On Internalizing Externalities: A Note

Von M. Jetzer

The problem of uncertainty inherent in non-separable externalities leads to the question if separability of the objective function is a necessary condition to ensure an unambigous individual decision.

In a seminal article, *Davis* and *Whinston* (1962) claimed that the classical tax-subsidy solution for internalizing externalities breaks down for the case of the non-separable type of externalities. This is because there arise problems of uncertainty and of the non-existence of equilibrium.

These authors defined separability of the utility function $u^k = f(x_1, \ldots, x_n)$ on the assumption that the utility of individual k provided by the consumption of one good is independent of the consumption of any other good, i. e.:

$$u^k = f_1(x_1) + f_2(x_2) + \ldots + f_n(x_n)$$

where f_i designates a function peculiar to commodity i ($f_i \neq \delta u^k / \delta x_i$). Inspection of this assumption quickly reveals that it implies independence of the marginal utility of good i from the consumption of any other good:

$$\frac{\partial^2 u^k}{\partial x_i \partial x_j} = 0, \nabla i \neq j$$

Davis and *Whinston* claim that the uncertainty inherent in non-separable externalities is due to the fact that the optimal strategy of one individual or firm depends upon the strategy selected by another individual or firm. To quote *Davis* and *Whinston* (1962, S. 255): "There seems to be *no a priori method* for determining the outputs (strategies) selected." And finally: "Non-separable externalities raise the possibility of the non-existence of equilibrium."

The formal demonstration of this result is not unassailable, but because of its substantial intuitive appeal, the presentation convinced many welfare theorists. So some unwarranted generalizations have been made and some deeper points overlooked. To certain of these I now turn.

Davis' and *Whinston's* theorem did give rise to some challenges which can only be touched upon briefly here. The first argument against the theorem concerns the problem of non-existence of equilibrium. In an important contribution in Economica, *Wellisz* (1964) suceeded in warding off the attack on the logic of the neoclassical theory of the State, as it originated in *Pigou* (1933). He set down a general proof of the existence of a determinate system of taxes and subsidies, which makes it possible to internalize non-separable externalities. *Davis* and *Whinston* (1966) rejoined that the differential equations which *Wellisz* employed fail to have a general solution method. Needless to say, this criticism in no way invalidates *Wellisz'* proof.

It is not the purpose of this note to offer a critical review of the merits of these various studies. Rather I will concentrate on the first point, namely the problem of uncertainty inherent in non-separable externalities, and prove that *Davis'* and *Whinston's* findings, based on their definition of separability, are not entirely general.

To this end let me bring *Meade's* (1952) well known article on external economies and diseconomies back to mind. There he drew a distinction between a factor of production and an "atmosphere" affecting production. Following *Meade's* example of an atmosphere, let us suppose "that afforestation schemes in one locality increase the rainfall in that district and that this is favourable to the production of wheat in that district. In this case the production of timber creates an atmosphere favourable to the production of wheat" (1952, S. 62). This situation is expressed by the following equation (x_1 stands for wheat, x_2 for timber, l for labour and c for capital):

$$x_1 = H_1 (l_1, c_1) A_1 (x_2)$$
$$x_2 = H_2 (l_2, c_2)$$

According to *Meade*, A_1 is always greater than zero: "there cannot be so powerful an external diseconomy that the output of the industry affected becomes negative" (1952, S. 63).

The first production function exhibits non-separable externalities in *Davis'* and *Whinston's* terminology. This is because of the multiplicative term $A_1 (x_2)$. Therefore the problems of uncertainty mentioned above arise.

I will show that this is not correct by proving that the separability of the objective function is not a necessary condition to ensure an unambigous individual decision. In this context, I refer to a decision being unambigous if, and only if, the set of the feasible strategies of an individual k, $s^k \in s_i^k$ contains a dominant strategy $s_*^k \in s^k$, written s_*^k dom s_i^k ·

Definition

$$s^1_*,\ s^1_i \in s^1,\ s^2_j \in s^2:\ s^1_* \text{ dom } s^1_i \Leftrightarrow u^1\left(s^1_*,\ s^2_j\right) \geqslant u^1\left(s^1_i,\ s^2_j\right),\ \nabla j$$

Theorem

The following utility function guarantees the existence of a dominant strategy for $k = 1$:

$$u^1\left(s^1_i,\ s^2_j\right) = ar\left(s^1_i\right) t\left(s^2_j\right) + br\left(s^1_i\right) + ct\left(s^2_j\right)$$

where a, b, and c are coefficients, r and t functions.

Conditions

1. $\exists\, s^1_*:\ \max r\left(s^1_i\right) = r\left(s^1_*\right)$, *i. e.* a maximum exists for $r\left(s^1_i\right)$
2. $t\left(s^2_j\right) \geqslant 0$
3. $a, b, c \geqslant 0$

Proof

$r\left(s^1_*\right) = \max r\left(s^1_i\right)$, *i. e.* $r\left(s^1_*\right) \geqslant r\left(s^1_i\right)$. Multiplied by $at\left(s^2_j\right)$:

$ar\left(s^1_*\right) t\left(s^2_j\right) \geqslant ar\left(s^1_i\right) t\left(s^2_j\right)$. Adding $br\left(s^1_*\right) + ct\left(s^2_j\right) \geqslant br\left(s^1_i\right) + ct\left(s^2_j\right)$ gives:

$ar\left(s^1_*\right) t\left(s^2_j\right) + br\left(s^1_*\right) + ct\left(s^2_j\right) \geqslant ar\left(s^1_i\right) t\left(s^2_j\right) + br\left(s^1_i\right) + ct\left(s^2_j\right)$, or

$u^1\left(s^1_*,\ s^2_j\right) \geqslant u^1\left(s^1_i,\ s^2_j\right),\ \nabla j$, *q. e. d.*

