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Calibration of Parametric CAT Bonds. A Case Study of Mexican Earthquakes*

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Abstract

The study of natural catastrophe models plays an important role in the prevention and mitigation of disasters. After the occurrence of a natural disaster, the reconstruction can be financed with catastrophe bonds (CAT bonds) or reinsurance. This paper examines the calibration of a real parametric CAT bond for earthquakes that was sponsored by the Mexican government, which is based on the estimation of the intensity rate of the arrival process of earthquake (which would trigger this particular CAT bond) from the two sides of the contract: the reinsurance and the capital markets. Additionally, the intensity rate from the historical data was estimated to conduct a comparative analysis. The results demonstrate that, under specific conditions, the financial strategy of the government, a mix of reinsurance and CAT bond, is optimal in the sense that it provides coverage of USD 450 million for a lower cost than the reinsurance itself.

JEL Classification: G19, G29, N26, N56, Q29, Q54

1. Introduction

By its geographical position, Mexico finds itself under a great variety of natural phenomena which can cause disasters, like earthquakes, eruptions, hurricanes, burning forest, floods and aridity (dryness). In case of disaster, the effects on financial and natural resources are huge and volatile. In Mexico the first risk to transfer is the seismic risk, because although it is the less recurrent, it has the biggest impact on the population and country. For example, an earthquake of magnitude 8.1 *Mw* Richter scale that hit Mexico in 1985, destroyed hundreds of buildings and caused thousands of deaths. Figure 1 depicts the number of earthquakes higher than 6.5 *Mw* occurred in Mexico during the years 1900 – 2003.

After the occurrence of a natural disaster, the reconstruction can be financed with catastrophe bonds (CAT bonds) or reinsurance. For insurers, rein-

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surers and other corporations CAT bonds are hedging instruments that offer multi year protection without the credit risk present in reinsurance by providing full collateral for the risk limits offered through the transaction. For investors CAT bonds offer attractive returns and reduction of portfolio risk, since CAT bonds defaults are uncorrelated with defaults of other securities.

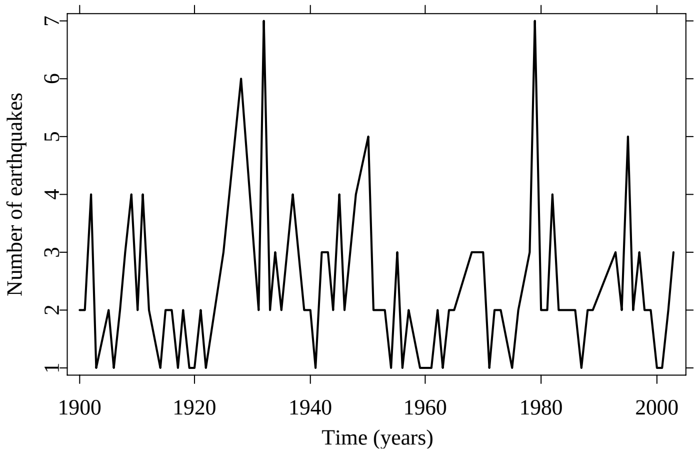

Figure 1: *Number of Mexican earthquakes occurred during 1900 – 2003*. In this figure, we plot the number of earthquakes higher than 6.5 *Mw* occurred in Mexico during the years 1900-2003. Earthquakes less than 6.5 *Mw* were not taken into account because of their high frequency and low loss impact. The data was provided by the National Institute of Seismology in Mexico, SSN (2006)

Baryshnikov / Mayo / Taylor (2001) present an arbitrage free solution to the pricing of CAT bonds under conditions of continous trading and according to the statistical characteristics of the dominant underlying processes. Instead of pricing, Anderson / Bendimerad / Canabarro / Finkemeier (2000) devoted to the CAT bond benefits by providing an extensive relative value analysis. Others, like Croson / Kunreuther (2000) focus on the CAT management and their combination with reinsurance. Lee / Yu (2002) analyze default risk on CAT bonds and therefore their pricing methodology is focused only on CAT bonds that are issued by insurers. Also under an arbitrage-free framework, Vaugirard (2003) valuates catastrophe bonds by Monte Carlo simulation and stochastic interest rates. Burnecki / Kukla (2003) correct and apply the results of Baryshnikov et al. (2001) to calculate non-arbitrage prices of a zero coupon and coupon CAT bond. Lee / Yu (2006) examine how a reinsurance company can increase the value of reinsurance contract and reduce its default risk by issuing CAT bonds.

