

# Inflation, Unemployment and Unemployment Benefits

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An unemployment compensation system is integrated into a short-run macroeconomic model of prices and employment. The dynamic properties are discussed and some qualifications are made with respect to the stabilizing effects of unemployment benefits.

## I. Introduction

The unemployment cum inflation phenomenon or “stagflation”, as politicians like to describe it, has gained much attention during the last decade from theorists and practitioners alike. Various remedies have been tried without significant success, and the gap of communication and understanding seems to widen between those who are in charge of the measures and those who believe they know what should be done. In this paper, we shall not try to add to these recommendations, but rather point out one aspect of unemployment which seems to have been pushed aside during large parts of the discussion: the inflationary effects of unemployment benefits.

Usually, the unemployment compensation system is regarded as some kind of built-in stabilizer (*Musgrave*, 1959, pp. 505 - 6). Authorities install income redistribution programs by which, during periods of “normal” employment, those who are employed have to support the jobless part of the labour force. If the unemployment ratio rises beyond its “normal” level, available funds fall short of the amounts needed and government steps in by extending credit to the employment agencies. These loans are to be repaid during the next recovery, when inflows exceed disbursements.

It remains questionable, however, whether we really experience such countercyclical switches from deficit spending to surplus saving, and even if this were the case, no straightforward conclusions should be drawn as to the stabilizing effects on employment. An extension of

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government transfers to households which is geared to a rise in the unemployment ratio may raise effective demand, but this does not necessarily mean that employment will be raised too. Moreover, the fact that unemployed workers are to receive transfers should encourage trade unions not to bother too much with the employment problem when bargaining over new wage rates. It is not surprising that during inflationary recessions only employers tend to stress the job situation, while unions concentrate on the income losses due to inflation.

In what follows we shall set up a short run macroeconomic model of prices and employment which is dynamically stable and we shall describe the adjustment processes which may or may not be supported by employment benefit programs. From the assumptions about the wage determination it follows that, except at the price of a running inflation, no monetary policy will be able to raise employment beyond what is considered as "normal" by the bargaining groups in the labour market. It also is to be shown that, if unions disregard the job situation during recessions (because members are "insured" against unemployment), a stable unemployment ratio above the normal level may exist which is accompanied by a positive rate of inflation.

## II. Assumptions and Notations

There are three sectors in the economy: firms, private households, and a central bank and government sector. Relative prices of goods and services produced by firms remain constant so that total real output ( $X$ ) can be conceived of as one conglomerate good, which may be either consumed or invested. Total output is a function of capital ( $K$ ) and labour ( $N$ ) employed, and is subject to diminishing marginal returns. We shall further assume that for the time period considered labour is the only variable factor, i. e.

$$(1) \quad X = X(N), X_N > 0, X_{NN} < 0.^1$$

Firms are price takers in the labour market, i. e. given the present output price ( $P$ ), at which entrepreneurs expect to be able to sell their products, and given the current nominal wage rate ( $W$ ), the demand for labour is determined by the equality of this money wage rate and the value of the marginal product of labour,

$$(2) \quad W = PX_N.$$

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<sup>1</sup> Throughout this paper the (partial) derivate of a function  $f$  with respect to a variable  $x$  is denoted by  $f_x$ .

Using  $w$  for the real wage rate,

$$(3) \quad w := \frac{W}{P} ,$$

from (1) and (2) we obtain a demand function for labour which falls with a rising real wage,

$$(4) \quad N^d = N(w), \quad N_w < 0 .$$

On the supply side of the labour market it is assumed that trade unions follow the concept of a “fair” real wage rate,  $w^*$ , from which they deduce the demanded money wage rate

$$(5) \quad W = w^* P^e ,$$

where  $P^e$  is the price level expected to be valid during the near future. At this wage rate supply is perfectly elastic, which means that labour demanded by firms is always equal to labour employed

$$(6) \quad N^d = N .$$

If, however, there is a deviation of the amount of labour demanded (and employed) from some “natural” or “normal” level,  $\bar{N}$ , unions correct their desired real wage rate according to the following differential equation:<sup>2</sup>

$$(7) \quad \frac{Dw^*}{w^*} = \lambda \frac{N - \bar{N}}{\bar{N}} , \quad \lambda > 0 \text{ and constant.}$$

From (5) and (7) we can derive a corresponding adjustment equation for the money wage rate,

$$(8) \quad \frac{DW}{W} = \pi - \lambda u ,$$

where

$$(9) \quad \pi := \frac{DP^e}{P^e}$$

and

$$(10) \quad u := \frac{\bar{N} - N}{N} .$$

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<sup>2</sup>  $D$  is the differential operator  $d/dt$ .

The unemployment ratio (10), which is based on normal employment rather than on full employment of the whole labour force, is taken to be the relevant figure for union policy. Trade unions do not feel responsible for any kind of unemployment regarded as “structural” or “frictional”, but if the ratio defined by (10) becomes positive, i. e. if employment is lower than regarded as normal, they would prefer to gain some more employment rather than to offset the expected rate of inflation completely. On the other hand, if the demand for labour is higher than  $\bar{N}$  ( $u < 0$ ), they raise the money wage rate by more than would be necessary to compensate for the expected real income losses due to rising output prices.<sup>3</sup>

Once the level of employment is determined, firms produce output according to production function (1) and pay out real income  $X$  to wage earners and shareholders. If total real sales fall short of  $X$ , unsold goods are stored. On the other hand, if excess demand for goods and services is positive, firms cut down inventories of finished products accumulated in the past.<sup>4</sup> To keep the argument simple, we shall assume that demand is always satisfied, i. e. any excess demand for goods and services will lead to passive disinvestment of exactly that amount.

The general idea is that in oligopolistic markets producers first of all try to keep profits in a certain relation to costs, i. e. they raise output prices whenever the money wage rates have gone up. Unintended investment, however, indicates that markets have not been cleared and prices should be checked and/or corrected. So any excess demand for goods together with the corresponding change in inventories should lead to a further correction of the prices. A price setting function which contains these two elements may be represented by the following differential equation

$$(11) \quad \frac{DP}{P} = \frac{DW}{W} + \mu v, \quad \mu > 0 \text{ and constant,}^5$$

<sup>3</sup> The fixed normal employment  $\bar{N}$  could also be conceived of as a classical supply curve for labour which is completely inelastic with respect to the real wage rate. It should be mentioned, however, that (7) and (8) allow for a more general interpretation, if  $\bar{N}$  is not taken as a *target* figure but simply indicates that level of employment at which the bargaining forces of employers and workers counterbalance each other, thus leaving the distribution of real income unchanged. See Stein (1974) for a similar version of this Phillips-Lipsey type of wage rate adjustment. In this connection, reference should also be made to the approach taken by Michael Mussa (1975), which was brought to my attention after the present paper had been completed.

