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M&A and the simulation-based valuation of companies with an uncertain exit price and special rights

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Abstract

This article presents a new methodological approach to value private equity investments based on simulation. The valuation relies on 'imperfect replication'. This method does not presuppose the perfection of the capital market and is essentially built on measuring the risk. The approach turns out to be easy to implement. Firm specific characteristics as well as and existing special rights can be depicted and modelled. The proposed methodology is of immediate practical usefulness as it can help to find decision support for concrete investment situations. Also, during the investment period it can be used for monitoring. The originality of the research lies in the combination of Monte Carlo simulation, multiple methods, relevant risk measures and risk-value models.

Keywords: Company valuation, share valuation, exit price, risk analysis, Monte Carlo simulation, incomplete replications

JEL Classification: G17, G24, G32, G33, G34

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I. Introduction and overview

A key challenge for private-equity and venture-capital companies is estimating a realistic range of possible future exit prices for an investment, particularly

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in small- and medium-sized enterprises (SMEs[1]). The current value or the maximum acceptable purchase price (the subjective decision value[2]) depends on this range of possible future sale prices, and specifically on the expected value of the sale price and the 'sale price risks', which are investment risk, expressed, for instance, in the standard deviation of this price. Uncertain future sale prices are relevant to the decision, e. g. regarding the purchase of a company. Therefore, traditional 'multiple methods', which deal with the current form of the valuation level (multiple) and profitability (EBIT: earnings before interest and taxes) of the company, are insufficient on their own due to their lack of future reference.[3]

In light of the empirically proven imperfections[4] of the capital market, the prices that can be achieved in an upcoming transaction (and also the stock-market prices) often deviate from the company value as calculated based on the model.[5] As many empirical studies show, transaction prices for non-listed companies, private equity (PE) and, especially, venture capital (VC) are determined using comparative methods, in particular multiple methods. The often-realised 'low' transaction multiples (related to EBIT or EBITDA) or the high 'return requirements' show that the valuation[6] takes into account:

- the above-average probability of insolvency, which is usually particularly high for SMEs and venture-capital investments, and/or
- company-specific (idiosyncratic) risks

Above-average earnings and insolvency risks in smaller companies, especially SMEs, are one explanation for the 'size effect' regularly found in empirical studies.[7] Everything else being equal, smaller companies with higher earnings and insolvency risk have higher costs of capital and lower enterprise value.[8]

The Business Judgment Rule[9] makes it important to consider the risks of a target company, including that of insolvency. When making a business decision

[1] On the peculiarities of valuating SMEs, see, for instance, *Coulon* (2022) and *Damodaran* (2011), who specifically deal with the importance of risk analysis and risk simulation.

[2] See *Matschke* (1972).

[3] See *Bhojraj/Lee* (2002) on criticisms of multiple methods.

[4] See, for instance, *Haugen* (2002); *Gromb/Vayanos* (2010); *Shleifer/Vishny* (1997); *Jegadeesh/ Titman* (2011); *Joyce/Mayer* (2012) and *Zhang* (2020).

[5] See *Calhoun* (2020).

[6] See, for instance, *Kerins* et al. (2004) and *Müller* (2004).

[7] See *Grabowski* (2018).

[8] See also *Blitz* (2020) for an overview of the current empirical research, as well as *Fama/French* (2015, 2018) and *Elgammal* et al. (2020).

[9] See e. g. *Leach* (2014). When buying or selling a shareholding, a 'business decision' within the meaning of the Business Judgment Rule (Section 93 of the German Stock Corporation Act) must generally be assumed. The application of this provision is subject to

to purchase a target company, the buyer's board of directors should be aware of how their own company's risk exposure changes if they acquire the target company. This requires much more than just due diligence or the consideration of stock-return fluctuations. It necessitates the analysis and aggregation of the risks of the target company, and the consideration of additional transaction-specific risks (e.g., those arising due to uncertain synergies and integration costs) and changes in the risk position as a result of the financing of the acquisition.[10] Methods for quantitative risk analysis, risk aggregation and simulation-based valuation can capture the implications of a decision to purchase an investment and the effects on the risk-return profile of the buyer's company.[11]

The simulation-based method we propose in this article, is suitable for adequately considering the 'special rights' of individual investors that are typical of venture-capital investments when valuing a share.[12] Based on the uncertain exit price, it is possible to determine individual investors' share of the exit price, taking into account existing special contractual regulations such as liquidity-preference regulations. The share of the exit price depends on the level of the exit price. This is the basis for a risk-adequate assessment of individual 'equity tranches,' including special rights. We show how realistic ranges of possible future sale prices can be determined based on risk analysis and simulation methods, and, in turn, how risk-adjusted discount rates for the valuation of a (potential) investment can be determined.[13] With the methods we present, it is possible, based on the findings of quantitative risk analysis, to derive risk-adjusted return requirements (discount interest rates) or cost-of-capital and fundamental values directly from the 'earning risks', without resorting to unavailable historical capital-market data as with the capital-asset pricing model (CAPM).[14] The effects of insolvency risks (probability of insolvency, rating) are also taken into account in the valuation calculation.[15]

When estimating prices, we utilize the multiple method to take into account the influence of market imperfections on the prices. Our valuation rationally

the condition that the VC company is a corporation, but this should not be a larger restriction in practice.

[10] On the methods, see *Gleißner/Ernst* (2019); *Dorfleitner/Gleißner* (2018); *Dorfleitner* (2022) and *Ernst* (2022a) on the assumptions and fundamentals of the methods used.

[11] See *Gleißner* (2019) on the risks of M&A transactions.

[12] See *Jenkinson* et al. (2019) and *Cederburg/Stoughton* (2018).

[13] The structure of the company in the case study is based on *Gleißner/Wolfrum* (2008), but the valuation methods used in this article, which were not known at the time, were not used.

[14] See, fundamentally, *Gleißner* (2011) for the method.

[15] See *Morris* (2009); *Gleißner* (2010); *Knabe* (2012); *Saha/Malkiel* (2012); *Lahmann* et al. (2019) and *Franken* et al. (2020).

values previously little-considered information about the risks of the valuation object (and does not assume the perfection of the capital market, unlike, for example, a valuation based on the CAPM). We also take up the concepts of semi-investment-theoretical valuation paradigm, which is based on the method of imperfect replication, as well as simulation-based valuation methods. Summarizing, the article is the first to

1. demonstrate the application of both concepts to purchase decisions in the context of mergers and acquisitions,
2. combine the simulation-based valuation method with standard market concepts for estimating possible transaction prices (multiple method),
3. compare the valuation based on the standard deviation risk measure with that based on the semi-standard deviation, and
4. integrate a consistent valuation of the special rights of individual shareholders.

Section II, discusses the theoretical underpinnings of our approach. Section III presents our model, including the concept of risk-based valuation using 'imperfect replication' and applies this to an uncertain exit price with standard deviation and semi-standard deviation as risk measures and a realistic model for the EBIT development over time. The new valuation approach is applied in a case study in Section IV. Based on certain valuation assumptions and, we undertake a risk-adequate assessment of the company as a whole and a consistent share valuation (taking special rights into account). A comparison with conventional valuation methods illustrates the superiority of the new approach. A general discussion in the last section concludes the paper.

II. Valuation with uncertain exit prices: Theory

1. *Existing approaches*

The standard theory of business valuation is represented by discounted cash flow methods, which involve taking the expected cash flows for each period and discounting them using a risk-adjusted discount rate, using the CAPM. This approach is well-documented in nearly every corporate finance textbook,[16] so we refrain from providing a traditional literature review on the conventional solutions to this problem.

However, to categorise our approach, it is important to distinguish between the various valuation functions, with particular emphasis on the argumentation

[16] See e.g. *Berk/DeMarzo* (2023) for a contemporary presentation and the references herein for further academic literature.

value and the decision value, as they are especially relevant to the field of application under consideration.[17] While the *argumentation value* is calculated for negotiation situations to justify one's own price expectations, the *decision value* represents the maximum acceptable purchase or sale price for the subject of valuation, for example, a marginal price.

Valuation methods rooted in financing theory are founded on the neoclassical hypothesis of perfect capital markets. These methods calculate discount rates based on historical fluctuations in share returns using the CAPM. The assumptions of the CAPM lead to an equivalence between price and value.[18] In contrast, factor modelling methods consider multiple factors – not just the beta factor of the CAPM – to explain the expected share returns.[19] The theoretical foundation for these methods often draws upon *Ross's* arbitrage pricing theory (1976).