As the study of natural catastrophe models plays an important role in the prevention and mitigation of disasters, the main motivation of this paper is the calibration of CAT bonds. In particular, we examine the calibration of a pure parametric CAT bond for earthquakes that was sponsored by the Mexican government. The advantage of the pure parametric CAT bond is that the CAT bond payment is based on some physical parameters of the underlying event, e.g. the magnitude *Mw* of the earthquake. The calibration of the CAT bond is based on the estimation of the intensity rate that describes the event process (earthquakes that trigger the CAT bond's payoff) from the two sides of the contract: from the reinsurance market that consists of the sponsor company (the Mexican government) and the issuer of reinsurance coverage (in this case Swiss Re) and from the capital market, which is formed by the issuer of the CAT bond (CAT-MEX Ltd.) and the investors. In addition to these intensity estimates, the historical intensity rate is computed to conduct a comparative analysis between the intensity rates to know whether the sponsor company is getting protection at a fair price or whether the CAT bond is sold to the investors for a reasonable price. Our results demonstrate that the reinsurance market estimates a probability of an earthquake lower than the one estimated from historical data. Under specific conditions, the financial strategy of the government, a mix of reinsurance and CAT bond is optimal in the sense that it provides coverage of USD 450 million for a lower cost than the reinsurance itself.

Our paper is structured as follows. In the next section we discuss fundamentals of CAT bonds. Section 3 is devoted to the calibration of the real pure parametric CAT bond for earthquakes in Mexico. Section 4 summarizes the article. All quotations of money in this paper will be in USD and therefore we will omit the explicit notion of the currency.

2. CAT Bonds

In the mid-1990's catastrophe bonds (CAT bonds), also named as *Act of God or Insurance-linked bond,* were developed to ease the transfer of catastrophe based insurance risk from insurers, reinsurers and corporations (sponsors) to capital market investors. CAT bonds are bonds whose coupons and principal payments depend on the performance of a pool or index of natural catastrophe risks, or on the presence of specified trigger conditions. They protect sponsor companies from financial losses caused by large natural disasters by offering an alternative or complement to traditional reinsurance.

The transaction involves four parties: the sponsor or ceding company (government agencies, insurers, reinsurers), the special purpose vehicle SPV (or issuer), the collateral and the investors (institutional investors, insurers, reinsurers, and hedge funds). The basic structure is shown in Figure 2. The sponsor sets up a SPV as an issuer of the bond and a source of reinsurance protection. The issuer sells bonds to capital market investors and the proceeds are deposited in a collateral account, in which earnings from assets are collected and from which a floating rate is paid to the SPV. The sponsor enters into a reinsurance or derivative contract with the issuer and pays him a premium. The SPV usually gives quarterly coupon payments to the investors. The premium and the investment bond proceeds that the SPV received from the collateral are a source of interest or coupons paid to investors. If there is no trigger event during the life of the bonds, the SPV gives the principal back to the investors with the final coupon or the generous interest; otherwise the SPV pays the ceding according to the terms of the reinsurance contract and sometimes pays nothing or partially the principal and interest to the investors.

There is a variety of trigger mechanisms to determine when the losses of a natural catastrophe should be covered by the CAT bond. These include the indemnity, the modeled loss, the industry index, the parametric index, the pure parametric and the hybrid trigger. Each of these mechanisms shows a range of levels of basis risks and transparency to investors.

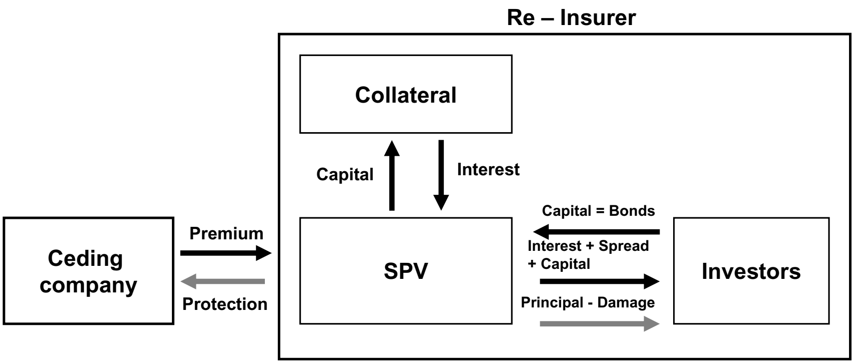

Figure 2: *Cash flows diagram of a CAT bond*. In case of the occurrence of an event (gray arrow), the SPV gives the principal back to the investors with the final coupon or the generous interest. In case of no event (black arrow), the SPV pays the ceding according to the terms of the reinsurance contract and sometimes pays nothing or partially the principal and interest to the investors

