

# Trend and Cycle in the Euro-Area: A Permanent-Transitory Decomposition Using a Cointegrated VAR Model

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## Summary

*This paper investigates the Euro-area business cycle using a multivariate autoregressive time series model with cointegration. The cointegration restrictions help to identify permanent and transitory shocks which form the stochastic part of trend and cyclical GDP, respectively. The identification allows for a historical decomposition of Euro-area GDP into trend and cycle. Further, the relative importance of both structural shocks is examined with forecast error variance decompositions. The results show that permanent shocks account for a significant fraction of output fluctuations so that the stochastic trend of Euro-area GDP has considerable variability.*

## 1. Introduction

The measurement of business cycles is a widely discussed topic in the literature. A wide range of alternative models stems from the fact that the business cycle is a purely theoretical concept and it is not clear how it should be measured unless appropriate definitions are made. In this sense, the business cycle is not a directly observable concept. This implies a variety of possible theories and empirical methods. To allow for comparisons, empirical contributions should make clear on which assumptions they rely on.

In this paper, a rather traditional definition of the business cycle is used. It is assumed that the economy fluctuates along a trend. The difference between observed output and such a trend is defined to be the cycle or the business fluctuations. The cyclical fluctuations are allowed to be persistent but have to be transitory so the cycle is a stationary time series. In the concept applied here, the trend and cycle of output are the result of shocks hitting the economy. The shocks are divided into two groups: permanent and transitory shocks. Following the widely known definition of Blanchard and Fisher (1989), the permanent shocks determine the trend whereas the transitory shocks form the cycle. The permanent-transitory decomposition (PT) employed here allows to identify these two types of shocks and derive the appropriate cycle plus stochastic trend decomposition. The method has also some background in the recent theoretical literature. For example,

Yun (1995) and Kimball (1996) propose rational expectation models with imperfect competition and price staggering that show business fluctuations around a stochastic trend due to imperfect price adjustment. The trend represents a situation where prices rigidities are absent. In this class of models the distinction of trend and cycle has strict microfoundations. In theory, it is not clear how high the variability of the trend is. For example, the Real Business Cycle (RBC) baseline model attributes almost all fluctuations to fluctuations of the trend because no rigidities or market imperfections are allowed. Hence, under this model the variance of the permanent part is nearly equal to the variance of output and there is no room for a cyclical component defined as above. Hence, the role of permanent shocks or the trend on the one hand and the transitory shocks on the other is an empirical question.<sup>1</sup> The permanent-transitory decomposition employed here is able to address this questions. The purpose of the paper is twofold: Euro-area GDP is decomposed into trend and cycle and the relative importance of permanent and transitory shocks is investigated.

From a methodological point of view, the method should be distinguished from other approaches. For example, the trend cycle decomposition does not take into account

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<sup>1</sup> The seminal papers treating these questions empirically are Blanchard and Quah (1989) and King et al. (1991).

business cycle asymmetries explicitly. Nonetheless, the cycle will not follow a strict cyclical pattern in the sense of a trigonometric function. As has been pointed out before, the output fluctuations are the result of shocks hitting the economy. Since these shocks are stochastic, partly asymmetric patterns of cyclical fluctuations may arise although asymmetries are not modeled explicitly. Moreover, the approach employed here doesn't consider multiple trends such as empirical models with regime shifts. The work presented here is most closely related to the VAR literature that uses structural identification schemes to identify potential output, for example Dupasquier et al. (1999), Astley and Yates (1999) and Funke (1998). In one way or another all these papers use a-priori long-run identifying restrictions to decompose output into a permanent and transitory part. In comparison with these papers the approach chosen here relies on only two assumptions: first, cointegration must hold, second, the groups of permanent and transitory shocks are uncorrelated. Hence, if cointegration is found in a multivariate setting, only one additional restriction is needed for a unique decomposition. The imposition of further a-priori restrictions can be avoided. This is an advantage over the existing methods especially when higher dimensional systems are investigated.

The paper proceeds as follows: In section 2 the method is explained step-by-step from estimation of the multivariate time series process to the derivation of the permanent and transitory shocks and the resulting historical decomposition of output into trend and cycle. The method is related to comparable approaches in section 3. Section 4 treats the empirical model and presents estimates of the Euro-area business cycle as well as robustness checks over the time axis. The role of permanent and transitory shocks is examined in section 5. The last section concludes.

## 2. The Permanent-transitory (PT) Decomposition

Behind the PT decomposition used here stands the general belief that behind short-run movements, the economy evolves along a growth path, which is interpreted as the trend. The economy is being affected by two types of shocks: permanent and transitory shocks. The permanent shocks are mainly alterations of technology and improvements of productivity that have a long-run effect on output. The PT decomposition defines that part of output as trend output that is due to the permanent shocks (see Blanchard and Fisher, 1989, 8). The short-run fluctuations of output are determined by the transitory shocks. These shocks have no long-run effect on output so that the transitory component is a stationary variable. The permanent and transitory shocks cannot be measured directly. Instead, the PT decomposition recovers them by identifica-

tion. The role of technological shocks is widely discussed in the theoretical literature. The Real-Business-Cycle (RBC) baseline model attributes most of the variations in output to permanent shocks, transitory shocks play no role. In recent models with optimizing behavior, monopolistic competition and price staggering, technology only determines the growth path. Short-run fluctuations are affected by imperfect price adjustments. The application of the PT decomposition can shed some light on the question of the relative importance of permanent and transitory shocks.

Starting point of the derivation is the estimation of a vector error correction model (VECM)

$$\Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \alpha \beta' X_{t-1} + \mu + \varepsilon_t,$$

where  $X_t$  is the  $m$ -dimensional vector of endogenous variables that include output as the variable of main interest. The variables are assumed to be integrated of order one and hence enter the model in first differences,  $\Delta X_t = X_t - X_{t-1}$ . The autoregressive lag order of the model is  $k-1$ .  $\mu$  is an unrestricted constant. The cointegration rank and the number of cointegration relations in the model is  $r < m$ . The cointegration property is modeled as a linear combination of the levels of  $X_t$ ,  $\beta' X_t$ , where  $\beta$  is the  $(m \times r)$  matrix of constants that forms the cointegration relationships. Since the variables are assumed to be integrated of order one, the linear combinations  $\beta' X_t$  should be stationary. In the  $(m \times r)$  matrix  $\alpha$  are the loadings that show how the system reacts to cointegration errors. The  $\varepsilon_t$  are the error terms of the system and are assumed to have mean zero and variance covariance matrix  $\Omega$ . The VEC model can be estimated with the reduced rank regression methods developed by Johansen (1988, 1991).

