

Performance of Bond Ladder Strategies: Evidence from a Period of Low Interest Rates

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Abstract

Based on German government bond yields, this paper analyses the performance of laddered strategies during a period of low interest rates. Relying on the REX, Germany's leading bond index, laddered cash flows are created, and maturity structures are systematically changed. A constructed rolling window of annual returns reveals that risk and return significantly increase with the length of maturity. Performance measures, such as return on risk-adjusted capital and the Sharpe ratio, show that long-term bond ladders significantly dominate short-term ladders. However, for upward movements in the average yield level, the dominance is reduced. The results imply that portfolio managers should consider performance characteristics in maturity decisions as well as expectations of changes in the yield level.

Performance gestaffelter Anlagestrategien in einer Niedrigzinsphase

Zusammenfassung

Die aktuelle Studie analysiert die Performance gestaffelter Anlagestrategien in einem Niedrigzinsumfeld. Die Auswertungen basieren auf der Renditestruktur von Bundeswertpapieren. In Anlehnung an die Laufzeitstruktur des Deutschen Rentenindex REX werden Bondportfolios als gleitende Durchschnitte unterschiedlicher Laufzeiten erzeugt und deren Fristigkeit systematisch verändert. Auf Basis einer rollierenden Ermittlung von Renditen wird gezeigt, dass Risiko und Ertrag mit zunehmender Maximallaufzeit steigen. Dies trifft auch signifikant auf die Performancemaße Return on Risk Adjusted Capital und Sharpe Ratio zu. Für steigende Renditeniveaus ist die Dominanz langfristiger Laufzeitstrukturen jedoch schwächer ausgeprägt. Portfoliomanager sollten daher bei der

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Auswahl der Laufzeiten neben der Performance auch die Erwartungen an Renditestrukturänderungen berücksichtigen.

Keywords: Bond Ladders, Return, Risk, Performance, RORAC, Sharpe Ratio, Maturity, Fixed-Income Portfolios, Period of Low Interest Rates

JEL Classification: E43, G11, G12, G21

I. Introduction

Since *Markowitz* (1952) developed the fundamental modern portfolio theory, various studies investigate portfolio investment strategies that address risk and return characteristics. A prominent example is *Fama/French* (1992), who analyse a broad dataset of nonfinancial stock returns with respect to market risk and variables such as size and book-to-market value.¹ Whereas a majority of corresponding studies rely on stock portfolios, the literature that focuses on fixed-income securities is rare. *Judd/Kubler/Schmedders* (2011) find that ‘there is surprisingly little reference to this subject in the economics and finance literature’, and *Ferson/Henry/Kisgen* (2006) state that ‘Active fixed-income fund management would seem to present a rich, undeveloped field for research’. However, management knowledge has established prominent bond portfolio benchmarks that are considered to be efficient in the context of risk and return. In Germany, the national bond index REX and its corresponding performance index REXP are well-known examples.² The index consists of German government bonds with maturities from 1 to 10 years. The present paper is the first to evaluate the efficiency of REX-related maturity strategies by systematically analysing the performance contribution of maturity changes. To create a homogeneous variation of maturity intervals, equally weighted cash flows (CFs) – denoted as bond ladders – are constructed. Using rolling windows of annual returns allows for testing of return, risk and performance differences. Whereas dominant maturity strategies can be identified during the current period of low interest rates, subsamples of upward and downward moving yield levels show that expectations of yield curve changes should also be considered in portfolio decisions.

The results provide valuable insights for managers of banking books, which are primarily addressed in the present article, and for managers of funds and insurance portfolios. Considering the institutional sector, *Hanson et al.* (2015) describe commercial banks as financial institutions that typically invest in assets that create stable CFs. In this regard, it is notable that, in addition to loan provisions, German institutions have invested more than one trillion Euros in

¹ Further examples are *Carhart* (1997), who addresses momentum strategies, and *Walkshäusl/Lobe* (2015), who analyse multiples in the context of fund performance.

² Börse Frankfurt provides definitions and daily information on the REX and REXP.

fixed-income securities.³ Based on the variability of fixed-income instruments, managers can strategically create portfolios that meet criteria such as risk-taking attitude and regulatory standards. Whereas government bonds, such as German government bonds, can be regarded as free from default risk, corporate bonds are exposed to default risk. For bond portfolio returns, *Fama/French* (1993) uncover the influence of two factors: the term structure, which strongly influences government bonds, and default risk, which is a main explanatory variable for corporate bond returns.⁴ In the absence of default risk, *Gultekin/Rogalski* (1985) state that bond portfolio risk purely depends on the term to maturity. To focus on maturity characteristics, German government bond yields are used in this study to evaluate CFs because they do not include default premiums. Whereas *Ferson/Henry/Kisgen* (2006) use stochastic discount factors from term structure models to evaluate US government bond fund performance, the present article relies on German government bond yields provided by Deutsche Bundesbank.⁵ The importance of government bonds can also be illustrated from the perspective of institutional managers. Although interest payments are low or even negative during the current period of low interest rates, investments in government bonds contribute to meeting regulatory standards, such as liquidity ratios or minimum capital requirements.⁶

