

## **Spread Risk Premia in Corporate Credit Default Swap Markets<sup>1</sup>**

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### **Abstract**

The spread risk premium component of credit default swap (CDS) spreads represents a compensation demanded by protection sellers for future changes in CDS spreads caused by unpredictable fluctuations in the reference entity's risk-neutral default intensity. This paper defines and estimates a measure of the spread risk premium component in CDS spreads of a sample of European investment-grade firms by using a stochastic intensity credit model. Our results show that, on average, investors demand a positive premium for such mark-to-market risks. After controlling for CDS market conditions, like liquidity and supply/demand effects, a panel data analysis of the estimated spread risk premia reveals a positive impact of event risk captured by the overall stock market volatility and a negative impact of investors' appetite for exposure to credit markets as reflected by the overall CDS market.

### **Marktrisikoprämien in Credit Default Swaps von Unternehmen**

#### **Zusammenfassung**

Die Marktrisikoprämie im Credit Default Swap (CDS) Spread ist eine Kompensation für den Protection Seller für zukünftige Schwankungen des CDS-Spreads, die durch unvorhersehbare Fluktuationen der risikoneutralen Ausfallintensität

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des Referenzunternehmens verursacht werden. Dieser Aufsatz definiert und schätzt ein Maß für die Marktrisikoprämienkomponente im CDS-Spread unter Verwendung eines Kreditmodells mit stochastischer Ausfallintensität für eine Stichprobe europäischer Investment-Grade-Unternehmen. Unsere Ergebnisse zeigen, dass Protection Seller im Durchschnitt eine positive Prämie für die Übernahme von Marktrisiken verlangen. Eine Paneldatenanalyse der geschätzten Marktrisikoprämien findet zwei signifikante gemeinsame Faktoren, in der Form von Event Risk, erfasst durch die Aktienmarktvolatilität und der Nachfrage der Marktteilnehmer nach Kreditinvestments im Corporate Investment-Grade-Sektor, verkörpert durch den iTraxx Europe Index, selbst nach Bereinigung um Effekte durch Liquidität und Angebotschocks im CDS-Markt.

*Keywords:* credit default swap; spread risk premium; mark-to-market risk premium; stochastic intensity model

*JEL classification:* G12, G13, G15

## I. Introduction

Fluctuations in CDS premia and credit spreads of corporate entities are driven by a wide range of different factors affecting the various spread components. Abstracting from possible nondefault-related effects, like liquidity or tax, CDS premia and credit spreads are made up of a component for the actuarial compensation for default risk and a *default risk premium component*, whereby the first is subject to changing firm-specific business conditions and the second to shifts in investors' risk appetite.

The default risk premium component itself is made up, as outlined in Yu (2002), of two distinct components, a *default event risk premium component* and a *spread risk premium component*. The default event risk premium, also called the jump-to-default risk premium, compensates investors for bearing the risk of a default. Instead of following the actuarial method and pricing a default-risky instrument using the so-called *physical default intensity* derived from historical observations of defaults, investors, who are risk-averse toward a default event, would insist on using a higher intensity, the so-called *risk-neutral default intensity*. The gap between these two different intensities expresses the premium demanded by an investor for bearing the default risk.

The spread risk premium, also called the mark-to-market risk premium, is a compensation for nondefault-related fluctuations in the risk-neutral default intensity leading to fluctuations in the market value of default-risky instruments. Besides demanding a higher default intensity in the form of the risk-neutral default intensity, an investor who is not

only risk-averse toward a default event but also toward unpredictable nondefault-related fluctuations in the risk-neutral default intensity, would insist on pricing a default-risky instrument using the risk-neutral distribution rather than the physical (i.e. empirical) distribution of the risk-neutral intensity. The difference between these two distributions of the risk-neutral default intensity, determined by the *market price of default risk*, expresses an investor's risk-aversion toward market value fluctuations, i.e. the demanded spread risk premium.

While numerous empirical studies have recently been devoted to the investigation of the default event risk premium in credit markets, the spread risk premium has received surprisingly little empirical attention so far. Therefore, the purpose of this paper is to estimate spread risk premia in corporate credit markets using market data on credit default swaps and to explain their time variation. While previous empirical studies on spread risk premia focused on swap markets and sovereign credit markets, to the best of our knowledge, this is the first paper to explain spread risk premia in corporate credit markets.

The use of CDS data instead of corporate bond data in estimating spread risk premia offers two major advantages. First, the CDS spread offers a cleaner measure of the reward for the exposure to a firm's credit risk than credit spreads of corporate bonds, which have been found to be contaminated by tax effects as found in *Elton et al. (2001)* and by liquidity effects as diagnosed in *Longstaff/Mithal/Neis (2005)*. Second, due to the constant availability of a firm-specific CDS curve, using CDS data to estimate a firm's spread risk premium yields more reliable estimates than using corporate bond data, since such data is very often characterised by a scarcity in the cross-sectional dimension.

We calibrate a stochastic intensity credit model to the time series of the ten-year CDS curve of a sample of European investment-grade firms, and infer both the physical distribution of a firm's risk-neutral intensity from the time series dimension of the data and its risk-neutral distribution from the cross-sectional dimension of the data, represented by the CDS curve. We calculate a firm's spread risk premium as a deviation between two different default probabilities, the one calculated using the risk-neutral distribution, called the *risk-neutral default probability* and the other using the physical distribution of the firm's risk-neutral intensity, called the *pseudo-physical default probability*. Using panel regression methods, we examine the influence of various factors on the variation of the estimated spread risk premia.

On average, we detect a positive spread risk premium, indicating that protection sellers in CDS contracts demand, and protection buyers in CDS contracts are willing to pay, a risk premium for unpredictable fluctuations in the reference entity's risk-neutral default intensity. We find that even after controlling for liquidity and supply/demand conditions in CDS markets, event risk, as captured by the overall stock market volatility, has a positive and investors' appetite for exposure to credit markets, as reflected by the overall CDS market, has a negative impact on spread risk premia. Further, we also detect a significant positive influence of the firm-specific equity volatility on spread risk premia, whereas the firm-specific equity return shows no impact. We observe mixed effects for macroeconomic risk variables.

The remainder of this paper is organised as follows: Section II. provides an overview on the related literature. Section III. specifies the credit model we calibrate to the CDS panel data and defines a measure that captures the spread risk premium. Section IV. describes the data and the credit model estimation methodology and illustrates the resulting spread risk premia. Section V. introduces proxies for various factors potentially influencing the spread risk premium and conducts a panel data analysis to determine their impact. Section VI. concludes.

## II. Related Literature

Numerous empirical studies have recently been devoted to the investigation of default risk premia. One of the first amongst them, *Elton et al. (2001)*, examines corporate bond yield spreads and detects a residual component beyond a compensation for expected losses and tax effects. The authors interpret this residual component as a risk premium and find it to be correlated with equity market factors. However, they do not further decompose this risk premium component into default event risk and spread risk premium components.

With this decomposition in mind, we can identify two main strands of research investigating default risk premia. The first strand of research, pioneered among others by *Berndt et al. (2005)*, *Driessen (2005)* and *Anderson (2008)*, explores default event risk premia, usually defined as the ratio between a firm's risk-neutral default intensity and its physical default intensity. A firm's risk-neutral default intensity is typically estimated from CDS or corporate bond data and its physical default intensity from expected default frequencies (EDFs) delivered by Moody's

KMV, observed default rates provided by rating agencies or financial ratios and equity market variables.

The second strand of research, the one to which we contribute here, investigates spread risk premia. The study conducted by *Liu/Longstaff/Mandell* (2006) examines spread risk premia in interest rate swap markets. Using a reduced-form credit model, they calculate instantaneous expected returns of zero-coupon bonds, whose credit spreads are derived from swap spreads. They compute lower and upper bounds for the default risk premium component (the sum of the default event risk premium component and the spread risk premium component) in these instantaneous expected returns, where the lower bound corresponds to the spread risk premium component. They find a flat term structure for the average spread risk premium component with -2 basis points at the one-year maturity and with -3 basis points for maturities from two to ten years. In contrast to their study, we examine corporate credit markets and estimate the spread risk premium of not only one but of several different reference entities and we also explain their variations.

While the study conducted by *Liu/Longstaff/Mandell* (2006) focuses on swap markets, the two other remaining studies, *Pan/Singleton* (2008) and *Zhang* (2008), examine spread risk premia in sovereign credit markets. *Pan/Singleton* (2008) calibrate a stochastic intensity credit model to the time series of the ten-year sovereign CDS curves of Mexico, Turkey, and Korea. Using the physical distribution instead of the risk-neutral distribution of the risk-neutral default intensity, they calculate a model CDS spread that does not contain a spread risk premium component. The authors investigate the percentage contribution of this spread risk premium component to the CDS spread for the three countries. They detect a significant explanatory power of (i) investors' appetite for global "event risk", proxied by the CBOE VIX volatility index, (ii) the desirability of carry trade positions (borrowing short-term in USD and investing in long-term emerging market bonds), captured by the spread between the U.S. Industrial 10-year BB Yield and the 6-month Treasury bill yield, and (iii) the extent of capital flows in and out of the country, captured by the implied country-specific currency option volatility.

Similar to *Pan/Singleton* (2008), *Zhang* (2008) estimates a reduced-form credit model using the time series of the ten-year sovereign CDS curve of Argentina. However, in contrast to them, instead of calculating the spread risk premium component in CDS spreads, he captures this component by defining the spread risk premium as the difference be-

tween the cumulative risk-neutral and the cumulative pseudo-physical default probability for the one-year maturity. He finds that proxies for the U.S. business cycle and for U.S. credit conditions have a significant impact on the time variation of the resulting spread risk premium in Argentine sovereign debt.

In contrast to *Pan/Singleton* (2008) and *Zhang* (2008) we examine spread risk premia in corporate rather than sovereign credit markets and base our analysis on a much higher number of reference entities. Contrary to *Pan/Singleton* (2008) and following *Zhang* (2008) we capture the spread risk premium component in CDS spreads by the difference between the one-year cumulative risk-neutral and pseudo-physical default probabilities. Further, rather than assuming a deterministic short rate like in *Pan/Singleton* (2008), we follow *Zhang* (2008) by considering a stochastic short rate process and allowing for correlation with the risk-neutral intensity process as specified in *Duffie* (1999). However, unlike *Zhang* (2008), who assumes continuous CDS premium payments, we follow *Pan/Singleton* (2008) and take into account contractual features like quarterly premium payment dates and accrued premia in case of the occurrence of a credit event between two such scheduled payment dates. Finally, in contrast to both these studies, we consider a wider range of potential explanatory variables and make use of panel data methods to explain our resulting spread risk premia.

### III. Credit Model

A standard CDS is a contract between a protection buyer and a protection seller. The protection buyer can gain insurance against the default of a specified firm by making fixed quarterly payments to the protection seller until the contract matures or a specified credit event is triggered, whichever occurs first. If a credit event is triggered prior to maturity, the protection buyer has the right to deliver a corporate bond of the defaulted firm to the protection seller and to receive the payment of the par value of the bond.

#### 1. Credit Default Swap Valuation

We choose the reduced-form concept in the sense of *Duffie* (1998), *Lando* (1998), *Duffie/Singleton* (1999), and others in order to value CDS. Therefore, we define a probability space  $(\Omega, \mathcal{F}, \mathbb{Q})$  endowed with a

filtration  $(\mathcal{F}_t)_{t \geq 0}$  satisfying the usual conditions, i.e.  $(\mathcal{F}_t)_{t \geq 0}$  is complete and right-continuous. The filtration describes the flow of information available to the market, i.e.  $\mathcal{F}_t$  represents all events observable up until time  $t$ .  $\mathbb{Q}$  is an equivalent martingale measure, in the sense that all discounted assets are  $\mathbb{Q}$ -martingales, where the discounting is defined with respect to a money market account that accumulates at a default-free short rate process  $r(t)$ . The physical probability measure, to which  $\mathbb{Q}$  is equivalent, is denoted by  $\mathbb{P}$ . A company's default time is characterised by the first jump time  $\xi$  of a Cox process. We denote the default indicator function by  $\mathbf{1}_{\{\xi \leq t\}}$  and the associated risk-neutral intensity process by  $h(t)$ . The value at time  $t$  of a promised payment of one unit of currency in  $s > t$  with a recovery of zero in the case of default is

$$(1) \quad \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^s r(u) du \right) \mathbf{1}_{\{\xi > s\}} \right] = \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^s [r(u) + h(u)] du \right) \right],$$

where we set  $\mathbb{E}_t^{\mathbb{Q}}[\cdot] := \mathbb{E}^{\mathbb{Q}}[\cdot | \mathcal{F}_t]$ . The value at time  $t < \min(s, \xi)$  of a recovery payment  $\omega(\xi)$  due to a default during the time period  $[t, s]$  is given by

$$(2) \quad \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{\xi} r(u) du \right) \mathbf{1}_{\{\xi \leq s\}} \omega(\xi) \right] = \text{REC}(t) \int_t^s \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du,$$

where  $\text{REC}(t)$  denotes the risk-neutral expectation in  $t$  of the recovery payment  $\omega(\xi)$ , which we assume to be independent of the processes  $r(t)$  and  $h(t)$ . Formal proofs of the identities (1) and (2) are presented in Duffie (2001). These two building blocks can be used to state both sides of the credit default swap contract, the premium leg and the default leg.