Many methods involve price estimation.[20] Valuation multiples are derived from the prices of comparable companies, and these multiples are subsequently used to estimate the value of the company in question (see II.2.).[21]

So far, none of the methods mentioned allow for the calculation of decision values; this is instead facilitated by investment-theoretical valuation methods. These methods consider the individual information, actionable options, and constraints of the asset being valued.[22] Notably, they do not rely on the assumption of perfect capital markets. Instead, valuation is based on *'valuation principles derived from subjective value theory'*[23], where value is understood as a *'subject-object-object relationship'*.[24, 25] Summarizing *Olbrich* et al. (2015, p. 34), subjective business valuation is grounded in the concept of valuation as an entity and its future orientation. These aspects provide a robust decision-making basis for economic subjects.

The solid theoretical foundation of these methods is offset by the significant calculation effort required for the 'total model,'[26] which allows for the consideration of numerous alternative investment options for the valuation subject. Con-

[17] See *Matschke* (1975) on the decision value and *Matschke* (1979) and *Olbrich* et al. (2015, pp. 29 – 32) on the 'functional business valuation theory'.

[18] For criticism of the approach, see also *Olbrich* et al. (2015).

[19] See *Fama/French* (1993, 2015, and 2018); *Swade* et al. (2023).

[20] See *Schüler* (2020); *Krolle* et al. (2005).

[21] For 'theory-based' multiplier methods, see e.g. *Richter* (2005); *Kelleners* (2004) and *Herrmann* (2002).

[22] See *Hering* (2000).

[23] See *Olbrich* et al. (2015, p. 17).

[24] See *Olbrich* et al. (2015, p. 18), with reference to *Matschke* et al. (2010).

[25] See also *Olbrich* et al. (2015, pp. 25 – 27) on the inadequate capture of uncertainty by the ß factor of the CAPM.

[26] See *Hering/Toll* (2013).

versely, simple heuristic variants, known as partial models[27], are easier to apply. However, these partial models rely on predetermined discount rates and do not provide a methodology that would, for example, incorporate insights from the company's risk analysis.

Klingelhöfer et al. (2009) provide an example of using these methods to determine a '*maximum affordable settlement*'[28], thereby establishing a decision value.[29] The calculation requires linear optimisation, which takes into account existing investment and financing options. For the first time, *Hering* et al. (2012) combined this method with a Monte Carlo simulation to value venture capital.[30] Building on this work, *Hering* et al. (2013a) presented a model for simulative company valuation that addresses capital market imperfections and ambiguous cash flows associated with the asset, ultimately determining a range of subjective marginal prices. This valuation also required linear optimisation. In response to critiques of traditional valuation methods based on DCF and CAPM, *Olbrich* et al. (2015) developed a similar method specifically designed for M&A valuations.

The semi-investment-theoretical valuation paradigm[31] draws inspiration from investment-theoretical valuation concepts and integrates them with risk-value models.[32] However, it also accepts certain simplifications that are common in financial valuation theory.[33]

In contrast to the strict focus on a specific business subject[34], semi-investment-theoretical valuation paradigm embraces simplifications regarding the options considered. For instance, it typically evaluates two alternative investment options for the asset: a risk-free investment and a broad stock market index. The Monte Carlo simulation aggregates the company's risks using quantitative analysis based on corporate planning (risk aggregation)[35], thereby accounting for financing restrictions and insolvency risks.[36] Based on the simplified assump-

[27] See *Hering/Toll* (2013).

[28] See *Klingelhöfer* et al. (2009, p. 302).

[29] This holds especially for the valuation of changes in voting rights.

[30] They utilize a simulation-based version of the state marginal quota model.

[31] See *Gleißner/Follert* (2022) and *Gleißner/Ernst* (2024).

[32] See *Dorfleitner/Gleißner* (2018), discusses below.

[33] See *Gleißner/Follert* (2022).

[34] See *Olbrich* et al. (2015) and note II.1.

[35] For risk aggregation using Monte Carlo simulation in risk management, see e.g. *Hunziker* (2021); *Gleißner* (2019) and *Vanini*/Rieg (2021).

[36] For the basic idea, see *Coenenberg* (1970) and *Gleißner* (2019); *Gleißner/Ernst* (2023) and *Ernst* (2022a, 2022b). Supplementary *Hering* et al. (2013a) on simulation in the context of investment theory and *Matschke/Brösel* (2013, pp. 277 – 253) on dealing with uncertainty.

tion of reliable environmental parameters, this approach yields a single reliable company value, rather than a range of values. The integration of a risk-value model[37], which employs a risk measure to capture the risk content of cash flows, facilitates a comprehensive transformation of uncertainty concerning those cash flows. Alongside the limitation of alternative investment options, this uncertainty transformation through the risk-value model is the key feature that distinguishes semi-investment-theoretical valuation paradigm from its predecessors.

Even independently of published case studies, surveys indicate that the use of simulation-based valuation methods has increased significantly in recent years, particularly for complex valuation problems. According to *Gleißner* et al. (2024), a survey of German valuation professionals revealed that 80 % of respondents already utilise such methods.

2. *Theoretical background to our approach*

Our approach for the risk-adequate valuation of an investment company to be sold at an uncertain exit price integrates valuation and price-estimation methods.[38] To ensure a strong connection between theoretical foundations and practical relevance, our approach focuses on three theoretical concepts: multiple valuation, risk-value model valuation, and Monte-Carlo simulation.

Multiple valuation is a straightforward approach that is widely used in practice. It relies on deriving the valuation from a similar or comparable company based on the principle of proportionality.[39] We chose to use this concept not for its theoretical elegance, but because provides an accurate representation of reality when selling private equity shares in a company. *Schüler* (2020) provides an overview of the conceptual requirements and implementation of company valuations using multiples, highlighting which multiples are suitable for accurate valuations and how the multiples used in practice are interrelated (e.g., enterprise value/sales or enterprise value/EBITDA). Additionally, empirical studies have been conducted by *Cheng/McNamara* (2000) and *Chullen* et al. (2015). Leveraged buyouts (LBOs) demonstrate that the total company value (enterprise value), based on median figures, is approximately seven times the EBITDA. This finding comes from an analysis of valuation multiples for European LBOs dur-

[37] See *Gleißner/Dorfleitner* (2018) on the derivation of the valuation equations using incomplete replication.

[38] See *Hering* et al. (2012) on the valuation of venture capital.

[39] See *Berk/DeMarzo* (2023, ch. 9).

ing the period from 2000 to 2003,[40] revealing significant fluctuations in valuation over time.[41]

Most older studies on multiple valuations primarily focus on industry multiples, often incorporating additional factors such as historical growth.[42] Previous empirical studies have demonstrated that the price-earnings ratio can be effectively explained by industry affiliation, as it provides a suitable representation of risk and earnings growth.[43] The estimation accuracy of industry-related valuation multiples can be further enhanced by using return on equity as an additional selection criterion alongside industry affiliation.[44] These investigations, which use controlled multiples based on fundamental factors, yield significantly improved results and indicate that industry affiliation does not contribute additional explanatory power compared to the fundamental explanatory factors.[45]

The second, more theoretically oriented, conceptual framework used in our analysis is the risk-value model of valuation, as outlined by *Dorfleitner/Gleißner* (2018). This approach involves comparing the investment to be valued with a reference investment that has equal risk and expected end value. Notably, this methodology avoids unrealistic assumptions and does not require extensive information. Its grounding in rational decision theory makes it particularly well-suited for addressing the valuation problem at hand.

Fundamental ideas for risk-adequate valuation, which do not rely on the assumption of a perfect capital market, stem from investment theory (see II.1.).[46] These methods have emerged in competition with valuation theories based on financing theory, which assume the perfection of the capital market (for example, the CAPM). Due to the high complexity associated with classical investment theory methods,[47] alternative valuation approaches, known as semi-in-

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[40] Accordingly, the ratio of the total enterprise value to the difference between EBITDA and investments in property, plant and equipment is around 10. See *Richter* (2005, p. 181).

[41] The financial investors finance the total company value with an average of 63 % debt, corresponding to 4.6 times the EBITDA, see also *Roosenboom* (2012).

[42] The best estimation results for market prices can probably be achieved via a combination of DCF company-valuation methods and market-oriented multiples (*Herrmann*, 2002, p. 31; *Kaplan/Ruback*, 1996, p. 45; *DeAngelo*, 1989, p. 93; *Bruner* et al., 1998, p. 13).

[43] See e.g., *Alford* (1992).

[44] See *Alford* (1992).

[45] See *Herrmann* (2002); *Richter* (2005) and *Kelleners* (2004).

[46] See *Matschke* et al. (2010); *Hering* et al. (2013a); *Hering* et al. (2014); *Matschke/Brösel* (2021).

[47] See *Matschke* et al. (2010) and *Matschke* et al. (2020).

vestment-theoretical methods, have been developed (see II.1.).[48] While these models are grounded in investment-theoretical valuation concepts, they explicitly account for the company's financial success and insolvency risks. They do not require a perfect capital market and also accommodate rating and financing restrictions. As with the CAPM, these methods allow for the derivation of simple valuation equations by considering only two alternative investment options.