Since we are interested in identifying shocks, it is necessary to find an expression for the VECM that is dependent of the residuals. Later those will be transformed into shocks. The VEC model can be inverted to obtain the moving average (MA) representation

$$\begin{aligned} \Delta X_t &= \tau + A(L)\varepsilon_t \\ &= \mu + A_0\varepsilon_t + A_1\varepsilon_{t-1} + A_2\varepsilon_{t-2} + \dots \end{aligned}$$

Here,  $\Delta X_t$  is linked to the error terms through the lag polynomial  $A(L) = \sum_{j=0}^{\infty} A_j L^j$ , where  $L$  is the lag operator so that  $LX_t = X_{t-1}$ . In the MA representation the vector of endogenous variables only depends on the residuals which will be transformed into permanent and transitory shocks later. Due to this similarity, each parameter matrix  $A_i$  can be interpreted as a multiplier. An error or shock in period  $t$ ,  $\varepsilon_t$ , has an impact on  $\Delta X_t$  of  $A_0$ . After one period the shock in  $t$  causes  $\Delta X_{t+1}$  to change with  $A_1$ , after two periods it has an impact on  $\Delta X_{t+2}$  of  $A_2$  and so on.

Since the vector  $X_t$  can be understood as the sum of cumulated differences starting from an initial value, that is

$$X_{t+n} = X_{t-1} + \Delta X_t + \Delta X_{t+1} + \dots + \Delta X_{t+n},$$

the long-run impact of a shock in  $t$  is the sum of the parameter matrices  $A_j$

$$\frac{\partial X_{t+n}}{\partial \varepsilon_t} = \sum_{j=0}^n A_j.$$

Letting the forecast interval  $n$  become very large, we get the long-run multiplier

$$\sum_{j=0}^{\infty} A_j = A(1) = A_0 + A_1 + A_2 + \dots$$

This long-run effect has a natural interpretation as it provides a time series measure of long-run equilibrium.  $A(1)$  is the value of  $X_t$  due to shocks that is reached after all transitional dynamics have died out. Since the permanent shocks are defined to have a non-zero long-run effect on output, their derivation starts at  $A(1)$ . To obtain the permanent part of the model, one divides the matrix polynomial into a long-run and a short-run part, that is

$$A(L) = A(1) + \tilde{A}(L)(1-L),$$

where  $\tilde{A}(L)$  is simply  $\tilde{A}(L) = (A(L) - A(1))(1-L)^{-1}$ . The first difference of the endogenous variables can then be expressed as

$$\begin{aligned} \Delta X_t &= \tau + A(L)\varepsilon_t \\ &= \tau + A(1)\varepsilon_t + \tilde{A}(L)(1-L)\varepsilon_t \\ &= \tau + \beta_{\perp}(\alpha'_{\perp}(I - \sum_{i=1}^{k-1} \Gamma_i)\beta_{\perp})^{-1}\alpha'_{\perp}\varepsilon_t + \tilde{A}(L)(1-L)\varepsilon_t, \end{aligned}$$

where  $\beta_{\perp}(\alpha'_{\perp}(I - \sum_{i=1}^{k-1} \Gamma_i)\beta_{\perp})^{-1}$  is the long-run effect of the  $(m-r)$  permanent shocks  $\alpha'_{\perp}\varepsilon_t$ ,<sup>2</sup>  $\beta_{\perp}$  and  $\alpha_{\perp}$  are full rank  $(m \times (m-r))$  orthogonal complements to the cointegration vectors  $\beta$  and the matrix of the loadings  $\alpha$ , respectively. The orthogonal complement is defined as  $\alpha'\alpha_{\perp} = 0$ . For later use, we call the permanent shocks  $\varepsilon_t^p = \alpha'_{\perp}\varepsilon_t$  with dimension  $(m-r)$ . It must be noted that the permanent shocks are identified as a group of shocks. Individual shocks are not identified because we only seek for their overall impact on output which was defined to be the trend.

One must find an expression where the MA representation is related to the permanent shocks. The moving average representation is to be decomposed into

$$\begin{aligned} \Delta X_t &= \tau + A(L)\varepsilon_t \\ &= \tau + A^p(L)\varepsilon_t^p + A^{tr}(L)\varepsilon_t^{tr}. \end{aligned}$$

<sup>2</sup> For a detailed derivation, see Johansen (1995, 41).

The aim is now to find the lag matrices  $A^p(L)$  and  $A^{tr}(L)$  as well as the transitory shocks  $\varepsilon_t^{tr}$ , while the permanent shocks are already identified in the VEC model. Yang (1998) shows how to obtain the unknown matrices. He defines

$$\Delta X_t = \tau + A(L)\bar{\alpha}_{\perp}\alpha'_{\perp}\varepsilon_t + A(L)\bar{\gamma}\gamma'\varepsilon_t$$

using the unknown matrices  $\bar{\alpha}_{\perp}\gamma$  and  $\bar{\gamma}$  with appropriate dimensions,  $(m \times (m-r))$ ,  $(m \times r)$  and  $(m \times r)$ , respectively. The matrix  $g$  transforms the residuals into the transitory shocks such that  $\varepsilon_t^{tr} = \gamma'\varepsilon_t$  is an  $r$ -dimensional vector. Again, the only things we know are the permanent shocks  $\alpha'_{\perp}\varepsilon_t$  and the MA lag polynomial  $A(L)$ . All unknown matrices must now be constructed so that the permanent and transitory part add up to the MA polynomial, that is

$$A(L)\varepsilon_t = A(L)\bar{\alpha}_{\perp}\alpha'_{\perp}\varepsilon_t + A(L)\bar{\gamma}\gamma'\varepsilon_t,$$

so that trend and cycle sum up to the whole time series process. After summarizing terms, this adding-up restriction implies

$$(\bar{\alpha}_{\perp} \quad \bar{\gamma}) \begin{pmatrix} \alpha'_{\perp} \\ \gamma' \end{pmatrix} = I \text{ or } (\bar{\alpha}_{\perp} \quad \bar{\gamma}) = \begin{pmatrix} \alpha'_{\perp} \\ \gamma' \end{pmatrix}^{-1}.$$