Assuming a notional value of one unit of currency, the premium leg of the contract at time  $t$  for an integer-valued contract maturity  $\tau$  is given by

$$(3) \quad \begin{aligned} PL(t, \tau) &= \delta \text{CDS}(t, \tau) \sum_{n=1}^{4\tau} \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{T_n} [r(u) + h(u)] du \right) \right] \\ &+ \delta \text{CDS}(t, \tau) \sum_{n=1}^{4\tau} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du, \end{aligned}$$

where  $\text{CDS}(t, \tau)$  is the fixed annual  $\tau$ -year CDS premium, and  $\delta := \frac{1}{4} \frac{365}{360}$  is the day count fraction for quarterly premium payments, approximating the ACT/360 day count convention used in CDS markets. The first

term of the premium leg is the present value of the quarterly-paid premia with the first payment being made in  $T_1$ . The second term is the present value of the accrued premia to be paid in case of the occurrence of a credit event between two premium payment dates  $T_{n-1}$  and  $T_n$ .

The default leg of the contract can be stated as

$$(4) \quad DL(t, \tau) = [1 - REC(t)] \int_t^{t+\tau} \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du,$$

where  $REC(t)$  denotes the fractional recovery of par and is assumed to be a deterministic function of the time  $t$ . Following the traditional “par spread” quotation instead of the recently introduced (economically equivalent) “upfront” quotation method for CDS, the two legs of the contract have to be equal at the inception date  $t$ . Consequently, the CDS premium at time  $t$  with contract maturity  $\tau$  is given by

$$(5) \quad CDS(t, \tau) = \frac{[1 - REC(t)] \int_t^{t+\tau} \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du}{\left\{ \delta \sum_{n=1}^{4\tau} \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{T_n} [r(u) + h(u)] du \right) \right] + \delta \sum_{n=1}^{4\tau} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \mathbb{E}_t^{\mathbb{Q}} \left[ h(u) \exp \left( - \int_t^u [r(z) + h(z)] dz \right) \right] du \right\}}.$$

### 2. Parameterisation of the Term Structure and Default Intensity Model

We follow the approach of *Liu/Longstaff/Mandell* (2006), *Entrop/Schiemert/Wilkens* (2013) and *Zhang* (2008) and choose a parameterisation of our credit model that allows for correlations between the short rate process and the risk-neutral default intensity process by following the idea of *Duffee* (1999). Hence, we specify the short rate  $r(t)$  as a three-factor version of the short rate process proposed by *Cox/Ingersoll/Ross* (1985) (CIR). Thus  $r(t)$  is defined as the sum of three economy-wide latent state variables,

$$(6) \quad r(t) = X_1(t) + X_2(t) + X_3(t),$$

with the three state variables following square-root processes, i. e.



$$(7) \quad dX_i(t) = \kappa_i(\theta_i - X_i(t))dt + \sigma_i\sqrt{X_i(t)}dW_i^{\mathbb{P}}(t), \quad i = 1, 2, 3,$$

where  $(W_i^{\mathbb{P}}(t))_{i=1,2,3}$  are independent standard Brownian motions under the physical measure  $\mathbb{P}$ . In order to capture credit risk, we define a company-specific distress variable  $Z(t)$  also following a square-root process,

$$(8) \quad dZ(t) = \kappa_z(\theta_z - Z(t))dt + \sigma_z\sqrt{Z(t)}dW_z^{\mathbb{P}}(t),$$

such that  $(W_i^{\mathbb{P}}(t))_{i=1,2,3}$  and  $W_z^{\mathbb{P}}(t)$  are independent Brownian motions under  $\mathbb{P}$ . We specify the market price of risk for the four variables as

$$(9) \quad \Gamma_i(t) = \frac{\lambda_i\sqrt{X_i(t)}}{\sigma_i} \quad \text{for } i = 1, 2, 3, \quad \text{and} \quad \Gamma_z(t) = \frac{\lambda_z\sqrt{Z(t)}}{\sigma_z},$$

and apply the Girsanov theorem to obtain their dynamics under the equivalent martingale measure  $\mathbb{Q}$  as

$$\begin{aligned} dX_i(t) &= [\kappa_i\theta_i - (\kappa_i + \lambda_i)X_i(t)]dt + \sigma_i\sqrt{X_i(t)}dW_i^{\mathbb{Q}}(t) \quad i = 1, 2, 3, \\ dZ(t) &= [\kappa_z\theta_z - (\kappa_z + \lambda_z)Z(t)]dt + \sigma_z\sqrt{Z(t)}dW_z^{\mathbb{Q}}(t), \end{aligned}$$

where  $(W_i^{\mathbb{Q}}(t))_{i=1,2,3}$  and  $W_z^{\mathbb{Q}}(t)$  are independent standard Brownian motions under  $\mathbb{Q}$ .

Finally, using the specification of *Duffee* (1999), we define the risk-neutral default intensity process  $h(t)$  as an affine-linear function of the four variables,

$$(10) \quad h(t) = \Lambda_0 + \Lambda_1[X_1(t) - \bar{X}_1] + \Lambda_2[X_2(t) - \bar{X}_2] + \Lambda_3[X_3(t) - \bar{X}_3] + Z(t),$$

where  $\Lambda_1, \Lambda_2,$  and  $\Lambda_3$  determine the correlation between the short rate and the risk-neutral default intensity process, and  $\bar{X}_1, \bar{X}_2,$  and  $\bar{X}_3$  stand for the time series averages of  $X_1(t), X_2(t),$  and  $X_3(t)$  respectively.

In order to ensure computational feasibility of the specified credit model, we require the conditional expectations on the right-hand side of the identities (1) and (2) in closed form. Therefore, we follow the methodology developed by *Duffie/Pan/Singleton* (2000) and define as in *Zhang* (2008) a conditional discounted characteristic function of  $h(t + \tau)$

$$(11) \quad \Phi(t, t + \tau; \phi) := \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{t+\tau} [r(u) + h(u)]du + \mathbf{i}\phi h(t + \tau) \right) \right],$$

with the boundary condition  $\Phi(t + \tau, t + \tau; \phi) = \exp(\mathbf{i}\phi h(t + \tau))$  and  $\mathbf{i} := \sqrt{-1}$  the imaginary unit. The exponentially affine closed-form solution of this characteristic function, given in *Entrop/Schiemert/Wilkens* (2013), can be used to obtain the desired closed-form expressions for the conditional expectations in (1) and (2) which yield a computationally feasible expression for the CDS premium in (5), as shown in Appendix A.

### 3. Defining a Measure of the Spread Risk Premium Component

Specifying the risk-neutral default intensity in (10) as being correlated with the short rate process precludes a direct calculation of the spread risk premium component in the CDS spread, as done in *Pan/Singleton* (2008) who specify these processes as uncorrelated. In order to calculate their spread risk premium component, they set the market price of risk parameters zero and calculate a model CDS spread that contains no spread risk premium and subtract it from the market CDS spread. In our case however, setting the market price of risk parameters  $(\lambda_i)_{i=1,2,3}$  and  $\lambda_z$  in (9) to zero, would lead to a change from the risk-neutral measure  $\mathbb{Q}$  to the physical measure  $\mathbb{P}$  not only for the intensity process but also for the short rate process.

Therefore, we follow the approach by *Zhang* (2008) and define a measure that proxies the spread risk premium component by capturing the effect of this change of measure solely for the intensity process. This measure is based on the deviation between two survival probabilities, the cumulative  $\tau$ -year *pseudo-physical survival probability*,

$$(12) \quad \mathcal{PS}^{\mathbb{P}}(t, \tau) := \mathbb{E}_t^{\mathbb{P}} \left[ \exp \left( - \int_t^{t+\tau} h(u) du \right) \right],$$

which uses the physical distribution of the risk-neutral intensity and the cumulative  $\tau$ -year *risk-neutral survival probability*

$$(13) \quad \mathcal{S}^{\mathbb{Q}}(t, \tau) := \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{t+\tau} h(u) du \right) \right],$$

which uses its risk-neutral distribution. Closed-form solutions for these two survival probabilities are given in Appendix B.

From these two survival probabilities we can calculate the according annualised cumulative  $\tau$ -year pseudo-physical and risk-neutral default probabilities as

$$(14) \quad \mathcal{PD}^{\mathbb{P}}(t, \tau) := [1 - \mathcal{PS}^{\mathbb{P}}(t, \tau)]^{\frac{1}{\tau}} \quad \text{and} \quad \mathcal{D}^{\mathbb{Q}}(t, \tau) := [1 - \mathcal{S}^{\mathbb{Q}}(t, \tau)]^{\frac{1}{\tau}}.$$

Following *Zhang (2008)*, we define our measure of the spread risk premium in CDS spreads as the difference between these two default probabilities and base our empirical analysis on the one-year maturity. Thus we define the one-year spread risk premium for company  $j$  at time  $t$  as

$$(15) \quad \Pi_{jt} := [\mathcal{D}_j^{\mathbb{Q}}(t, \tau) - \mathcal{PD}_j^{\mathbb{P}}(t, \tau)]_{\tau=1}.$$

#### IV. Data and Estimation Methodology

##### 1. Data Description

*Credit Default Swap Dataset.* We download our credit default swap panel data and recovery rates from the Markit database. Markit composes its CDS quotes and recovery rates from pricing information contributed by a broad range of market makers on a daily basis. Markit quotes are widely used by financial institutions for mark-to-market and risk management purposes. In order to obtain a representative sample of the entire credit market and at the same time to facilitate model implementation, our data sample comprises 29 constituent companies of the iTraxx Europe index from four different sector groups: (i) financials, (ii) consumers, (iii) technology, media, and telecommunications (TMT), and (iv) industrials and utilities. We download the CDS pricing information for our 29 companies from January 2001 to January 2005 for EUR-denominated contracts on senior unsecured debt. The restructuring clause of the contracts is complete restructuring (CR), under which any restructuring type qualifies as a credit event and senior unsecured bonds with maturities up to 30 years count as deliverable obligations. The CDS dataset consists of daily CDS spreads for the one-, two-, three-, five-, seven-, and ten-year maturities and the expected recovery rate for every company. Since the CDS time series during the first two years are characterised by a large proportion of missing or stale spreads and in order to guarantee a meaningful analysis, we limit our analysis to the sample period from January 2003 to January 2005. This sample period displays a significant improvement in the CDS data quality, characterised by a sharp drop in the proportion of stale spreads, which was probably brought about by the 2003 ISDA Credit Derivative Definitions. More precisely, during our

sample period none of our 29 companies had more than two consecutive daily stale spreads in any of their six available contract maturities, except for year-end holidays. We treat these few remaining daily stale spreads during our sample period as missing values and fill these gaps by linear interpolation. Finally, since we use a stochastic intensity credit model in our analysis, we reduce our data sample to a weekly frequency by choosing the observations on every Wednesday between January 1, 2003 and January 5, 2005, resulting in 106 data points. Table 1 provides some summary statistics of our CDS data sample at the company level for the six contract maturities.

During our sample period the average CDS spread across all 29 companies and all six available contract maturities is 49.27 basis points. At the company level the average CDS spread across the six maturities ranges from 15.66 basis points for ING to 90.03 basis points for DaimlerChrysler. For all companies we find a positive slope of the CDS curve. The average one-year CDS spread across all companies is 31.34 basis points and the average ten-year CDS spread is 66.60 basis points. The coefficient of variation, defined as the ratio of the standard deviation of the CDS spread to the mean of the CDS spread for the same contract maturity, is highest for the one-year maturity for all companies. The coefficient of variation decreases with longer contract maturities for all companies except for Danone where we find a slight increase between the seven-year and the ten-year maturity. At the sector level we find the lowest average CDS spread in the financial sector with 28.70 basis points. For all the other three sector groups the average CDS spread is approximately twice as high, with 54.67 basis points for consumers, 53.88 basis points for TMTs, and 55.09 basis points for industrials and utilities.