<sup>4</sup> The concept of inventories seems a bit strange if reference is made to services produced by firms. Here one would have to think of producers who are able to switch between “routine” and “project” work, i. e. if they face a positive excess demand for their services, they delay any routine work and concentrate their employees on the projects ordered.

where  $v$  is the (relative) excess of planned investment ( $I$ ) and consumption ( $C$ ) as explained further below over output produced:

$$(12) \quad v = \frac{I + C - X}{X}.$$

It should be noted that *Walras' Law* has been expelled partly from the model economy. First, we excluded the labour market by determining *realized* employment and, consequently, *realized* income. Second, the goods and services market was settled in a similar way and we had a spillover to the financial markets. We now assume that there are only two financial assets in the economy: equities or ownership of real capital ( $K$ ) and fiat money ( $M$ ). On the markets for these assets we shall abandon the above non-tâtonnement process and allow for any kind of recontracting until a (portfolio) stock equilibrium is reached.<sup>6</sup>

A necessary assumption for this outcome would be that financial markets are perfectly organized and immediately react to disturbances created by shifts in demand and/or supply. The demand for money in real terms may be given by a conventional liquidity preference function<sup>7</sup>

$$(13) \quad L = g(r, \pi, X), \quad g_r < 0, g_\pi < 0, g_X > 0$$

where  $r$  is the expected real rate of return on capital (equities),  $\pi$  again is the expected rate of inflation (the negative rate of return on real balances), and  $X$  is the level of income, which may be taken as determining the transactions demand for money. Permanent stock (portfolio) equilibrium in the money and equities market insures that we always have

$$(14) \quad L = \frac{M}{P},$$

where  $M$  is the amount of fiat money existing in the economy.

Another feature of the model is that we shall allow the valuation of existing capital goods ( $P_K$ ) to differ in the short run from their reproduction cost ( $P$ ), i. e. although real capital is to be conceived of as one

<sup>5</sup> The adjustment coefficient  $\mu$  is taken as a constant in (11). Usually, it should be expected to vary with the market rate of interest since passive investment enlarges the need for new funds. We shall omit this aspects for convenience.

<sup>6</sup> In the standard IS-LM context this means that we are always on the LM curve, which is a bond (equity) and money market equilibrium curve.

<sup>7</sup> Wealth does not appear as an explicit variable in (13). Hence, it must be assumed implicitly that along a moving equilibrium households would be willing to put all their savings into equities, i. e. they are not diversifiers on the margin although, according to (13), they diversify their average portfolio.

homogeneous good (which, for convenience, may be undestructable), the relative price  $p_K (\equiv P_K/P)$  may be different from one.<sup>8</sup>

If, for example, the demand of households for real balances rises relative to the demand for equities, sales on the stock market will force share prices down and at the same time raise the imputed rate of return on these shares. Because newly created equities will also have to be served at an interest rate equal to this higher rate of return, investment projects which seemed profitable up to this point will have to be cancelled and the level of investment will go down in the next period (next instant). *For the moment*, i. e. until the slowing down of investment has raised the price of existing capital goods to its portfolio equilibrium level,  $p_K$  will stay below one or, equivalently, the marginal efficiency of capital will be lower than the market rate of interest.

There will be an inverse relation between the market valuation,  $p_K$ , of existing capital and the real rate of return upon it,  $r$ , i. e. if the real marginal efficiency of capital,  $X_K$ , is expected to remain constant in the future, the market return on equities follows from the present value formula

$$(15) \quad r = X_K/p_K$$

It should be noticed that (15) is a definition of either  $r$  or  $p_K$ . Once  $p_K$  is determined in the market, the value of  $r$  follows and vice versa. We assume that a difference between the two rates  $X_K$  and  $r$  is the major force governing investment decisions, i. e. taking  $R$  as an exogenous variable the short run investment function is

$$(16) \quad I = I(r), \quad I_r < 0.$$

The demand for consumption goods (C) is assumed to depend on current disposable income which, as long as government activities are not considered, is simply the factor income created during the production process:

$$(17) \quad C = c Y, \quad 1 > c > 0 \text{ and constant.}$$

Finally, expectations about the rate of inflation are assumed to be changed according to past experience:

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<sup>8</sup> By assumption, financial markets are cleared at any moment in time, while this is not necessarily the case in the goods markets. Consequently, the valuation of capital should only in very special cases be the same as the output price. Cf. *Brainard and Tobin* (1968), *Tobin* (1969), and *Stein* (1971, 1974). *Frenkel and Rodriguez* (1975) arrive at essentially the same point by arguing in terms of adjustment costs. The basic reference for this second approach would be *Gould* (1968).

$$(18) \quad D\pi = \varepsilon \left( \frac{DP}{P} - \pi \right), \quad \varepsilon > 0 \text{ and constant}$$

The above formula has two limiting cases for  $\varepsilon \rightarrow 0$  and  $\varepsilon \rightarrow \infty$ , respectively. The first one implies  $D\pi = 0$  and is often referred to as “static” expectations, because economic agents do not learn from or react to their own experience. The second one implies “perfect myopic foresight”, i. e. the observed rate of inflation is expected to remain the same in the near future. For reasons, which will become clear in the next section, we shall assume that  $\varepsilon$  is fairly small, i. e. people react slowly to any discrepancies between realized and expected rates of inflation.

### III. Stability and Adjustment Processes

The model to be discussed consists of three differential equations (8), (11) and (18), one stock demand function (13), and a market clearing condition (14), which is continuously maintained by appropriate changes in the market rate of interest or, equivalently, in the price of equities. Assuming that (13) can be solved explicitly for  $r$ , we can insert (14) and derive the following equilibrium condition for the money and equities markets

$$(19) \quad r = G(m, \pi, X),$$

$$G_m = \frac{1}{g_r} < 0, \quad G_\pi = -\frac{g_\pi}{g_r} < 0, \quad G_X = -\frac{g_x}{g_r} > 0,$$

where

$$(20) \quad m := \frac{M}{P}.$$

From equations (1) - (4) and (6) it follows that output is a function of the real wage rate only,

$$(21) \quad X = h(w), \quad h_w = X_N N_w < 0.$$

We then may insert (16), (17) and (21) into (12) and, substituting  $r$  by equation (19), rewrite the relative excess demand for goods and services,  $v$ , as a function of  $m$ ,  $\pi$ , and  $w$ :

$$(22) \quad v = v(m, \pi, w), \quad v_m > 0, \quad v_\pi > 0, \quad v_w > 0,$$

where the signs of the partial derivatives follow directly from the functions involved.

It should be remembered that, since planned saving always equals realized saving, there are no influences on  $v$  from the consumption side. The above variables work on  $v$  only via realized output,  $X$ , and planned investment,  $I$ . Planned investment (and, consequently,  $v$ ) shrinks whenever the real rate of interest rises, which is the case if either  $m$  or  $\pi$  (or both) decline. A decline in the real wage rate also leads to a decline in  $v$ , because output, and therefore supply, is increased while at the same time a higher demand for transaction balances will raise the required rate of return on equities, thereby reducing planned investment demand.

By means of (4) and (6) we can interpret the unemployment ratio (10) as a function of the real wage rate,

$$(23) \quad u = u(w), \quad u_w > 0.$$

Next we insert (23) and (22) into equations (8) and (11) and, together with the adaptive expectations function (18), arrive at the following differential equations:

$$(24) \quad DW/W = \pi - \lambda u(w),$$

$$(25) \quad DP/P = DW/W + \mu v(m, \pi, w),$$

$$(26) \quad D\pi = \varepsilon [DP/P - \pi].$$

At first sight one might have the impression that this system is capable of producing a steady state inflation, where

$$DP/P = DW/W = \pi = \text{some positive constant}$$

at full employment and zero excess demand for goods. As long as the government remains passive, however, the amount of fiat money does not change, and a positive rate of inflation will drive real balances down. Lower real balances in turn via the interest rate mechanism lead to a lower rate of investment which again slows down the rate of price inflation. In other words: regardless of the forces, which may have set an inflationary process into motion, it will peter out, if it is not fed by an appropriate monetary expansion.