A risk-based valuation method transforms the scope of valuation-relevant risks associated with the asset – expressed, for example, as a frequency distribution of the cash flows – into a number (risk value model). It is straightforward to assess the risks of the relevant cash flows directly, determining a risk measure based on a cash-flow figure expressed in monetary units. When deriving typified decision values, the risk information of the valuation subject (including insider information) is incorporated into the valuation through quantitative risk analysis and simulation-based risk aggregation. This process results in a frequency distribution of the cash flow to be valued. For simplification, this can be represented as a risk measure, such as the standard deviation or (relative) value-at-risk of the cash flows. However, when valuing an entire company, the uncertain cash flows (or dividends) are fundamentally important to the valuation, as they determine the 'risk of changes in value' (investment risk) – specifically, the potential extent of value changes over time. Our approach, therefore, aims to establish a specific value rather than a range of values. To achieve this, we must abstract from uncertainty regarding the parameters of the valuation environment.[49]

The third concept, Monte Carlo simulation, is a mathematical method that employs repeated random draws of variables to achieve numerical results. It is a well-established and reliable technique for deriving valuations, even in cases where non-linear relationships between variables exist.[50] Initially, such procedures were often applied without a foundation in valuation theory, primarily to estimate price ranges.[51] The Monte Carlo simulation can be traced back to the 1960s[52], with its first application to company valuation occurring in 1970[53], al-

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[48] See *Gleißner* (2011); *Dorfleitner/Gleißner* (2018); *Dorfleitner* (2022); *Gleißner/Ernst* (2023) and *Gleißner* (2023).

[49] See *Fama* (1977).

[50] See, for instance, *Oh/Cho* (2013); *Khalkar/Mehta* (2020); *Beaton/Sawyer* (2019); *Coenenberg* (1970).

[51] Often but not precisely, the outcomes were referred to as 'value'.

[52] See *Hertz* (1964).

[53] See *Coenenberg* (1970).

though it was not yet integrated with a risk-value model. Today, Monte Carlo simulation is frequently used in conjunction with risk-value model valuation.[54]

The interaction of the three concepts works as follows: the multiple method is used to estimate the uncertain exit price, taking into account both (1) future valuation levels and (2) the company's future profitability. The company's value at the valuation point t = 0 (present) is derived from this uncertain future exit price. This assessment utilises risk-value models and incorporates insolvency risks arising from rating and financing constraints.[55] The valuation risk stems from the uncertainty surrounding the exit price, which is influenced by both the future valuation level and company's earnings situation.[56]

In addition to corporate planning, insights from a risk analysis are utilised to determine the uncertain future earnings of the company at the time of exit. A Monte Carlo simulation is necessary to capture future risks associated with corporate planning.[57] In recent years, researchers have advocated for simulation-based valuations[58] grounded in risk-value models and the 'incomplete replication' method, which does not depend on a perfect capital market or on comprehensive capital-market data pertaining to the company being valued.[59]

One significant advantage of this method is that it eliminates the need to optimise the subject's investment programme or to have complete knowledge of all alternative investment opportunities.[60] In this article, we apply this valuation method to assess the shares of individual equity providers (shareholders) sepa-

[54] For instance, see *Ernst* (2022a); *Gleißner/Ernst* (2023); *Gleißner/Kamarás* (2023) and *Ernst/Kamarás* (2023) and in the application for the valuation of football players in *Follert/Gleißner* (2024).

[55] See *Gleißner* (2010); *Knabe* (2012); *Saha/Malkiel* (2012); *Friedrich* (2016); *Lahmann* et al. (2018, 2019) *and Franken* et al. (2020).

[56] The derivation of discount interest rates through the statistical valuation of historical fluctuations in the return on shares of a company or the companies in a peer group, as is usual with a valuation based on the CAPM, is therefore not necessary.

[57] Such a risk analysis in Germany is available as an informational basis for the assessment if companies have an early risk detection system in accordance with the requirements of Section 91 AktG and Section 1 StaRUG that can identify possible 'developments that threaten the existence of the company' at an early stage. See the new IDW PS 340 new version (2020), with the requirements for risk aggregation. See also *Schwartz/Moon* (2001); *Behm* (2003) and *Klobucnik/Sievers* (2013). For the basics of using a stochastic simulation in the context of company valuation, see also *Coenenberg* (1970); *Ungemach/Hachmeister* (2019) and *Damodaran* (2018).

[58] See *Ernst* (2022a).

[59] See *Gleißner* (2019); *Gleißner/Ernst* (2019); *Ernst* (2022a); *Dorfleitner/Gleißner* (2018) and *Gleißner/Ernst* (2023).

[60] A utility function as a basis for valuation is also not necessary. See *Schosser/Grottke* (2013), on utility-based company valuation. See also *Gleißner/Follert* (2022) for the classification.

rately, alongside the risk-appropriate valuation of the company as a whole. We show that simulation-based valuation allows for a share valuation that accounts for the specific rights of individual shareholders, such as earn-out agreements.

Capital investment companies particularly require exit price-estimation methods that consider the uncertainty of future earnings. They also need valuation techniques that convert the uncertain future sale price into a current risk-appropriate value, effectively serving as a marginal price.

Based on the estimates of the possible return from the sale of a company, two perspectives emerge. Firstly, if a potential purchase price for the company is provided, it is possible to evaluate whether buying the company generates a positive increase in value, thereby making the decision worthwhile. Secondly, the marginal price at which a purchase remains beneficial (i. e., the increase in value is 0) can also be calculated.

The marginal price, interpreted as a decision value,[61] is determined in the following way: using a Monte Carlo simulation, we assess the range of possible sale proceeds at the end of the planning period. This includes calculating the expected value and the risk measure associated with the stochastic exit price, often referred to as the 'price estimation function' (for example, through 'multiples'). Following this, we conduct a risk-based assessment to identify the riskless value at the outset that is equivalent to the stochastic future exit price at the end of the holding period.

III. Simulation-based valuation with uncertain exit prices: The model

1. General framework

The risk of future cash flows ($\tilde{Z}$), in the sense of possible deviations from the expected value ($E(\tilde{Z})$), can be taken into account in two ways:[62] with a risk discount from the expected value of the cash flows or by assessing the expected value using a risk-adjusted interest rate (cost-of-capital rate).[63] The derivation of the valuation equation does not require the assumption of perfect markets, as is the case with the CAPM. With the method of 'incomplete replication' and risk-value models,[64] valuation equations can be derived based on less restrictive

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[61] See *Matschke* (1972) and *Matschke/Brösel* (2021) for the basics.

[62] For the derivation of the valuation equations using the 'incomplete replication' method, which does not depend on the hypothesis of a perfect market, see *Dorfleitner/Gleißner* (2018) and *Dorfleitner* (2022), as well as *Gleißner/Ernst* (2019) and *Gleißner/Follert* (2022).

[63] See *Gleißner* (2019).

[64] See *Gleißner/Wolfrum* (2008); *Gleißner* (2011); *Dorfleitner/Gleißner* (2018).

assumptions: two cash flows at the same point in time have the same value if their expected value and the selected risk measure match. In addition to this assumption, only two alternative investment options must be specified for the valuation – for example, a risk-free investment with an interest rate r_f and a stock market index with an uncertain return $\tilde{r}_m$.[65]

Often, only the case in which the cost-of-capital rate is calculated from the information of the first period[66] (especially about the earnings risk) is considered. In principle, however, any future period t can also be assumed to be the 'representative period' and the basis for calculating the cost of capital, as shown below.[67] The valuation according to the risk-value model is formulated as:

$$V(\tilde{Z}_t) = \frac{E(\tilde{Z}_t)}{(1+c)^t} = \frac{E(\tilde{Z}_t) - \lambda_t^R R(\tilde{Z}_t) \cdot d}{(1+r_f)^t}. \tag{1}$$

where $E\left(\widetilde{Z_t}\right)$ is the expected value of the cash flows, $R\left(\widetilde{Z_t}\right)$ is the risk quantity expressed by the risk measure $R(\ldots)$, and λ_t^R is the multi-period risk price in period t.

The variable d expresses the proportion of the risk of the valuation subject borne by the risk object due to the risk-diversification options in its portfolio. *Dorfleitner/Gleißner* (2018) initially derived (1) without explicitly specifying d, which can be solved by taking risk-diversification options into account when determining the cash flow to be valued $\widetilde{Z_t}$ or implies that the risk cannot be reduced by any diversification. In practice, however, this assumption is often too restrictive, which is represented by a value of d smaller than 1. This implies that the valuation subject only has to bear part d of the risk due to diversification effects. Various methods can be used to derive the risk-diversification factor d (e.g., statistical valuation of historical earnings fluctuations of the company or risk-factor models).