*Interest Rate Dataset.* Since *Liu/Longstaff/Mandell* (2006) find a non-zero spread risk premium component in interest rate swap spreads, we choose, contrary to *Zhang* (2008), interest rates of EUR-denominated German government bonds as our risk-free rates instead of the swap curve. Therefore, we use the ten-year spot rate curve, calculated by the Deutsche Bundesbank from German government bond yields using the methodology proposed by *Svensson* (1994). The spot rate curve consists of the ten spot rates for the one-, two-, three-, ..., and ten-year maturities.

Table 1: Summary Statistics of the CDS Data

Sector	Contract Maturity	Company	1Y		2Y		3Y		5Y		7Y		10Y		Curve Mean	Sector Mean
			Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev		
FINANCIAL	Abbey National		8.21	4.79	10.96	5.41	13.28	5.69	18.99	6.73	21.85	6.92	25.06	6.64	16.39	
	Allianz		17.92	17.05	23.97	19.13	28.60	20.24	36.58	20.59	41.16	21.16	46.20	20.79	32.40	
	Hypo-Vereinsbank		25.22	24.88	34.90	29.69	42.93	33.57	53.91	36.16	58.32	36.36	62.84	36.25	46.35	
	Commerzbank		25.22	27.96	32.64	31.41	37.59	33.14	48.40	36.79	53.07	38.22	58.01	39.27	42.49	
	Deutsche Bank		9.30	5.69	13.14	6.43	15.79	7.10	21.86	7.76	24.93	8.07	28.48	8.29	18.92	
	ING		7.82	4.56	10.71	4.78	12.92	5.04	17.62	5.47	20.77	6.26	24.11	6.77	15.66	28.70
	Allied Domecq		31.61	16.74	38.14	16.44	43.38	16.58	52.38	15.53	58.77	14.89	65.14	14.25	48.24	
	BAT		50.92	27.35	62.07	25.67	70.65	25.52	82.68	23.48	89.20	22.37	96.87	21.62	75.40	
	Carrefour		16.27	8.46	20.02	8.15	24.84	8.76	32.64	8.01	37.63	7.85	43.31	7.26	29.12	
	Casino Guichard		50.63	36.59	60.15	36.47	69.01	34.63	81.39	33.62	88.11	34.06	96.32	35.84	74.27	
CONSUMER	DaimlerChrysler		51.45	25.89	70.17	28.85	81.44	27.56	102.11	23.59	111.75	22.38	123.27	19.52	90.03	
	Danone		11.94	4.10	14.65	3.84	17.24	3.96	23.00	4.24	26.50	4.82	30.09	5.94	20.57	
	Gallaher		35.59	16.71	43.92	16.10	52.26	17.12	64.05	15.22	69.63	15.47	76.87	14.86	57.05	
	Investor		38.55	33.91	46.50	35.45	50.85	35.21	61.40	35.31	68.06	37.10	73.44	36.87	56.47	
	LVMH		35.84	25.07	42.47	24.70	49.79	27.02	60.06	27.46	65.45	27.83	71.58	28.33	54.20	
	Volvo		25.91	15.02	32.12	15.90	37.58	15.72	45.42	16.19	50.72	16.38	56.18	15.09	41.32	54.67
	France Télécom		56.46	64.74	66.80	65.95	77.07	65.92	91.51	62.41	101.08	60.03	113.01	56.59	84.32	
	KPN		33.07	24.70	42.63	27.94	50.62	29.94	62.52	27.50	69.72	26.09	77.73	24.48	56.05	
	Telefónica		24.12	15.57	30.39	16.86	35.81	17.65	46.66	16.80	54.71	15.96	63.36	15.16	42.51	
	Vodafone		20.71	11.66	25.84	12.13	30.69	12.63	39.31	11.89	45.94	11.34	53.94	10.71	36.07	
INDUSTRIAL	Wolters Kluwer		32.21	18.00	39.27	17.58	45.42	18.01	55.23	16.19	61.15	15.44	69.31	14.16	50.43	53.88
	Arecol		57.36	42.38	68.70	44.12	75.33	44.42	87.00	44.71	91.81	45.30	99.01	45.32	79.87	
	St. Gobain		32.50	28.46	39.53	30.45	44.61	31.24	53.02	31.29	57.55	31.38	63.98	30.76	48.53	
	Iberdrola		19.49	10.05	23.02	10.66	26.13	11.45	33.23	12.08	37.62	12.04	43.14	12.07	30.44	
	Lafarge		39.73	23.05	49.92	25.80	57.40	26.70	66.01	24.35	71.26	24.06	78.52	22.84	60.47	
	Repsol		44.29	47.44	52.43	45.97	59.41	47.23	67.08	41.43	72.59	40.71	79.59	39.54	62.56	
	Rolls-Royce		62.59	66.69	72.73	68.87	80.84	70.06	92.55	70.89	99.37	71.13	107.32	70.47	85.90	
	RWE		24.73	20.39	30.73	22.15	34.91	23.59	43.07	23.93	48.46	23.77	55.30	23.65	39.53	
	Vattenfall		19.22	9.90	24.32	10.10	28.24	10.53	36.92	10.86	42.34	11.47	49.41	12.19	33.41	55.09
	All Companies		31.34		38.72		44.64		54.36		59.98		66.60		49.27	

This table reports summary statistics of the CDS data sample at the company level for the six available contract maturities from January 2003 to January 2005. The statistics of the CDS spreads, i.e. mean and standard deviation are measured in basis points. The curve mean of a company is defined as the mean of its CDS spreads across all six contract maturities. The sector mean is defined as the average curve mean of all companies within the sector.

## 2. Estimation of the Credit Model

We calibrate our credit model in two steps: In the first step [Section a)] we calibrate the short rate process to our interest-rate data using the linear Kalman filter combined with a quasi-maximum likelihood (QML) estimation. In the second step [Section b)] we take these results as inputs and carry out a QML estimation using the inversion method for each of the 29 companies separately in order to obtain their default intensity parameters from their CDS data.

The decision for these two different estimation methods for the interest rate and the default intensity parameters is motivated by two reasons. The first reason lies in the differing characteristics of the interest rate and CDS data. The ten different spot rates are all subject to a certain estimation error resulting from the *Svensson* (1994)-method, whereas among the six CDS spreads the five-year maturity is by far the most heavily traded one and can therefore be assumed to be more reliable than the others. The Kalman filter can model all the ten observation errors contained in the ten spot rates, whereas the inversion method assumes that the five-year CDS spread is observed without error while the other maturities are observed with a certain error.

The second reason lies in the differing functional relationships between the observable market data and the driving latent variables. Since in the three-factor CIR interest rate model the ten spot rates are assumed to be affine-linear functions of the three latent state variables  $(X_i(t))_{i=1,2,3}$ , the use of the linear Kalman filter can be assumed to yield reliable results. The CDS spread however is a nonlinear function of the company-specific variable  $Z_j(t)$ . Therefore, the use of the Kalman-filter would involve a linear approximation of this nonlinear functional relationship, which is not always reliable. Since the default intensity estimation has to be carried out for 29 different cases, the inversion method is the more robust choice in this case.

Since we use panel data both for the interest rate and for the CDS calibration, we are able to estimate the market price of risk parameters  $(\lambda_i)_{i=1,2,3}$  and  $\lambda_{z,j}$  for each company  $j$  in equations (9). Therefore we are able to identify both the physical and the risk-neutral distributions of the three state variables  $(X_i(t))_{i=1,2,3}$  and the company-specific variable  $Z_j(t)$ . Finally, using the specification of the risk-neutral default intensity in (10), we can identify for each of our 29 sample companies both the risk-neutral and the physical distribution of their risk-neutral default

intensity and therefore calculate in Section IV.3. their spread risk premium according to the definition in (15).

a) Estimation of the Interest Rate Parameters

The Kalman filter technique has been applied to the calibration of multi-factor CIR models, among others by *Duan/Simonato* (1999), *Geyer/Pichler* (1999), *de Jong* (2000), *Chen/Scott* (2003), and *Hördahl/Vestin* (2005). The details of the calibration methodology for the three-factor CIR model are described in Appendix C. Table 2 shows the estimates of the interest rate parameters with their standard errors in parentheses.

The parameter estimates are all in line with previous studies, i.e. they all have the expected sign and a realistic magnitude. The rate of mean reversion parameter estimates  $\kappa_i$  indicate a mean half life of 0.55, 1.20, and 3.31 years for the three according state variables under the physical measure  $\mathbb{P}$ . The long-term mean parameter estimates  $\theta_i$  add up to about 1.6. The expected negative sign and the magnitude of the market price of risk parameter estimates  $\lambda_i$  lead to a negative speed of mean reversion

*Table 2*  
**Interest Rate Parameter Estimates of the Three-Factor CIR Model**

$i$	$\kappa_i$	$\theta_i$	$\sigma_i$	$\lambda_i$
1	1.2743 (0.0892)	0.0037 (0.1519)	0.1562 (0.0680)	-0.0007 (0.2133)
2	0.5748 (0.0719)	0.0078 (0.0951)	0.3447 (0.0117)	-0.7706 (0.0604)
3	0.2091 (0.0798)	0.0051 (0.0792)	0.1356 (0.0103)	-0.3543 (0.0382)

This table shows the estimates of the interest rate parameters of the three-factor CIR model using the Kalman filter combined with a QML method. The logarithm of the estimated likelihood value is 7878.93. In the three-factor CIR model the short rate  $r(t)$  is defined as the sum of three economy-wide latent state variables

$$r(t) = X_1(t) + X_2(t) + X_3(t),$$

where each of them follows a square-root process, i.e.

$$dX_i(t) = \kappa_i(\theta_i - X_i(t))dt + \sigma_i\sqrt{X_i(t)}dW_i^{\mathbb{P}}(t), \quad i = 1, 2, 3,$$

under the physical measure  $\mathbb{P}$ .  $\kappa_i$  is the speed of mean reversion,  $\theta_i$  is the long-term mean level,  $\sigma_i$  is the volatility parameter, and  $\lambda_i$  is the market price of risk parameter of the according state variable  $X_i$ . The standard errors of the point estimates in the parentheses are calculated as proposed by *Bollerslev/Wooldridge* (1992).

Table 3

**Fit of the Three-Factor CIR Model**

	Maturity in Years										Mean
	1	2	3	4	5	6	7	8	9	10	
MAE in bp	3.96	0.39	0.67	0.43	0.32	0.41	0.43	0.36	0.26	0.48	0.77
MAPE in %	1.79	0.17	0.25	0.15	0.11	0.12	0.12	0.10	0.07	0.12	0.30

This table reports the fit of the three-factor CIR model. MAE stands for the mean absolute error and MAPE stands for the mean absolute percentage error.

$(\kappa_i + \lambda_i)$  and therefore to an explosive behaviour under the risk-neutral measure  $\mathbb{Q}$  for the second and third state variables. Parallels to previous studies also include the fact that most estimates lack statistical significance. While the volatility parameters  $\sigma_i$  are statistically significant, especially the long-term mean parameter estimates are only fractions of their estimated standard errors. However, as we do not intend to forecast future movements of the spot rate curve, but to capture the interest rate process during our sample period, the lack of statistical significance is not an issue of concern. Table 3 demonstrates that our interest rate model is capable of performing this task quite well.

The average mean absolute error (MAE) over all ten maturities is 0.77 basis points. Except for the MAE of the one-year maturity (3.96 basis points), where affine term structure models like the three-factor CIR usually show a poorer fit due to a higher volatility at the short end of the interest rate curve, all MAEs are well below 1 basis point. In relative terms, the mean absolute percentage error (MAPE) of the one-year maturity is 1.79%, whereas the MAPEs for all other maturities are less or equal to 0.25%.

### b) Estimation of the Default Intensity Parameters

We use the estimates of the interest rate parameters and the associated time series of the three state variables as inputs and calibrate the default intensity process of our credit model to the CDS data for each of the 29 companies separately. We use the inversion methodology also used in *Zhang* (2008), under which we assume that five-year CDS spreads are observed without error and that the one-, three-, and ten-year CDS spreads are observed with errors. The technical details of the QML estimation method are presented in Appendix D. Table 4 reports the default intensity parameter estimates.