Hence, the first individual prevents the opponent of exerting an influence on the relevant marginal conditions. In this case, only the *level* of the payoff of individual 1 is subject to control by individual 2. Formally:

$$\frac{d}{ds^2_j}\left[\frac{du^1}{ds^1_i}\right]_{s^1_i = s^1_*} = 0$$

This theorem yields a clear view of the special features of the objective function underlying *Davis*' and *Whinston*'s analysis. As I mentioned above, they alluded to the fact that there were no problems of decision if the crossderivatives were zero over the whole range of the function. This is shown not to be a necessary condition. It is sufficient for the crossderivatives to be zero subject to the optimal, i. e. dominant strategy. In order to distinguish the two functions, I will characterize them as strongly separable and weakly separable, respectively. And by setting $a = 0$, $b = 1$, and $c = 1$, strong and weak separability coincide. While there is no problem of decision-making in the separable case, non-separable externalities in the weak sense do affect the margin. There, both the problem of misallocation and the problem of decision-making arise. This means that the policy-maker will not be able to determine the strategy which individual units might be following because no dominant strategies exist. Therefore "it would seem necessary, in the absence of a priori methods, to obtain information concerning the psychologies of the managers, their 'taste' for risk, and so on" (*Davis, Whinston* 1962, S. 256).

I believe I have pinpointed one of the difficulties encountered in many contentions based on the essay by *Davis* and *Whinston*. One example of a false inference by the two authors refers to the illustration of economic externalities given above. It is now clear that *Meade's* example conforms with the notion of weak separability; the term $A_1(x_2) \geq 0$ is identical with the term $t(s_j^2)$ of the utility function of my theorem. I therefore stick to my assumption (2). In other words, no obstacle prevents the affected individuals from making a rational decision. *No problems of uncertainty arise.* Contrary to the assertion of *Davis* and *Whinston, Meade's* example is *not* a "non-dominance case" (*Davis, Whinston* 1962, S. 257).

While the analysis originating in *Davis* and *Whinston* has these weak points, the present theorem too is restrictive. It is assumption (2), namely that any strategy individual 2 takes does not reduce the utility of individual 1, which makes my model not wholly general in representing a game strategic situation. In this respect *Davis*' and *Whinston*'s theorem is the more general one, for they do not introduce any restrictions on individual strategies. This has to be seen clearly. Otherwise, there is the risk of attaching a spurious generality to the present conclusion. The only possibility of achieving generality in the sense of dropping condition (2) rests upon the introduction of a third player, i. e. the State, whose function would be to prevent negative external effects. The most simple way would be for the State to compensate individual 1 by the lump sum $M = \sup |t(s_j^2)|$, the externality creating term being then transformed to $T(s_j^2) = t(s_j^2) + M \geq 0$. The point of this remark is to show that by suitable reinterpretation, negative externalities pose no problems of decision, and all the formal theory remains valid. This way out should, however, not be stressed too much. There is serious doubt regarding the applicability of this strategy. The actual information needed for diagnosing and curing the ills of the market is so great that an attempt to correct externality levels will be a success only by chance. And to postulate a government with total information is no more a real solution than to postulate markets in all commodities in all continguencies at all future dates. These points have been discussed enough in the literature on the internalization of external effects, they do not require further elaboration here.

Notwithstanding the above qualification, the substance of my argument is not affected. For I was able to point out the limitations of *Davis*' and *Whinston*'s analysis as well as wrong conclusions following from their criticism of *Meade*.

I hope this present formulation helps to clarify certain aspects of the contribution of the above mentioned authors and at the same time provides a basis for a more precise understanding of the problems of internalizing externalities. The literature on external effects, while enormous, still lacks the elegance and generality which makes the greater part of microeconomic theory so logically powerful.

Zusammenfassung

Meades bekanntes Beispiel einer positiven Externalität im Economic Journal von 1952 ist nach Ansicht von *Davis* und *Whinston* deshalb problematisch, weil es sich um eine untrennbare Externalität handelt. Es sollen deshalb besonders schwerwiegende Probleme der Ungewißheit auftreten, Probleme, welche eine rationale Entscheidung der betroffenen Individuen unmöglich machen. Es wird bewiesen, daß dies nicht stimmt. Die Trennbarkeit einer Nutzenfunktion ist keine notwendige Bedingung für rationales Handeln — die behaupteten Probleme treten in *Meades* Illustration nicht auf.

Summary

The note corrects an assertion of *Davis* and *Whinston* concerning *Meade's* illustration of external economies in the Economic Journal, 1952. Because of the multiplicative term, *Meade's* production function on p. 62 exhibits in *Davis*' and *Whinston*' terminology non-separable externalities. Therefore problems of uncertainty arise preventing the affected individuals from making a rational decision. This is shown not to be correct by proving that separability of the objective function is not a necessary condition to ensure an unambigous individual decision. Contrary to the assertion of *Davis* and *Whinston, Meades's* example is *not* a non-dominance case; no problems of uncertainty arise.

Literatur

Davis, O. A. and *A. Whinston* (1962), Externalities, Welfare, and the Theory of Games, Journal of Political Economy 70 (1962), S. 241—261.

— (1966), On Externalities, Information, and the Government-Assisted Invisible Hand, Economica 30, S. 303—318.

DOI https://doi.org/10.3790/schm.96.2.183 | Generated on 2024-07-06 02:22:06

Meade, J. E. (1952), External Economies and Diseconomies in a Competitive Situation, Economic Journal 62 (1952), S. 54—67.

Pigou, A. C. (1932), The Economics of Welfare, 4th ed., London 1932.

Wellisz, S. (1964), On External Diseconomies and the Government-Assisted Invisible Hand, Economica 31 (1964), S. 345—362.