Hence, if we know the matrix  $\gamma$ , we know the left hand side and can determine the rest of the unknown matrices. Because there is no further information left to identify  $\gamma$ , one has to impose a further restriction. Following the majority of the shock hunting literature, Yang (1998) assumes that permanent and transitory shocks are uncorrelated, that is

$$E \left[ \varepsilon_t^p \varepsilon_t^{tr'} \right] = E \left[ (\alpha'_{\perp} \varepsilon_t) (\gamma' \varepsilon_t)' \right] = 0$$

This restriction is fulfilled by the matrix

$$\gamma = \alpha - \alpha_{\perp}(\alpha'_{\perp}\Omega\alpha_{\perp})^{-1}\alpha'_{\perp}\Omega\alpha,$$

which is only in terms of known matrices. Given this matrix, the different structural groups of shocks and their multipliers are identified. One can now derive the permanent part of output

$$\Delta X_t^p = \tau + A^p(L)\varepsilon_t^p = \tau + A(L)\bar{\alpha}_{\perp}\alpha'_{\perp}\varepsilon_t,$$

which is interpreted as the trend. In the PT decomposition used here, output minus trend output is the transitory part. In terms of the structural multipliers and shocks it is

$$X_t^{tr} = \sum_{i=1}^t A^{tr}(L)\varepsilon_t^{tr} = \sum_{i=1}^t A(L)\bar{\gamma}\gamma'\varepsilon_t,$$

ignoring a starting value for simplicity and redefining the lag polynomial. To summarize, the PT decomposition de-

scribed here is based on two main assumptions: First, cointegration is a valid restriction of the empirical model and second, the groups of structural shocks are assumed to be uncorrelated. With these assumptions it is possible to uniquely identify permanent and transitory shocks which can be used to decompose a vector of time series into trend and cycle.

### 3. Comparison with other Methods

The proposed method is compared with other trend and cycle decompositions in the literature. Especially its relationship to other measures based on VAR models is worth mentioning. Evans and Reichlin (1994), for example, propose the so-called multivariate Beveridge-Nelson (MBN) decomposition. In the MBN, the trend is restricted to be a random walk. This implies that shocks that have a permanent effect on output immediately alter trend output with their full long-run impact measured by the long-run multiplier  $A(1)$ . This definition of trend ignores possible partial adjustments after a permanent shock occurred. This assumption is in stark contrast to the widely held view that technological innovations have transitional dynamics. Lippi and Reichlin (1994) declare the random walk assumption of trend output as inconsistent with standard views about the dynamics of productivity shocks that are justified with adjustment costs on capital and labor, learning-by-doing processes and time to build. The PT decomposition applied here allows for more general adjustment processes after the occurrence of a structural shock.

Another widely applied tool to decompose output into trend and cycle is the structural VAR (SVAR) approach.<sup>3</sup> Here, a VAR model without cointegration is estimated. The model is also inverted into MA form. Then, often restrictions on the long-run matrix of shocks  $A(1)$  must be implemented. For example, a structural shock has no long-run effect on an endogenous variable and hence is a transitory shock.<sup>4</sup> The structural shocks are usually assumed to be mutually uncorrelated. But in higher dimensional systems it is problematic to identify shocks and find an economic meaning for them. Moreover, the identification schemes are not unique. When cointegration is found, no such identifying restrictions are needed and the PT decomposition above should be applied without the need for further identification. Since the data set we will use later in the empirical application shows common trends, the PT decomposition seems to be the appropriate method. In other VAR based trend-cycle decompositions, the cointegration restrictions are sometimes not fully taken into consideration. The approach of Dumasquier et al. (1999) uses VAR models to determine the permanent part of output under consideration of the transitory dynamics of permanent shocks, too. They call their approach LRRO, be-

cause long-run restrictions are imposed on shocks to output. This is in general also in accordance with the PT decomposition, but what differs from this paper is the way in which the long-run restrictions are imposed. Dumasquier et al. (1999) suggest to estimate the VAR in a restricted form when cointegration is present. In their paper, a two-step strategy is used. At the first step, the cointegration vectors are determined using for example preliminary estimations. In the second step, output in first differences, the cointegration errors and other variables enter a new vector of endogenous stationary variables that is used to form a VAR model. Then, after the inversion direct restrictions on the long-run matrix of shocks serve to identify permanent and transitory shocks. Here, Dumasquier et al. (1999) use a triangularization of the multiplier matrix  $A(1)$  so it has full rank. One objection can be stated against this identification scheme. If there is cointegration in the set of variables, there are less permanent shocks than the number of variables and the long-run multiplier matrix has reduced rank. Hence, the cointegration restrictions of the first step of the LRRO approach is not correctly taken over into the second step. In the PT decomposition applied here, the cointegration restrictions are fully taken into account. Once the cointegration vectors are estimated, the assumed non-correlation of permanent and transitory shocks leads to a uniquely defined permanent part of output. The two-step procedure of the LRRO approach is less efficient than the PT decomposition applied here, because the explicit restrictions in the first step model are not taken into account in the restricted VAR estimation. Although it is possible to restrict the long-run matrix of a restricted VAR correctly in principle, the PT method applied here is more direct. Of course, this advantage holds only if cointegration can be found. If not, a-priori restrictions have to be used to identify permanent and transitory shocks as in the SVAR approach.

Another group of models that provide useful decompositions into trend and cycle is the group of state-space models with unobserved components (UC). These models can be analyzed using the Kalman filter and estimated with maximum likelihood where trend output and the cycle are unobserved components. In UC models, an additional equation to define trend output must be supplied. Trend output is often restricted to follow a random walk, sometimes with noise. The UC approach in general has the potential to implement richer trend dynamics. But multivariate trends as in VAR or VEC models are not possible due to identification problems. Another difference in comparison with the PT approach is the more restricted modeling strategy, since a general-to-specific procedure is not applicable due to the computational burden of the

<sup>3</sup> A recent application is Astley and Yates (1999).