Several notes on bond portfolio strategies appeared in the 1970s to analyse portfolio choice optimisation techniques, including bonds and equities. Generally, bonds or bond portfolios were investigated that represented a variety of risk free (in the context of default risk), investment grade, and speculative grade, or varied in the maturity structure. *Crane* (1971) presents a stochastic programming model that allows for dynamic portfolio decision making. For optimal portfolios, he shows that the composition of short- and long-term bonds depends on the magnitude of the tax rates and the maximum loss allowed. Fur-

³ Deutsche Bundesbank (2016) provides statistical data on the financial institutions sector.

⁴ Besides maturity and default as explanatory variable for bond returns, *Ludvigson/Ng* (2009) identify the forecasting power of macroeconomic factors that influence the future excess returns of US government bonds.

⁵ German government bond yield structures as provided by Deutsche Bundesbank rely on, for example, *Nelson/Siegel* (1987) estimation techniques as stated in Deutsche Bundesbank (1997). A further example of the application of the *Nelson/Siegel* (1987) framework is found in *Diebold/Li* (2006), who forecast government bond yields using US Treasuries.

⁶ In European countries, international regulatory standards for credit institutions and investment firms initiated by the Basel Committee on Banking Supervision (BCBS) are transferred into European legislation via REGULATION (EU) No 575/2013 (2013). Investments, such as in European government bonds, are privileged with respect to minimum capital requirements. Additionally, European government bonds meet the highest liquidity standards.

thermore, he experimentally demonstrates that optimal portfolios commonly consist of short and/or long maturities. Models addressing portfolio optimisation that include bonds are provided by *Breadly/Crane* (1972), who illustrate an efficient salvation procedure of multistage decision problems, and *Merton* (1971), who applies time-dynamic models including assets such as common stocks and 'risky' bonds.

Examples of maturity strategies are in *Fogler/Groves/Richardson* (1977), who define ladders, dumbbells and long maturity portfolios. A ladder strategy requires investing equal portions of assets in different maturities. When the shortest maturity expires, the CF is reinvested in the longest maturity. A dumbbell strategy allocates parts of an investment to short-term bonds and the remaining portion to long-term bonds. Thereby, a long maturity strategy focuses on holding long-term bond investments until a predefined time span before the expiration date. According to their study, dumbbell strategies are outperformed by long maturity portfolios with respect to risk and return. A survey of the relevant literature in the field of fixed-income strategies is provided by *Bierwag/Kaufman* (1978). Regarding dumbbell and ladder portfolios, the authors conclude that the results are heterogeneous or even conflict with respect to risk and return efficiency.

In recent years, several studies in the field of fixed-income management advisory were published. An interesting example is *Frère/Reuse/Svoboda* (2008), who recommend combining ladder structures of 10 and 15 years to manage banking books. Their results are based on a descriptive analysis of 5-, 10- and 15-year ladders during 1986 and 2007. *Bohlin/Strickland* (2004) argue that bond laddering provides an opportunity to simultaneously increase returns and reduce risk. In the early nineties, *Benke* (1993) states that Germany's bond index is a widely used benchmark for the treasury of financial institutions. In this context, it must be mentioned that the construction of the REX is also comparable to a ladder structure, which is illustrated in greater detail in section 2. However, the financial literature lacks the empirical evidence that supports the efficiency of bond ladders and of the REX, accordingly. One step in that direction can be found in *Fama/French* (1993), who examine the performance of US government bonds, corporate bonds and shares. For government bond excess returns, the term structure contribution of a sample of 6- to 10-year maturities exceeds that of a sample of 1- to 5-year maturities. A further step in analysing bond maturity structures is provided in *Judd/Kubler/Schmedders* (2011), who address the question of creating optimal portfolios by combining bond ladders and equity assets. Based on a general equilibrium model, their findings suggest that an increasing bond ladder length positively corresponds to welfare.