[65] According to the market portfolio of the CAPM. The 'market price of the risk' (λ) can be derived from the risk/return profile of the alternative investments, which indicates the additional return per unit of risk that can be expected from the alternative investments (on the capital market).

[66] Between time $t = 0$ and $t = 1$.

[67] With this simplified procedure, the correct cost-of-capital rate for the 'representative period' is determined and is also used for the other periods. The cost-of-capital rate for a specific period is used as the period-independent cost-of-capital rate. The procedure can be carried out on the basis of period-dependent cost-of-capital rates.

$$V\left(\tilde{Z}_t\right) = \frac{E\left(\tilde{Z}_t\right)}{\prod_{n=1}^{t}\left(1+c_t\right)^n} = \frac{E\left(\tilde{Z}_t\right) - \lambda_t \cdot R\left(\tilde{Z}_t\right) \cdot d}{\left(1+r_f\right)}.$$

However, since this makes the formulas considerably more complex, the simplified equation is used here for the presentation of the didactics.

With the standard deviation as the risk measure [that is, $R(.) := \sigma(.)$], solving (1) results in the following equation based on period t for the time-invariant assumed risk-based capitalisation rate c:[68]

$$c = \frac{1+r_f}{\sqrt[t]{1-\lambda_t^{\sigma} \cdot \frac{\sigma(\tilde{Z}_t)}{E(\tilde{Z}_t)} \cdot d}} - 1 = \frac{1+r_f}{\sqrt[t]{1-\lambda_t^{\sigma} \cdot CV(\tilde{Z}_t) \cdot d}} - 1, \tag{2}$$

where $CV(\tilde{Z}_t) = \frac{\sigma(\tilde{Z}_t)}{E(\tilde{Z}_t)}$ is the variation coefficient.

The function λ_t^e is now required for the calculation of c, which is explained below. The multi-period risk price can be determined from the repayment distribution of a market portfolio compared to a risk-free investment (with a suitable term, T) depending on the risk measure $R(.)$ as:[69]

$$\lambda_t^R = \frac{E\left(\prod_{i=1}^{t}(1+\tilde{r}_{M,i})\right) - (1+r_{f,t})^t}{R\left(\prod_{i=1}^{t}(1+\tilde{r}_{M,i})\right)}. \tag{3}$$

For $t = 1$, this parameter is simply the 'Sharpe Ratio'. For any t, under the assumptions of (i) a (time-invariant) interest rate r_f for the risk-free alternative investment and (ii) a log-normally distributed return $\tilde{r}_m \sim LN(\mu_m; \sigma_m)$ of an empirical market portfolio, the risk price can be expressed as:[70]

$$\lambda_t^{\sigma} = \frac{e^{t\cdot\mu_m + t\cdot\frac{\sigma_m'^2}{2}} - (1+r_f)^t}{e^{t\cdot\mu_m + t\cdot\frac{\sigma_m'^2}{2}} \cdot \sqrt{e^{t\cdot\sigma_m'^2} - 1}}. \tag{4}$$

2. *Uncertain exit price as a basis for valuation*

For private-equity investments a valuation based on the final-value distribution is appropriate, if during the holding period no cashflows are obtained. It is assumed that the company will be sold (or, if cheaper, liquidated) at the end of the planning period (T). The value – or, better, the estimated price – is based on

[68] For $\lambda_t^{\sigma} \cdot \frac{\sigma(\tilde{Z}_t)}{E(\tilde{Z}_t)} \cdot d < 1$ for all t.

[69] See *Dorfleitner/Gleißner* (2018).

[70] The parameters μ_m and σ_m are not the expected value and standard deviation of the return, but the parameters of the log-normal distribution. See also *Gleißner* (2022, pp. 705 – 713); *Dorfleitner/Gleißner* (2018) and *Dorfleitner* (2022).

the probability distribution of the cash flows that can be obtained in period T, the exit price being $\tilde{P}^{Exit}$.

Applying equation (1) we have:

$$V\left(\tilde{P}^{Exit}\right) = \frac{E\left(\tilde{P}^{Exit}\right) - \lambda_T^R \cdot R\left(\tilde{P}^{Exit}\right) \cdot d}{\left(1 + r_f\right)^T}. \tag{5}$$

A specification of the valuation equation (5) consists in using the standard deviation as a risk measure. Then, λ_T^R is determined according to equation (4).

Given that $\tilde{P}^{Exit}$ represents an uncertain market price, valuation and a price-estimation procedure are combined. The valuation subject therefore assumes that the attainable price of the unlisted investment at the planned uncertain exit date is determined using the multiple method, in accordance with standard practice. Both the future earning power (EBIT or EBITDA) and the valuation multiple (m) are uncertain and particularly dependent on future market conditions (valuation level). Estimating the value from today's perspective ($t = 0$) requires an estimate of future attainable prices. The uncertainty about the future attainable sale price determines the value from the point of view of a private-equity investor, in contrast to that of a 'forever' invested valuation subject, for which only uncertainty about future cash-flow surpluses is relevant. This marginal price can be interpreted as a fundamental value from the point of view of the assessor (risk-adequate decision value).

A major advantage of the procedure explained here is that the high insolvency probabilities of SMEs and, especially, venture-capital investments are considered in this valuation calculation. The use of the Monte Carlo simulation allows the distribution of $\tilde{P}^{Exit}$ to represent the possibility of insolvency. A higher probability of insolvency leads accordingly to ceteris paribus a lower expected value of the exit price. In light of the well-known fact that venture capital combines high chances of profit with a high probability of insolvency,[71] recording the insolvency risk is important. Neglecting the possibility of insolvency leads to a significant overestimation of the market value.

To determine the distribution of $\tilde{P}^{Exit}$, 'stochastic' multiple valuation can be used, which includes the uncertainty of (1) future multiples and (2) future corporate earnings (EBIT) and ranges of the possible exit price $\tilde{P}^{EXIT}$ with realistic minimum revenues.[72] The extent of possible losses is limited when acquiring a stake. The worst-case scenario is total loss, that is, an exit price of zero (and,

[71] This is reflected in the high 'required returns'.

[72] For multiples, see *Richter* (2005) and *Kelleners* (2004). On the basics of 'theory-based' multiples, see *Herrmann* (2002) and *Timmreck* (2003).

therefore, actual bankruptcy).[73] Because the exit price is paid on the equity, the debt capital $\tilde{D}_T$ must be repaid in accordance with the company's value. This results in the following equation:

$$\tilde{P}_T^{EXIT} = \max\left(0; \tilde{m} \cdot \widetilde{EBIT}_T - \tilde{D}_T\right), \tag{6}$$

where $\tilde{m}$ represents the stochastic multiple. This is a 'price estimate' in an imperfect market, and this exit price may deviate from a 'reasonable' fundamental value (although, as is well known, there is no such thing as a 'true' value).[74]

The exit price estimate is obtained from capital-market data (e.g., stock-market prices) or realised transaction prices of 'comparable' companies using a comparison method. The comparison method determines a potential market price ('stock-exchange price') likely to be achievable on the market. Such methods are therefore also referred to as market-oriented valuation methods. In the multiplier method, the potential market price $\tilde{P}$ is determined by multiplying a specific (size-related) parameter X of the company to be valued by a factor, m, that depends on the selected reference value and is usually industry-specific.[75]

$$\tilde{P}(X) = m \cdot X - D,$$

where $\tilde{P}$ is the estimated market price, X is the parameter,[76] m is the industry-specific factor[77] and D is debt capital. EBITDA, EBIT, cash flow or sales are commonly used to operationalize X. In terms of suitability for calculating multiples, *Liu* et al. (2002) rank the possibilities as follows: (1) earnings forecasts, (2) historical earnings, (3) cash flow and book value of equity, and (4) sales.[78]

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[73] See *Gleißner* (2010, 2019) and *Franken* et al. (2020) on the significance of the probability of insolvency, which is only touched upon here.

[74] Although this should not exist in perfect markets, see *Shleifer/Vishny* (1997) for such incorrect valuations; *Haugen* (2002) and *Campbell/Shiller* (1998).

[75] The debt capital FK is deducted if an enterprise value is calculated with '$m \cdot X$'.

[76] Examples are EBITDA or EBIT.

[77] One example is the quasi-reciprocal discount factor. See *Cochrane* (2011).

[78] *Baker/Ruback's* (1999) empirical study of industry multiples for the S&P 500 index in 1995 shows that multiples based on EBITDA lead to better estimates of market prices than those based on EBIT or sales. In addition, in this empirical study, the formation of a harmonic mean turns out to be a suitable method for calculating the valuation multiple; see also *Kaboth* et al. (2022); *Herrmann/Richter* (2003); *Yoo* (2006) and *Schreiner* (2007).