The *Indemnity trigger* involves the actual loss of the ceding company. The ceding company receives reimbursement for its actual losses from the covered event, above the predetermined level of losses. This trigger closely replicates the traditional reinsurance, but it is exposed to catastrophic and operational risk of the ceding company. In a *Modeled loss trigger* mechanism, after a catastrophe occurs the physical parameters of the catastrophe are used by a modelling firm to estimate the expected losses to the ceding company's portfolio. Instead of dealing with the company's actual claims, the transaction is based on the estimates of the model. If the modeled losses are above a specified threshold, the bond is triggered. With an *Industry index trigger,* the ceding company recovers a proportion of total industry losses in excess of a predetermined point to the extent of the remainder of the principal. The *Parametric index trigger* uses different weighted boxes to reflect the ceding company's exposure to events in different areas. The *Pure parametric index* payouts are triggered by the occurrence of a catastrophic event with certain defined physical parameters, e.g. wind speed and location of a hurricane or the magnitude or location of an earthquake. A *Hybrid trigger* uses more than one trigger type in a single transaction.

The pricing of CAT bonds reveals some similarities to the defaultable bonds, but CAT bonds offer higher returns because of the unfixed stochastic nature of the catastrophe process. The similarity between catastrophe und default in the log-normal context has been commented in Kau / Keenan (1996).

3. Calibrating a Mexican Parametric CAT Bond

In 1996, the Mexican government established the Mexico's Fund for Natural Disasters (FONDEN) in order to reduce the exposure to the impact of natural catastrophes and to recover quickly as soon as they occur. However, FONDEN is funded by fiscal resources which are limited and have been insufficient to meet the government's obligations. Faced with the shortage of the FONDEN's resources and the high probability of earthquake occurrence, in May 2006 the Mexican government sponsored a parametric CAT bond against earthquake risk. The decision was taken because the instrument design protects and magnifies, with a degree of transparency, the resources of the trust. The CAT bond payment is based on some physical parameters of the underlying event (e.g. the magnitude *Mw*), thereby there is no justification of losses. The parametric CAT bond helps the government with emergency services and rebuilding after a big earthquake.

The CAT bond was issued by a SPV Cayman Islands CAT-MEX Ltd. and structured by Swiss Reinsurance Company (SRC) together with Deutsche Bank Securities. The 160 million CAT bond pays a tranche equal to the London Inter-Bank Offered Rate (LIBOR) plus 235 basis points. The CAT bond is part of a total coverage of 450 million against earthquake risk for three years and the total premium paid by the government is equal to 26 million. The payment of losses is conditional upon confirmation by a leading independent consulting firm which develops catastrophe risk assessment. This event verification agent (Applied Insurance Research Worldwide Corporation – AIR) modeled the seismic risk and detected nine seismic zones, see Figure 3. Given the federal governmental budget plan, just three out of these nine zones were insured in the transaction: zone 1, zone 2 and zone 5, with coverage of 150 million in each case, Secretaría de Hacienda y Crédito Público México (2004). The CAT bond payment would be triggered if there is an *event,* i.e. an earthquake higher or equal than 8 *Mw* hitting zone 1 or zone 2, or an earthquake higher or equal than 7.5 *Mw* hitting zone 5.

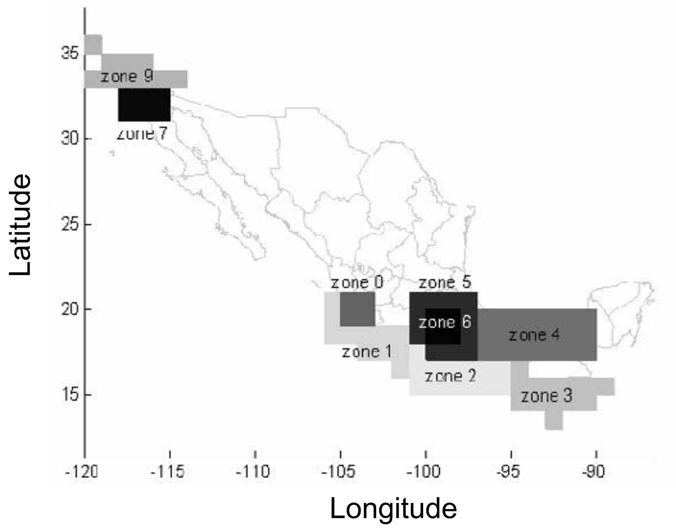

Figure 3: *Map of seismic regions in Mexico, source: SHCP (2004)*. The event verification agent modeled the seismic risk and detected nine seismic zones in Mexico. Only zone 1, zone 2 and zone 5 were insured in the transaction with coverage of 150 million in each case