Whereas *Fama/French* (1993) explain the influence of term structure changes on bond excess returns, the present paper additionally addresses the influence

of maturity strategies on return, risk and performance. The model of *Judd/Kubler/Schmedders* (2011) is based on an infinite time period, while the present study relies on a discrete, specifically chosen period of low interest rates. This period is of special interest to the institutional sector because low or negative yield structures reduce earnings opportunities as published by Deutsche Bundesbank (2017).⁷ A comparable situation is described in *Ketzler/Wiener* (2017) for insurances. In addition, *Rehm* (2008) illustrates that financial institutions face high intensity competition. In the present paper, dominant maturity structures are identified which can help bond portfolio managers optimise their strategies and increase earnings opportunities. The results show that return and risk significantly increase with the maximum maturity of bond ladders from 1 year to 10 years. Similar results are observed for the performance measures return on risk-adjusted capital (RORAC) and the Sharpe ratio (SR) when long maturities dominate short maturities. An interesting observation is that the dominance of long maturities still holds for subsamples of upward and downward moving yield levels. However, for upward movements, the dominance is strongly reduced. As long as yield levels remain stable or increases are moderate, long maturity bond ladders can be regarded as a dominant strategy. The findings also indicate that expectations of changes in future yield levels should be incorporated in management decisions.

The remainder of the article is structured as follows. Section II. describes the benchmark REX. Section III. illustrates the data. Section IV. presents the methodology and empirical results of the performance analysis, and section V concludes.

II. Benchmark

REX, Germany's national bond index, replicates the development of government bonds including maturities from 1 to 10 years. The index consists of 30 selected bonds with coupons of 6%, 7.5% and 9%. Table 1 illustrates the share of each bond maturity derived from the market share of the past 25 years. Detailed information on the REX is provided in, for example, an information prospectus of Deutsche Börse Group (2017). The composition of the REX is exemplarily illustrated in Table 1.

Because fractions of the REX are unequally distributed, the composition of the index is of limited suitability for a systematic analysis of varying maturity structures. Hence, I construct laddered bond portfolios with equally weighted CFs. In addition to the REX, Table 1 shows the maturity weights of a 5-year and

⁷ Deutsche Bundesbank (2017) analyses the profitability of financial institutions with respect to a period of low interest rates.

a 10-year laddered structure. Whereas a 10-year structure consists of 10 % fractions of CFs for each maturity between 1 and 10 years, the 5-year structure consists of 20 % fractions of CFs between 1 and 5 years. The analysis includes portfolio ladders from 1 to 10 years. An average maturity of 5.5 years indicates that a 10-year ladder compares well with the REX with an average of 5.49 years. The average maturity of the bond ladders systematically decreases by 0.5 years with a 1-year decrease in the maximum term. As an example, the average maturity of a 9-year ladder amounts to 5 years and that of a 5-year ladder amounts to 3 years (see also Table 1).

Table 1
Composition of the REX and 5-Year and 10-Year Bond Ladders

Maturity (years)	REX (%)	10-year laddered structure (%)	5-year laddered structure (%)
1	7.39	10.00	20.00
2	8.80	10.00	20.00
3	10.25	10.00	20.00
4	11.95	10.00	20.00
5	12.04	10.00	20.00
6	11.25	10.00	
7	11.63	10.00	
8	10.58	10.00	
9	9.65	10.00	
10	6.46	10.00	
Sum	100.00	100.00	100,00
Average maturity	5.49	5.50	3.00

III. Data Description

The analysis focuses on a period of low interest rates. To define this period, this study relies on the development of the key interest rate provided by the European Central Bank (ECB). The starting point of April 2009 is selected, when the key interest rate declined to lower than 1.5 %. The performance analysis is based on a 5-year interval during a period of low interest rates from April 2009 to March 2014. Given risk and return calculation techniques, the total sample includes German government bond yields between April 1999 and March 2015.



Figure 1: Yield Structure of German Government Bonds With 1-Year, 5-Year and 10-Year Maturities and ECB's Key Interest Rates Between April 1999 and March 2015

Figure 1 depicts monthly yield structures of German government bonds for 1-year, 5-year and 10-year maturities, as well as the ECB's key interest rates between April 1999 and March 2015.

Table 2 provides a descriptive analysis of German government bond yields with maturities from 1 to 10 years for the total sample between April 1999 and March 2015.⁸ A review of the minimum values reveals that the sample also includes bonds with negative returns. Mean as well as median values continuously increase from 1-year to 10-year bonds, which is again typical for an overall 'normal' yield structure. Means, for example, increase from 2.15 % for 1-year bonds

Table 2

**German Government Bond Yields with Maturities from 1 Year to 10 Years
(as Percentage) from April 1999 to March 2015**

Years	1	2	3	4	5	6	7	8	9	10
Minimum	-0.24	-0.24	-0.23	-0.18	-0.12	-0.04	0.02	0.09	0.16	0.22
Maximum	5.17	5.26	5.31	5.33	5.34	5.36	5.45	5.52	5.58	5.63
Median	2.27	2.53	2.77	2.99	3.20	3.37	3.56	3.68	3.76	3.81
Mean	2.15	2.32	2.51	2.69	2.86	3.02	3.17	3.29	3.41	3.51
Volatility ^o	1.63	1.62	1.60	1.57	1.53	1.48	1.44	1.39	1.35	1.32

^o Volatility is measured as the standard deviation of monthly returns.