Table 4  
**Default Intensity Parameter Estimates**

Sector	Company	$\kappa_z$	$\theta_z$	$\sigma_z$	$\lambda_z$	$\Lambda_1$	$\Lambda_2$	$\Lambda_3$
FINANCIAL	Abbey National	0.0240	0.0050	0.1910	-0.4331	0.0034	-0.0167	0.1199
	Allianz	-0.0041	0.0031	0.1610	-0.1928	0.0757	0.0399	0.5607
	Hypo-Vereinsbank	0.0021	0.0021	0.1935	-0.3259	0.1631	0.1765	0.3699
	Commerzbank	0.0731	-0.0020	0.1554	-0.2999	0.1699	0.1319	0.5501
	Deutsche Bank	-0.0105	0.0059	0.1730	-0.2727	0.0819	0.0293	0.4311
	ING	0.0036	-0.0049	0.1583	-0.2811	0.1204	0.0568	0.2316
CONSUMER	Allied Domecq	0.1123	0.0228	0.2281	-0.1968	-0.0741	-0.0890	-0.1854
	BAT	0.2034	0.0277	0.2388	-0.3104	0.2569	-0.3105	-0.5821
	Carrefour	0.0801	0.0258	0.2800	-0.3168	0.1064	-0.1579	-0.1950
	Casino Guichard	0.2118	0.0227	0.0503	-0.1999	-0.1501	-0.0332	-0.8568
	DaimlerChrysler	0.1809	0.0215	0.3611	-0.4571	0.3969	0.1578	-0.0598
	Danone	-0.0689	0.0020	0.1222	-0.1606	0.0189	-0.0658	0.5875
	Gallaher	0.0555	0.0595	0.2852	-0.2103	-0.1366	-0.1621	-0.2195
	Investor	0.0321	-0.0009	0.1050	-0.0969	0.0839	0.0971	0.8499
	LVMH	-0.0023	-0.0004	0.0925	-0.0725	-0.0924	-0.0723	1.2798
	Volvo	-0.0219	-0.0106	0.1177	-0.0503	-0.0201	-0.1510	1.1999
TMT	France Télécom	0.0400	0.0334	0.1361	-0.0847	-0.2599	-0.0999	1.0999
	KPN	0.0796	0.0131	0.2001	-0.2722	0.0499	-0.2616	1.2846
	Telefónica	0.0664	0.0341	0.2233	-0.2268	-0.0696	-0.1291	-0.1699
	Vodafone	0.0865	0.0075	0.1641	-0.3072	0.0384	-0.3197	1.0767
	Wolters Kluwer	0.0529	0.0360	0.2148	-0.0883	0.0499	0.0812	0.0601
INDUSTRIAL AND UTILITIES	Arcelor	0.0058	-0.0139	0.1071	-0.0573	0.0099	0.1190	1.0999
	St. Gobain	0.0141	0.0063	0.1108	-0.0788	-0.0933	-0.0448	0.8920
	Iberdrola	0.0075	0.0195	0.1384	-0.1611	-0.0299	-0.1245	0.6904
	Lafarge	0.1651	0.0110	0.2301	-0.2939	-0.0999	0.0674	-0.1311
	Repsol	0.0606	0.0069	0.0779	-0.0662	-0.0163	-0.0883	1.1999
	Rolls-Royce	-0.0249	-0.0081	0.1012	-0.0292	0.0501	0.0109	1.1949
	RWE	0.0452	0.0054	0.1321	-0.1578	0.0256	-0.0883	0.7899
	Vattenfall	-0.0281	0.0003	0.1177	-0.1285	0.0349	-0.0174	0.7381

This table shows the estimates of the default intensity parameters of the credit model calibrated to the CDS data, where the short rate is defined as the sum of three economy-wide state variables  $X_1(t)$ ,  $X_2(t)$ , and  $X_3(t)$ , and where the risk-neutral default intensity of company  $j$  is defined as

$$h_j(t) = \Lambda_{0,j} + \Lambda_{1,j}[X_1(t) - \bar{X}_1] + \Lambda_{2,j}[\bar{X}_2(t) - \bar{X}_2] + \Lambda_{3,j}[X_3(t) - \bar{X}_3] + Z_j(t),$$

with the name-specific distress variable  $Z_j(t)$  of company  $j$  following a square-root process

$$dZ_j(t) = \kappa_{z,j}(\theta_{z,j} - Z_j(t))dt + \sigma_{z,j}\sqrt{Z_j(t)}dW_{z,j}^p(t)$$

under the physical measure  $\mathbb{P}$ . In order to reduce the dimensionality of the optimisation problem, we set  $\Lambda_{0,j} = 0$  for all companies. The optimisation procedure minimises the sum of the MAEs (MAE stands for mean absolute error) of the one-, three, and ten-year contracts, in order to find the optimal default intensity parameters.

Overall, the default intensity parameter estimates are in line with our expectations. The estimates of the mean reversion parameter  $\kappa_z$  are positive, except for seven companies where they are negative leading to an explosive behaviour of the risk-neutral intensity under the physical measure  $\mathbb{P}$ . All estimates for the market price of risk parameter  $\lambda_z$  have the expected negative sign. The speed of mean reversion  $\kappa_z + \lambda_z$  under the risk-neutral measure  $\mathbb{Q}$  is negative for all but one company, Casino Guichard, resulting in an explosive behaviour of the risk-neutral intensity for these 28 companies under this measure. For seven companies the estimates of the long-term level parameter  $\theta_z$  have a negative sign, albeit their absolute magnitude is very close to zero. The volatility parameter estimates  $\sigma_z$  range between 0.05 and 0.36. Taking all companies into account, the estimates of the interest rate sensitivity parameters  $\Lambda_i$  do not confirm the expected negative correlation between the short rate and the default intensity processes as found in *Duffee (1999)*. In fact, our parameter estimates show that for financial firms all three of the sensitivity parameters are positive with the exception of the second parameter for Abbey National, which is however very close to zero. On the other hand, for seven of the ten companies in the consumer sector we obtain two or three negative sensitivity parameters. Within the two remaining sector groups the resulting estimates for  $\Lambda_i$  are more balanced.

It is also noteworthy that although the estimated time series of the economy-wide state variables  $(X_i(t))_{i=1,2,3}$  and the company-specific distress variable  $Z(t)$  are nonnegative, the risk-neutral default intensity defined in (10) as an affine-linear function of these four variables can admit negative values for sufficiently negative values of the coefficients  $(\Lambda_i)_{i=1,2,3}$ . The estimated time series of the risk-neutral intensity of four of the 29 companies, Commerzbank, France Télécom, Gallaher and RWE, become slightly negative for a very few number of observations towards the end of our sample period. This however is not a concern as long as the credit model is able to reproduce the market data with a reasonable precision. Table 5 reports the errors of the credit model and shows that it fits the market data quite well.

The average MAE over all companies and all maturities (except for the five-year maturity which is assumed to be observed without error) is 2.22 basis points and the according average MAPE is 6.47%. As in general, the weakest fit is found at the two ends of the curve. The average MAE over all companies for the one-year maturity is 2.91, for the two-year maturity it is 2.46 and for the ten-year contracts it is 2.18 basis points.

Table 5: Fit of the Credit Model to the CDS Data

Sector	Contract Maturity Company	1Y		2Y		3Y		7Y		10Y		Mean	
		MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE	MAE	MAPE
FINANCIAL	Abbe National	1.45	19.37	0.88	7.45	0.61	4.76	1.18	5.58	0.95	3.86	1.01	8.20
	Allianz	2.48	20.84	2.20	9.29	1.55	5.12	1.15	3.08	1.06	2.49	1.69	8.16
	Hypo-Vereinsbank	4.14	19.23	3.33	8.85	2.65	6.10	2.58	4.20	1.75	3.17	2.89	8.31
	Commerzbank	3.78	18.20	2.23	7.60	1.47	5.06	2.02	3.95	1.72	2.80	2.24	7.52
	Deutsche Bank	1.17	13.58	1.05	8.95	0.72	4.99	1.17	4.73	0.78	2.62	0.98	6.97
	ING	1.16	16.14	1.15	10.95	0.76	5.88	0.87	4.35	1.04	3.92	1.00	8.25
	Allied Domecq	2.59	9.86	2.20	6.00	1.48	3.67	1.16	1.98	2.60	3.96	2.01	5.09
	BAT	5.86	17.36	4.65	9.12	3.42	5.62	2.18	2.68	3.66	3.54	3.95	7.66
	Carrefour	1.58	12.92	1.37	7.62	1.13	5.20	1.23	3.56	1.92	4.54	1.45	6.77
	Casino Guichard	5.63	20.15	4.63	10.84	3.00	5.37	1.90	2.44	3.37	3.30	3.71	8.42
CONSUMER	DaimlerChrysler	4.43	8.55	4.70	6.24	2.69	3.57	2.99	3.00	3.33	2.63	3.63	4.80
	Danone	1.65	16.48	1.40	10.28	1.23	7.41	1.36	5.46	1.83	6.32	1.49	9.19
	Gallagher	4.38	15.62	3.11	8.97	2.09	4.97	2.53	4.02	3.57	4.75	3.14	7.67
	Investor	2.93	11.10	2.53	5.62	1.50	3.38	1.94	2.72	3.29	4.09	2.44	5.38
	LVMH	2.90	9.91	2.48	6.41	1.71	4.14	1.59	2.53	1.86	2.79	2.11	5.16
	Volvo	1.66	7.61	1.30	4.52	1.30	3.79	1.11	2.34	1.76	3.35	1.43	4.32
	France Télécom	5.35	11.22	5.38	11.29	4.10	7.00	1.81	1.88	1.93	1.70	3.71	6.62
	KPN	2.50	9.97	3.07	9.50	3.60	8.57	1.97	2.95	3.26	4.79	2.88	7.16
	Telefonica	2.13	10.06	1.70	6.47	1.47	5.49	1.15	2.25	3.02	5.20	1.89	5.89
	Vodafone	2.37	15.44	1.93	8.22	1.61	5.68	1.23	2.64	2.57	4.68	1.94	7.33
INDUSTRIAL AND UTILITIES	Wolters Kluwer	2.74	11.02	2.08	5.73	1.88	4.43	1.77	3.05	2.32	3.35	2.16	5.52
	Arcelor	3.19	11.55	3.74	8.64	1.70	2.89	2.62	3.53	2.64	2.67	2.78	5.86
	St. Gobain	1.64	6.88	1.62	5.03	1.21	3.45	1.69	3.37	1.92	3.59	1.62	4.46
	Iberdrola	1.44	9.19	1.05	5.09	0.91	3.70	1.05	2.82	1.14	2.93	1.12	4.75
	Lafarge	2.19	6.33	2.15	4.94	2.47	4.17	2.08	3.12	3.02	3.42	2.38	4.40
	Repsol	5.09	16.23	3.56	7.29	4.32	7.45	1.88	2.73	2.05	2.61	3.38	7.26
	Rolls-Royce	3.42	9.36	3.04	7.77	1.59	2.81	1.74	2.14	1.96	2.27	2.35	4.87
	RWE	2.35	12.71	1.67	6.91	1.05	3.22	1.17	2.56	1.09	2.39	1.47	5.56
	Vattenfall	2.17	14.09	1.24	5.59	0.96	3.62	1.39	3.41	1.84	3.62	1.52	6.07
	Mean	2.91	13.14	2.46	7.63	1.87	4.88	1.67	3.21	2.18	3.49	2.22	6.47

This table reports the pricing errors between the credit model and the CDS data, where the five-year contract is assumed to be measured without error. MAE stands for the mean absolute error (in basis points) and MAPE stands for the mean absolute percentage error (in %).

### 3. Estimation of the Spread Risk Premium

We use our interest rate and default intensity parameter estimates from Tables 2 and 4 and calculate the one-year spread risk premium as defined in equation (15) for our 29 sample companies. Table 6 reports some descriptive statistics of our results measured in basis points.

Across all 29 companies we obtain an average spread risk premium of 8.25 basis points. This positive estimate suggests that on average investors demand a risk premium for bearing the risk of unpredictable fluctuations in CDS spreads caused by unpredictable nondefault-related fluctuations in the reference entity's risk-neutral default intensity. At the company level however, we obtain a slightly different picture. In fact, for seven of our 29 companies the spread risk premium turns negative during our sample period. This is similar to the findings of previous studies. *Liu/Longstaff/Mandell* (2006) detect an average spread risk premium component (the lower bound of their default risk premium component) of -2 basis points in the expected returns of one-year zero-coupon bonds derived from swap rates and *Pan/Singleton* (2008) obtain a negative spread risk premium component in the one-year sovereign CDS spreads of Mexico, Turkey, and Korea during the later period of their sample.