<sup>4</sup> See the famous example from Blanchard and Quah (1989) for a bivariate VAR model.

iterative Maximum Likelihood estimation. The major advantage of the UC approach is that it is the only method at the moment that can implement economic theory and the estimation of trend and cycle components in one step. Although it is possible to estimate VEC models with economic content, the PT decomposition is essentially a two-step procedure that needs an estimation step and after that an identification step where the permanent and transitory parts are derived from the model's parameters as shown above.

To conclude, in relationship to the VAR methods discussed above, the PT decomposition employed here relies on weaker assumptions concerning the time series properties of the trend part and a more direct identification strategy when cointegration is given. These advantages over the existing VAR based trend measures motivate the measurement of Euro-area trend output with the PT decomposition in this paper. In comparison with the UC models it is not clear whether the higher flexibility of the VEC models overcompensates the deficiencies of the two-step identification procedure. These approaches can hardly be compared because of their different modeling philosophies. Moreover, the unobservability of the trend and cyclical components in general should lead one to use these models as complements.

#### 4. Estimation of a Euro-Area VECM

We now follow closely Evans and Reichlin (1994) and Dumasquier et al. (1999) who decompose U.S. output into trend and output gap with similar methods, to estimate the PT decomposition for the Euro-area empirically. In both of these papers, VAR models with cointegration restrictions are estimated. Evans and Reichlin (1994) include time series into the data set which are expected to be good forecasters of GDP. This stems from the fact that the long-run value of permanent shocks, as represented by the long-run multiplier matrix  $A(1)$ , can be interpreted as the long-run forecast of the underlying series (see Evans and Reichlin, 1994, 234). The use of variables that help to predict output movements may therefore improve the PT decomposition. In their paper, Evans and Reichlin (1994) use preliminary Granger causality tests to identify possible variables that help to explain GDP fluctuations. As a result, they find consumption, the unemployment rate, a composite leading indicator and a coincident indicator as suitable indicators.<sup>5</sup> Because of data limitations, some of these variables are not available for an investigation of Euro-area trend output. Nonetheless, we apply the same modeling strategy using a set of Euro-area variables and perform partial Granger causality tests to check for the explanatory power of various time series for output. Hence, the selection of variables in the mentioned paper is not based on theoretical grounds explicitly. The reason for that

stems from the fact that a lot of theoretical models have a good fit for US data but not for European data (see Neusser, 1991). For example, preliminary testing shows that the long-run restrictions of the neoclassical growth model don't hold for the Euro-area data set employed here. This is in line with earlier tests for single countries, but certainly not satisfying from a theorist's point of view.

To investigate the predictive power of various variables, one-sided granger causality tests are performed. It is tested whether a given number of lags of a possible indicator variable helps to explain a portion of output variability significantly. For this purpose, we use a quarterly data set which has been compiled from various sources because no official data with a sufficient sample size is available at the moment. Detailed information about the data is given in the data appendix. Table 1 presents the results of the causality tests.

Table 1

**Granger causality tests**

| Variable | Lag order |      |      |      |
|----------|-----------|------|------|------|
|          | 2         | 3    | 4    | 5    |
| Dlc      | 0.51      | 0.68 | 0.92 | 0.97 |
| Dlgfcf   | 0.34      | 0.25 | 0.55 | 0.43 |
| Dsr      | 0.83      | 0.28 | 0.01 | 0.03 |
| Dlli     | 0.00      | 0.00 | 0.00 | 0.00 |

Notes: Figures in the table are P-values for the null of no explanatory power of the various indicators. Dependent variable in each case is output. The explanatory variables in first differences are Dlc = private consumption in logs, Dlgfcf = gross fixed capital formation in logs, Dsr = short-term interest rate, Dlli = OECD leading indicator in logs.

The tests suggest that only the short-term interest rate as well as the OECD leading indicator have a significant impact on output. Other variables such as investment have no explanatory power for output. Hence, in the following a trivariate VAR model with output, the short-term interest rate and the leading indicator is estimated. Now, information criteria as well as goodness-of-fit statistics are used to determine the lag length of the VAR model. The information criteria indicate different lag lengths. The Akaike criterion gives 5 lags, whereas the Schwarz criterion indicates two and the Hannan-Quinn criterion 3 lags. Since the criteria show no unique result, additionally the residual statistics of the VAR models are investigated. A model with four lags has the best overall properties. Its residual properties are presented in the following table 2. For the goodness-of-fit statistics, the VAR is estimated with full rank.

<sup>5</sup> The data set used by Dumasquier et al. (1999) combines GDP, consumption and a short-term interest rate to decompose output.



Table 2

**Goodness-of-fit statistics**

| Multivariate tests                                |  |                                     |
|---|--|-------------------------------------|
| Autocorrelation<br>LM(1 or 4), CHISQ <sub>9</sub> |  | Normality<br>DH, CHISQ <sub>6</sub> |
| Lag 1, 10.20 (0.33)                               |  | 4.63 (0.59)                         |
| Lag 4, 13.78 (0.13)                               |  |                                     |
| Univariate tests                                  |  |                                     |
|   | Heteroscedasticity<br>ARCH, CHISQ <sub>4</sub> | Normality<br>DH, CHISQ <sub>2</sub> |
| Lgdp  | 1.40 (0.85)                                    | 2.35 (0.31)                         |
| Sr  | 4.85 (0.30)                                    | 1.47 (0.48)                         |
| Lli   | 2.14 (0.71)                                    | 1.81 (0.41)                         |

Notes: P-values are in parentheses. Each test is chisquared distributed where the index denotes the degrees of freedom. The VAR in levels is estimated using 4 lags and an unrestricted constant as well as the following impulse dummies: dum791, dum812, dum911, dum922, dum923 which are one in the quarter mentioned where the first to digits assign the year and the last digit the corresponding quarter. The dummies are zero elsewhere in the sample. The variables are Lgdp = output in logs, Sr = short-term interest rate and Lli = OECD leading indicator in logs.