⁸ Data are provided by Deutsche Bundesbank and are publicly available.

Table 3
German Government Bond Yields with Maturities from 1 Year to 10 Years
(as Percentage) During a Period of Low Interest Rates from April 2009
to March 2014

Years	1	2	3	4	5	6	7	8	9	10
Minimum	-0.10	-0.08	0.00	0.13	0.31	0.51	0.71	0.89	1.07	1.23
Maximum	1.46	1.86	2.17	2.44	2.69	2.98	3.23	3.45	3.62	3.78
Median	0.26	0.51	0.80	1.02	1.28	1.51	1.72	1.91	2.08	2.22
Mean	0.42	0.63	0.88	1.14	1.39	1.63	1.84	2.04	2.22	2.37
Volatility [°]	0.44	0.60	0.71	0.78	0.81	0.83	0.83	0.83	0.81	0.80

[°] Volatility is measured as the standard deviation of monthly returns.

to 3.51 % for 10-year bonds. Volatility, measured as the standard deviation of monthly returns, slightly decreases with maturity from 1.63 % to 1.32 %.

Table 3 depicts the returns of the defined 5-year period of low interest rates between April 2009 and March 2014. Mean values increase from 0.42 % for 1-year bonds to 2.37 % for 10-year bonds. The medians qualitatively confirm these results. In line with the total sample, the analysed period of low interest rates displays a ‘normal’ yield structure. In contrast to the total sample, the volatility of the returns tends to increase slightly with maturity.

IV. Methodology and Empirical Analysis

1. Calculation of Returns, Risk and Performance

The construction of bond ladders is based on annual CFs, as illustrated in Table 1. For a 10-year bond ladder, the first capital flow occurs after 1 year and the latest after 10 years. Practical applications of annual bond ladders lead to annually revolving CFs. For example, a 10-year bond ladder strategy requires reinvestments of free annual CFs in 10-year bonds at the beginning of each year. This strategy is denoted as a rolling window of investments.

The value of a bond or a laddered bond structure equals the present value (PV) of the corresponding CFs.⁹ The calculation of PVs is based on zero bond discount rates (ZBDRs) derived from the yield structure of German government bonds. Techniques to determine ZBDRs are discussed in Gruber/Overbeck

⁹ In the context of corporate bonds, Elton et al. (2002) illustrate the valuation of CFs and the factors that influence bond values.

(1998). Formula 1 illustrates the calculation of PVs using ZBDRs. The availability of German government bonds allows for the use of monthly time intervals t :

$$(1) \quad PV_{s,t} = \sum_{j=1}^s CF_{j,t} \cdot ZBDR_{j,t}$$

The PV of bond ladder s at time t , $PV_{s,t}$, is determined by multiplying annual CFs of maturity j and time t , $CF_{j,t}$, and the corresponding discount factor, $ZBDR_{j,t}$. The maximum number s of annual CFs j is determined by the longest maturity of a bond ladder. For example, for a 5-year bond ladder, index s equals 5 (years) and consists of $j = 1, \dots, 5$ discounted annual CFs.

Consistent with annual CF structures, annual periods are used to calculate the returns. The reasons for defining 1-year intervals can be found in *Baltussen/Post/Vliet* (2012). The authors argue that financial statements must be elaborated each year. Similarly, these arguments apply from a regulatory perspective, as described in Deutsche Bundesbank (2013). To calculate returns, the PV of bond ladder s at time $t + 12$, $PV_{s,t+12}$, is determined:

$$(2) \quad PV_{s,t+12} = \sum_{j=1}^s CF_{j,t} \cdot ZBDR_{j-1,t+12}$$

Investments of month t with annual CFs of maturity j , $CF_{j,t}$, are discounted by $j-1$ years at time $t+12$. Accordingly, ZBDRs are determined for $j-1$ years. Because they rely on the yield structure at time $t+12$, they are denoted as $ZBDR_{j-1,t+12}$.

