81.45 per cent proportion of the population contribute to total poverty of only 56.55 per cent.

In Table II the whole population is disaggregated according to the age of the head of the household. Family units with age of the head between 35 - 44 years contribute to total poverty of 8.8 per cent. The poorest family units are those with age of head of the household below 24 and above 70 years, which together contribute to 43 per cent of the total poverty (19.48 per cent and 23.12 per cent respectively).

Table III gives the breakdown of family units to metropolitan, other cities, urban and rural sectors. Metropolitan areas with populations 63.8 per cent of the total contribute to total poverty of 53.42 per cent. On the other hand the rural sector with a proportion of the population of 18.6 per cent has a contribution to total poverty of 24.94 per cent.

The various indices for different provinces are presented in Table IV. British Columbia with mean income higher than the national mean income contributes to total poverty by higher percentage than the Atlantic provinces with mean income significantly below the national one. The contribution of the Atlantic provinces and British Columbia to total poverty is almost the same (10.2 per cent and 12.18 per cent respectively). Ontario with 36.75 per cent of the population has a contribution to total poverty of 30.61 per cent.

The national poverty index is 0.067, which implies that 6.7 per cent of the total income has to be transferred from non-poor to poor so that no family units in Canada would have income less than $ 4,000.

V. Concluding remarks

In this study we have introduced two new poverty indices from grouped observations of income distribution. We have also attempted an application by employing Canadian data. The results seem to support both measures of poverty. In all cases there are differences between

P_1 and P_2 indicating the superiority of P_2 which is sensitive to inequality among the poor.

The war on poverty is designed to alter the distribution of income by raising everyone above the poverty line. Given that the existing systems have not resolved the poverty problem, various antipoverty programs and proposals have emerged for a "guaranteed annual income" for every family unit. The programs are based on social, economic, humanitarian and sometimes political motives. Generally speaking, there are two broad categories of techniques or policies by which poverty can be reduced. One is the market approach which embraces a series of measures of investment in human capital designed to make the distribution of income more nearly equal. The other is the tax-transfer approach which causes the after taxes and transfers distribution of income to be more nearly equal than the before taxes and transfers distribution. According to the market approach, an increase in "training and skills" of low-income workers, it will move on these workers to higher-paying positions; hence, the supply of unskilled low-income workers will fall and the supply of skilled higher-income workers will rise. Thus, the wages for low-income workers will tend to rise and, conversely, the wages of higher-income workers will tend to fall. Since many people in poverty such as the aged, the mentally and physically infirm and mothers with young children cannot be expected to participate in the labor market, the market approach alone cannot eliminate poverty. The tax-transfer approach entails the establishment of a guaranteed minimum income which can be integrated with the existing tax systems by making provisions for a "negative income". *Friedman* (1962), who is one of the proponents of the negative income tax, has argued that the existing tax systems should be modified to provide income transfers (negative tax payments) to families whose incomes fall below the poverty line[7].

The proposed index of poverty can help point out to policy makers how to measure the success of antipoverty programs. The effectiveness of alternative programs against poverty can be measured by integrating the poverty index with the antipoverty technique or policy. The potency of each program can be evaluated by studying the corresponding variation in the poverty measure.

Although this paper is not concerned with policy recommendations, once an index of poverty can be derived for comparative studies among mutually exclusive regions or social groups, the economic and social implications for developed or developing countries are quite clear.

---

[7] A full flavor of antipoverty measures can be found in *Green* and *Lampman* (1965 - 67).

Table I: **Poverty Indices by Sex of the Head**

Sex	Proportion of Population	Mean Income	Mean Income of Poor	Proportion of Poor	Poverty Index P_1	Poverty Index P_2	% Contribution of each sex to Total P_1	% Contribution of each sex to Total P_2
Male	81.45	9882	2189	0.1852	0.0339	0,0408	53.99	56.55
Female	18.55	4288	1835	0.5789	0.2923	0.3172	46.01	43.45
Total	100.00	8845	2042	0.2582	0.0571	0.0656	100.00	100.00

Table II: **Poverty Indices by Age of the Head**

Age of Head	Proportion of Population	Mean Income	Mean Income of Poor	Proportion of Poor	Poverty Index P_1	Poverty Index P_2	% Contribution of each age to Total P_1	% Contribution of each age to Total P_2
24 years and under	10.65	5379	1713	0.4107	0.1746	0.2027	19.85	19.48
25 - 34 years	21.15	9353	2034	0.1355	0.0285	0.0338	11.19	11.21
35 - 44 years	19.59	10970	2137	0.1202	0.0204	0.0244	8.71	8.80
45 - 54 years	18.33	10909	1981	0.1564	0.0289	0.0343	11.47	11.51
55 - 64 years	14.58	9210	1984	0.2658	0.0582	0.0689	15.51	15.52
65 - 69 years	5.98	6268	2164	0.4719	0.1382	0.1648	10.33	10.36
70 years and over	9.72	4404	2254	0.6807	0.2699	0.3219	22.94	23.12
Total	100.00	8845	2047	0.2577	0.0569	0.0674	100.00	100.00