For all the presented tests, the null hypothesis is absence of specification error.<sup>6</sup> Hence, high P-values in brackets tend to support the absence of misspecification. The tests indicate sufficient statistical properties of the VAR model. There is no sign of autocorrelation, non-normality or heteroscedasticity. However, as denoted in the table, several impulse dummies must correct for outliers to fulfill especially the requirement of normal distributed residuals although the results concerning the cycle and trend decomposition are only slightly altered by the inclusion of these additional variables. We can now perform a cointegration rank test to determine the number of cointegration vectors in the system. Table 3 shows the trace statistic and the corresponding critical values.<sup>7</sup>

Table 3

**Cointegration rank**

| H0: r | Trace<br>Statistic | 95 percent critical values |           |
|-------|--------------------|----------------------------|-----------|
|       |                    | Asymptotic                 | Bootstrap |
| 0     | 55.55              | 29.68                      | 32.42     |
| 1     | 19.23              | 15.41                      | 17.20     |
| 2     | 2.85               | 3.76                       | 4.95      |

Notes: The asymptotic critical values are from Osterwald-Lenum (1992), the bootstrap critical values are generated with 1,500 replications of the model.

The test procedure begins to assume that there are zero cointegration restrictions under the null. If the null is rejected, the cointegration rank is increased and the new null of a cointegration rank of one has to be tested. The testing procedure proceeds this way and stops until the null cannot be rejected the first time. The critical values don't follow standard distributions and have been simulated in the literature using asymptotic distributions (see, for example, Osterwald-Lenum, 1992). The corresponding critical values are determined for alternative specifications of the deterministic part of the VAR model, because the asymptotic distributions are altered by the inclusion of time trends, constants or intervention dummies. Because our model includes various impulse dummies, the critical values may be different from the tabulated ones in the literature. Though, their impact may be small since each impulse dummy eliminates only one point of information from a relatively large data set. Nonetheless, in addition to the asymptotic critical values, a bootstrap simulation exercise is performed where the critical values are derived on the basis of their observed empirical distribution. This procedure has the advantage of considering the different deterministic terms of the models as well as the finite sample size of the data set. The asymptotic values instead are derived under the assumption of an infinite sample size. However, there is not a big difference between the critical values as the table of the trace statistic shows. The critical values obtained from the bootstrap exercise are only slightly larger than their asymptotic counterparts without the inclusion of dummies and only an unrestricted constant. The trace statistics indicate in both cases the existence of two cointegration relationships. So we can conclude that a cointegration rank of two is a statistically supported restriction for the model. Moreover, the important requirement of cointegration for the applicability of the PT decomposition is given.

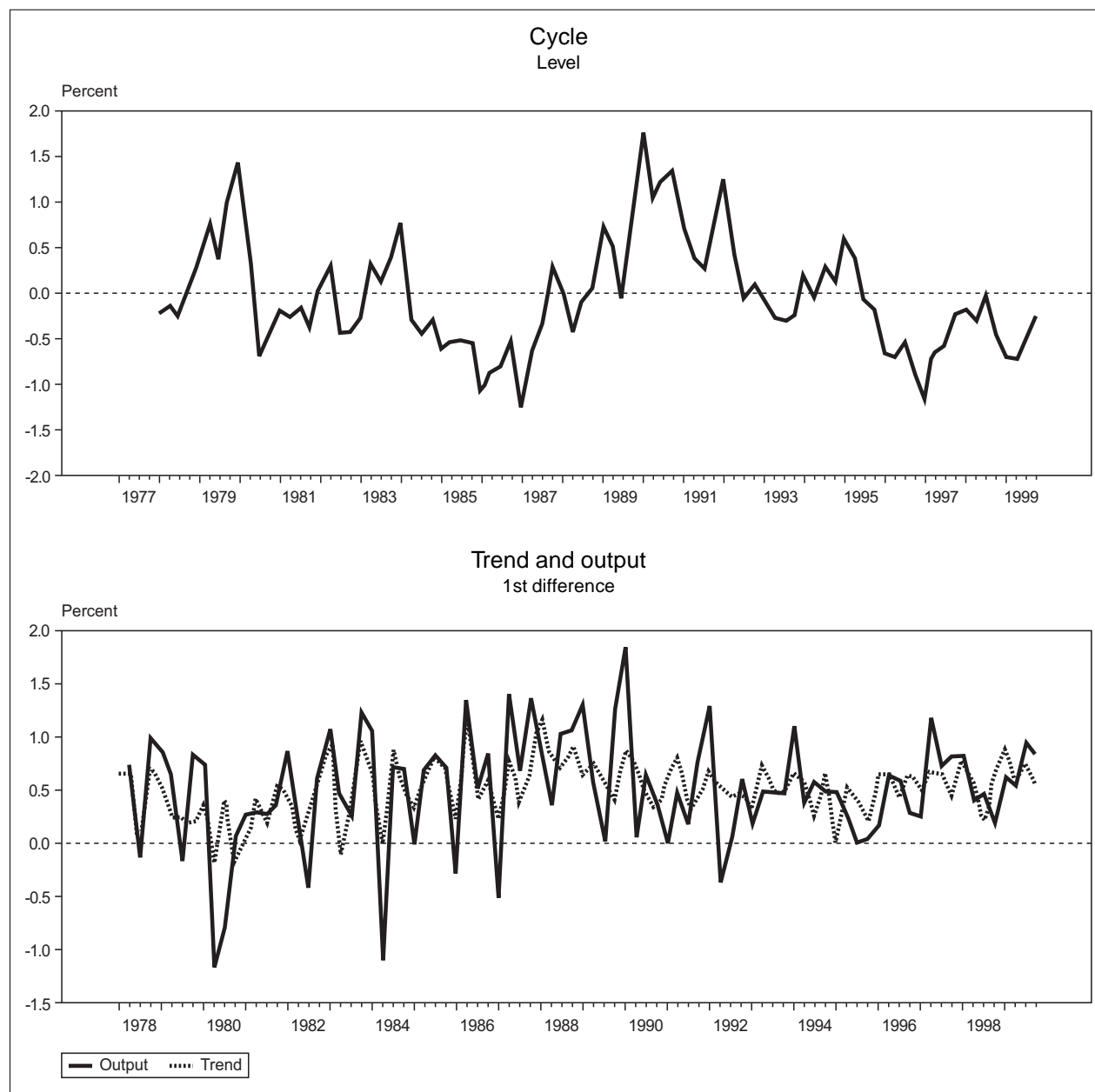
## 5. Trend and Cycle in the Euro-Area

With the estimated VEC model we can now derive the permanent and transitory part of output. Therefore, in addition to the cointegration restrictions the second assumption of the uncorrelatedness of the structural shocks is imposed, so the groups of permanent and transitory shocks can be identified as well as their multipliers. According to the definition of Blanchard and Fisher (1989), the trend of output for the Euro-area is derived as that part of output that is due to the permanent shocks and the cycle as that part determined by the transitory shocks. In figure 1, the cycle in levels and the trend with output in first differences are displayed.