Discrete returns for each bond ladder s at time t are determined as follows:

$$(3) \quad R_{s,t} = \frac{PV_{s,t+12} - PV_{s,t}}{PV_{s,t}}$$

According to *Fama/McBeth* (1973), *Merton* (1980) and more recently *Hirtle/Stiroh* (2007), risk is calculated as the standard deviation of returns, $\sigma_{s,t}$. In line with annual planning periods, 1-year returns are used to determine risk at time t for each bond ladder s . Consistent with the longest bond ladder, 10 years of preceding returns are included to calculate the return volatility. Also relying on *Merton* (1980), risk is determined at each time t on the basis of a rolling window of returns. *Ghysels/Santa-Clara/Valkanov* (2005) identify that the length of a rolling window can influence risk results. Hence, for robustness purposes, the analysis additionally includes a 5-year period of annual return volatility.¹⁰

¹⁰ An example of a 5-year period of return volatility can be found in *Poshakwale/Courtis* (2005).

Commonly used performance measures are RORAC and SR.^{11, 12} The calculation of $RORAC_{s,t}$ of bond ladders at month t is given by:

$$(4) \quad RORAC_{s,t} = \frac{R_{s,t}}{\sigma_{s,t}}$$

Regarding SR, returns are adjusted for a one-period reference which is free from default risk (see, e.g., *Munk* 2013). Because institutions are required to apply annual planning periods, 1-year government bond returns $R_{1,t}$ are chosen as an appropriate reference. Equation (5) provides the calculation of $SR_{s,t}$ of bond ladders and time t as:

$$(5) \quad SR_{s,t} = \frac{R_{s,t} - R_{1,t}}{\sigma_{s,t}}$$

2. Regression Models

Returns of a bond or a bond ladder can be derived from relative changes in PVs. Because the yield level affects discount rates, it has a strong influence on bond returns. In the context of US bonds, *Ferson/Harvey* (1991) and *Campbell* (1996), for example, illustrate the contribution of interest rate factors on bond returns. Another example is *Fama/French* (1993), who show the influence of term structure changes on bond excess returns. Figure 1 highlights yields of selected German government bonds between April 2009 and March 2014. Both upward and downward movements are observed for different maturities. To show an unbiased effect of bond ladder structures, regression models control for overall yield levels, \bar{Y}_t , calculated as the mean of government bond returns in month t .

This paper uncovers the influence of each bond ladder on return, risk and performance. Regression models include monthly variables during the time span between April 2009 and March 2014. As robustness checks, subsamples of increasing and decreasing return levels are analysed. Upward (downward) movements generally affect long maturity returns more negatively (positively) than short maturity returns. In this context, the question arises as to what extent return, risk and performance results are influenced during a period of low interest rates.

¹¹ *Hirtle/Stiroh* (2007) apply the RORAC measure when evaluating the performance of banks. RORAC is also used in *Buch/Dorfleitner/Wimmer* (2011) and *Burghof/Dresel* (2002).

¹² A significant contribution to fund performance analysis is provided by *Sharpe* (1966).

First, annual returns $R_{s,t}$ are explained by the maturity structure. The ordinary least squares (OLS) regression specification is expressed as:

$$(6) \quad R_{s,t} = \alpha + \beta \cdot \overline{Y}_t + C' \cdot M_s + \varepsilon_{s,t}$$

The return $R_{s,t}$ of bond ladder s and month t is defined as a dependent variable. Coefficient α represents the constant term. The average yield level \overline{Y}_t at time t is captured by β . M_s is a vector of maturity dummies, where s represents the maximum maturity of each bond ladder. The 1-year maturity is omitted as a reference. As a vector of coefficients, C captures maturity-specific return effects. The dimension of vector C is equal to the dimension of vector M_s . To correct for heteroscedasticity and autocorrelations in the sample's residuals, $\varepsilon_{s,t}$, the method of *Newey/West* (1987) is applied.

Similar to returns, the influence of the maturity structure on return volatility $\sigma_{s,t}$ is estimated according to:

$$(7) \quad \sigma_{s,t} = \alpha + \beta \cdot \overline{Y}_t + C' \cdot M_s + \varepsilon_{s,t}$$

In the following, performance is analysed using RORAC and SR. Both are appropriate measures to simultaneously account for return and risk. Practical applications of RORAC in the financial sector, for example, in the context of consultancy or regulation, are described in *Milne/Onorato* (2012). *Schuhmacher/Eling* (2012) provide a theoretic foundation of SR and related measures. The following regression specification captures the influence of the maturity structure on RORAC:

$$(8) \quad RORAC_{s,t} = \alpha + \beta \cdot \overline{Y}_t + C' \cdot M_s + \varepsilon_{s,t}$$