The functional form of the drift adjustment on the risk-neutral intensity process, which results from shifting from its physical to its risk-neutral distribution, can provide an indication why the spread risk premia for these seven firms turn negative during our sample period. Given the specification of the market prices of risk in equation (9) and of the risk-neutral default intensity in equation (10), the drift adjustment on company  $j$ 's risk-neutral default intensity  $h_j(t)$  can be written as

$$(16) \quad \pi_j(t) := \Lambda_{1,j}[-\lambda_1 X_1(t)] + \Lambda_{2,j}[-\lambda_2 X_2(t)] + \Lambda_{3,j}[-\lambda_3 X_3(t)] + [-\lambda_{z,j} Z_j(t)].$$

The terms in the brackets are always non-negative since  $\lambda_i < 0$  for all  $i = 1, 2, 3$ ,  $\lambda_{z,j} < 0$  for all firms  $j = 1, \dots, 29$ , and  $X_i(t) \geq 0$  for all  $i = 1, 2, 3$  and  $Z_j(t) > 0$  for all firms  $j = 1, \dots, 29$  during the whole sample period  $t = 1, \dots, 106$ . Thus, the drift adjustment  $\pi_j(t)$  for the firm  $j$  can become negative only with sufficiently negative estimates for its parameters  $\Lambda_{i,j}$ , which determine the correlation between the short rate process and its intensity process. A negative value for  $\pi_j(t)$  implies a lower drift term for the risk-neutral intensity under its risk-neutral distribution than under its physical distribution and can therefore result in negative spread risk premia.

Table 6: Spread Risk Premia

Sector	Company	Mean	MAD	StDev	Skew	Kurtosis	Min	1. Quartile	Median	3. Quartile	Max
FINANCIAL	Abbey National	3.48	3.48	1.40	0.91	0.34	1.47	2.44	3.31	4.05	7.41
	Allianz	9.55	9.55	2.68	1.26	1.06	5.16	8.38	8.87	9.57	17.03
	Hypo-Vereinsbank	14.68	14.68	5.58	1.14	0.20	8.45	10.58	12.31	16.47	30.42
	Commerzbank	14.78	14.78	6.12	1.46	0.86	8.42	11.30	12.54	14.69	32.23
	Deutsche Bank	6.98	6.98	1.21	0.60	0.45	4.57	6.52	6.90	7.24	10.18
	ING	5.57	5.57	0.81	-0.25	0.28	3.74	5.23	5.66	6.01	7.61
	Allied Domecq	0.55	2.15	2.87	1.05	0.81	-3.37	-1.86	0.01	1.69	10.50
	BAT	0.15	5.14	6.39	1.15	0.94	-9.03	-4.16	-2.50	4.52	22.95
	Carrefour	-1.29	2.16	2.29	0.52	-0.03	-5.11	-3.06	-1.56	0.20	5.06
	Casino Guichard	-1.67	4.72	5.50	1.48	1.54	-8.16	-4.87	-3.77	-0.01	14.66
CONSUMER	DaimlerChrysler	19.95	19.95	7.48	0.04	-0.80	7.13	15.66	19.49	25.39	36.48
	Danone	5.87	5.87	1.39	-1.44	2.44	1.11	5.33	6.14	6.69	7.77
	Gallaher	-0.76	2.89	3.11	0.61	-0.93	-5.34	-2.90	-1.75	2.38	5.62
	Investor	14.11	14.11	3.21	-0.20	0.18	6.19	12.81	14.35	15.56	21.43
	LVMH	13.37	13.37	3.85	-1.03	1.21	1.48	12.17	14.06	15.54	19.95
	Volvo	9.65	9.78	3.72	-1.31	1.75	-2.56	8.21	10.64	12.15	14.75
	France Télécom	12.10	12.10	5.32	0.64	0.58	0.75	9.66	11.39	13.91	25.58
	KPN	13.26	13.26	6.11	0.84	-0.06	3.66	9.09	12.16	16.12	28.41
	Telefonica	-0.68	2.66	3.21	1.30	0.92	-4.53	-3.16	-1.48	0.58	8.27
	Vodafone	8.02	8.02	3.74	0.27	-0.77	0.75	5.01	7.27	10.97	16.20
INDUSTRIAL AND UTILITIES	Wolters Kluwer	4.83	4.83	1.16	0.62	0.61	2.71	4.08	4.72	5.38	8.24
	Areclor	16.87	16.87	3.40	-1.01	0.98	6.73	15.31	17.60	18.98	23.74
	St. Gobain	10.02	10.02	3.11	-0.28	0.44	1.70	8.77	10.08	11.44	16.61
	Iberdrola	6.36	6.36	2.49	-0.04	-0.36	0.41	4.84	6.21	8.30	11.35
	Lafarge	8.92	8.92	5.29	1.22	0.85	2.72	5.48	7.17	10.40	25.30
	Repsol	12.40	12.40	4.10	-0.50	0.89	0.72	11.05	12.70	14.39	22.34
	Rolls-Royce	14.13	14.13	3.63	1.83	2.29	13.14	14.99	16.09	20.61	26.61
	RWE	8.93	8.93	3.62	0.27	-0.48	1.56	6.70	8.36	11.96	17.14
	Vattenfall	9.00	9.00	1.88	-0.97	1.09	3.27	8.53	9.27	10.08	12.12
	All Companies	8.25	9.06	7.05	0.31	0.35	-9.03	4.07	8.09	12.60	36.48

This table shows one-year spread risk premia statistics in basis points for the 29 sample companies. The one-year spread risk premium for company  $j$  at time  $t$  is defined as

$$\Pi_{j,t} := [D_j^0(t, \tau) - PD_j^F(t, \tau)]_{r,t=1}$$

where  $D_j^0$  is the cumulative one-year risk-neutral default probability and  $PD_j^F$  is the cumulative one-year pseudo-physical default probability.

Therefore, it is not surprising that six of these seven companies, whose spread risk premia turn negative during our sample period, are from the consumer sector, where, as reported in Table 4, seven of the ten companies have two or three negative estimates for  $\Lambda_i$ . Since we find the implication of this result, – i. e. that protection sellers would rather pay than demand a premium for bearing the risk of CDS spread fluctuations caused by nondefault-related fluctuations in the risk-neutral intensity – economically implausible, we exclude these seven companies from further analysis; these are: Allied Domecq, BAT, Carrefour, Casino Guichard, Gallaher, Volvo, and Telefónica.

For the remaining 22 companies we obtain an average spread risk premium of 10.60 basis points, going from 3.48 for Abbey National to 19.95 basis points for DaimlerChrysler. At the sector level, the average spread risk premium ranges from 9.17 basis points for financials to 13.33 basis points for consumers. It is 9.55 basis points for TMTs and 10.83 basis points for industrials and utilities. Table 6 reveals substantial variation also in the time series dimension. We find the highest standard deviation of 7.48 basis points for DaimlerChrysler, with its spread risk premium fluctuating between 7.13 and 36.48 basis points. We detect the lowest standard deviation of 0.81 basis points for ING, with its spread risk premia ranging from 3.74 to 7.61 basis points.

In order to put our results into perspective, we compare them with the results obtained by *Berndt* (2014), who estimates the spread risk premium component in five-year U.S. corporate CDS spreads by using the *Chen/Collin-Dufresne/Goldstein* (2009) adaptation of the *Campbell/Cochrane* (1999) pricing kernel. For investment grade firms she finds a median spread risk premium component of 8 basis points in 2003 and 7 basis points in 2004, respectively 10% and 13% expressed as a fraction of the CDS spread.

As discussed in Section III.3., we can only calculate an approximative value for the spread risk premium component in the CDS spread due to our specification of the risk-neutral default intensity as being correlated with the short rate process. We calculate, similar to *Pan/Singleton* (2008), a one-year model CDS spread that contains no spread risk premium by setting the market price of risk parameters  $(\lambda_i)_{i=1,2,3}$  and  $\lambda_z$  in (9) to zero and subtract it from our fitted one-year model CDS spread containing the spread risk premium component. Across our 22 companies we find a median spread risk premium component of 7 basis points in 2003 and 4.8

basis points in 2004, respectively 22 % and 30 % expressed as a fraction of our fitted model CDS spread.

While our median spread risk premium component in basis points is very close to the results obtained by *Berndt* (2014), expressed as a fraction of the CDS spread it is more than twice as high. This difference however is explained by the fact that she finds a significant median residual component, due to supply/demand and liquidity effects, amounting to 42 % in 2003 and 53 % in 2004 of the CDS spread, whereas our credit model specification implicitly assumes this residual component to be zero.

## V. Empirical Analysis of the Spread Risk Premium

### 1. Determinants

In order to explain the resulting spread risk premia for our 22 remaining sample companies by conducting a panel data analysis, we first present a set of potential determinants that could have an impact on their variation. The explanatory variables we include in our analysis can be divided into five groups:

#### a) Credit Risk

The first group of explanatory variables attempts to measure credit risk at two different levels. The first proxy, the widely watched Markit iTraxx Europe CDS index for the five-year maturity, reflects the Eurozone-wide level of credit risk. We view this index as a measure of investors' appetite for exposure to the EUR-denominated investment-grade corporate credit class, whereby higher (lower) index levels indicate a decreasing (increasing) appetite. Since the index was launched only in the second half of 2004, we compute the missing index values as the arithmetic average of the five-year CDS spreads of those 121 of the 125 index companies for which we have reliable data, leaving out Dresdner Bank, Endesa, Philips Electronics and TDC A/S. Since the relative change between our last calculated index value and the first official index value calculated by Markit is less than half the standard deviation of the complete time series, we make no further adjustments to our calculated index values.

The second proxy, the one-year swap spread, aims to mirror the credit risk in the Eurozone's financial sector. It is defined as the difference be-

tween the one-year EUR-swap rate and the one-year spot rate estimated from EUR-denominated German government bond yields by the Deutsche Bundesbank using the method proposed by *Svensson* (1994). We expect a rise in these two credit risk measures to indicate a higher level of uncertainty in credit markets or a lower level of appetite for credit exposure, which should lead to increasing spread risk premia.

#### b) Overall Stock Market

The second group of explanatory variables, consisting of two proxies, aims to capture the effect of changing uncertainty as expressed by the overall Eurozone stock market. The first proxy is the weekly return of the widely watched Euro Stoxx 50 stock market index. We expect higher stock market index returns to mirror a decrease in the level of perceived uncertainty and hence to lead to lower spread risk premia in CDS markets.

The second proxy is the Euro Stoxx 50 Volatility Index (VSTOXX). The VSTOXX index measures the 30-day rolling implied volatility of the Euro Stoxx 50 stock market index, calculated by the index provider STOXX from index options. This is similar to *Pan/Singleton* (2008), who include the CBOE VIX volatility index measuring the rolling 30-day implied volatility of the S&P 500 stock market index calculated from index options. They find the VIX to be a key ingredient in investors's appetite for global *event risk* in credit markets, where "event risk" is the risk of an unexpected major event that could trigger sudden and wild fluctuations in market prices. Therefore, similar to them, we interpret the VSTOXX as a Eurozone-specific measure of investors' appetite for event risk. Hence we expect higher levels of the VSTOXX index to result in higher spread risk premia in CDS, since protection sellers would demand a higher compensation for event risk.

#### c) Firm-Specific Uncertainty

With the third group of variables we would like to explore whether there is an idiosyncratic impact on the spread risk premium. Following *Pan/Singleton* (2008), who include a country-specific proxy, the implied currency option volatility, in their regression analysis of spread risk premia in sovereign CDS of Mexico, Turkey and Korea, we include proxies reflecting firm-specific uncertainty through the stock market. These are



the equity return and its one-month historical volatility. In case we detect a statistically significant effect, we would expect a higher equity return to indicate lower company-specific uncertainty, which should translate into a lower spread risk premium. A higher volatility on the other hand should result in a higher spread risk premium. These assumptions are based on the results of *Norden/Weber* (2009) who detect a leading role of the stock market over the CDS market in the price discovery process.

#### d) Macroeconomic Uncertainty

The fourth group of variables deals with macroeconomic uncertainty. In order to capture uncertainty concerning central bank decisions, we calculate the one-month historical volatility of the three-month EURIBOR. In order to measure inflation risk, we use the one-month historical volatility of the ten-year spot rate. We expect these volatilities to translate into higher uncertainties regarding unexpected nondefault-related fluctuations in CDS spreads, and hence to result in higher spread risk premia.

#### e) CDS Market Conditions

Finally, with the last group of variables we aim to control for CDS market conditions regarding liquidity and supply/demand effects. In order to control for liquidity conditions, we include for every firm the time series of the number of financial institutions that provide CDS quotes for its five-year contract from our Markit dataset. We hypothesise that a higher number of available quotes reduces the uncertainty about liquidity-driven spread fluctuations, which should result in lower spread risk premia. In order to control for supply/demand conditions, we take into account the possible impact of synthetic CDO transactions. As highlighted by *O'Kane/McAdie* (2001) and *Hjort* (2003), these transactions create a massive additional supply of protection, leading to a tightening of the CDS spreads of the names in the underlying portfolio. We download the monthly time series of the global synthetic CDO issuance volumes from the CreditFlux database. In order to convert the data into a weekly frequency, we assume that the monthly issuance volumes are uniformly distributed over time. We expect this downward pressure on CDS spreads to lead to a compression in spread risk premia.