The cash flows diagram for the Mexican CAT bond is described in Figure 4, Secretaría de Hacienda y Crédito Público México (2004). CAT-MEX Ltd. issues the bond that is placed among investors and invests the proceeds in high quality assets within a collateral account. Simultaneous to the issuance of the bond, CAT-MEX Ltd. enters into a reinsurance contract with SRC. The proceeds of the bond will also serve to provide SRC coverage for earthquakes in Mexico in connection with an insurance agreement that FONDEN has entered with the European Finance Reinsurance Co. Ltd., an indirect wholly-owned subsidiary of SRC. A separate Event Payment Account was established with the Bank of New York providing FONDEN the ability to receive loss payments directly from CAT-MEX Ltd., subject to the terms and conditions of the insurance agreement. In case of occurrence of a trigger event, an earthquake with a certain magnitude in any of the three defined zones in Mexico, SRC pays the covered insured amount to the government, which stops paying premiums at that time and investors sacrifices their full principal and coupons.

Assuming perfect financial market, the calibration of the parametric CAT bond is based on the estimation of the intensity rate that describes the flow process of events (earthquakes that trigger the CAT bond's payoff) from the two sides of the contract: from the reinsurance and the capital markets.

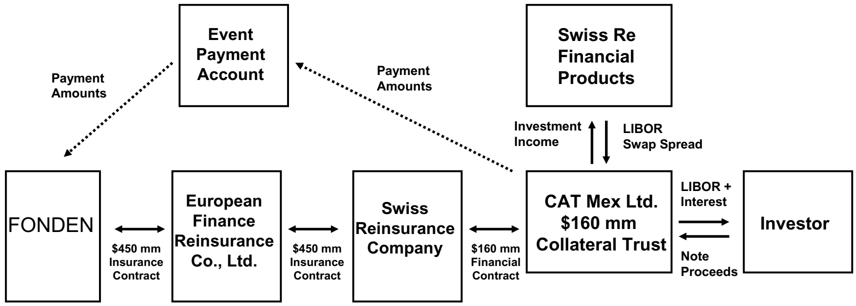

Figure 4: *The cash flows diagram for the Mexican CAT bond, source: SHCP (2004).* CAT-MEX Ltd. issues the bond and invests the proceeds in a collateral account. Simultaneous, CAT-MEX Ltd. enters into a reinsurance contract with SRC. The proceeds of the bond will also serve to provide SRC coverage for earthquakes in Mexico in connection with an insurance agreement that FONDEN has entered with the European Finance Reinsurance Co. Ltd. A separate Event Payment Account was established providing FONDEN the ability to receive loss payments directly from CAT-MEX Ltd. In case of occurrence of a trigger event, an earthquake with a certain magnitude in any of the three defined zones in Mexico, SRC pays the covered insured amount to the government and investors sacrifices their full principal and coupons

The arrival process of earthquakes or the number of earthquakes in the interval $(0,t]$ is described by the process $N_t, t \geq 0$. This process uses the times T_i when the ith earthquake occurs or the times between earthquakes $\tau_i = T_i - T_{i-1}$. The earthquake process N_t in terms of τ_i's is defined as:

(1) $$N_t = \sum_{n=1}^{\infty} 1(T_n < t)\,.$$

Since earthquakes can strike at any time during the year with the same probability, the traditional approach in seismology is to model earthquake recurrence as a random process, in which the earthquakes suffer the *loss of memory property* $\mathrm{P}(X > x + y | X > y) = \mathrm{P}(X > x)$, where X is a random variable. The arrival process of earthquakes N_t can be characterized with a Homogeneous Poisson Process (HPP), with intensity rate $\lambda > 0$ if N_t is a point process governed by the Poisson law and the waiting times τ_i are exponentially distributed with intensity λ. Hence, the probability of occurrence of an earthquake is:

(2) $$\mathrm{P}(\tau_i < t) = 1 - \mathrm{P}(\tau_i \geq t) = 1 - e^{-\lambda t}\,.$$

In fact, we are interested in the occurrence of the first event. We define the first waiting time as the *stopping time* equal to $\tau = \min\{t : N_t > 0\}$, with cdf $F_\tau(t) = \mathrm{P}(\tau < t) = \mathrm{P}(N_t > 0) = 1 - e^{-\lambda t}$ and $f_\tau(t) = \lambda e^{-\lambda t}$.