The foregoing argument becomes clear, if we replace (24) and (25) with two differential equations in the real wage rate and real balances, respectively. Taking account of relationships (3) and (20) we may derive from (24) - (26)

$$(27) \quad Dm = -m[\pi + \mu v(m, \pi, w) - \lambda u(w)].$$

$$(28) \quad Dw = -w\mu v(m, \pi, w),$$

$$(29) \quad D\pi = \varepsilon [\mu v(m, \pi, w) - \lambda u(w)].$$

A stationary solution to this system of differential equations implies zero excess demand for goods ( $v = 0$ ), full employment in the labour market ( $u = 0$ ), and constant prices with no inflationary expectations ( $\pi = 0$ ).

Let us expand the system around the equilibrium values  $w^*$ ,  $\pi^*$ , and  $m^*$  and linearize. We then have

$$(30) \begin{bmatrix} Dw \\ D\pi \\ Dm \end{bmatrix} = \begin{bmatrix} -\mu w^* v_w & -\mu w^* v_\pi & -\mu w^* v_m \\ \varepsilon (\mu v_w - \lambda u_w) & \varepsilon \mu v_\pi & \varepsilon \mu v_m \\ -m^* (\mu v_w - \lambda u_w) & -m^* (1 + \mu v_\pi) & -m^* \mu v_m \end{bmatrix} \begin{bmatrix} w - w^* \\ \pi \\ m - m^* \end{bmatrix}$$

If we assume that the following two inequalities are fulfilled in the neighbourhood of  $w^*$ ,  $\pi^*$ , and  $m^*$ ,

$$(31) \quad a_1 = \mu (w^* v_w - \varepsilon v_\pi + m^* v_m) > 0$$

and

$$(32) \quad \frac{a_1 - \varepsilon}{a_1} > \frac{\varepsilon (\lambda w^* u_w v_\pi - m^* v_m)}{m^* \lambda w^* u_w v_m},$$

the Routh-Hurwitz conditions will be met, i. e. (31) and (32) together with our previous assumptions imply local stability of system (30) and vice versa.<sup>9</sup>

These two inequalities deserve some comment. The first and third term on the right hand side of (31) both are positive, while the second is not. For (31) to hold, therefore, either  $\varepsilon$  or  $v_\pi$  or both have to be sufficiently small. The value of  $\varepsilon$ , the adjustment speed of inflationary expectations, always plays a crucial role in determining the stability of monetary models.<sup>10</sup> In the present case, however, the influence of  $\pi$  on portfolio decisions and, via the interest rate mechanism, on excess demand for goods also may lead to an unstable behaviour the system.

A rise in the rate of inflation raises  $\pi$ , and this change in expected returns on real money balances will induce owners of wealth to shift from money into equities. Consequently, firms will carry out higher real investments, which again will raise the rate of price inflation. On the other hand, the upward move in  $DP/P$  lowers the value of real balances, thereby partly offsetting the portfolio shift described above. Also, the rise in  $DP/P$  leads, via a decline in real wage rates to higher output and real income and thus forces the rate of inflation down both “directly” and “indirectly”. A necessary condition for this latter effect to dominate the first one is given by (31).

<sup>9</sup> A more detailed analysis may be found in Appendix 1.

<sup>10</sup> Cf. *Cagan* (1956).

Stability condition (32) is somewhat more complex and does not offer a straightforward interpretation. Again,  $\varepsilon$  and  $v_\pi$  play a crucial role, i. e. if both are sufficiently small, the inequality holds. We shall not get further into any details and summarize that for system (30) to be locally stable, inflationary expectations must not be so strong as to offset the main stabilizing forces in the goods and labour markets.

To get an idea of how the adjustment process works, let us inspect system (30) more closely. For  $Dw = 0$ ,  $D\pi = 0$  and  $Dm = 0$  three tangential planes in the phase space may be derived. It will be more convenient, however, to argue in terms of  $Dw = 0$ ,  $D\pi = 0$  and  $Dm = 0$  lines which are projections of the three dimensional phase diagram onto the  $m, w$  plane for given levels of  $\pi$ . For three different situations ( $\pi = \pi_1 < 0$ ,  $\pi = 0$ ,  $\pi = \pi_2 > 0$ ) such sets of lines are shown in Figure 1, where the slope along  $D\pi = 0$  and  $Dm = 0$  is arbitrarily assumed to be positive, although the sign is ambiguous. If it is negative, the slope will still be smaller than the slope of  $Dw = 0$  in absolute terms, and no change in the interpretation would result.

As one possible point of departure, let us assume that a low real wage rate has raised employment beyond  $\bar{N}$  (i. e.  $u < 0$ ) and the expected rate of inflation is negative for some reason ( $\pi = \pi_1 < 0$ ), while cash balances have the equilibrium value  $m^*$ . In Figure 1, this situation is characterized by  $Q$  and by  $Dm = 0$ ,  $D\pi = 0$  and  $Dw = 0$  lines which run through points  $A$  and  $B$ .

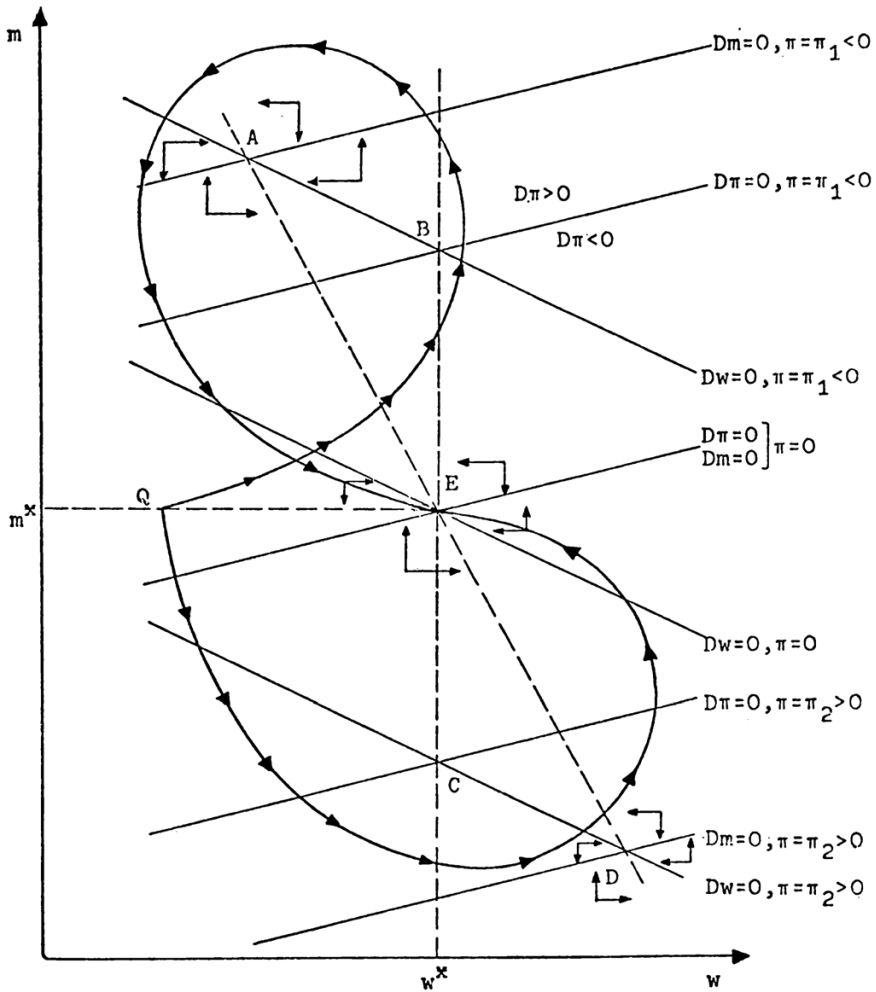
As may be seen from equations (27) and (29), at point  $A$  ( $Dw = 0 = Dm$ ) the effects of “overemployment” and the expected rate of deflation on the wage bidding process exactly outweigh each other ( $\pi = \lambda u$ ). Since employment at  $Q$  is higher than at  $A$ , it follows from (24) that the initial situation is characterized by a rising money wage rate. On the other hand, there is not only a negative excess demand for goods ( $v < 0$  and therefore  $Dw > 0$ ), but  $Q$  also lies below the  $Dm = 0$  line. In terms of (29) this means that