In regression specifications (6), (7) and (8), only the dependent variable has to be exchanged. The analysis of SR additionally requires an adjustment of the vector of maturity dummies. Because the 1-year return is used as a benchmark, the 1-year SR becomes zero and is therefore excluded in the regression specification. As a reference category, the 2-year SR is omitted in the vector of maturity dummies, M_s . Coefficient vector D captures maturity-specific effects. Again, *Newey/West* (1987) is applied to correct for heteroscedasticity and autocorrelations in the sample's residuals, $\varepsilon_{s,t}$.

$$(9) \quad SR_{s,t} = \alpha + \beta \cdot \overline{Y}_t + D' \cdot M_s + \varepsilon_{s,t}$$

3. Return, Risk and Performance Results

Table 4 shows the regression results of maturity structures on return and risk as dependent variables. The 600 observations are based on a monthly rolling window of 1-year to 10-year bond ladders during a 5-year period of low interest rates between April 2009 and March 2014. Both models control for the influence of the overall yield level, \bar{Y}_t . The model 'return' exhibits a positive and significant coefficient of 2.0 % (0.020) for \bar{Y}_t . A positive coefficient of \bar{Y}_t indicates that high yield levels lead to high annual returns of bond ladders and vice versa. The result is in line with the method of calculating PVs and discrete returns according to Equations (1) and (3). Coefficients M_2 (0.003) to M_{10} (0.041) significantly increase with maturity, except for the coefficient of the 2-year bond ladder M_2 , which does not significantly differ from the 1-year reference. In summary, increasing maturity structures lead to significant increases in returns. This result supports the descriptive analysis of mean returns given in Table 3.

The results for the model 'risk' show that risk significantly increases with the maturity structure. The risk contribution of M_2 is 0.2 % (0.002) and increases to 2.1 % (0.021) for the coefficient of a 10-year bond ladder M_{10} . The average bond yield exhibits no significant influence. Altogether, an increase in the maximum maturity of a bond ladder leads to a simultaneous increase in returns and risk. For the German equity market, *Walkshäusl* (2012) observes a volatility anomaly, where high return equities show low volatility levels. This anomaly does not hold for portfolios of German government bonds.

Using the risk and return results, performance measures such as RORAC and SR can be calculated. Because negative values of RORAC and SR can hardly be interpreted, OLS regressions are estimated for two samples. Whereas Sample I comprises all observations, Sample II solely includes positive values of RORAC and SR. The performance results are depicted in Table 5. In both samples, the coefficients of \bar{Y}_t exhibit significant positive values for RORAC and SR, implying that performance increases with an average level of bond yields. For model 'RORAC' of Sample I, M_2 shows a positive but insignificant value of 0.143. This result indicates that the performance of the 2-year bond ladder does not significantly differ from that of the 1-year bond ladder. However, highly significant maturity coefficients can be observed from M_3 (0.284) to M_{10} (1.014), for which performance increases with the length of the bond ladder. For Sample I and SR as a dependent variable, again significant maturity coefficients can be registered from M_3 (0.194) to M_{10} (1.034). Hence, the performance results of RORAC and SR are qualitatively comparable.

The maturity coefficients of Sample II slightly exceed those of Sample I because of the exclusion of negative RORACs and SRs. In Sample II, the coefficient of the 2-year bond ladder significantly differs from the reference. The per-

Table 4
Return and Risk of Bond Ladders

	Return			Risk		
	Coeff. ^o		t-Stat.	Coeff. ^o		t-Stat.
<i>const.</i>	-0.025	***	-5.993	0.012	***	13.211
\bar{Y}_t	0.020	***	10.532	0.001		1.148
M_2	0.003		0.713	0.002	***	2.743
M_3	0.007	*	1.764	0.005	***	5.813
M_4	0.012	***	2.843	0.008	***	7.936
M_5	0.017	***	3.698	0.011	***	9.387
M_6	0.022	***	4.29	0.013	***	10.485
M_7	0.027	***	4.681	0.015	***	11.395
M_8	0.032	***	4.933	0.018	***	12.205
M_9	0.036	***	5.088	0.020	***	12.942
M_{10}	0.041	***	5.176	0.021	***	13.6
<i>adj. R²</i>	0.609			0.616		
<i>Obs.</i>	600			600		

^o Significance levels are 10% = *, 5% = ** and 1% = ***.

formance results of Sample I and Sample II indicate that long maturity bond ladders significantly dominate short maturity bond ladders during a period of low interest rates. With respect to management advisory, empirical findings provide evidence for the application of the REX as a portfolio benchmark. The results are in line with *Judd/Kubler/Schmedders* (2011), who find that welfare increases with the length of bond ladders. Based on a descriptive analysis, *Fama/French* (1993) show that excess returns of long maturities (sample of 6- to 10-year bonds) exceed those of short maturities (sample of 1- to 5-year bonds).