3.1 Calibration in the Reinsurance Market

Let the random variable $J = 450 \cdot \mathbf{1}(\tau < 3)$ with density function $f_\tau(t)$ be the payoff of the covered insured amount to the government in case of occurrence of an event over a three years period $T = 3$. Denote H as the total premium paid by the government equal to 26 million. Suppose a flat term structure of continuously compounded discount interest rates and a HPP with intensity λ_1 that describes the arrival process of earthquakes which would trigger the CAT bond's payoff. Under the risk neutral pricing measure, a compounded discount *actuarially fair insurance price at time* $t = 0$ in the reinsurance market is defined as:

$$
\begin{aligned}
H &= \mathrm{E}\left[J e^{-\tau r_\tau}\right] \\
&= \mathrm{E}\left[450 \cdot \mathbf{1}(\tau < 3) e^{-\tau r_\tau}\right] \\
&= 450 \int_0^3 e^{-r_t t} f_\tau(t) dt \\
&= 450 \int_0^3 e^{-r_t t} \lambda_1 e^{-\lambda_1 t} dt
\end{aligned}
\tag{3}
$$

i.e. the insurance premium is equal to the value of the expected discounted loss from earthquake. Substituing the value of H and assuming an annual continously compounded discount interest rate $r_t = \log(1.0541)$ constant and equal to the LIBOR in May 2006, we get:

$$
26 = 450 \int_0^3 e^{-\log(1.0541)t} \lambda_1 e^{-\lambda_1 t} dt \tag{4}
$$

where $1 - e^{-\lambda_1 t}$ is the probability of occurrence of an event. The estimation of the intensity rate of events from the reinsurance market λ_1 is equal to 0.0214. That means that the premium paid by the government to the insurance company considers a probability of occurrence of an event in three years equal to 0.0624 or the insurer expects 2.15 events in one hundred years.

3.2 Calibration in the Capital Market

For computing the intensity rate of events in the capital market λ_2, we suppose that the contract structure defines a coupon CAT bond that pays to the investors the principal P equal to 160 million at time to maturity $T = 3$ and gives coupons C every 3 months during the bond's life in case of no event. If there is an event, the investors sacrifice their principal and coupons. These coupon bonds offered by CAT-MEX Ltd. pay to the investors a fixed spread rate z equal to 235 basis points over LIBOR. We consider the annual discretely compounded discount interest rate $r_t = 5.4139\%$ to be constant and equal to LIBOR in May 2006. The fixed coupons payments C have a value of:

$$C = \left(\frac{r_t + z}{4}\right)P = \left(\frac{5.4139\% + 2.35\%}{4}\right)160 = 3.1055\,. \tag{5}$$

Let the random variable G be the investors' gain from investing in the bond, which consists of the principal and coupons. Moreover, assume that the arrival process of earthquakes, which would trigger this particular bond, follows a HPP with intensity λ_2. Under the risk neutral pricing measure, the discretely discount *fair bond price at time* $t = 0$ is given by:

$$\begin{aligned} P &= \mathrm{E}\left[G\left(\frac{1}{1+r_\tau}\right)^{\tau}\right] \\ &= \mathrm{E}\left[\sum_{t=1}^{12} C\cdot\mathbf{1}(\tau > \frac{t}{4})\left(\frac{1}{1+r_t}\right)^{\frac{t}{4}} + P\cdot 1(\tau > 3)\left(\frac{1}{1+r_t}\right)^{3}\right] \\ &= \sum_{t=1}^{12} Ce^{-\lambda_2 \frac{t}{4}}\left(\frac{1}{1+r_t}\right)^{\frac{t}{4}} + Pe^{-3\lambda_2}\left(\frac{1}{1+r_t}\right)^{3}. \end{aligned} \tag{6}$$

In this case, the investors receive 12 coupons during 3 years and its principal P at maturity $T = 3$. Hence, substituting the values of the principal $P = 160$ million and the coupons $C = 3.1055$ million in equation (6), it follows:

$$160 = \sum_{t=1}^{12} 3.06\left(\frac{e^{-\lambda_2}}{1.0541}\right)^{\frac{t}{4}} + \frac{160e^{-3\lambda_2}}{(1.0541)^3}\,. \tag{7}$$

Solving equation (7), the intensity rate of events from the capital market λ_2 is equal to 0.0241. In other words, the capital market estimates a probability of occurrence of an event equal to 0.0699, equivalently to 2.4 events in one hundred years.