3. *An alternative risk measure: the semi-standard deviation*

While the standard deviation measures positive and negative fluctuations around the expected value, the possibility of negative deviations can be regarded as more relevant to the decision to buy or sell a company, as these determine the company's equity requirements and, thus, the utilisation of the (scarce) risk-coverage potential.[79] Therefore, the semi-standard deviation

$$\sigma^{-}\left(\tilde{Z}\right)=E\left(\max\left(E\left(\tilde{Z}\right)-\tilde{Z};0\right)^{2}\right)^{\frac{1}{2}}. \tag{7}$$

Could be more appropriate as a measure of risk.[80] The importance of possible negative deviations from the plan and, especially, of possible losses in the valuation is well documented in research,[81] which justifies the use of the semi-standard deviation risk measure.[82] This is particularly true in the context of private-equity investments, where downward and upward deviations are not symmetrically distributed.

The marginal price (decision value) at which a purchase becomes worthwhile from the point of view of a potential investor can again be determined using Equation (5) with $R(.)=\sigma^{-}(.)$. In the case of a log-normal distribution, $\lambda_T^{\sigma^-}$ can be determined by simulation (see the case study in Section IV). For simplicity, it is assumed here that the investor does not have to spend any supplemental equity for the company in addition to the purchase price in order to provide it with equity that is in line with the risk. If this were the case, the required equity capital would reduce the marginal price. This equity requirement can also be estimated directly from the simulation. Furthermore, the probability of insolvency is implicitly considered in this marginal price given that the total loss of the investment is also taken into account in the simulation in the event of unfavourable business development.

4. *Special rights and share valuation*

With the procedure outlined here, a risk-adequate assessment is possible not only for the entire company but also for individual 'equity tranches' with their specific rights. Knowing the complete frequency distribution of the uncertain

[79] See *Gleißner* (2022, pp. 428 – 488) on the so-called risk-coverage approach.

[80] For the importance of downside risks and the 'skewness' of a distribution of results, see *Kraus/Litzenberger* (1976).

[81] See, for instance, *Kahnemann/Tversky* (1979) on the basics of psychology.

[82] And it is precisely this risk measure that can – better than the standard deviation – capture the implication of the limitations of liability and the 'clipping' of the possible losses.

exit price makes it easy to depict existing contractual agreements with individual investors (equity providers) and thereby independently determine the range of the 'uncertain return flow' for each equity provider. According to the given rules, the total exit price is split into the exit price shares of the n individual investors P_i^{Exit} (here, $i = 1, \ldots, n$):

(8) $$P^{Exit} = \sum_{i=1}^{n} P_i^{Exit}.$$

With the methods for a risk-based assessment outlined above, considering the expected value and risk of the returns to each investor $I = 1, \ldots, n$, the investors' subjective value can be calculated, taking all special rights into account. All that is necessary is a breakdown of the uncertain exit price in accordance with the contractual provisions.

If, for example, an investor i, who financed the last equity increase of a company, has a 'liquidity preference' such that they are initially served by the uncertain exit price until his or her capital investment I_i is reached and then proportionately with a share α_i of the exit price, the following payoff applies:

(9) $$P_i^{Exit} = \max\left\{\min\left\{P^{Exit}, I_i\right\}; \alpha_i \cdot P^{Exit}\right\}.$$

5. *A model for the EBIT development over time*

In practice, the collection of company-specific data for the valuation is based on a detailed due-diligence (including supported) detection of further risks that determine the planning security. The most important results are estimates of the expected value and risk of the stochastic sales growth rates $g_1, \ldots, g_{T-1}$, and an estimate of the expected EBIT margin and the typical extent (standard deviation) of risk-related deviations from this forecast.

Based on sales of the previous period R_0 of €15 million and sales growth g_0 of 20 % in the previous period, TextitAI, Inc. expects a linear decrease in the sales growth rate of 2 % from the terminal value period (meaning that planning is done with $R_5^{Plan} = €25$ million).[83] The coefficient of variation of the sales growth rate (i.e., the ratio of the standard deviation to the expected value) should be constant at 25 % annually, with deviations from the expected sales growth rate assumed to be normally distributed.

(10) $$\tilde{R}_t = \tilde{R}_{t-1}\left(1+\tilde{g}_t\right) = \tilde{R}_{t-1}\left(1+E\left(\tilde{g}_t\right)+\tilde{\varepsilon}_t^g\right), \text{ where } \tilde{\varepsilon}_t^g \sim N\left(0;\sigma_t^g\right).$$

[83] For other models, see, for example, *Schwartz/Moon* (2000, 2001); *Behm* (2003) and *Klobucnik/Sievers* (2013).

At the beginning (i.e., at $t = 0$), the EBIT margin (EBITM) is –1 %; in the long term, a value of 7.5 % is expected ($\overline{EBITM} = 7.5\%$).

The traditional EBIT planned for t = T = 5 without simulation is therefore:

$EBIT_T^{Plan} = R_T^{Plan} \cdot EBITM_T^{Plan} = 25 \cdot 7.5\%$ = € 1.875 million.

A mean-reverting process is assumed for the development of the EBITM. Such a risky process trends towards the $\overline{EBITM}$. The further the value from the previous period $\widetilde{EBITM}_{t-1}$ deviates from this mean, the stronger the mean-reverting tendency. Formally, this can be described recursively as follows:

$$\widetilde{EBITM}_t = \overline{EBITM} + a\left(\widetilde{EBITM}_{t-1} - \overline{EBITM}\right) + \tilde{\varepsilon}_t^{EBITM}. \tag{11}$$

Deviations from the expected trend value $E\left(\tilde{\varepsilon}_t^{EBITM}\right)$ are assumed to be normally distributed with a standard deviation σ_t^{EBITM}. The standard deviation of the EBIT margin should decrease linearly in the planning period, from 3 % in the past to 1 % in the terminal value period.[84]

As an alternative to the simple stochastic process of the EBIT margin, more detailed planning can be carried out. The fluctuation range of the EBIT margin can be determined using simulation-based risk aggregation methods. Starting from identified and valuated individual risks, these methods enable the direct calculation of the typical risk-related range of EBIT development by calculating a representative number of risk-related future scenarios. The interest rate on borrowed capital i for interest on borrowed capital (D) should remain constant[85] over the period under consideration and is assumed to be 5.1 %. The simulation is carried out recursively, that is, progressively from t–1 to t.

The profit before tax in period t is determined according to the following equation:

$$\widetilde{EBT}_t = \tilde{R}_t \cdot \widetilde{EBITM}_t - \tilde{D}_{t-1} \cdot i. \tag{12}$$

For the sake of simplicity, taxes are neglected here. Growth is to be financed by retaining the profit before tax, so no payouts are made. It is assumed that the capital employed at the beginning of the planning period, namely $CE_0 = E_o + D_0$, develops analogously to sales. In other words, the capital turnover remains constant, leading to the following equation:

[84] Various data sources such as comparative industry values, the results of risk analyses or subjective estimates by experts can be used to estimate these parameters.

[85] Assume that the company has a 5-year line of credit with a constant, deterministic interest rate. Strictly speaking, the interest cash flows (I) from the point of view of $t = 0$ are a conditional distribution, the form of which depends on the risks realised up to t – 1.

(13) $$\widetilde{CE}_t = \frac{\tilde{R}_t}{\tilde{R}_{t-1}} \widetilde{CE}_{t-1} = \frac{\tilde{R}_t}{R_0} CE_0.$$

If the EBT (earnings before taxes) is not sufficient for financing, additional outside capital is taken out. If the EBT exceeds the necessary investment amount, borrowed capital is repaid. This results in the following equation:

(14) $$\tilde{D}_t = \tilde{D}_{t-1} - \widetilde{EBT}_t + \widetilde{CE}_t - \widetilde{CE}_{t-1}.$$

The (net) borrowed capital can therefore also assume negative values. These can be regarded as liquid funds. Interest income is then generated with these, where for simplicity no distinction is made between the interest rate on borrowed capital and the interest on the credit balance (each of which = i). The debt capital D can thus be interpreted as a net bank liability. Given that distributions and capital increases are excluded, the equity at the end of period t is the sum of the equity at the beginning of the period and the EBT:

(15) $$\tilde{E}_t = \tilde{E}_{t-1} + \widetilde{EBT}_t.$$

To simulate $\tilde{P}^{EXIT}$ according to formula (6), the EBIT at time $t = T$ must be calculated. This is done according to the relationship already used in formula (13):

(16) $$\widetilde{EBIT}_t = \tilde{R}_t \cdot \widetilde{EBITM}_t.$$

The triangular distributed multiple m is stochastically simulated independently of the other variables.