## 2. Panel Data Regression

Since, in contrast to the studies of *Liu/Longstaff/Mandell* (2006) and *Zhang* (2008), we estimate the spread risk premium of more than only one reference entity, we are able to conduct our analysis using panel data methods rather than simple OLS. By capturing both the time series dimension (consisting of 106 weekly observations) and the cross-sectional dimension (consisting of 22 firms) of our spread risk premia, we can not only control for unobservable firm-specific heterogeneities but also obtain more efficient regression estimates than those produced by OLS.

We choose a random-effects specification for our 22 firm-specific effects and assume them to be independent of our explanatory variables. We account only for firm-specific heterogeneities, since the inclusion of sector dummy variables indicates no systematic differences between our four sector groups. The choice of a random-effects specification is supported by the Hausman test, however a fixed-effects specification yields almost identical estimates. Thus, in order to examine the influence of our explanatory variables on the spread risk premium  $\Pi_{jt}$  of firm  $j$  at time  $t$ , we estimate the following random-effects model

$$(17) \quad \Pi_{jt} = \alpha + \beta^T x_{jt} + \alpha_j + \varepsilon_{jt},$$

where the vector  $x_{jt}$  represents our explanatory variables,  $\alpha_j$  is the firm-specific random effect,  $\varepsilon_{jt}$  is the unsystematic residual and the scalar  $\alpha$  and the vector  $\beta$  are the regression coefficients. Table 7 displays the results of seven random-effects regressions.

In the first five regression models of columns (1) to (5) we estimate the influence of our various explanatory variables on the spread risk premium after controlling for potentially distorting effects induced by certain CDS market conditions, i.e. CDS liquidity conditions proxied by the depth of the five-year CDS quote and supply/demand conditions proxied by the issuance volume of synthetic CDOs. While the first two regression estimates examine the isolated effects of event risk respectively investors' appetite for exposure to the investment-grade corporate credit class, the following three regressions combine explanatory variables from all five groups. The last two regression models of columns (6) and (7) test the robustness of our estimated spread risk premia to our specific credit model choice.

Table 7: Panel Data Analysis of the Spread Risk Premium  $\Pi_{jt}$

Explanatory Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<b>CREDIT RISK</b>							
iTraxx Europe 5Y CDS index (+) (in bps)		0.109 (0.000)	0.094 (0.000)		0.116 (0.000)		
1Y Swap Spread (+) (in bps)		0.002 (0.934)		-0.014 (0.568)	-0.027 (0.282)		
<b>OVERALL STOCK MARKET</b>							
Euro Stoxx 50 Index Return (-) (in %)			0.010 (0.768)	0.026 (0.374)	0.008 (0.800)		
VSTOXX 30-day Implied Volatility Index (+) (in %)	0.239 (0.000)		0.046 (0.018)	0.226 (0.000)			
<b>FIRM-SPECIFIC UNCERTAINTY</b>							
Equity return (-) (in %)			0.012 (0.551)		0.007 (0.727)		
Equity volatility (+) (in %)	0.422 (0.002)			0.285 (0.036)	0.178 (0.173)		
<b>MACROECONOMIC UNCERTAINTY</b>							
3-month EURIBOR volatility (+) (in %)			1.202 (0.000)	1.321 (0.000)			
10Y Spot Rate volatility (+) (in %)			-1.119 (0.000)		-1.237 (0.000)		
<b>CDS MARKET CONDITIONS</b>							
Depth of 5Y CDS quote (-) (in number of banks)	0.112 (0.000)	0.132 (0.000)	0.141 (0.000)	0.113 (0.000)	0.133 (0.000)		
Synthetic CDO issuance (-) (in USD bn)	-0.022 (0.000)	-0.029 (0.000)	-0.030 (0.000)	-0.023 (0.000)	-0.031 (0.000)		
<b>MODEL ERROR</b>							
Interest Rate Model Error (+/-) (in bps)						0.337 (0.000)	
CDS Model Error (+/-) (in bps)							0.070 (0.011)
Intercept	3.810 (0.000)	4.346 (0.000)	5.304 (0.000)	4.109 (0.000)	5.784 (0.000)	10.820 (0.000)	10.622 (0.000)
<hr/>							
Number of observations	2120	2332	2120	2120	2120	2332	2332
R <sup>2</sup> within	0.5901	0.5920	0.6205	0.5946	0.6163	0.0000	0.0075
R <sup>2</sup> between	0.1071	0.1024	0.0481	0.0966	0.0785	0.0000	0.0346
R <sup>2</sup> overall	0.3204	0.2982	0.3271	0.3194	0.3292	0.0804	0.0018

This table reports the results of the random-effects panel data regressions of the one-year spread risk premium  $\Pi_{jt}$  (measured in bps). Explanatory variables are presented with their expected signs in parentheses. The coefficient estimates are reported with robust *p*-values in parentheses, using the *Huber (1967)-White (1980)*-sandwich estimator implemented in STATA which accounts for heteroskedasticity and serial correlation in the residuals. The inclusion of firm-specific equity-related proxies in models (1), (3), (4), and (5) reduces the number of observations since Abbey National and Vattenfall are not listed companies. Coefficients of determination are defined according to STATA.

In column (1) we investigate the influence of the overall stock market volatility in combination with the firm-specific equity volatility after controlling for CDS market conditions. Both equity volatility measures are statistically significant with the expected positive sign. This means that a higher degree of uncertainty concerning future price fluctuations in the equity market translates into a higher degree of uncertainty concerning nondefault-related fluctuations in risk-neutral default intensities. Furthermore, after controlling for CDS market conditions and firm-specific equity volatility, we find spread risk premia in CDS to be susceptible to changes of a common factor in the form of event risk proxied by the VSTOXX index. This result is similar to that of *Pan/Singleton* (2008), who find that their measure of event risk, the CBOE VIX volatility index, has a statistically significant positive effect on the estimated spread risk premia in the case of Mexico and Turkey, i.e. for two of their three sample countries.

Concerning the CDS market control variables, the synthetic CDO issuance volume is statistically significant with the expected negative sign. Thus an excess supply in credit protection leads to a compression of the spread risk premia in CDS spreads. The other control variable which captures CDS liquidity conditions through the depth of five-year CDS quote has an unexpected positive sign. Hence, contradictory to our expectation, a higher number of financial institutions providing CDS quotes leads to an increase rather than a reduction of the risk premium for fluctuations in CDS spreads.

In column (2) we examine the effect of our two credit risk proxies on spread risk premia after controlling for CDS market conditions. The two control variables are statistically significant with the same signs as in column (1). While our proxy for the market-wide credit risk in the form of the iTraxx index has the expected statistically significant positive effect on spread risk premia, the proxy for credit risk in the financial sector in the form of the one-year swap spread is not statistically significant. The latter result suggests that during our sample period there was no material deterioration in the credit quality of the financial sector that could have resulted in growing concerns about future fluctuations in CDS premia. On the other hand, the significance of the iTraxx index reveals the existence of an additional common factor on spread risk premia, beyond the event risk diagnosed in model (1), in the form of investors' appetite for exposure to the EUR-denominated investment-grade corporate credit class. This is not surprising given the high correlation of

93% between the iTraxx index and the VSTOXX index during our sample period and suggests that growing concerns about event risk and lower appetite for credit exposure go hand in hand and result in higher spread risk premia in the CDS of individual reference entities.

In column (3) we combine, again after controlling for CDS market conditions, explanatory variables from all the four other factor groups. The two common factors found in columns (1) and (2), the VSTOXX and the iTraxx, remain statistically significant with the expected positive sign. However, the two included equity return measures, the return of the Euro Stoxx 50 index and the firm-specific equity return, are not statistically significant. This finding for corporate credit markets is in line with the results for the sovereign credit market of Argentina by *Zhang* (2008), who finds no significant relationship between the return of the Argentine Merval stock index and the spread risk premium implied in CDS on Argentine sovereign debt. So while changing equity volatility, whether market-wide or firm-specific seems to impact spread risk premia in CDS spreads as diagnosed in column (1), the same is not true for equity returns, whether market-wide or firm-specific. Thus higher equity returns per se do not result in diminishing concerns about future market price fluctuations in CDS spreads.

Turning our attention to the two included macroeconomic variables, we see that they are both statistically significant. While the volatility of the three-month EURIBOR has the expected positive sign, the coefficient estimate for the volatility of the ten-year spot rate has an unexpected negative sign. A closer examination using two separate univariate regressions for these two macroeconomic variables shows a positive sign for the volatility of the ten-year spot rate, however in contrast to the volatility of the three-month EURIBOR, with almost no explanatory power. Thus, while spread risk premia in CDS are not affected by changing uncertainties regarding spot rates at the long end of the curve, they are susceptible to uncertainties regarding the short end.

In the remaining two regressions (4) and (5) we experiment with different combinations of our explanatory variables. They seem to confirm our results found so far. We see that both indices, the iTraxx and the VSTOXX, remain statistically significant, whereas in column (5) the firm-specific equity volatility lacks statistical significance, presumably due to the high correlation of over 64% with the included iTraxx index. Further, we see that both equity return measures, the market-wide and the firm-specific, remain statistically not significant. The same is true

for the one-year swap spread. Finally, we see that both our macroeconomic variables have a similar impact as detected in column (3).

Finally, in order to assess the robustness of our estimated spread risk premia to our specific credit model choice, we carry out a univariate random-effects regression of the spread risk premium on the interest rate model error in column (6) and on the CDS model error in column (7). The interest rate model error is defined as the difference between the one-year market and the one-year model spot rate. Likewise, the CDS model error is the difference between the one-year market and the one-year model CDS spread. Neither of the two model errors have any explanatory power, which indicates that the calculated spread risk premia are not systematically distorted by the quality of the model fit.

## VI. Conclusion

In this paper we estimate and explain spread risk premia implied in credit default swaps of a sample of European investment-grade companies. The spread risk premium component in the CDS spread represents a compensation protection sellers demand for unpredictable changes in CDS spreads caused by unexpected fluctuations in the reference entity's risk-neutral default intensity. Calibrating a stochastic intensity credit model to the time series of a firm's ten-year CDS curve, we are able to estimate the market price of default risk parameters, which determine the difference between the risk-neutral and the empirical distributions of the risk-neutral default intensity. The difference between these two distributions is an expression for the demanded spread risk premium and can be captured by the difference between the one-year cumulative risk-neutral and the one-year cumulative pseudo-physical default probabilities.

We find an average spread risk premium of 10.60 basis points, which indicates that on average investors demand a risk premium for unpredictable fluctuations in the risk-neutral intensity. Using panel data analysis, we gain the following main insights: First, with event risk proxied by the VSTOXX volatility index and with investor's appetite for exposure to corporate credit proxied by the iTraxx CDS index, we detect two different common factors, which both have a statistically significant positive impact on the time series variation of the spread risk premium, even after controlling for CDS market conditions like liquidity and supply/demand effects. The high correlation between these two indices also

revealed that a higher event risk as perceived through the overall stock market goes hand in hand with investors’s declining appetite for exposure to the investment-grade credit class. Second, in addition to the market-wide equity volatility, we find a significant positive influence of the firm-specific equity volatility as well. This result suggests that a higher degree of uncertainty concerning future firm-specific equity prices translates into a higher degree of uncertainty concerning nondefault-related changes in firm-specific risk-neutral default intensities. Third, in contrast to their volatilities, we observe no significant effect for market-wide or firm-specific equity returns. Finally, we obtain mixed results for variables capturing macroeconomic uncertainties. For the volatility of the three-month EURIBOR, which captures uncertainty concerning central bank decisions, we detect a significant positive effect on spread risk premia. By contrast, our proxy for inflation risk, the volatility of the ten-year spot rate, has almost no explanatory power.