3.3 Calibration via Historical Data

In addition to the intensity estimates in the Reinsurance and Capital Market, the historical intensity rate λ_3 that describes the flow process of events (earthquakes that trigger the CAT bond's payoff) is calculated to conduct a comparative analysis between them. The data was provided by the National Institute of Seismology in Mexico, Servicio Sismológico Nacional Instituto de Geosifísica

(2006). It describes the time t, the depth d, the magnitude Mw and the epicenters of 192 earthquakes higher than 6.5 Mw occurred in the country during 1900 to 2003. Earthquakes less than 6.5 Mw were not taken into account because of their high frequency and low loss impact. Table 1 shows that almost 50 % of the earthquakes has occurred in the insured zones that were defined in the CAT bond contract, mainly in zone 2.

Table 1

Frequency of the earthquake location for the 1900 – 2003 earthquake data

Zone	Frequency	Percent	% Cumulative
1	30	16 %	16 %
2	42	22 %	38 %
5	18	9 %	47 %
Other	102	53 %	100 %

Let Y_i be i.i.d. random variables, indicating the magnitude Mw of the ith earthquake at time t. Define $\bar{u}$ as the threshold magnitude for a specific location. The estimation of the historical intensity rate λ_3 is based on the *intensity model*. This model assumes that there exist i.i.d. random variables ε_i called *trigger events* that characterize earthquakes with magnitude Y_i higher than a defined threshold $\bar{u}$ for a specific location, i.e. $\varepsilon_i = \mathbf{1}(Y_i \geq \bar{u})$. Then the trigger event process B_t is characterized as:

$$B_t = \sum_{i=1}^{N_t} \varepsilon_i \tag{8}$$

where N_t is a HPP describing the arrival process of earthquakes with intensity $\lambda > 0$. B_t is a process which counts only earthquakes that trigger the CAT bond's payoff. However, the dataset contains only three such events, what leads to the calibration of the intensity of the process B_t be based on only two waiting times. Therefore in order to compute λ_3, consider the process B_t and define p as the probability of occurrence of a trigger event conditional on the occurrence of the earthquake. Then the probability of no event up to time t is equal to:

$$\begin{aligned} P(B_t = 0) &= \mathrm{P}(N_t = 0) + \mathrm{P}(N_t = 1)(1-p) + \mathrm{P}(N_t = 2)(1-p)^2 + \dots \\ &= \sum_{k=0}^{\infty} \mathrm{P}(N_t = k)(1-p)^k = \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!} e^{(-\lambda t)} (1-p)^k \\ &= \sum_{k=0}^{\infty} \frac{\{\lambda(1-p)t\}^k}{k!} e^{(-\lambda t)} e^{-\lambda(1-p)t} e^{\lambda(1-p)t} = e^{-\lambda p t} = e^{-\lambda_3 t} \end{aligned} \tag{9}$$

by definition of the Poisson distribution and since $\sum_{k=0}^{\infty} \frac{\{\lambda(1-p)t\}^k}{k!} e^{-\lambda(1-p)t}$ $= 1$. Now the calibration of the λ_3 can be decomposed into the calibration of the intensity of all earthquakes with a magnitude higher than 6.5 *Mw* and the estimation of the probability of the trigger event.

Since the historical data contains three earthquakes with magnitude *Mw* higher than the defined thresholds by the modelling company, the probability of occurrence of the trigger event is equal to $p = \left(\frac{3}{192}\right)$. The estimation of the annual intensity is obtained by taking the mean of the daily number of earthquakes times 360 i.e. $\lambda = (0.005140)(360) = 1.8504$. Consequently the annual historical intensity rate for a trigger event is equal to $\lambda_3 = \lambda p = 1.8504\left(\frac{3}{192}\right) = 0.0289$. This means that approximately 2.89 events are expected to occur in the insured areas of the country within one hundred years. The magnitude of earthquakes above 6.5 *Mw* that occurred in Mexico during the period 1900 to 2003 are illustrated in Figure 5. It also indicates earthquakes that occurred in the insured zones and trigger events.

Table 2 summarizes the values of the intensity rates $\lambda's$ and the probabilities of occurrence of a trigger event in one and three years. Whereas the reinsurance market expects approximately 2.15 events to occur in one hundred years, the capital market anticipates 2.42 events and the historical data predicts 2.89 events. In other words, the CAT bond has approximatly 6.99 % chance of default (or event) within 3 years, what makes the bond to be rated in the range of "BB" by Standard and Poor's (2007) or "Ba" by Moody's Investors Services (2007). Table 3 shows the one-to-ten year cumulative default rates from two sources of data.