6. *Model discussion*

Our approach can be used to determine decision values or argumentation values (see II.1.[86]). To establish an argumentation value in a purchase negotiation, parameters are set cautiously, so that they result in a very low valuation that acts as the starting point for the price negotiations. When realistic parameters are applied from the perspective of the valuation object (the buyer), a decision value is obtained, indicating the maximum acceptable purchase price. In a negotiation context, our approach has the advantage that only a consensus on a probability distribution is necessary concerning uncertain assumptions. For instance, this may pertain to the future sales growth rate or the price multiple achievable at the exit time.

[86] See also *Follert* et al. (2018).

When determining company values for private equity or venture capital investments, it is advantageous that the procedure is based on the familiar method of assessing adequate prices using EBIT or EBITDA multiples. Extending this existing approach to account for future uncertainties, such as the sales growth rate and the future valuation levels, represents a significant benefit.

It is also advantageous that, unlike valuations based on financing theory using the CAPM, this method does not assume a perfect capital market; instead, it considers existing rating and financing restrictions. The combination of a Monte Carlo simulation for estimating an uncertain exit price, combined with the results of a quantitative analysis of the company's risks, and the application of the semi-investment-theoretical valuation paradigm based on imperfect replication, further enhances the robustness of this approach.

In the simple model presented in Chapter III, it may be seen as a disadvantage that the valuation does not fully incorporate integrated corporate planning. While adding this element would increase complexity, it is certainly feasible to implement.

From the perspective of financial valuation theory, a notable disadvantage is the absence of the CAPM, which remains widely used in practice, particularly in legal company valuations. However, there is a growing demand for simulation-based methods in demanding valuation cases,[87] and the CAPM is increasingly subject to critical scrutiny in legal contexts, such as when calculating compensation for minority shareholders in Germany.[88]

From the perspective of investment valuation theory, the model can be critiqued for its simplifications, particularly regarding alternative investment options, which do not account for potentially relevant information about the valuation subject's opportunities and restrictions.[89] For example, the model is not designed to evaluate a large number of simultaneously feasible investment opportunities within a contextual framework, making it unsuitable for determining an optimal portfolio that considers the interdependencies between individual VC or PE investments.

For the sake of simplicity, the valuation assumes that all other valuation issues have been resolved and that only one investment opportunity needs to be evaluated in isolation. However, this limitation can be relaxed by determining the respective net present value rates for multiple investment alternatives, i. e., calculating the enterprise value (V) in relation to the initial investment (Io), which facilitates prioritisation. From this perspective, the calculated enterprise value is not a decision value does not serve as a decision value in the narrower sense, as

87 See the empirical survey in *Gleißner* et al. (2024).
88 For example, see *Lauber* (2014); *Follert* (2019); *Quill* (2020) and *Gleißner/Follert* (2022).
89 See again *Hering/Toll* (2013) and *Hering* et al. (2013b).

it does not incorporate all information relevant to the valuation subject. Instead, the valuation result can be viewed as a 'typified decision value' rather than a 'subjective decision value'.[90]

In contrast to the approaches of *Hering* et al. (2013a) and other frameworks in investment-theoretical valuation theory, our model, inspired by the ideas of *Fama* (1977), determines a specific value rather than a range of values. This value results from a complete transformation of uncertainty, achieved through the application of a risk-value model. The risks associated with cash flows are converted into a safe figure via the risk measure, while uncertainties related to environment parameters (e.g. the risk-free interest rate) are neglected in line with the principles outlined by *Fama* (1977). Therefore, semi-investment-theoretical valuation paradigm, similar to financial-theoretical valuation based on CAPM, aims to determine a definite value rather than a value range, provided the environment parameters are assumed to be stable.

IV. The valuation of a PE investment: A case study with an uncertain exit price

1. Basics and valuation assumptions

The following simple case study of a non-listed participation in an investment fund shows how a simulation-based valuation can be carried out.[91] In the case study, the marginal price P^* is to be determined for TextitAI, Inc., with an assumed exit in $T = 5$ years. The operating capital of €7.5 million is financed with equity at the level of 20% (= €1.5 million). The current borrowed capital D_0 is thus €6 million.

Two partners own the company. The founder (G) of the company and a venture-capital (VC) fund that joined after the company's founding each own 50% of the shares. With its entry into the company and the acquisition of the 50% shareholder share, the VC fund contributed one million euros in equity. It has agreed to a special right (liquidity preference) for the intended exit, the sale to a strategic investor, and will receive the first million euros from the uncertain future exit price $\left(p^{EXIT}\right)$ exclusively. The excess amount is divided in half.

The owners have two alternative investment options to choose from: quasi-risk-free government bonds with an interest rate of $r_f = 3\%$, or a broadly diversified stock index with an uncertain market return $\tilde{r}_m$. Their annual distribution is i.i.d. (independent and identically distributed) assumed to be log-normally

[90] A distinction that is also to be made in Germany with the revision of the IDW S1 valuation standard (IDW ES 1, 2024).

[91] See also *Gleißner/Ernst* (2019) and *Ernst* (2022a).

distributed, with an expected value $r_m^e = 8\,\%$ and standard deviation $\sigma_{r_m} = 20\,\%$. Equation (4) produces market price of the risk λ_T^σ results for the risk-measure standard deviation as shown in Table 1.

Table 1

Multi-period risk price depending on the period length T using the standard deviation as a risk measure and log-normally distributed market returns

T	1	2	3	4	5
λ_T^σ	0.25	0.34	0.41	0.45	0.49

These values (as well as the ones in Table 2) result from a Monte Carlo simulation with the parameters given above. With the same parameters, the $\lambda_T^{\sigma-}$ results shown in Table 2 are obtained for the risk measure of the (lower) standard deviation (σ).

Table 2

Multi-period risk price depending on the period length T using the semi-standard deviation as a risk measure and log-normally distributed market returns

T	1	2	3	4	5
$\lambda_T^{\sigma-}$	0.33	0.44	0.51	0.57	0.60

In the example case, it is fixed that

- the predicted exit takes place in year $T = 5$,
- the risk diversification factor is assumed to be $d = 0.5$,[92]
- at the time of exit, a valuation multiple (m) based on EBIT with a minimum of 6, the most likely value of 8 and a maximum of 11 is assumed for the rated company (modelled with a triangular distribution).

The most likely value is considered as the traditional planned value for the EBIT multiple m: $\bar{m} = 8$. In addition, a sales multiple typical for the industry is known, namely $m_R = 0.7$.

Finally, the additional funding obligations of equity investors are excluded, so the exit cannot take negative values. With these specifications, the uniform val-

[92] This means that half of the overall risk is assumed to be relevant to the assessor after diversification effects. In view of the imperfect diversification of most valuation subjects, this use of an average risk-diversification degree of $d = 0.5$ seems acceptable.

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uation equation for the marginal price (fundamental value) of the company is derived according to equation (5), whereby only two variables have to be estimated through using company-specific simulations: the expected value of the exit price, using a (stochastic) multiplier method; and the semi-standard deviation of the exit price, as a measure of the overall (operational) risk exposure. That is:

$$\sigma^{-}\left(\tilde{P}^{Exit}\right) = E\left(\max\left(E\left(\tilde{P}^{Exit}\right) - \tilde{P}^{Exit}; 0\right)^{2}\right)^{\frac{1}{2}} = \tag{17}$$

$$= E\left(\max\left(E\left(\max\left(0; \tilde{m}\cdot\widetilde{EBIT}_{t=T} - \tilde{D}_{t=T}\right)\right) - \max\left(0; \tilde{m}\cdot\widetilde{EBIT}_{t=T} - \tilde{D}_{t=T}\right); 0\right)^{2}\right)^{\frac{1}{2}}.$$

2. Risk simulation and exit price range

Figure 1 shows the course of the expected values for the capital items. This graphic was created using the values given above via simulation.[93]

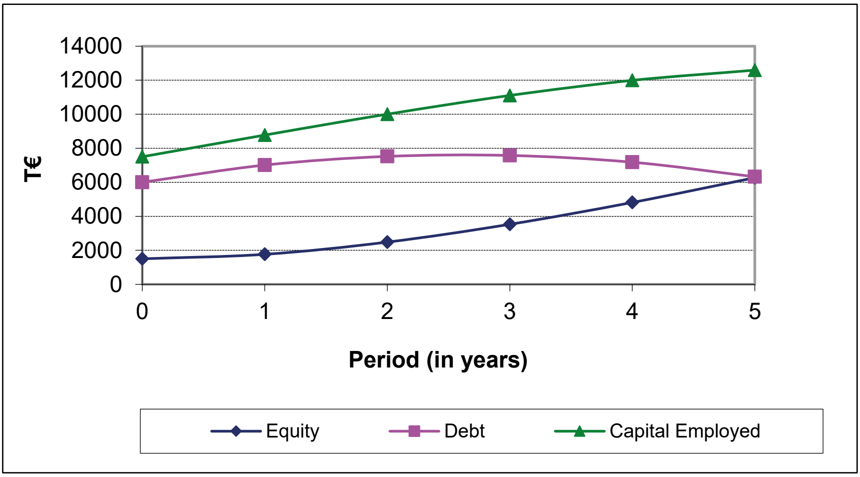

Figure 1: Expected course of the capital positions

Based on the (manageable) informational input from the analysis, we perform a simulation followed by a valuation. Figure 2 shows the range of EBIT development. The values result from a Monte Carlo simulation that we carried out in Microsoft Excel with a supplemental add-in.