$$\pi - \lambda u + \mu v < 0$$

or

$$(33) \quad DW/W = \pi - \lambda u < -\mu v ,$$

i. e. the deflationary pressure of the negative excess demand on output prices is stronger than the “cost push” element of rising labour costs. Consequently, the real wage rate as well as real balances will rise in the beginning and the time path will be directed towards the northeast of  $Q$ .



Figur 1

If  $\pi$  were exogenous to the system and remained constant, the adjustment path would finally move to  $A$  as indicated by the arrows. In the given situation, however, the actual fall in output prices exceeds the expected rate of deflation and therefore leads to a further decline in  $\pi$ , which shifts points  $A$  and  $B$  along the dotted lines away from equilibrium  $E$ . This is the general problem of the adaptive expectations hypothesis: if conditions (31) and (32) did not hold, the actual rate of de-

flation might pull  $\pi$  down continuously, while  $\pi$  would direct the time path away from  $E$  thus leading to a permanent disequilibrium development of overemployment (as defined above) and deflation.

The next phase of the adjustment process is initiated, when because of declining employment and the rise in  $|\pi|$  declining money wage rates lead to a stronger pressure on prices and a corresponding slowdown in the rise of the real wage rate. Our qualitative analysis does not make it possible to say exactly when expectations will catch up with actual development, but if  $\varepsilon$  and  $v_\pi$  are small, one should conclude that this will not be the case until employment has fallen below  $\bar{N}$  and the excess demand for goods has become positive. This will lead to a decline in the real wage rate and, consequently, in the rate of deflation, which will finally fall to the value of expectations.

Now the process is reversed insofar as  $\pi$  will take the "lead" and shift points  $A$  and  $B$  towards equilibrium  $E$  again. After some time, although  $DW$  still is negative, the rising excess demand for goods will end the deflationary phase of the process and push output prices up again, while rising prices together with falling money wage rates will raise employment beyond  $\bar{N}$ . We then have overemployment accompanied by inflation and the adjustment path will be directed towards the stationary equilibrium at  $E$  as indicated in Figure 1.

We may conclude from the foregoing that in most cases the adjustment path from an initial disequilibrium to the final stationary solution will lead the economy through (at least) one complete cycle of inflation/deflation, overemployment/underemployment and positive as well as negative excess demand for goods. If, for example, the initial constellation of the state variables is given by points  $Q$ ,  $C$  and  $D$  in Figure 1, the process will be inflationary at the beginning, but again there is a complete cycle, which terminates after a phase of deflation and unemployment.

Because the economy in this model always is stabilized at a predetermined level of employment,  $\bar{N}$ , the comparative statics are somewhat different from the usual short run Keynesian interpretation. In Figure 2 the well known macroeconomic "demand" and "supply" curves are drawn. The first one is an ISLM-curve derived jointly from

$$(12') \quad v = \frac{I(\tau)}{X} - (1 - c) = 0$$

and

$$(19') \quad \tau = G\left(\frac{M}{P}, \pi, X\right)$$

for  $\pi = 0$ . The second one is derived from equation (21) for a constant level of  $W$ . Let  $A$  be an equilibrium situation with  $W = W^*$ . If an autonomous rise in the demand for goods occurs, the standard constant wage rate approach would shift the DD-curve to the right and identify  $B$  as the new equilibrium point. In the present model, however, trade unions will raise the wage rate both because employment has gone up and because  $\pi$  has become positive in response to rising output prices. The final equilibrium point,  $C$ , is characterized by higher prices, while employment has fallen to the old level.

However, this is only part of the story. The excess demand for goods and services is not independent of the expected rate of inflation, since  $\pi$ , via the financial markets, influences  $r$  and, accordingly, the demand for new investment. From the above equations (12') and (19') we have

$$\frac{\partial P}{\partial \pi} = \frac{G_{\pi} P^2}{X G_m M} > 0 ,$$

i. e. the macroeconomic demand curve will shift beyond  $D'D'$  when, immediately after the initial disturbance,  $\pi$  becomes positive in response to rising output prices. If the expectations coefficient,  $\varepsilon$ , is not too small, the initial phase of inflationary expansion (which continuously shifts both the supply and demand curves) will not simply be neutralized by a counterbalancing contraction but produce a complete cycle, including a phase of underemployment and deflation. A graph is given in Figure 2.

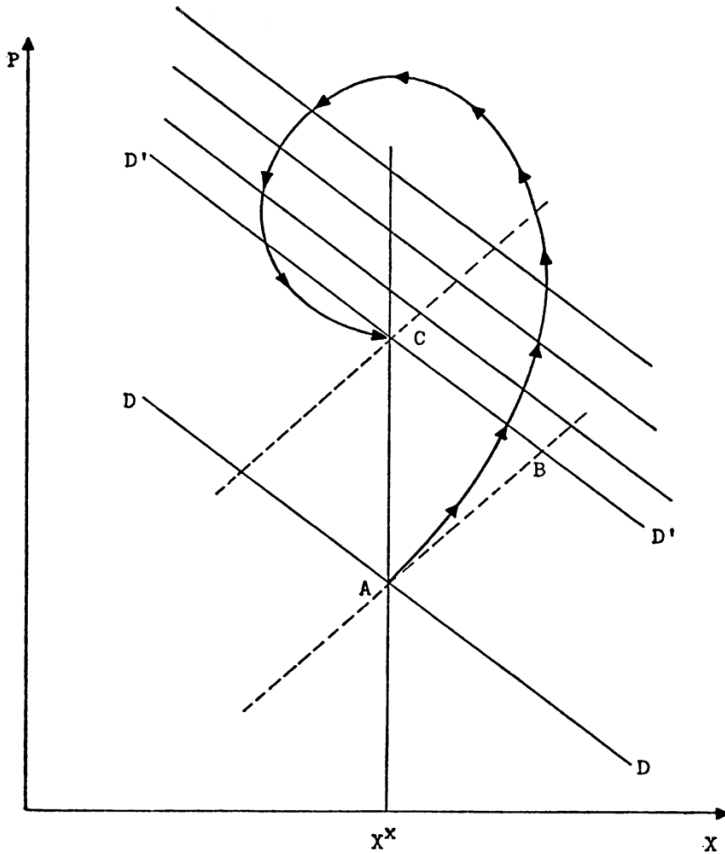
While the constant money supply in this model does not allow for a permanent steady state rise of output prices, it should be noted that stabilization of the  $\bar{N}$  employment level is entirely due to the wage adjustment concept followed by trade unions. If the central authorities fixed the *growth rate* of money,