### Appendix A Computational Feasibility of the CDS Formula

As also presented in *Entrop/Schiemert/Wilkens* (2013), by evaluating the characteristic function and its partial derivative with respect to  $\phi$  in  $\phi = 0$ , we can obtain closed-form expressions for the conditional expectations in (1) and (2) and find a computationally feasible expression for the CDS valuation formula in (5). We first consider the default-risk-adjusted discount factor in  $t$  for the maturity  $\tau$ , which is given by

$$(A.1) \quad \Phi(t, t + \tau; \phi = 0) = \exp\left(\mathcal{A}(\tau; 0) - \sum_{i=1}^3 \mathcal{B}_i(\tau; 0)X_i(t) - \mathcal{B}_z(\tau; 0)Z(t)\right),$$

with

$$(A.2) \quad \mathcal{A}(\tau; 0) = \sum_{i=1}^3 \mathcal{A}_i(\tau; 0) + \mathcal{A}_z(\tau; 0) - \tau(\Lambda_0 - \sum_{i=1}^3 \Lambda_i \bar{X}_i),$$

where

$$(A.3) \quad \mathcal{A}_i(\tau; 0) = \frac{2\kappa_i\theta_i}{\sigma_i^2} \log\left(\frac{\gamma_i \exp\left(\frac{(\kappa_i + \lambda_i)\tau}{2}\right)}{\gamma_i \cosh\left(\frac{\gamma_i\tau}{2}\right) + (\kappa_i + \lambda_i) \sinh\left(\frac{\gamma_i\tau}{2}\right)}\right),$$

$$(A.4) \quad \mathcal{A}_z(\tau; 0) = \frac{2\kappa_z\theta_z}{\sigma_z^2} \log\left(\frac{\gamma_z \exp\left(\frac{(\kappa_z + \lambda_z)\tau}{2}\right)}{\gamma_z \cosh\left(\frac{\gamma_z\tau}{2}\right) + (\kappa_z + \lambda_z) \sinh\left(\frac{\gamma_z\tau}{2}\right)}\right),$$

and

$$(A.5) \quad \mathcal{B}_i(\tau; 0) = \frac{2(1 + \Lambda_i) \sinh\left(\frac{\gamma_i \tau}{2}\right)}{\gamma_i \cosh\left(\frac{\gamma_i \tau}{2}\right) + (\kappa_i + \lambda_i) \sinh\left(\frac{\gamma_i \tau}{2}\right)},$$

$$(A.6) \quad \mathcal{B}_z(\tau; 0) = \frac{2 \sinh\left(\frac{\gamma_z \tau}{2}\right)}{\gamma_z \cosh\left(\frac{\gamma_z \tau}{2}\right) + (\kappa_z + \lambda_z) \sinh\left(\frac{\gamma_z \tau}{2}\right)},$$

with

$$\gamma_i := \sqrt{(\kappa_i + \lambda_i)^2 + 2\sigma_i^2(1 + \Lambda_i)}, \quad \text{for } i = 1, 2, 3, \quad \text{and} \quad \gamma_z := \sqrt{(\kappa_z + \lambda_z)^2 + 2\sigma_z^2}.$$

Second, we obtain the discounted density function of the risk-neutral default probability for  $t + \tau$  as

$$(A.7) \quad \frac{1}{\mathbf{i}} \frac{\partial \Phi(t, t + \tau; \phi)}{\partial \phi} \Big|_{\phi=0} = \Phi(t, t + \tau; \phi = 0) \left[ \frac{1}{\mathbf{i}} \frac{\partial \mathcal{A}(\tau; \phi)}{\partial \phi} \Big|_{\phi=0} - \sum_{i=1}^3 \frac{1}{\mathbf{i}} \frac{\partial \mathcal{B}_i(\tau; \phi)}{\partial \phi} \Big|_{\phi=0} \mathbf{X}_i(t) - \frac{1}{\mathbf{i}} \frac{\partial \mathcal{B}_z(\tau; \phi)}{\partial \phi} \Big|_{\phi=0} \mathbf{Z}(t) \right],$$

where

$$(A.8) \quad \frac{1}{\mathbf{i}} \frac{\partial \mathcal{A}(\tau; \phi)}{\partial \phi} \Big|_{\phi=0} = \sum_{i=1}^3 \frac{1}{\mathbf{i}} \frac{\partial \mathcal{A}_i(\tau; \phi)}{\partial \phi} \Big|_{\phi=0} + \frac{1}{\mathbf{i}} \frac{\partial \mathcal{A}_z(\tau; \phi)}{\partial \phi} \Big|_{\phi=0} + \left( \Lambda_0 - \sum_{i=1}^3 \Lambda_i \bar{\mathbf{X}}_i \right)$$

with

$$(A.9) \quad \frac{1}{\mathbf{i}} \frac{\partial \mathcal{A}_i}{\partial \phi} \Big|_{\phi=0} = \frac{2\kappa_i \theta_i \Lambda_i \sinh\left(\frac{\gamma_i \tau}{2}\right)}{\gamma_i \cosh\left(\frac{\gamma_i \tau}{2}\right) + (\kappa_i + \lambda_i) \sinh\left(\frac{\gamma_i \tau}{2}\right)},$$

$$(A.10) \quad \frac{1}{\mathbf{i}} \frac{\partial \mathcal{A}_z}{\partial \phi} \Big|_{\phi=0} = \frac{2\kappa_z \theta_z \sinh\left(\frac{\gamma_z \tau}{2}\right)}{\gamma_z \cosh\left(\frac{\gamma_z \tau}{2}\right) + (\kappa_z + \lambda_z) \sinh\left(\frac{\gamma_z \tau}{2}\right)},$$

and where

$$(A.11) \quad \frac{1}{\mathbf{i}} \frac{\partial \mathcal{B}_i}{\partial \phi} \Big|_{\phi=0} = \frac{-\Lambda_i \left[ \gamma_i^2 \cosh^2\left(\frac{\gamma_i \tau}{2}\right) - (\kappa_i + \lambda_i)^2 \sinh^2\left(\frac{\gamma_i \tau}{2}\right) \right] + 2\Lambda_i(1 + \Lambda_i) \sigma_i^2 \sinh^2\left(\frac{\gamma_i \tau}{2}\right)}{\left[ \gamma_i \cosh\left(\frac{\gamma_i \tau}{2}\right) + (\kappa_i + \lambda_i) \sinh\left(\frac{\gamma_i \tau}{2}\right) \right]^2},$$

$$(A.12) \quad \frac{1}{\mathbf{i}} \frac{\partial \mathcal{B}_z}{\partial \phi} \Big|_{\phi=0} = \frac{-\left[ \gamma_z^2 \cosh^2\left(\frac{\gamma_z \tau}{2}\right) - (\kappa_z + \lambda_z)^2 \sinh^2\left(\frac{\gamma_z \tau}{2}\right) \right] + 2\sigma_z^2 \sinh^2\left(\frac{\gamma_z \tau}{2}\right)}{\left[ \gamma_z \cosh\left(\frac{\gamma_z \tau}{2}\right) + (\kappa_z + \lambda_z) \sinh\left(\frac{\gamma_z \tau}{2}\right) \right]^2}.$$



Making use of the expressions in (A.1) and (A.7), we can express both sides of the CDS contract, its premium leg in (3) and its default leg in (4), yielding a computationally feasible form for the CDS premium in (5), given by

$$(A.13) \quad CDS(t, \tau) = \frac{[1 - REC(t)] \int_t^{t+\tau} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du}{\delta \sum_{n=1}^{4\tau} \Phi(t, T_n; \phi = 0) + \delta \sum_{n=1}^{4\tau} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{1}{i} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du}.$$

### Appendix B Formulas for Risk-Neutral and Pseudo-Physical Survival Probabilities

The  $\tau$ -year cumulative risk-neutral survival probability at time  $t$  can be expressed in closed form, as

$$(B.1) \quad S^{\mathbb{Q}}(t, \tau) := \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^{t+\tau} h(u) du \right) \right]$$

$$(B.2) \quad = \exp \left( \mathcal{A}^{\mathbb{Q}}(\tau) - \sum_{i=1}^3 \mathcal{B}_i^{\mathbb{Q}}(\tau) X_i(t) - \mathcal{B}_z^{\mathbb{Q}}(\tau) Z(t) \right),$$

with

$$(B.3) \quad \mathcal{A}^{\mathbb{Q}}(\tau) = \sum_{i=1}^3 \mathcal{A}_i^{\mathbb{Q}}(\tau) + \mathcal{A}_z^{\mathbb{Q}}(\tau) - \tau(\Lambda_0 - \sum_{i=1}^3 \Lambda_i \bar{X}_i),$$

where

$$(B.4) \quad \mathcal{A}_i^{\mathbb{Q}}(\tau) = \frac{2\kappa_i \theta_i}{\sigma_i^2} \log \left( \frac{\gamma_i^{\mathbb{Q}} \exp \left( \frac{(\kappa_i + \lambda_i) \tau}{2} \right)}{\gamma_i^{\mathbb{Q}} \cosh \left( \frac{\gamma_i^{\mathbb{Q}} \tau}{2} \right) + (\kappa_i + \lambda_i) \sinh \left( \frac{\gamma_i^{\mathbb{Q}} \tau}{2} \right)} \right),$$

$$(B.5) \quad \mathcal{A}_z^{\mathbb{Q}}(\tau) = \frac{2\kappa_z \theta_z}{\sigma_z^2} \log \left( \frac{\gamma_z^{\mathbb{Q}} \exp \left( \frac{(\kappa_z + \lambda_z) \tau}{2} \right)}{\gamma_z^{\mathbb{Q}} \cosh \left( \frac{\gamma_z^{\mathbb{Q}} \tau}{2} \right) + (\kappa_z + \lambda_z) \sinh \left( \frac{\gamma_z^{\mathbb{Q}} \tau}{2} \right)} \right),$$

and

$$(B.6) \quad \mathcal{B}_i^{\mathbb{Q}}(\tau) = \frac{2\Lambda_i \sinh \left( \frac{\gamma_i^{\mathbb{Q}} \tau}{2} \right)}{\gamma_i^{\mathbb{Q}} \cosh \left( \frac{\gamma_i^{\mathbb{Q}} \tau}{2} \right) + (\kappa_i + \lambda_i) \sinh \left( \frac{\gamma_i^{\mathbb{Q}} \tau}{2} \right)},$$

$$(B.7) \quad \mathcal{B}_z^{\mathbb{Q}}(\tau) = \frac{2 \sinh\left(\frac{\gamma_z^{\mathbb{Q}} \tau}{2}\right)}{\gamma_z^{\mathbb{Q}} \cosh\left(\frac{\gamma_z^{\mathbb{Q}} \tau}{2}\right) + (\kappa_z + \lambda_z) \sinh\left(\frac{\gamma_z^{\mathbb{Q}} \tau}{2}\right)},$$

with

$$\gamma_i^{\mathbb{Q}} := \sqrt{(\kappa_i + \lambda_i)^2 + 2\sigma_i^2 \Lambda_i}, \quad \text{for } i = 1, 2, 3, \quad \text{and} \quad \gamma_z^{\mathbb{Q}} := \sqrt{(\kappa_z + \lambda_z)^2 + 2\sigma_z^2}.$$

Similarly, the  $\tau$ -year cumulative pseudo-physical survival probability at time  $t$  can be written as

$$(B.8) \quad \mathcal{P}\mathcal{S}^{\mathbb{P}}(t, \tau) := \mathbb{E}_t^{\mathbb{P}} \left[ \exp\left(-\int_t^{t+\tau} h(u) du\right) \right]$$

$$(B.9) \quad = \exp\left(\mathcal{A}^{\mathbb{P}}(\tau) - \sum_{i=1}^3 \mathcal{B}_i^{\mathbb{P}}(\tau) X_i(t) - \mathcal{B}_z^{\mathbb{P}}(\tau) Z(t)\right),$$

with

$$(B.10) \quad \mathcal{A}^{\mathbb{P}}(\tau) = \sum_{i=1}^3 \mathcal{A}_i^{\mathbb{P}}(\tau) + \mathcal{A}_z^{\mathbb{P}}(\tau) - \tau(\Lambda_0 - \sum_{i=1}^3 \Lambda_i \bar{X}_i),$$

where

$$(B.11) \quad \mathcal{A}_i^{\mathbb{P}}(\tau) = \frac{2\kappa_i \theta_i}{\sigma_i^2} \log\left(\frac{\gamma_i^{\mathbb{P}} \exp\left(\frac{\kappa_i \tau}{2}\right)}{\gamma_i^{\mathbb{P}} \cosh\left(\frac{\gamma_i^{\mathbb{P}} \tau}{2}\right) + \kappa_i \sinh\left(\frac{\gamma_i^{\mathbb{P}} \tau}{2}\right)}\right),$$

$$(B.12) \quad \mathcal{A}_z^{\mathbb{P}}(\tau) = \frac{2\kappa_z \theta_z}{\sigma_z^2} \log\left(\frac{\gamma_z^{\mathbb{P}} \exp\left(\frac{\kappa_z \tau}{2}\right)}{\gamma_z^{\mathbb{P}} \cosh\left(\frac{\gamma_z^{\mathbb{P}} \tau}{2}\right) + \kappa_z \sinh\left(\frac{\gamma_z^{\mathbb{P}} \tau}{2}\right)}\right),$$

and

$$(B.13) \quad \mathcal{B}_i^{\mathbb{P}}(\tau) = \frac{2\Lambda_i \sinh\left(\frac{\gamma_i^{\mathbb{P}} \tau}{2}\right)}{\gamma_i^{\mathbb{P}} \cosh\left(\frac{\gamma_i^{\mathbb{P}} \tau}{2}\right) + \kappa_i \sinh\left(\frac{\gamma_i^{\mathbb{P}} \tau}{2}\right)},$$

$$(B.14) \quad \mathcal{B}_z^{\mathbb{P}}(\tau) = \frac{2 \sinh\left(\frac{\gamma_z^{\mathbb{P}} \tau}{2}\right)}{\gamma_z^{\mathbb{P}} \cosh\left(\frac{\gamma_z^{\mathbb{P}} \tau}{2}\right) + \kappa_z \sinh\left(\frac{\gamma_z^{\mathbb{P}} \tau}{2}\right)},$$

with

$$\gamma_i^{\mathbb{P}} := \sqrt{\kappa_i^2 + 2\sigma_i^2\Lambda_i}, \quad \text{for } i = 1, 2, 3, \quad \text{and} \quad \gamma_z^{\mathbb{P}} := \sqrt{\kappa_z^2 + 2\sigma_z^2}.$$