[93] We use Excel and Crystal Ball.

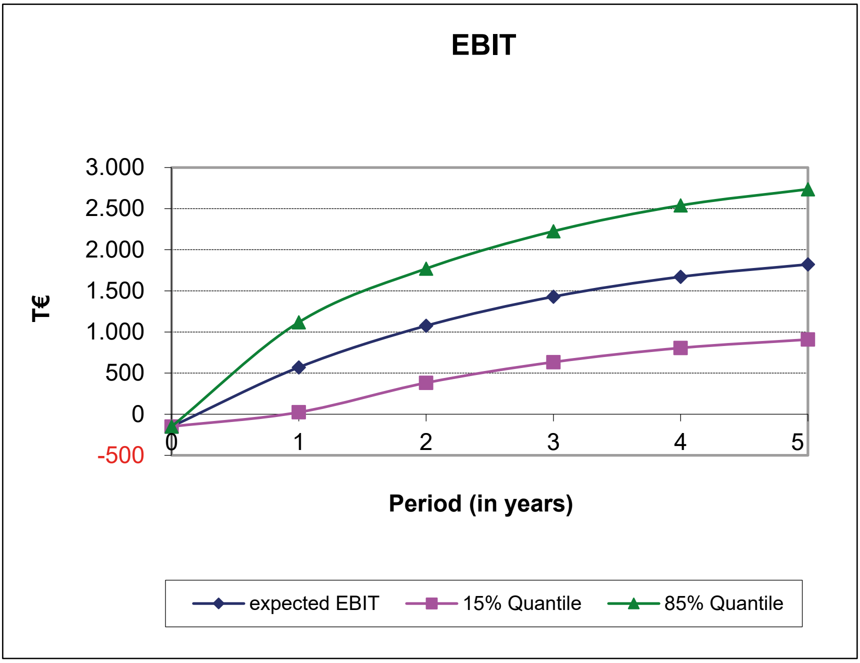

Figure 2: Range of possible EBIT developments

The initial objective is to perform a risk-based assessment of the company as a whole and assess the shares of the two shareholders, considering the special rights of the venture-capital investor. The immediate result of the simulations is the distribution of $\tilde{P}^{Exit}$ at $T = 5$, calculated according to (6), from which the expected value and (semi-)standard deviation can be calculated. Figure 3 shows the results of the simulation.

To determine a marginal price P^*, the range of the uncertain exit price must be estimated. The simulation of the cash flow in the planning period (and the implicit underlying risk assessment) is mainly used to determine the possible states of the company (and the environment) at the time of exit and to derive an exit price from them. The Monte Carlo simulation provides the following frequency distribution of $\tilde{P}^{EXIT}$.

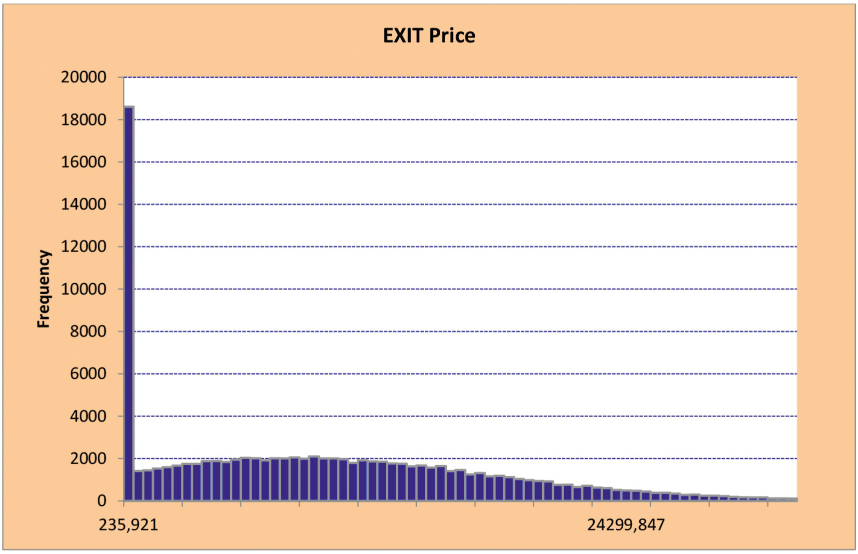

Figure 3: Exit price distribution

The expected exit price, $E(\tilde{P}^{Exit})$, is therefore approximately €9.7 million. The standard deviation of the exit price is €8.2 million, while the lower semi-standard deviation is only €5.2 million. The high relative frequency of the 0 value reflects the fact that an unfavourable course of business (the occurrence of negative risks, or dangers) can also lead to a total failure of the investment (i.e., insolvency).[94] Given that the owners' additional cash-flow obligations are excluded, the cash flow cannot be assumed to have negative values. Thus, the values that would actually result in an exit price lower than 0 are compressed to 0.

3. *Simulation-based valuation of the company*

a) Risk measure: standard deviation

With the information obtained through this simulation method, it is now easy to determine the fundamental value V, which directly depends on operational risks and, thus, the risk analysis of the company.

[94] The probability of insolvency (the rating) is an important value driver; see *Franken* et al. (2020) and *Saha/Malkiel* (2012). The probability of survival of 99.1 % calculated in the case study over the period under consideration (or cumulative probability of insolvency of 0.9 %) corresponds to an average probability of insolvency per year of 0.2 % (corresponding to a BBB rating).

The risk price λ_T^σ resulting from the above assumptions is 0.49. With an assumed risk-diversification factor d of 0.5, Equation (5) gives the marginal price (i.e., fundamental value) when using the standard deviation as a risk measure:

$$V_0 = P_0^* = \frac{E\left(\tilde{P}^{EXIT}\right) - \lambda_T^\sigma \cdot \sigma\left(\tilde{P}^{EXIT}\right) \cdot d}{\left(1 + r_f\right)^T} = \frac{9.7 - 0.49 \cdot 8.2 \cdot 0,5}{(1 + 3\%)^5} = 6.6. \tag{18}$$

Equation (2) can be used to determine an average expected return of 8.0 % from this marginal price of €6.6 million, which, in this case, also corresponds to the cost-of-capital rate that is applied when using the discounted cash-flow risk-premium method. In other words, an investor is willing to invest a maximum of €6.6 million today as the expected value of €9.7 million then results given the required risk-adequate return of 8.0 %.

b) Risk measure: Semi-standard deviation

The lower semi-standard deviation can be used as an alternative risk measure for the assessment with $\lambda_T^{\sigma^-} = 0.6$. The marginal price according to (5) is thus approximately €7 million. Equation (2) results in a value of 6.6 % as the cost of capital (that is, the average expected risk-adequate return).

With the semi-standard deviation as a risk measure, there are two opposing effects. On the one hand, the market price of the risk at 0.6 is approximately 20 % higher than the standard deviation as a risk measure at 0.49. On the other hand, the semi-standard deviation of €5.1 million is almost 40 % lower than the standard deviation of €8.2 million in the clearly asymmetrical distribution, which is cut off downwards at 0. The risk discount for the semi-standard deviation is therefore lower overall, and the value of the company is correspondingly higher. This shows that the choice of risk measure, which depends on the assessor's understanding of risk, can have a significant impact on the results of the assessment, especially when the distribution of the cash flows is asymmetric.

In the case study, the semi-standard deviation (5.1), which is significantly lower than the standard deviation (8.2), is the result of the loss limitation (which is the limitation of liability). By using the semi-standard deviation as a measure of risk, the main advantage of this limitation of loss liability is captured in the assessment more clearly than with the standard deviation.

c) Special rights and share valuation

Finally, the shares of the two shareholders are also valued. A determination of the uncertain returns of individual equity providers with different (special) rights is made straightforward if the existing rights and contractual conditions are mapped in the simulation model (see Section III.4).