<sup>6</sup> The tests are described in detail in Hansen and Juselius (1995, 72–76).

<sup>7</sup> The trace statistic is derived for example in Johansen (1988).

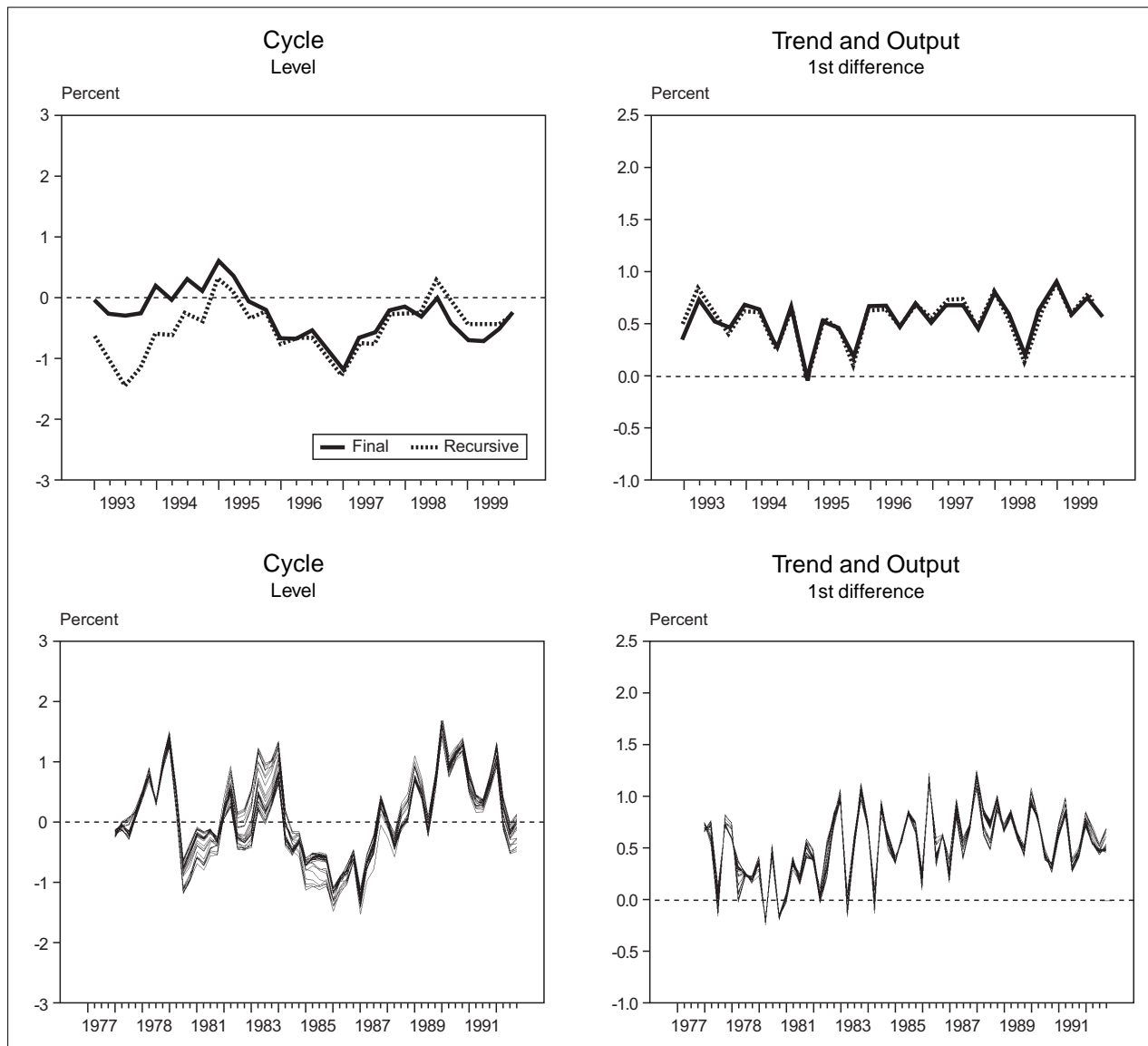
Figure 1



The cyclical component as shown in the upper panel lies between  $-1.5$  percent and  $1.8$  percent. It is not a smooth function of time but with a clearly persistent shape. But as the use of the PT decomposition suggests, the cycle is stationary over the sample. The second panel of the figure shows the first differences of output and the trend. Here, trend variability is considerable. In some periods the trend follows output fluctuations in the same direction, although it is clearly less volatile. The relative variance of the trend in relation to the variance of output is 28 percent. Hence, one fourth of output fluctuations are due to movements of the trend.

For applied business cycle usage, it is important whether the obtained estimates of trend and cycle are sufficiently stable when new time series information becomes available. When longer time series are published, both components shouldn't change dramatically after a reestimation of the model because this would distort their usefulness as indicators for monetary policy (see Camba-Méndez and Palenzuela, 2001). To check the stability of the trend output and cyclical estimators, we now perform a recursive estimation where the sample is divided into various subsamples starting from 1993:1. The sample size is increased step-by-step by one quarter, the whole

Figure 2



VEC model is estimated and the structural components are derived for each subsample. Each time the last estimate of trend output growth and the cyclical components are saved. This gives time series of most recent PT components which can be compared with the final estimate.

Moreover, for each of the subsamples trends and cycles are stored for the past time span. These estimates indicate the variability of past business cycle estimates and whether the judgement about past business cycles varies with the occurrence of new information. Theoretically, the above measures of reliability overestimate the robustness of the estimators, because in reality, not only the model's parameter change, but also the recent data points are revised by the statistical offices. Since this is a problem each trend-cycle measure has to face and usually such

revisions are not regularly documented for European time series, this is not further investigated here.