$$\frac{DM}{M} = \Theta > 0 ,$$

and if the expansion were performed by means of open market purchases of equities, only differential equation (29) would have to be modified:

$$(29') \quad Dm = m [\Theta - \pi + \lambda u(w) - \mu v(m, \pi, w)] .$$

The dynamic properties of the model would be maintained, and the equilibrium solution would exhibit full employment ( $u = 0$ ) and zero excess demand for goods ( $v = 0$ ) together with a constant and fully anticipated rate of price inflation ( $\Theta = \pi$ ).



Figur 2

#### IV. Unemployment Benefits

Next consider a government unemployment program, which may be such that at an "average" level of unemployment that amount of insurance premiums paid exactly equals the volume of unemployment benefits. Let this level be  $\tilde{N}$ , i. e. if employment equals  $\tilde{N}$ , there will be a pure redistribution of wage income without any additional fiscal or monetary policy measures.

If employment falls below  $\tilde{N}$ , disbursements of employment agencies will exceed money inflows, and it shall be assumed that the deficit is financed through loans from the central authorities. It seems plausible to conceive of these loans as taking the form of unconsolidated government debt, for if employment is expected to be stabilized at  $\tilde{N}$  by en-

ogenous forces (as, perhaps, experienced in the past), deviations from  $\tilde{N}$  should not only be considered transitory but also “symmetrical” in the sense that periods of low employment ( $\tilde{N} > N$ ) will be followed by periods of high employment ( $\tilde{N} < N$ ), during which loans are to be refunded.

To simplify matters further, it is assumed that unemployment transfers are based on *potential* rather than on past income, i. e. every unemployed worker is to receive a certain percentage  $\alpha$ , of the wage income, which he would be able to earn. Consequently, total unemployment benefits, measured in real terms, will amount to  $\alpha w (\bar{N} - N)$ , where  $\bar{N}$  stands for full employment of the whole labour force. On the other hand, if insurance premiums are to be certain fraction,  $\beta$ , of *actual* income, real payments to the insurance agencies will be  $\beta w N$  leading to a *net cash flow* of

$$(34) \quad \beta w N - \alpha w (N - \bar{N}) .$$

Because of the above assumption that during “normal” employment the compensation program simply consists of a redistribution of wage income, expression (34) must vanish for  $N = \tilde{N}$ , i. e.  $\alpha$  and  $\beta$  have to be fixed by the government in such a way as to ensure the following balance equation:

$$(35) \quad \beta w \tilde{N} - \alpha w (\bar{N} - \tilde{N}) = 0 .$$

Using (35) we may substitute for  $(\bar{N})$  in (34) and rewrite the net cash flow to insurance agencies as

$$(36) \quad (\alpha + \beta) w (N - \tilde{N}) ,$$

which will be positive (negative) whenever employment rises beyond (falls below) its “average” level,  $\tilde{N}$ . By assumption, these net cash inflows or outflows are part of the government budget. Disregarding, as before, all other government activities, we may set (36) equal to the change in the supply of money (in real terms):

$$(37) \quad (\alpha + \beta) w (N - \tilde{N}) = - \frac{DM}{P} .$$

Let us further assume that government sets  $\tilde{N}$  equal to  $\bar{N}$ , i. e. the insurance program is based on exactly that level of employment which indicates a standstill in the real wage bargaining process.<sup>11</sup> From (10),

<sup>11</sup> From what has been said at the beginning of this section it should be clear that unemployment benefits are not considered a policy instrument. The central authorities just try to find an *average level* of employment such that they will be able to stay out the insurance business in the long run.

(6) and (4) we may then substitute for the unemployment ratio,  $u(w)$ , and rewrite (37) as

$$(38) \quad \frac{DM}{P} = B(w) ,$$

where

$$(39) \quad B(w) = (\alpha + \beta) \bar{N} w u(w) , \quad B_w > 0 .$$

The real flow supply of money, being identical to a real income transfer from the government to private households, is an increasing function of the real wage rate, and it is positive (negative) whenever  $u$  is positive (negative).

There are two channels through which unemployment benefits affect excess demand. They are represented by the left and right hand side of (39), respectively. First, a change in the money supply influences the demand for commodities via the interest rate mechanism. Second, government transfers are part of disposable income and therefore affect demand via the consumption function.

Let  $v$  indicate, as before, the relative excess demand for goods and services, when government remains absent. The general expression for excess demand,

$$(40) \quad \tilde{v} = \frac{I + c(X + B) - X}{X}$$

may then be written

$$(41) \quad \hat{v} = v + c b ,$$

where

$$(42) \quad b = \frac{B(w)}{X(w)} \quad \text{and} \quad b_w > 0 .$$

It should be understood that the relevant figure for price decisions of firms is now given by  $\tilde{v}$  instead of  $v$ , i. e. instead of differential equation (11) we have

$$(43) \quad \frac{DP}{P} = \frac{DW}{W} + \mu [v + c b] .$$

From (43) and (38) together with (24) and (26) we may derive the following system of differential equations:

$$(44) \quad Dw = -w \mu (v + c b) ,$$

$$(45) \quad D\pi = \varepsilon [\mu (v + c b) - \lambda u] ,$$

$$(46) \quad Dm = B - m [\pi - \lambda u + \mu (v + c b)] .$$

The stationary solution to these equations is the same as that of (27) - (29). From  $Dw = 0 = D\pi$  it follows that  $u$  would have to be zero, and if  $u$  vanishes, so will both  $B$  and  $b$ , and therefore, the difference between systems (27) - (29) and (44) - (46).