4. Performance Attribution to Yield Level Changes

As mentioned, *Fama/French* (1993), for example, identify an influence of term structure changes on bond returns. To disentangle the effect of yield level changes on performance during annual planning periods, two additional samples are constructed. For RORAC, one sample comprises 190 annual upward movements of \bar{Y}_t . An upward movement is registered when \bar{Y}_t increases from time t to $t+12$. Hence, the results can be interpreted in the context of future yield expectations. The sample of analogously measured downward movements

Table 5
Performance of Bond Ladders (10-Year Return Volatility)

Sample I (all observations)				Sample II (positive RORAC and SR)							
RORAC		SR		RORAC		SR					
	Coeff.°	t-Stat.	Coeff.°	t-Stat.	Coeff.°	t-Stat.	t-Stat.				
const.	-0.752	***	-7.164	-0.614	***	-5.377	-0.775	***	-6.869	***	-5.188
\bar{Y}_t	0.76	***	13.435	0.565	***	9.717	0.735	***	12.541	***	9.456
M_2	0.143		1.56				0.212	***	2.662		
M_3	0.284	***	2.645	0.194	*	1.747	0.383	***	4.074	***	2.674
M_4	0.423	***	3.342	0.368	***	2.974	0.54	***	4.813	***	4.458
M_5	0.553	***	3.809	0.521	***	3.702	0.685	***	5.274	***	5.393
M_6	0.67	***	4.126	0.655	***	4.15	0.809	***	5.566	***	5.783
M_7	0.775	***	4.339	0.772	***	4.429	0.925	***	5.754	***	5.513
M_8	0.867	***	4.475	0.874	***	4.599	1.014	***	5.797	***	5.679
M_9	0.946	***	4.551	0.96	***	4.695	1.100	***	5.841	***	5.763
M_{10}	1.014	***	4.587	1.034	***	4.741	1.119	***	5.059	***	5.794
adj. R ²	0.575			0.482			0.574			0.512	
Obs.	600			540			530			477	

° Significance levels are 10% = *, 5% = ** and 1% = ***.

comprises 410 values. Because SR does not include the 1-year ladder, 171 upward and 369 downward movements are registered. Although \bar{Y}_t generally decreases during the 5-year period of low interest rates, the sample also includes increases, such as an increase from 1.37% to 2.63% between August 2010 and March 2011. However, increases can be regarded as moderate. According to Equation (1), increasing yields are disadvantageous for long maturity returns (versus short maturity returns), whereas decreasing \bar{Y}_t is beneficial for long maturities. Table 6 illustrates the performance results for upward and downward movements of \bar{Y}_t during annual planning periods.

It is interesting to observe that the coefficients for RORAC and SR still increase from short to long maturities during upward movements. However, the maturity coefficients for RORAC (from -0.038 to 0.122) and SR (from 0.024 to 0.236) are clearly lower than the level of the performance results in Table 5. Significant increases in performance can be found for 8-, 9- and 10-year ladders for RORAC and for 6- to 10-year ladders for SR. Although to a weaker extent, long maturity strategies still dominate short maturity strategies. In line with *Crane* (1971), optimal bank bond portfolio strategies include investments in long maturities, even though interest rates might increase. However, after significant increases, the author recommends to sell long maturity portfolios and invest in short maturities. As expected, during downward movements of \bar{Y}_t , the coefficients for RORAC (from 0.226 to 1.427) and SR (from 0.272 to 1.404) considerably exceed those of upward movements and the results in Table 5. Performance increases from short to long maturities are highly significant for each maturity coefficient.

The results of increasing and decreasing levels of \bar{Y}_t provide important implications for bond portfolio managers. The dominance of long maturity structures, even during upward movements, can be attributed to moderate changes in bond yield levels. If interest levels are expected to remain stable, investing in long maturity bond ladders can be regarded as an efficient strategy during a period of low interest rates. For large increases in yield level, performance outcomes might reverse. Thus, considering the expectations of future yield level changes is crucial for sustainable portfolio decisions.