In the upper panel, the two graphs show end-of-the-sub-sample estimators of the cycle and trend. The trend and cycle components are quite stable at the end of each subsample. In 1993, the cyclical downturn was somewhat overestimated in absolute terms. The final estimate for the full sample is not that pessimistic in that time span. The trend estimation is relatively more reliable than the cycle. For the whole time span under consideration the recursive estimates don't deviate far from the final one. Concerning the judgement of past business cycles the recursive estimates are quite stable and give an impression of a quite stable business cycle measure. But again, the trend measures show less variability than their cyclical counterparts.



## 6. The Relative Importance of Permanent and Transitory Shocks

To treat the question of how important the two groups of structural shocks are, a forecast experiment is undertaken. The model predicts future outcomes of the variables based on past information. Future shocks that hit the economy lead to forecast errors. The statistic derived below shows how much variance can be attributed to permanent and transitory shocks. It provides a natural measure of the relative importance of shocks. Starting point to derive the forecast error decomposition is the MA representation of the VEC model

$$\Delta X_t = \tau + A(L)\varepsilon_t.$$

A projection  $h$  periods into the future gives the forecast error

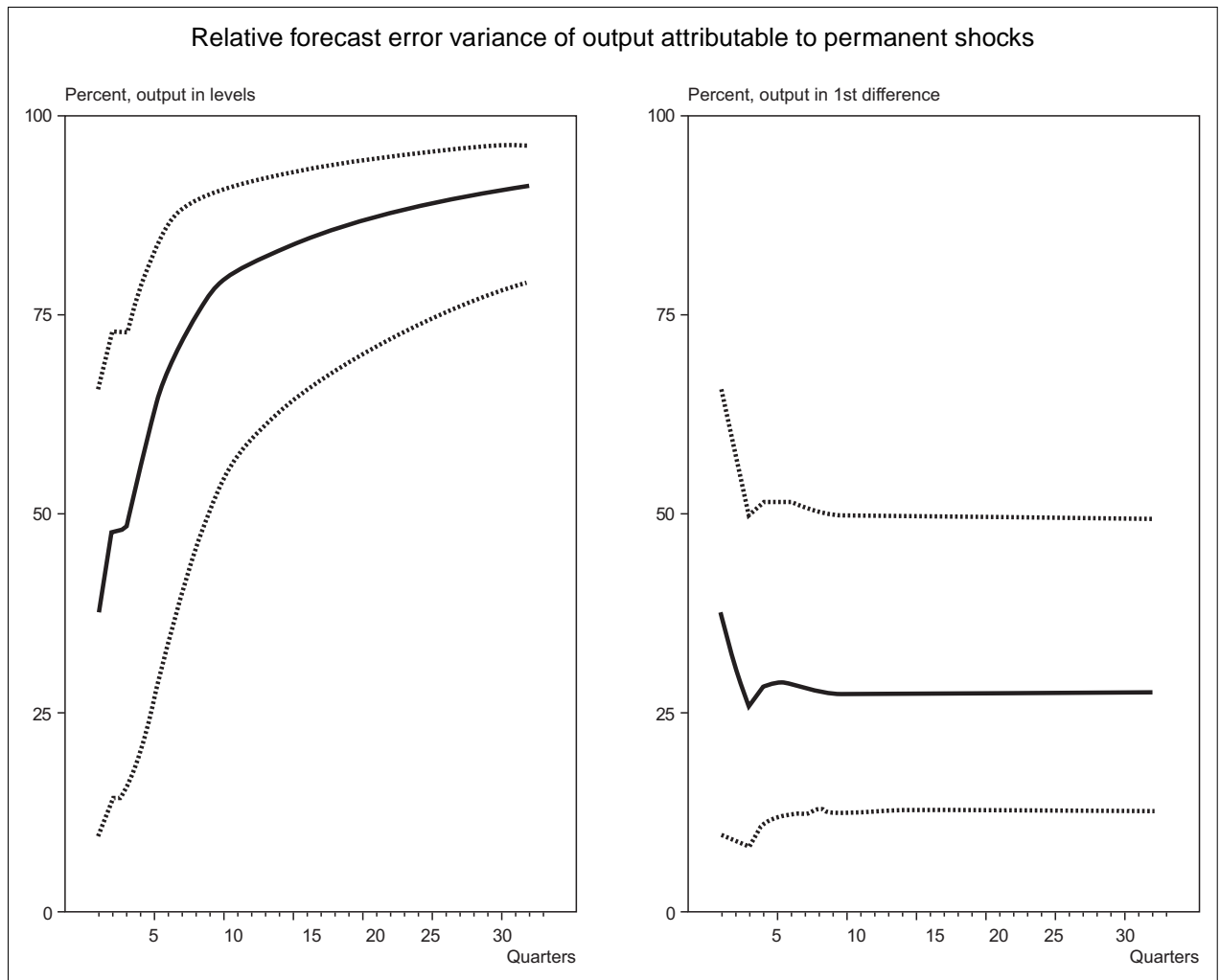
$$FE_{\Delta X_{t+h|t}} = \Delta X_{t+h|h} - \Delta X_{t+h} = A_0 \varepsilon_{t+h} + A_1 \varepsilon_{t+h-1} + \dots + A_{h-1} \varepsilon_{t+1},$$

so forecast errors are functions dependent on future shocks that hit the economy. The variance of this forecast error and its decomposition into a permanent and transitory part is given in the following.

$$\begin{aligned} FEV_{\Delta X_{t+h|t}} &= E(\Delta X_{t+h|t} - \Delta X_{t+h})(\Delta X_{t+h|t} - \Delta X_{t+h})' \\ &= \sum_{i=0}^{h-1} A_i E(\varepsilon_{t+i-1} \varepsilon'_{t+i-1}) A_i' \\ &= \sum_{i=0}^{h-1} A_i (\bar{\alpha}_{\perp} \bar{\gamma}) \begin{pmatrix} \alpha_{\perp} \\ \gamma \end{pmatrix} E(\varepsilon_{t+i-1} \varepsilon'_{t+i-1}) \begin{pmatrix} \alpha_{\perp} \\ \gamma \end{pmatrix}' (\bar{\alpha}_{\perp} \bar{\gamma})' A_i' \\ &= \sum_{i=0}^{h-1} (A_i^p A_i^{tr}) E \begin{pmatrix} \Omega^p & 0 \\ 0 & \Omega^{tr} \end{pmatrix} (A_i^p A_i^{tr})' \end{aligned}$$