As to the dynamic behaviour of the economy, we shall again disregard all but small disturbances and linearize around the stationary solution:

$$(47) \quad \begin{bmatrix} Dw \\ D\pi \\ Dm \end{bmatrix} = \begin{bmatrix} -w^* \mu (v_w + c b_w) & -w^* \mu v_\pi & -w^* \mu v_m \\ \varepsilon \{ \mu (v_w + c b_w) - \lambda u_w \} & \varepsilon \mu v_\pi & \varepsilon \mu v_m \\ B_w - m^* \{ \mu (v_w + c b_w) - \lambda u_w \} - m^* (1 + \mu v_\pi) - m^* \mu v_m \end{bmatrix} \begin{bmatrix} w - w^* \\ \pi \\ m - m^* \end{bmatrix}$$

Since (47) and (30) are both evaluated at the same point in the phase space, the values of partial derivatives must be the same. The only difference then between the two systems consists of  $b_w$  and  $B_w$  terms in the first columns of (47). It is easily shown that conditions (31) and (32), while still being sufficient, are no longer necessary for stability, i. e. after introduction of unemployment benefits weaker restrictions would suffice.<sup>12</sup>

The unemployment compensation program outlined above thus may be termed "stabilizing" in the general sense that it enlarges the set of parameter distributions, which ensure dynamic stability of the system. This does not necessarily mean, however, that the adjustment process will be speeded up. If  $x_i$  and  $y_i$  ( $i = 1, 2, 3$ ) are the roots of characteristic equations corresponding to (30) and (47), respectively, the following results may be obtained:

$$(48) \quad |x_1 + x_2 + x_3| < |y_1 + y_2 + y_3|$$

and

$$(49) \quad x_1 x_2 x_3 = y_1 y_2 y_3,$$

i. e. at least one but at most two of the terms on the right hand side will be greater in absolute value than the corresponding  $x_i$  roots. Therefore, it cannot be stated that unemployment benefits always increase stability in the sense that a certain neighbourhood around the equilibrium point will be reached in shorter time.

<sup>12</sup> This and the following results are obtained in Appendix 2.

So far, we have described the economy by means of  $m$ ,  $w$  and  $\pi$ . In terms of these variables, stationary solutions to (27) - (29) and (44) - (46) are identical, and disequilibrium dynamics, as given by (30) and (47), do not exhibit characteristic differences. However, there remains one important modification with respect to the money supply. While  $M$  was fixed in the model of section III, in the present framework it is being changed throughout the adjustment process. As a result, it is not possible to determine the final price level in the economy without first having evaluated the change of  $M$  along the time path.

A graphic exposition may be given by means of standard macro-economic “demand” and “supply” schedules. In Figure 3, all curves are drawn under the assumption of zero inflationary expectations. Let  $A$  represent an initial equilibrium identical to the starting point of Figure 2, and let the autonomous shift in the demand for goods be of the same size in both diagrams. We then may identify two conceptual differences.

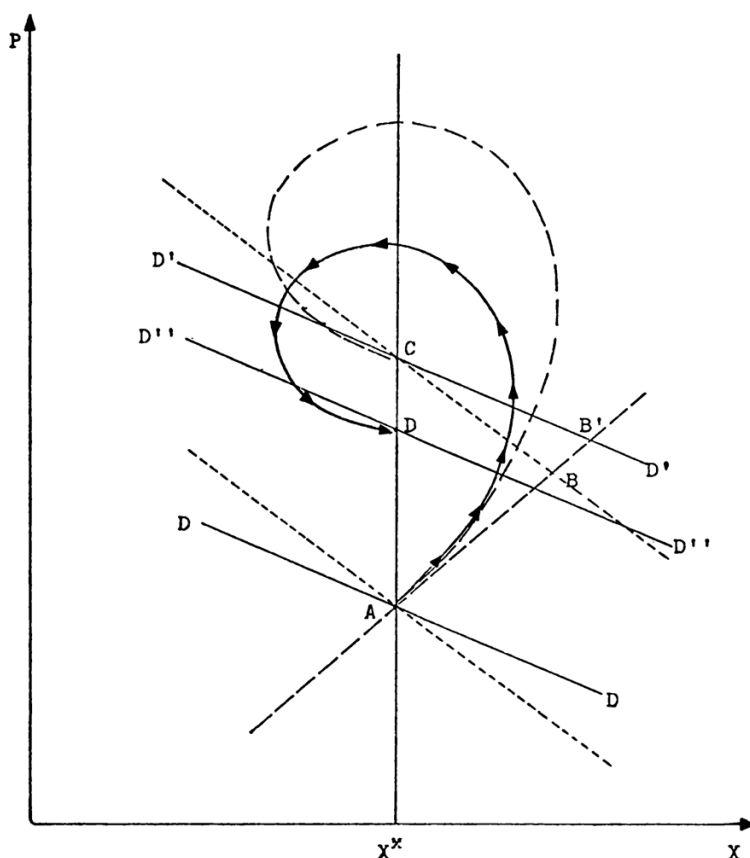
First, the “demand” curves in Figure 3, being calculated from  $v + cb = 0$  instead of  $v = 0$ , are flatter than the corresponding ones in Figure 2, while each respective pair has one point in common at  $X = X^*$  (i. e.  $b = 0$ ). Accordingly, a short run “Keynesian” solution under a fixed money supply would represent both a higher output and price level than in the case of Figure 2. (For comparison, demand curves from Figure 2 repeated and appear as dotted lines in Figure 3.)

Second, in the present model the pull which carries income away from  $X^*$  will create a government surplus and thus, via a decline in  $M$ , shift the demand schedule downwards. Without further quantitative specification it cannot be told whether this shift fully or partly compensates the upward move in the demand curve due to rising inflationary expectations. As long as the time path shows a cyclical pattern, there will be at least one period of unemployment accompanied by monetary expansion and a corresponding (partial) upward shift in the demand curve.

In Figure 3, the overall net effect on  $M$  is assumed to be negative, i. e. the final equilibrium point,  $D$ , represents lower nominal money balances than at the outset and, consequently, a lower output price level.<sup>13</sup>

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<sup>13</sup> Taking into account the assumption about stability and linearity as well as the direction of the initial demand shift, the above solution seems most plausible. However, a rigorous proof has not yet been carried out.



Figur 3

### V. Unemployment plus Inflation

In this last section consider the case where labour unions are reluctant to accept any wage rates that would lower their members' real income, i. e. they restrict the wage bargain by

$$(50) \quad DW/W \geq \pi .$$

There may be times during which unions are neither willing nor able to follow the rule given by (50). The aftermath of the oil crisis provides a fairly good example of their not being certain as to which wage raise they should demand (and which might be feasible). During the course of a normal trade cycle, however, restriction (50) seems quite plausible

with respect both to the trade unions' target of defending what they have achieved in the past and the general asymmetry of the bargaining process.

In terms of the wage adjustment equation (24) it follows from (50) that  $\lambda$  should become zero whenever employment falls below  $\bar{N}$ , i. e.

$$(51) \quad \lambda \begin{bmatrix} > \\ = \end{bmatrix} 0 \quad \text{if} \quad u \begin{bmatrix} \leq \\ > \end{bmatrix} 0 .$$

The model now consists of two distinct parts. Up to the full employment real wage rate, which may be termed  $w^0$ , equations (44) - (46) are still effective while for real wage rates higher than  $w^0$  the coefficient  $\lambda$  would vanish:

$$(52) \quad Dw = -w \mu (v + c b) ,$$

$$(53) \quad D\pi = \varepsilon \pi (v + c b) ,$$

$$(54) \quad Dm = B - m [\mu (v + c b) + \pi] .$$

At the full employment wage rate, where  $u$ ,  $B$  and  $b$  all become zero, both systems of differential equations coincide, and we shall argue that  $w = w^0$  as a limiting case is contained in either part of the model.