### Appendix C Interest Rate Parameter Estimation with the Kalman Filter

In order to estimate the interest rate parameters with the linear Kalman filter, we first transform the three-factor CIR model into the state space formulation, as also illustrated in *Entropy/Schiemert/Wilkens* (2013), consisting of the transition and measurement equation. The transition equation, which describes the evolution of the three state variables over our 106 observation points, has the following form:

$$(C.1) \quad X(t + 1) = \mathbf{c} + FX(t) + \boldsymbol{\eta}(\mathbf{t} + \mathbf{1}),$$

where  $\mathbf{X}(\mathbf{t})$  is a  $3 \times 1$  vector with  $i$ -th element  $X_i(t)$ . Since we replace the exact transition density of the state variables by a normal density,  $\mathbf{c}$  is a  $3 \times 1$  vector with  $i$ -th element  $\theta_i(1 - e^{-\kappa_i\Delta t})$ ,  $\mathbf{F}$  is a  $3 \times 3$  diagonal transition matrix with  $i$ -th element  $e^{-\kappa_i\Delta t}$ , and  $\boldsymbol{\eta}(\mathbf{t} + \mathbf{1})$  is a  $3 \times 1$  disturbance vector with zero mean and a  $3 \times 3$  diagonal covariance matrix with  $i$ -th element

$$(C.2) \quad \xi_i(t + 1) = \frac{\theta_i\sigma_i^2}{2\kappa_i}(1 - e^{-\kappa_i\Delta t})^2 + \frac{\sigma_i^2}{\kappa_i}(e^{-\kappa_i\Delta t} - e^{-2\kappa_i\Delta t})X_i(t).$$

The measurement equation determines the relationship between the ten spot rates and the three state variables, as

$$(C.3) \quad R(t) = d + HX(t) + \boldsymbol{\epsilon}(t),$$

where  $\mathbf{R}(t)$  is a  $10 \times 1$  vector, containing the spot rates  $R_{\tau_1}(t), \dots, R_{\tau_{10}}(t)$ , with  $\tau_j = j$  years.  $d$  is a  $10 \times 1$  vector with  $j$ -th element  $\sum_{i=1}^3 \frac{-A_i(\tau_j)}{\tau_j}$ , and  $H$  is a  $10 \times 3$  matrix of factor loadings with  $(j, i)$  element  $\frac{B_i(\tau_j)}{\tau_j}$ , where

$$(C.4) \quad A_i(\tau_j) = \frac{2\kappa_i\theta_i}{\sigma_i^2} \log \left( \frac{\tilde{\gamma}_i \exp\left(\frac{(\kappa_i + \lambda_i)\tau_j}{2}\right)}{\tilde{\gamma}_i \cosh\left(\frac{\tilde{\gamma}_i\tau_j}{2}\right) + (\kappa_i + \lambda_i) \sinh\left(\frac{\tilde{\gamma}_i\tau_j}{2}\right)} \right) \quad (\equiv A_i(\tau_j; \phi = 0, \Lambda_i = 0)),$$

$$(C.5) \quad B_i(\tau_j) = \frac{2 \sinh\left(\frac{\tilde{\gamma}_i\tau_j}{2}\right)}{\tilde{\gamma}_i \cosh\left(\frac{\tilde{\gamma}_i\tau_j}{2}\right) + (\kappa_i + \lambda_i) \sinh\left(\frac{\tilde{\gamma}_i\tau_j}{2}\right)} \quad (\equiv B_i(\tau_j; \phi = 0, \Lambda_i = 0)),$$

and

$$\tilde{\gamma}_i = \sqrt{(\kappa_i + \lambda_i)^2 + 2\sigma_i^2},$$

are given by *Cox/Ingersoll/Ross* (1985). Finally  $\epsilon(t + 1)$  is a normally distributed  $10 \times 1$  disturbance vector with zero mean and a  $10 \times 10$  diagonal covariance matrix  $\mathbf{RHO}$  with  $j$ -th element  $\rho_j^2$ . The implementation of the Kalman filter yields the logarithm of the quasi-likelihood function for the  $m = 10$  spot rates over the  $T = 106$  observation points

$$(C.6) \quad \mathcal{L}(\Psi) = -\frac{mT}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log |\mathbf{\Omega}_t| - \frac{1}{2} \sum_{t=1}^T \zeta_t^T \mathbf{\Omega}_t^{-1} \zeta_t,$$

with parameter vector  $\Psi = (\kappa_1, \kappa_2, \kappa_3, \theta_1, \theta_2, \theta_3, \sigma_1, \sigma_2, \sigma_3, \lambda_1, \lambda_2, \lambda_3, \rho_1, \dots, \rho_{10})$ , forecast error  $\zeta_t$  and mean squared error  $\mathbf{\Omega}_t$  of the measurement equation. We maximise  $\mathcal{L}(\Psi)$  with the Nelder-Mead simplex search method without constraining the parameter space and whenever we obtain a negative value for one of the resulting state variables, we set it to zero like in *Chen/Scott* (2003). For our optimal parameter set this was the case only once for  $X_2$  and for  $X_3$ .

### Appendix D

#### Default Intensity Parameter Estimation with the Inversion Method

In order to estimate the default intensity parameters of the 29 sample companies, we take the interest rate parameter estimates and the associated time series of the three state variables as inputs and make use of the inversion method, as also done in *Entrop/Schiemert/Wilkens* (2013), where we assume that five-year CDS spreads are observed without error and that the one-, three- and ten-year CDS spreads are observed with errors. We assume the error vector for these three CDS spreads  $\mathbf{v}_t = (\nu_t^1, \nu_t^3, \nu_t^{10})^T$  to have zero mean and to be serially uncorrelated but jointly normally distributed with the time-invariant covariance matrix  $\mathbf{\Omega}_v$ . To guarantee its regularity, the covariance matrix  $\mathbf{\Omega}_v = CC^T$  satisfies the Cholesky decomposition, where  $C$  is a lower triangular matrix with non-zero elements  $C_{11}, C_{22}, C_{33}, C_{21}, C_{31}$  and  $C_{32}$ . In order to carry out the QML estimation, we substitute the exact transition density of  $Z(t)$  with a normal density. Thus, the logarithm of the quasi-likelihood function is

$$(D.1) \quad \begin{aligned} \mathcal{L}_{CDS} = & -\frac{T-1}{2} \log(2\pi) - \frac{1}{2} \sum_{t=2}^T \log Q_z(t) - \frac{1}{2} \sum_{t=2}^T \frac{[Z(t) - \mu_z(t)]^2}{Q_z(t)} - \sum_{t=2}^T \log(|J_t|) \\ & - \frac{3(T-1)}{2} \log(2\pi) - \frac{T-1}{2} \log(|\mathbf{\Omega}_v|) - \frac{1}{2} \sum_{t=2}^T \mathbf{v}_t^T \mathbf{\Omega}_v^{-1} \mathbf{v}_t, \end{aligned}$$

with the conditional expectation and variance of the transition density of  $Z(t)$ ,

$$(D.2) \quad \mu_z(t) := \mathbb{E}^{\mathbb{P}}[Z(t)|Z(t-1)] = \theta_z(1 - e^{-\kappa_z \Delta t}) + e^{-\kappa_z \Delta t} Z(t-1),$$

$$(D.3) \quad Q_z(t) := \text{Var}^{\mathbb{P}}[Z(t)|Z(t-1)] = \frac{\theta_z \sigma_z^2}{2\kappa_z} (1 - e^{-\kappa_z \Delta t})^2 + \frac{\sigma_z^2}{\kappa_z} (e^{-\kappa_z \Delta t} - e^{-2\kappa_z \Delta t}) Z(t-1),$$

and the time-dependent Jacobian term  $J_t$  of the variable transformation.  $J_t$  is the partial derivative of the CDS pricing function in (A.13) with respect to  $Z(t)$ :

$$(D.4) \quad J_t = \frac{\partial CDS(t, \tau = 5)}{\partial Z(t)} = \frac{\mathcal{N}}{\left[ \delta \sum_{n=1}^{20} \Phi(t, T_n; \phi = 0) + \delta \sum_{n=1}^{20} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{1}{\mathbf{i}} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \right]^2},$$

with the numerator

$$\begin{aligned} \mathcal{N} = & \left[ 1 - REC(t) \int_t^{t+5} \frac{\partial}{\partial Z(t)} \left[ \frac{1}{\mathbf{i}} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} \right] du \right] \\ & \times \left[ \delta \sum_{n=1}^{20} \Phi(t, T_n; \phi = 0) + \delta \sum_{n=1}^{20} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{1}{\mathbf{i}} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \right] \\ & - \left[ 1 - REC(t) \int_t^{t+5} \frac{1}{\mathbf{i}} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} du \right] \\ & \times \left[ \delta \sum_{n=1}^{20} \frac{\partial \Phi(t, T_n; \phi = 0)}{\partial Z(t)} + \delta \sum_{n=1}^{20} \int_{T_{n-1}}^{T_n} \frac{u - T_{n-1}}{T_n - T_{n-1}} \frac{\partial}{\partial Z(t)} \left[ \frac{1}{\mathbf{i}} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} \right] du \right], \end{aligned}$$

where

$$(D.5) \quad \frac{\partial \Phi(t, T_n; \phi = 0)}{\partial Z(t)} = -\Phi(t, T_n; \phi = 0) \mathcal{B}_z(T_n - t; 0),$$

and

$$(D.6) \quad \begin{aligned} \frac{\partial}{\partial Z(t)} \left[ \frac{1}{\mathbf{i}} \frac{\partial \Phi(t, u; \phi)}{\partial \phi} \Big|_{\phi=0} \right] = & -\Phi(t, u; \phi = 0) \mathcal{B}_z(u - t; 0) \left[ \frac{1}{\mathbf{i}} \frac{\partial \mathcal{A}(u - t; \phi)}{\partial \phi} \Big|_{\phi=0} \right. \\ & - \sum_{i=1}^3 \frac{1}{\mathbf{i}} \frac{\partial \mathcal{B}_i(u - t; \phi)}{\partial \phi} \Big|_{\phi=0} \mathbf{X}_i(t) - \frac{1}{\mathbf{i}} \frac{\partial \mathcal{B}_z(u - t; \phi)}{\partial \phi} \Big|_{\phi=0} Z(t) \Big] \\ & - \Phi(t, u; \phi = 0) \frac{1}{\mathbf{i}} \frac{\partial \mathcal{B}_z(u - t; \phi)}{\partial \phi} \Big|_{\phi=0}. \end{aligned}$$

Since maximising the quasi-likelihood function in (D.1) 29 times by using standard optimisation routines turns out to be an infeasible task, we make use of an iterated simulated QML procedure, similar to that used in *Saita* (2006) and *Anderson* (2008). We minimise the sum of the MAEs of one-, three, and ten-year maturities with respect to the parameter vector  $\Psi_{CDS} = (\kappa_z, \theta_z, \sigma_z, \lambda_z, \Lambda_1, \Lambda_2, \Lambda_3)$ , using a multi-level grid search procedure. In order to reduce the dimensionality of

our problem, we eliminate the parameter  $\Lambda_0$  by setting it to zero. Further, we constrain  $\sigma_z$  to be positive, since negative values lead generally to a very poor fit. Finally, we also discard all parameter combinations that result in a negative value for  $Z(t)$ . We terminate the grid search when the sum of the three MAEs cannot be reduced by more than  $\varepsilon = 10^{-4}$  basis points, which leads us to our optimal default intensity parameters.

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