Table 2

Calibration of intensity rates: the intensity rate from the reinsurance market λ_1, the intensity rate from the capital market λ_2 and the historical intensity rate λ_3

	λ_1	λ_2	λ_3
Intensity	0.0214	0.0241	0.0289
Probability of event in 1 year	0.0212	0.0238	0.0284
Probability of event in 3 year	0.0624	0.0699	0.0830
No. expected events in 100 years	2.1482	2.4171	2.8912

In Table 2, we also observe that the value of the λ_3 depends on the time period of the historical data and it is not very accurate since it is based only on three events. The intensity rate λ_3 could be interpreted as the real intensity rate describing the flow process of events, however its confidence intervals in Table 4 are very large that its value might change with a different period of the historical data. Nevertheless, the estimation of the risk neutral intensity rates of event λ_1 or λ_2 are reliable since their values lie within the confidence intervals of λ_3.

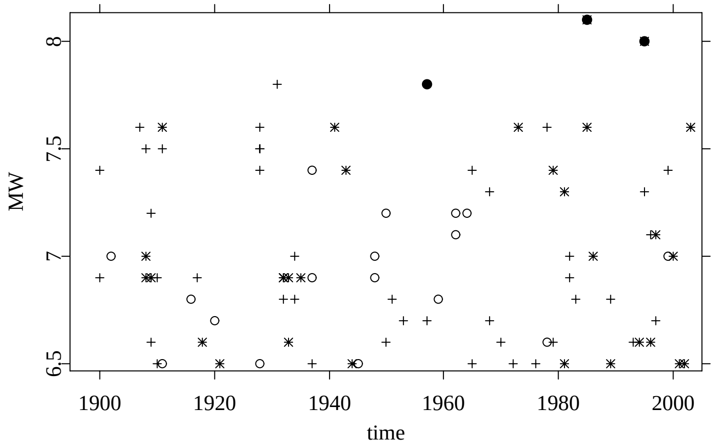

Figure 5: *Magnitude of trigger events and Mexican earthquakes occurred during the years 1900-2003*. The plot shows the magnitude of trigger events (filled circles) occurred in Mexico during 1900-2003, earthquakes occurred in zone 1 (stars), in zone 2 (crosses), in zone 5 (circles) and earthquakes out of insured zones (triangles). Observe that there were more earthquakes with high magnitude but did not occurred in insured zones

Table 3

Cumulative Default Rate comparison (in % for up to 10 years)

Rating	Data	1	2	3	4	5	6	7	8	9	10
AAA / Aaa	Moody's	0.00	0.00	0.00	0.04	0.08	0.13	0.19	0.19	0.19	0.19
	S&P	0.00	0.00	0.09	0.19	0.29	0.43	0.50	0.62	0.66	0.70
AA / Aaa	Moody's	0.01	0.02	0.05	0.12	0.18	0.23	0.26	0.29	0.31	0.37
	S&P	0.01	0.05	0.10	0.20	0.32	0.43	0.56	0.68	0.78	0.89
A / A	Moody's	0.20	0.10	0.25	0.38	0.51	0.64	0.76	0.89	1.01	1.09
	S&P	0.06	0.17	0.31	0.47	0.68	0.91	1.19	1.41	1.64	1.90
BBB / Baa	Moody's	0.21	0.57	1.00	1.53	2.06	2.57	3.70	3.51	3.92	4.37
	S&P	0.24	0.71	1.23	1.92	2.61	3.28	3.82	4.38	4.89	5.42
BB / Ba	Moody's	1.27	3.50	6.20	8.89	11.26	13.37	15.26	16.95	18.44	19.83
	S&P	1.07	3.14	5.61	7.97	10.10	12.12	13.73	15.15	16.47	17.49
B / B	Moody's	5.26	11.44	17.31	22.41	27.26	31.59	35.50	38.80	41.59	43.80
	S&P	4.99	10.92	15.90	19.76	22.55	24.72	26.54	28.00	29.20	30.42
CCC / Caa	Moody's	17.14	28.13	37.62	45.34	50.89	55.00	57.76	60.65	64.79	71.27
	S&P	26.29	34.73	39.96	43.19	46.22	47.49	48.61	49.23	50.95	51.83

Apparently the difference between the intensity rates for a trigger event λ_1, λ_2 and λ_3 seems to be insignificant, but for the government it has a financial and social repercussion since the intensity rate of the flow process of events influences the price of the parametric CAT bond that will help the government to obtain resources after a big earthquake. Particularly after a catastrophic event occurred, the reinsurance market suffers from a shortage of capital but this gives reinsurance firms the ability to gain more market power that will enable them to charge higher premiums than expected. Our estimation of intensity rates, contrary to the theory predictions, show that the Mexican government paid total premiums of 26 million that is 0.75 times the real actuarially fair one (34.605 million), which is obtained by substituting the historical intensity rate λ_3 in equation (4). At first glance, it appears that either the government saves 8.605 million (24.86) % from transferring the seismic risk with a reinsurance contract, but in fact this difference is explained by the market price of risk, which in this case is negative. Since λ_3 is only 50 % confident, no further analysis about market price of risk will be done in our analysis.