As to equations (44) - (46), we know from the last section that a stationary solution would require stable prices as well as full employment, i. e. the equilibrium point would lie on the common border just mentioned. A solution of (52) - (54) would require zero excess demand for goods

$$(55) \quad v(m, \pi, w) + c b(w) = 0$$

and

$$(56) \quad B(w) - m\pi = 0 .$$

These equations hold not only if both  $B$  and  $\pi$  vanish. If employment is normal and prices are stable<sup>14</sup>, they are also compatible with a whole array of pairs of positive  $B$  and  $\pi$ <sup>15</sup> or, less formally, with many different combinations of unemployment and inflation.

Let us inspect system (52) - (54) more closely. From the first two of these equations we may derive

$$(57) \quad Dw/w = -D\pi/\varepsilon .$$

<sup>14</sup> A zero inflation rate would follow from  $D\pi_0 = 0 = \pi$ .

<sup>15</sup> Since (54) - (56) are only valid for  $w \geq w^0$ , net transfers  $B$  cannot be negative, and the same follows for  $\pi$  in (56), since  $m$  should be positive for obvious reasons.

After integrating, this becomes

$$(58) \quad \ln w = -\pi/\varepsilon + A,$$

where  $A$  is a constant. In that part of the phase space, which is governed by (52) - (54), variables  $w$  and  $\pi$  obviously have to stay in a certain relation. Moreover, time does not explicitly appear in (58), so the relation must be entirely determined by the values of  $w$  and  $\pi$  which happen to be realized when the phase path "crosses the border", i. e. when the unemployment ratio becomes positive. The reason for this is quite simple. From the definition of the real wage rate it follows that

$$Dw = w [DW/W - DP/P],$$

while the adaptive expectations rule is

$$D\pi = \varepsilon [DP/P - \pi].$$

Both of the bracketed terms above are equal in absolute value if

$$DW/W = \pi,$$

i. e. if the change in the money wage rate is equal to inflationary expectations, as assumed in (51) for employment levels lower than  $\bar{N}$ .

As to the determination of the constant of integration in (58), once the value of  $\pi$  is known (call it  $\pi^0$ ) at which the phase path goes through the full employment coordinate,  $w = w^0$ , it follows from (58) that

$$\ln w^0 = -\pi^0/\varepsilon + A$$

and, consequently, for all values of  $w \geq w^0$ :

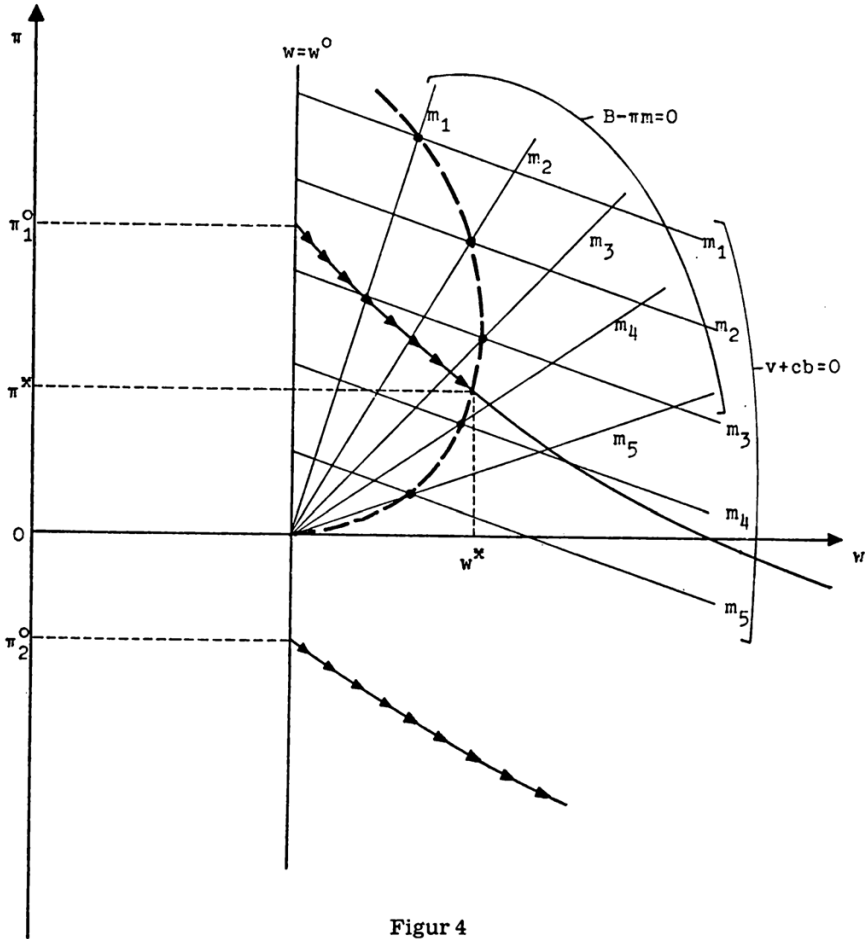
$$(59) \quad \pi = \pi^0 - [\ln w - \ln w^0].$$

Because of the negative slope of (59) the initial value  $\pi^0$  is the maximum rate of inflation that may be realized under system (52) - (54). On the other hand, equation (56) requires stationary values of  $\pi$  to be non-negative. Therefore, if  $\pi^0$  happens to be negative, an equilibrium solution for (52) - (54) does not exist within the range  $w^0 \leq w < \infty$ .

This can be demonstrated by means of Figure 4, which is a projection of parallel cuts through the phase space at different levels of  $m$ . Equilibrium condition (55) is represented<sup>16</sup> by the set of lines running from the  $w^0$  coordinate to the southeast, while (56) is a bunch of rays from a common origin at  $w = w^0$  and  $\pi = 0$ . Note that the  $w^0$  coordinate and the abscissa itself are the limiting cases for  $m \rightarrow 0$  and  $m \rightarrow \infty$ ,

<sup>16</sup> For convenience, both (55) and (56) are linearized.

respectively. Intersections of corresponding pairs of these curves are connected by the dotted line, which starts from  $w = w^0$  and  $\pi = 0$  with a positive slope and later bends back towards the  $w^0$  axis. Points on this curve are candidates for a stationary equilibrium with both unemployment and inflation.



Figur 4

Finally, two graphs of equation (59) are plotted down in Figure 4, each starting at a different initial level of  $\pi$ . If  $\pi^0 = \pi_1^0 > 0$ , a unique equilibrium is given by intersection point E, while in the second case, where  $\pi^0 = \pi_2^0 < 0$ , no stationary solution will exist. This last outcome (as well as the uniqueness of the above equilibrium) is mainly due to the

negative slope of (59). At present, however, we shall not discuss this phenomenon any further and shall confine ourselves to the statement that the partitioned system as given by (44) - (46) and (52) - (54) has at least one and at most two stationary solutions, where the first is always the equilibrium developed in previous sections.

Stability conditions for the partial system (52) - (54) are given in Appendix 3. Assuming that they are met we still cannot say very much about the actual time path. If the economy starts at a high employment level (i. e.  $N > \bar{N}$ ) and follows a cyclical path towards the stationary equilibrium, unemployment will become positive at some point in time and thus set behavioural system (52) - (54) into effect. If on the common boundary inflationary expectations are positive (i. e.  $\pi^0 = 0$ ), there may be a stable path towards a stationary solution characterized by *unemployment cum inflation*, but if this part of the model also produces cycles, the time path might swing back to the high employment region, and the above process would start all over anew, with a different value of  $\pi^0$  and a different unemployment equilibrium.