Table III: **Poverty Indices by Areas**

Areas	Proportion of Population	Mean Income	Mean Income of Poor	Proportion of Poor	Poverty Index P_1	Poverty Index P_2	% Contribution of each area to Total P_1	% Contribution of each area to Total P_2
Metropolitan	63.2	9760	2055	0.2188	0.0436	0.0519	52.62	53.42
Other Cities	6.3	8034	1822	0.2877	0.0780	0.0913	7.96	7.71
Small Urban	11.9	7728	2003	0.2966	0.0766	0.0908	14.20	13.93
Rural	18.6	6851	2100	0.3539	0.0981	0.1173	25.22	24.94
Total	100.0	8845	2043	0.2575	0.0561	0.0678	100.00	100.00

Table IV: **Poverty Indices by Provinces**

Provinces	Proportion of Population	Mean Income	Mean Income of Poor	Proportion of Poor	Poverty Index P_1	Poverty Index P_2	% Contribution of each province to Total P_1	% Contribution of each province to Total P_2
Atlantic Provinces	8.41	6860	2186.0	0.3339	0.0883	0.1062	10.10	10.20
Quebec	26.77	8532	2007.4	0.2590	0.0605	0.0718	27.37	27.29
Ontario	36.76	9853	1994.6	0.2101	0.0428	0.0508	30.71	30.62
Praire Provinces	16.78	7914	2112.0	0.3127	0.0746	0.0892	19.62	19.71
British Columbia	11.28	9165	2001.0	0.2733	0.0596	0.0708	12.20	12.18
Total	100.00	8845	2043.9	0.2579	0.0571	0.0679	100.00	100.00

Summary

This paper introduces two measures of poverty based on the approach of transfer of income from rich to poor in order to eliminate the existence of families having income less than the poverty line. Their derivation from grouped observations of income distribution is relatively simple permitting practical use. The first index ignores distribution among the poor. Introducing the Gini coefficient, the second index is sensitive to the pattern of the distribution among the poor. An application of the measures is employed based on Canadian data and the results provide a perspective on poverty among mutually exclusive regions or social groups.

Zusammenfassung

Es werden zwei Armutsmaße vorgestellt, die beide von dem Einkommenstransfer ausgehen, der notwendig wäre, jedes Familieneinkommen auf ein bestimmtes Minimaleinkommen (= Armutsgrenze) anzuheben. Ihre Herleitung aus der beobachteten Besetzung der Einkommensklassen ist relativ einfach. Während der erste Index die Einkommensverteilung zwischen den Armen nicht berücksichtigt, ist der zweite Index durch die Einführung des Gini-Koeffizienten hiervon abhängig. Beide Maßzahlen werden zur Illustration mit Hilfe kanadischer Daten berechnet, die Ergebnisse geben einen Einblick in die Verteilung der Armut zwischen einzelnen Regionen und/oder sozialen Gruppen.

References

Atkinson, A. B. (1970), On the Measurement of Inequality, Journal of Economic Theory (1970), pp. 244 - 263.

Batchelder, A. B. (1971), The Economics of Poverty, New York 1971.

Friedman, M. (1962), Capitalism and Freedom, Chicago 1962.

Green, C., and R. *Lampman* (1966 - 67), Schemes for Transferring Income to the Poor, Industrial Relations 6 (1966 - 67), pp. 121 - 137.

Love, R., and G. *Oja* (1975), Low Income in Canada, Paper on Session 5 on Definition and Measurement of Poverty of the Fourteenth General Conference of the International Association for Research in Income and Wealth, Finland 1975.

Sen, A. K. (1973), Poverty, Inequality and Unemployment: Some Conceptual Issues in Measurement, Economic and Social Weekly 8 (1973), pp. 1457 - 1464.

— (1976), Poverty: An Ordinal Approach to Measurement, Econometrica 44 (1976), pp. 219 - 231.

Statistics Canada (1973), Revision of Low Income Cut-Offs, Research and Analysis Section of Consumer Income and Expenditure Division, Ottawa 1973.

Statistics Canada (1974), Income Distribution by Size in Canada, 1971, Catalogue Number 13 - 207, Ottawa 1973.

Townsend, P. (1954), Measuring Poverty, British Journal of Sociology 5 (1954), pp. 130 - 137.

Weisbrod, B. A. (1965): The Economics of Poverty, New Jersey 1965.