Table 4

Confidence Intervals for the intensity rate of events from the historical data λ_3

$100\,(1-\alpha)$ %	Confidence intervals
99 %	0 – 0.0791
95 %	0 – 0.0666
90 %	0.0083 – 0.0594
50 %	0.0183 – 0.0392

The difference between λ_1 and λ_2 might be explained by the absence of the public and liquid market of earthquake risk in the reinsurance market, since just limited information is available. This might cause the pricing in the reinsurance market to be less transparent than pricing in the capital market. Another argument to this difference might be that contracts in the capital market are more expensive than contracts in the reinsurance market, e.g. when one considers the default risk or the cost of risk capital (the required return necessary to make a capital budgeting project): the cost of risk capital in the capital market is usually higher than that in the reinsurance market and a CAT bond presents no credit risk as the proceeds of the bond are held in a SPV, a transaction off the insurer's balance sheet.

Since the insured loss faced by the government is independent of the state of insolvency of the reinsurer, our results indicate that the probability that the resinsurer will default in this transaction over the next three years could be approximately equal to the price discount that the government gets in the risk transfer of earthquake risk (≈ 10.7) %, which is the difference in premiums computed with the corresponding intensity rates in the reinsurance and capital markets, λ_1 and λ_2. Transferring the earthquake risk with reinsurance in the three years implies that the exposure at default (the amount the reinsurer will loose as a result of the default) is approximately equal to 10.7 % of 450 million coverage and the expected loss is equal to 10.7 % of 450 million coverage times the loss given default (1-recovery rate). Since recovery rates are not very accurately estimated, we neglect the computation of the expected loss.

However, the best explanation of the low premiums for covering the seismic risk paid by the government might be the mix of the reinsurance contract and the CAT bond. Since the 160 million CAT bond is part of a total coverage of 450 million, the reinsurance company transfers 35 % of the total seismic risk to the investors, who effectively are betting that a trigger event will not hit specified regions in Mexico in the next three years. If there is an event, the reinsurer must pay to the government 290 million from the reinsurance part and 160 million from the CAT bond to cover the insured loss of 450 million. The value of the premium for 290 million coverage with intensity rate of event λ_2 is $\int_0^3 290\lambda_2 e^{-t(r_t+\lambda_1)}dt = 18.799$. Therefore the total paid premium of 26 million might consist of 18.799 million premium from the reinsurance and the CAT bond layers and 7.221 million for transaction costs or the management added value or for coupon payments. This time the exposure at default is approximately equal to 10.7 % of 290 million coverage and the expected loss is equal to 10.7 % $\times$ 290 million $\times$ (1-recovery rate). This government's financial strategy is optimal in the sense that it provides coverage of 450 million against seismic risk for a lower cost and lower exposure at default than the reinsurance itself. However, this financial strategy does not eliminate completely the costs imposed by market imperfections. Lee / Yu (2006) get similar results, when examining how a reinsurance company can increase the value of reinsurance contract and reduce its default risk by issuing CAT bonds.

4. Conclusion

The occurrence of disfavoured extreme natural events like earthquakes, hurricanes, long cold winter, heat, drought, freeze, etc. may cause substantial financial losses. In the presence of this, many sponsor companies have turned to the capital markets to cover costs of potential catastrophes by issuing CAT bonds that passes the risk on to investors. This paper calibrates a real parametric CAT bonds that was sponsored in 2006 by the Mexican government. The decision of the government to issue a parametric CAT bond relies on the fact that it triggers immediately when an earthquake meets the defined physical parameters. The parametric CAT bond especially helps the government with fast emergency services and rebuilding after a big earthquake.

Under the assumption of perfect markets, the calibration of the bond is based on the estimation of the intensity rate that describes the flow process of events (earthquakes that trigger the CAT bond's payoff) from the two sides of the contract: from the reinsurance and the capital markets. This intensity rate reflects the CAT bond's spread rate. Additionally, we estimate the historical intensity rate using the *intensity model* that accounts only earthquakes that trigger the CAT bond's payoff. The results indicate that in presence of catastrophe risk, the default risk is substantial for the valuation of the reinsurance premium. However, the argument to the low premiums payed by the government for covering the seismic risk might be the mix of reinsurance and CAT bond, where 35 % of the total seismic risk is transferred to the investors and the exposure at default is reduced.

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