These remarks may suffice for the present until an analysis of joint stability of the two partial systems has been carried through. What seems remarkable at this point is the tendency towards a path of self-sustained inflation and unemployment. Of course, no government would tolerate such a constellation in the long run; new policy tools might be activated and, more important, the old tools would have to be checked and corrected. In the present model, the principal source of inflation is the growth in the money supply, which could be stopped by an appropriate raising of unemployment insurance premiums (as it was done in Germany at the beginning of 1976, when premiums jumped by fifty percent).

However, it does not become clear from the model at which employment level authorities should fix  $\bar{N}$ . If the economy has reached a stable position in the unemployment region, a decline of  $\bar{N}$  to the prevailing level of employment brings the growth of money down to zero immediately, but a bit later the fact that  $\bar{N}$  exceeds  $\tilde{N}$  may create new difficulties of its own kind. The problem does not seem little enough to justify its being neglected. At least the traditional fiction of the stabilizing forces of unemployment benefits should deserve a more critical interpretation.

## Appendix

### A.1 Stability of (30)

If the characteristic equation of the linearized system (30) is written as

$$(A.1) \quad x^3 + a_1 x^2 + a_2 x + a_3 = 0 ,$$

the Routh-Hurwitz theorem states that necessary and sufficient conditions for all roots  $x_i$  ( $i = 1, 2, 3$ ) to have negative real parts are

$$(A.2) \quad a_1 > 0 ,$$

$$(A.3) \quad a_1 a_2 - a_3 > 0 ,$$

$$(A.4) \quad a_3 > 0 ,$$

where  $a_1$  is the negative of the trace of the system matrix in (30),  $a_2$  is the sum of all second order principal minors, and  $a_3$  is the negative of the determinant.

Inequality (A.2) is the same as (31) in the text and is not repeated here. Expansion of the determinant gives us

$$(A.5) \quad a_3 = \varepsilon \lambda \mu m^* w^* u_w v_m > 0 ,$$

i. e. condition (A.4) will be fulfilled. The sum of the second order principal minors is

$$(A.6) \quad a_2 = \mu [m^* v_m (\varepsilon + \lambda w^* u_w) - \varepsilon \lambda w^* u_w v_\pi] .$$

This expression is not necessarily positive, but it must be if (A.3) holds. Inserting the proper expressions for  $a_1$ ,  $a_2$  and  $a_3$  in (A.3) we get

$$(A.7) \quad (a_1 - \varepsilon) \lambda m^* w^* u_w v_m > a_1 \varepsilon (\lambda w^* u_w v_\pi - m^* v_m) ,$$

which is the equivalent of condition (32).

### A.2 Stability of (47)

Write the characteristic equation of (47) as

$$y^3 + b_1 y^2 + b_2 y + b_3 = 0$$

and obtain the following results:

$$(A.8) \quad \begin{aligned} b_1 &= \mu [w^* (v_w + c b_w) - \varepsilon v_\pi + m^* v_m] \\ &= a_1 + \mu w^* c b_w > a_1 , \end{aligned}$$

$$(A.9) \quad b_2 = a_2 + B_w w^* \mu v_m > a_2 ,$$

$$(A.10) \quad b_3 = a_3 .$$

If  $a_1$  and  $a_3$  are positive, so will be  $b_1$  and  $b_3$ .

Moreover, we have

$$(A.11) \quad b_1 b_2 - b_3 = (a_1 + \mu w^* c b_w) (a_2 + B_w w^* \mu v_m) - a_3 ,$$

which is positive if (A.3) holds.

According to the Vieta Theorem on relations between roots and coefficients of an algebraic equation the following equalities must hold:

$$(A.12) \quad b_1 = - (y_1 + y_2 + y_3) , \quad a_1 = - (x_1 + x_2 + x_3) ,$$

$$(A.13) \quad b_3 = - y_1 y_2 y_3 , \quad a_3 = - x_1 x_2 x_3 .$$

Next insert (A.12) and (A.13) into (A.8) and (A.10), respectively, and arrive at (48) and (49).

### A.3 Stability of (52) - (54)

We linearize the system around a potential equilibrium point and at the same time substitute (53) by (58). We then have

$$(A.14) \quad \begin{bmatrix} Dw \\ Dm \end{bmatrix} = \begin{bmatrix} -\mu w^* (v_w + c b_w - v_\pi \varepsilon / w^*) & -\mu w^* v_m \\ B_w - m^* [\mu (v_w + c b_w) - \frac{\varepsilon}{w^*} (1 + \mu v_\pi)] - l \frac{B(w^*)}{m^*} + \mu m^* v_m \end{bmatrix} \begin{bmatrix} w^* - w \\ m^* - m \end{bmatrix}$$

This system is asymptotically stable if trace and determinant vary in sign, the first one being negative. The trace is

$$(A.15) \quad \text{tr} = -\mu [w^* v_w - \varepsilon v_\pi + m^* v_m] - [\mu w^* c b_w + B/m^*] .$$

This expression will be negative if  $a$  and  $v_\pi$  are sufficiently small or, in terms of the analysis of Appendix A.1, if  $a_1$  evaluated at this equilibrium point is positive.

The determinant of (A.14) is

$$(A.16) \quad \det = \mu w^* \left( \frac{\varepsilon}{w^*} m^* v_m + v_m B_w + \frac{B}{m^*} (v_w + c b_w) - \frac{\varepsilon B}{w^* m^*} v_\pi \right) .$$

Here the crucial term is the last one. Again, small values of  $\varepsilon$  and  $v_\pi$  might ensure a positive sign of the determinant. It should be noted, however, that (A.15) and (A.16) are evaluated at only one single equilibrium point and that results cannot be generalized in the same way as it might be done with systems (30) or (47).

### Summary

Because of their countercyclical swings unemployment benefits generally are regarded as being "stabilizing" with respect to employment, income and prices. In a short-run macro model which explicitly includes the financial part of the system, this conception is shown to be correct if "stabilizing" re-

presents a growth in the set of parametric configurations which insure stability. However, if because of the benefit system a rise in unemployment no longer induces wage rate changes to fall below the rate of inflation, the economy will tend towards a certain rate of unemployment accompanied by inflation. In the model presented, changes in employment can then be realized only by exogenous shifts of policy parameters.

### Zusammenfassung

Wegen ihrer antizyklischen Ausschläge wird die Arbeitslosenunterstützung allgemein als „stabilisierend“ in Bezug auf Beschäftigung, Einkommen und Preise betrachtet. In einem kurzfristigen Makromodell, das die Finanzierung explizit berücksichtigt, kann gezeigt werden, daß diese Vorstellung korrekt ist, wenn „stabilisierend“ heißt, daß die Menge der Parameterkonstellationen, die Stabilität des Systems sichern, wächst. Wenn allerdings wegen des Unterstützungssystems ein Anstieg der Unterbeschäftigung nicht mehr zu einem Absinken der Lohnsatzsteigerung unter die Inflationsrate führt, tendiert die Wirtschaft zu einer festen Unterbeschäftigungsrate bei Inflation. In dem vorgestellten Modell kann dann eine Änderung der Beschäftigung nur noch herbeigeführt werden durch erneute Eingriffe von außen.

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