From the second to the third line, the add-up restriction of the permanent and transitory components was imposed. In the third line it is again assumed that permanent and transitory shocks are uncorrelated. Writing this out gives

Figure 3



$$\begin{aligned} \text{FEV}_{\Delta X_{t+h|h}} &= \sum_{i=0}^{h-1} \left( A_i^p \Omega^p A_i^{p'} + A_i^{\text{tr}} \Omega^{\text{tr}} A_i^{\text{tr}'} \right) \\ &= \text{FEV}_{\Delta X_{t+h|h}}^p + \text{FEV}_{\Delta X_{t+h|h}}^{\text{tr}} \end{aligned}$$

Hence, the uncorrelatedness of the structural shocks implies that the forecast error variance can be decomposed into the forecast error variance of the permanent and transitory part additively. In the literature, often the relative importance of the permanent and transitory shocks is investigated. This is simply the proportion of the forecast error variance due to permanent shocks from the overall output variance. So far, the forecast error variance has been decomposed for the first difference of the endogenous variables. These values converge to the variances of the trend and cycle in the long-run so the forecast error variance decomposition should tend to replicate the variance ratios of the historical decomposition of output in the second panel of figure 1 asymptotically. In comparable studies, often variance decompositions are often applied for a further investigation of the relative importance of permanent and transitory shocks. In detail, it is tested how high the relative forecast error variance of the permanent shocks is at business cycle frequencies, for example up to 20 quarters. The forecast error variances in levels are obtained by simply cumulating the appropriate multiplier matrices. The empirical forecast error decompositions for the Euro-area output in levels and first differences are presented in figure 3. The figures show the proportion of forecast error variance of output that is due to a permanent shock in the initial period. Confidence bands are calculated via bootstrap with 1,500 replications.

The relative forecast error variance of output in levels attributable to the permanent shocks is displayed in the left graph. Since the permanent shocks are by definition the only ones that have a long-run effect on output, the forecast error variance should be explained fully by the permanent shocks in the long-run. Hence, at the end of the simulation horizon, the relative forecast error variance

should be 100 percent. This is the case in the figure, where the relative variance has a tendency towards the upper bound. But for business cycle analysis the higher short- or medium run frequencies are more of interest. Here, within a time span up to five years or 20 quarters, the permanent shocks account for a lot of the forecast error variance. For example, after five years the relative forecast error variance of the permanent shock is approximately 80 percent increasing from nearly 30 percent in the initial period. Hence, in accordance with the above results from the historical decomposition, the permanent shock plays a significant role at business cycle frequencies. The relative forecast error variance in first differences converges to the relative variances of the permanent part of output very fast. The results show that after a short time period of adjustment, the permanent shocks account for 28 percent of output variability so one fourth of output variations can be attributed to fluctuations of the trend and three fourths are attributable to cyclical fluctuations.

## 7. Conclusions

The empirical results give an impression of Euro-area business cycle fluctuations. The measured trend of output in the Euro-area has some variability. According to the empirical results, more than one fourth of output variability is due to variability of the trend. When output decreases in a cyclical downturn the trend may move in the same direction. Hence, the variability of the trend is higher than that of a simple linear deterministic trend. On the other hand, permanent shocks don't account for all of the output fluctuations. So the results obtained here doesn't support extreme views of the business cycle. If one equals the permanent or trend part of output with the supplied production of the economy, one has to conclude the supply side of the economy is quite flexible but far away from fully explaining output fluctuations.

## Data Appendix

In the data set used in this paper, time series for GDP, private consumption, gross fixed capital formation, a short-term interest rate and a leading indicator are employed. Since data limitations are present especially for the Euro-area, this appendix explains how the various time series are obtained.

The used time series are quarterly and seasonally adjusted except the interest rate. The sample range is from 1977: 1 to 1999: 4. The main data source for aggregated

Euro-area national accounts data is Eurostat. This institution provides time series in accordance with the European System of National Accounts. Unfortunately, the time series of the quarterly national accounts are only available from 1991 up to now. This data series would imply only a very small sample size for econometric testing.

To get longer time series and therefore increase the degrees of freedom, one solution of this problem is to aggregate national time series. For the national accounts

time series, we use data from OECD, Main Economic Indicators. Since these time series are measured in national currencies, the time series of the member countries must be converted. Aggregation requires a conversion of each member countries' GDP into a single currency. Here, the method of the ECB that uses fixed exchange rates to aggregate past money data is employed (ECB, 1999, 42). Output at PPP exchange rates for 1997 of the OECD are used to construct the weights of the national series. A detailed discussion of the alternative weighting schemes can be found in Fagan and Henry (1999). The resulting series is then linked with the GDP series of Eurostat. For estimation, the GDP and consumption time series are

transformed into natural logarithms. The OECD leading indicator for the Euro-area is already available for a sufficient time span and is directly used for estimation after taking logarithms. The money market rate is the 3-month deposits interest rate provided by the ECB. The series starts in the first quarter of 1994. A longer time series can be generated by aggregation again. National short-term interest rates are provided by the IMF, International Financial Statistics. We use the money market rate (line 60B). The weights for aggregation are the same as for the national accounts data. The resulting series is linked with the 3-month deposits interest rate.

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## **Zusammenfassung**

### **Trend und Zyklus im Euroraum:**

#### **Eine permanent-transitorische Zerlegung unter Verwendung eines kointegrierten VAR-Modells**

*Der Beitrag untersucht den Konjunkturzyklus im Euroraum unter Verwendung eines multivariaten Zeitreihenmodells mit Kointegration. Die Kointegrationsrestriktionen helfen, permanente und transitorische Schocks zu identifizieren. Die permanenten Schocks bilden den stochastischen Trend, während die transitorischen Schocks den zyklischen Teil der Produktion bestimmen. Die Identifikation erlaubt eine historische Zerlegung der Produktion im Euroraum in Trend und Zyklus. Zudem wird mit Hilfe von Prognosefehler-Varianzzerlegungen die Bedeutung der strukturellen Schocks untersucht. Die Ergebnisse zeigen, dass die permanenten Schocks einen signifikanten Anteil an den Schwankungen in der Produktion haben, so dass der Trend der Produktion im Euroraum eine deutliche Variabilität aufweist.*