In the example above, another result of the Monte Carlo simulation for the venture-capital investor's capital investment *I* of €1 million and a share α_2 of 67% is expected exit proceeds of approximately €6.5 million, with a lower semi-standard deviation of €3.4 million. The value of investor *II*'s investment, V_2, is €4.7 million, which is calculated as follows:

$$(19) \qquad V_2 = \frac{E\left(\tilde{P}_1^{Exit}\right) - \lambda_T^{\sigma^-} \cdot \sigma^-\left(\tilde{P}_1^{Exit}\right) \cdot d}{\left(1+r_f\right)^T} = \frac{6.5 - 0.6 \cdot 3.4 \cdot 0.5}{(1+3\%)^5} = 4.7.$$

Analogously, the value V_1 for investor *I* is:

$$V_1 = \frac{3.2 - 0.6 \cdot 1.7 \cdot 0.5}{(1+3\%)^5} = 2.3.$$

4. Comparison with traditional valuation methods

In order to better assess the new methodology, we also carry out a valuation using traditional methods for the purpose of comparison. First, we conduct a simple valuation of the valuation object using the multiple model (see section 2.2.2 above) at time $t = 0$; second, we apply a CAPM-based valuation model.

In contrast, a comparable valuation using an investment theory model, e.g. based on the valuation of a VC investment as in *Hering* et al. (2012 and 2013b), is not possible based on the information used in our case study. The valuation result would essentially be the result of a large number of additional assumptions about the valuation subject, which makes comparability practically impossible. This possibility of mapping the decision field and the restriction of a valuation subject more precisely than in our model is an advantage, but it is generally offset by an increased workload and, at least in the determination of argumentation values, also has disadvantages due to increased complexity and more difficult communicability (see Section III.6.).

In the simple valuation, future prospects are ignored completely and only information at the starting time $t = 0$ (the present) is used. The exit price based on the multiple valuation and EBIT is therefore 0 because of the negative EBIT.[95] So, in this case, the application of the sales multiple could be more appropriate, which with a value of $m_R = 0.7$ (see IV.1) leads to

$$V_{A1} = m_R \cdot R_0 - D_0 = 0.7 \cdot 15 - 6 = 4.5\ .$$

[95] Or 1.5 million if you also use equity as a lower limit.

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While the value is lower, as with our new method, there are disadvantages to this approach: it ignores

- future prospects (corporate planning) and
- all valuation-relevant information about earnings risks (and insolvency risk).

A valuation of the expected future exit price is generally possible using the CAPM and assuming perfect markets. For the case study, we therefore additionally assume that a beta factor of β = 1.8 can be derived from a peer group[96] via $ß = \rho \cdot \frac{\sigma_i}{\sigma_{r_m}} = 0.6 \cdot \frac{0.6}{0.2} = 1.8.$

This beta value results in capital costs (as a WACC value) amounting to:

$$c = r_f + ß \cdot \left(r_m^e - r_f\right) = 3\% + 1.8 \cdot 5\% = 12\%.$$

With these costs of capital, the planned exit price $EBIT_5^{Plan} \cdot \bar{m}$ is discounted as:

$$V_{A2} = \frac{EBIT_5^{Plan} \cdot \bar{m}}{(1+\mathrm{c})^T} - D_0 = \frac{1.875 \cdot 8}{(1+0.12)^5} - 6 = 2.5.$$

This process also has disadvantages compared to the method we suggest. In contrast to the multiple method, the company's future prospects are taken into account, but only with regard to planning and its consequences for the planned exit price. Here, too, information about the risks (specifically, the earnings risks) is not taken into account when discounting with the interest rate c = 12 %. In addition, the use of the CAPM to calculate c always assumes that investors are perfectly diversified, which is frequently inappropriate (as it is in this example).

The results of both standard methods differ significantly from the value calculated in Section III. 5. Both valuations are too low, which means that relying on these low values could lead to a wrong decision. The valuation is too low because earnings and insolvency risks relevant to the valuation are not (adequately) taken into account in either model (and in particular, opportunities are ignored). Instead, the high risks from the capital market (fluctuations in stock returns from 'peers') are used as a 'substitute'. The inside information about the earnings risks of the company itself is not considered at all, since the beta factor only valuated the risks of the returns (i. e., the stock returns) of the companies in the peer group. However, in real imperfect markets, no conclusions can be drawn about the risks of future profits or cash flows – and thus about the exit price – from historical (valuation-relevant) stock return risks.

[96] Here, we take into account the financing structure (via the asset beta). A correlation ρ =0.6 and a standard deviation of the return of the representative shares from the peer group of σ_i = 60 % are assumed. σ_{r_m} is again the standard deviation of the stock market return.

V. Final discussion

The application of the method outlined in this article for a risk-adequate valuation of participations and special rights based on an estimate of the uncertain exit price is associated with some challenges for users who have previously used traditional DCF (discounted cash flow) methods or simple, non-stochastic multiple methods. What is required is a departure from the idea of perfect markets and a clear conceptual separation between the model-based calculated earnings value and a currently realisable market price. This amounts to a paradigm shift.[97] Even though the imperfect replication valuation methodology used is now based on a secure axiomatic foundation,[98] and the Monte Carlo simulation required for the simulation-based valuation can be implemented easily with standard software,[99] many potential users still lack the necessary knowledge and experience. Methods for quantitative risk analysis and simulation-based risk aggregation are still often used only by risk management specialists, although they are fundamentally helpful for every business decision, especially regarding the sale or purchase of investments, which are always associated with risks. It can also be argued that the simulation-determined derivation of decision values is based on subjective assumptions, but, ultimately, all planning and decisions depend on subjective assumptions, and the method outlined here has the advantage of showing the uncertainty of the assumptions explicitly and transparently and thus avoiding apparent but false accuracies.

The relative novelty of the methods outlined here and the consequent lack of experience of many potential practitioners appear to be an obstacle to their practical implementation. Nonetheless, as our case study shows, the effort associated with the application of the method can be managed without problems. The advantages of a simulation-based valuation, which does not require capital-market perfection and consistently enables the recording of special rights in one step, encourage the increased use of such instruments. This use may initially target particularly important decisions, especially about the purchase or sale of investments or the associated determination of acceptable marginal prices.

Without historical capital-market data, which is often unavailable or of little meaning, investment-specific, risk-appropriate return requirements and risk-adjusted values can be derived from the transparent presentation of the risks of an investment. The great advantage of our procedure lies in its consistent reference to the future and explicit consideration of the risks ahead. The procedure also allows the investment values to be continuously monitored during ongoing investment controlling and in the context of a value-oriented management ap-

97 See *Ernst/Gleißner* (2022).

98 See *Dorfleitner/Gleißner* (2018) and *Dorfleitner* (2022).

99 Such as Microsoft Excel in conjunction with @Risk or Crystal Ball.

proach by assessing the consequences of strategic options for action on the company's value.

The Monte Carlo simulation, and thus the simulation-based valuation, is the basis of the valuation methodology we outline because essential requirements cannot be met by other methods. Machine learning processes (i. e., artificial intelligence) require a large amount of data from comparable companies in order to learn the essential connections.[100] They are therefore suited to estimating market prices[101] if many comparable data are available, meaning that they can therefore be viewed as an alternative or further development to the multiplier methods used in this article, which use significantly less information. In this respect, machine learning methods can improve the valuation methods outlined here with additional data input (concerning the possible exit prices).[102] However, the actual valuation – that is, the calculation of a model-based company value as a basis for decision-making – cannot be carried out by a machine learning process because values, unlike prices, are not observable: in real markets, prices and values differ from one another.[103] Machine learning methods are also usually based on publicly available data and are specifically unable to take into account insider information about the company's risks and corporate planning, which are vital to a company's future forecasts. The Monte Carlo simulation, on the other hand, is able to systematically aggregate identified and quantified risks, which can also be described by different probability distributions, with reference to corporate planning. There is no formal analytical solution to such a risk aggregation problem. In the future, we expect that machine learning methods will be used as a supplement to simulation-based valuation (and Monte Carlo simulation) in order to improve the data input and thus arrive at better estimates of the range of market prices that can be achieved.

A clear limitation lies in the fact that the method suggested here cannot be proven to deliver the 'correct' price, as there is no such thing as a correct price. Moreover, it has become clear that the outcome depends on the specification of the internal processes of the company to be valued. Consequently, the application of the method requires a very diligent modelling of these internal processes.

Our case study demonstrates that a simulation of the cash flows is also possible by depicting the complete corporate planning and taking into account the results of the analysis of all the company's opportunities and threats, especially considering the mostly uncertain planning assumptions. In the case of an equi-

[100] See *Chen* et al. (2019).

[101] See *Herrmann/Richter* (2003).

[102] Bayesian methods are also possible for data input, but they do not replace the Monte Carlos simulation here, but rather supplement it, see *Wieczorek/Nickert* (2023).

[103] See *Gromb/Vayanos* (2010); *Shleifer/Vishny* (1997).

ty-investment company that buys shares in a company today with the aim of selling it at a higher price in the future, the uncertain future sale price is relevant to the valuation. Our methodology combines appropriate valuation methods and future-oriented price-estimation methods in an integrative model that can consistently and simultaneously value the company as a whole and the shares in the company held by different owners with different special rights.

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