5. Robustness

Relying on *Ghysels/Santa-Clara/Valkanov* (2005), the length of a rolling window can influence risk results. For robustness purposes, volatility is calculated using an alternative, 5-year period of preceding annual returns, as applied in *Poshakwale/Courtis* (2005). Table 7 illustrates risk and performance results. For the 'risk' model, maturity coefficients increase slightly but significantly from 0.003 for M_1 to 0.023 for M_{10} . The results qualitatively confirm the maturity

Table 6
Performance of Bond Ladders (10-Year Return Volatility) for Upward and Downward Movements of Overall Yield Levels

upward movements				downward movements			
RORAC	SR	RORAC		SR	RORAC		SR
		Coeff.°	t-Stat.	Coeff.°	t-Stat.	Coeff.°	t-Stat.
const.	***	-0.202	-6.096	-0.167	-3.282	-0.716	-7.174
\bar{Y}_t	***	0.313	11.572	0.134	4.851	0.725	13.506
M_2		-0.038	-0.936			0.226	3.311
M_3		-0.038	-0.705	0.024	0.354	0.433	5.488
M_4		-0.013	-0.224	0.065	0.938	0.626	6.670
M_5		0.018	0.287	0.108	1.561	0.801	7.475
M_6		0.048	0.792	0.147	2.167	0.959	8.028
M_7		0.075	1.27	0.179	2.717	1.100	8.405
M_8	*	0.097	1.702	0.205	3.166	1.224	8.639
M_9	**	0.112	2.061	0.224	3.477	1.333	8.757
M_{10}	**	0.122	2.319	0.236	3.626	1.427	8.812
adj. R^2		0.460		0.210		0.740	
Obs.	190			171		410	

° Significance levels are 10% = *, 5% = ** and 1% = ***.

Table 7
Performance of Bond Ladders (5-Year Return Volatility)

	Risk			RORAC			SR		
	Coeff. ^o		t-Stat.	Coeff. ^o		t-Stat.	Coeff. ^o		t-Stat.
<i>const.</i>	0.013	***	8.973	-0.648	***	-6.575	-0.528	***	-4.953
\bar{Y}_t	0.001		1.063	0.669	***	12.096	0.492	***	8.672
M_2	0.003	**	2.129	0.121		1.560			
M_3	0.007	***	4.152	0.244	***	2.643	0.171	*	1.770
M_4	0.010	***	5.808	0.371	***	3.312	0.329	***	2.995
M_5	0.013	***	7.100	0.493	***	3.731	0.472	***	3.681
M_6	0.015	***	8.118	0.606	***	4.001	0.599	***	4.079
M_7	0.017	***	8.943	0.708	***	4.180	0.711	***	4.323
M_8	0.019	***	9.638	0.798	***	4.299	0.810	***	4.476
M_9	0.021	***	10.233	0.877	***	4.374	0.895	***	4.572
M_{10}	0.023	***	10.747	0.945	***	4.419	0.968	***	4.627
<i>adj. R²</i>	0.496			0.542			0.452		
<i>Obs.</i>	600			600			540		

^o Significance levels are 10% = *, 5% = ** and 1% = ***.

coefficients of the 10-year volatility, as illustrated in Table 4, showing a range from 0.002 to 0.021. Additionally, the RORAC and SR results in Table 7 qualitatively confirm the maturity coefficients and significance levels of Sample I, Table 5. The same is valid for the Sample II results (not illustrated), excluding negative RORAC and SR values. Altogether, the risk and performance results are robust with respect to the volatility measure.

To test the robustness of the performance attribution to yield level changes, RORAC and SR regression models are estimated for upward and downward movements by applying the 5-year return volatility. The results are illustrated in Table 8. For upward movements of \bar{Y}_t , the maturity coefficients for RORAC significantly increase for 8-, 9- and 10-year structures and for SR for 6- to 10-year ladders. Analogous results can be observed in Table 6. For downward movements, performance significantly increases from short to long for all maturities for the 5-year volatility, as illustrated in Table 8. This result coincides with the performance results for the 10-year volatility in Table 6.

Table 8
Performance of Bond Ladders (5-Year Return Volatility) for Upward and Downward Movements of Overall Yield Levels

upward movements				downward movements			
RORAC		SR		RORAC		SR	
	Coeff.°	t-Stat.	Coeff.°	t-Stat.	Coeff.°	t-Stat.	t-Stat.
const.	-0.175	***	-0.141	***	-0.598	***	***
\bar{Y}_t	0.278	***	0.115	***	0.630	***	***
M_2	-0.032		-0.808		0.192	***	
M_3	-0.034		-0.694		0.366	***	
M_4	-0.016		-0.291		0.935	***	
M_5	0.011		0.196		1.549	***	
M_6	0.041		0.715		2.151	***	
M_7	0.072		1.217		2.691	***	
M_8	0.100	*	1.659		3.118	***	
M_9	0.126	**	2.005		3.407	***	
M_{10}	0.147	**	2.240		3.564	***	
adj. R ²	0.420		0.195		0.686		
Obs.	190		171		410		

° Significance levels are 10 % = *, 5 % = ** and 1 % = ***.

Table 9
Return, Risk and Performance of Bond Ladders (10-Year Return Volatility) Including AR Terms

	Return			Risk			RORAC			SR		
	Coeff. ^o	t-Stat.	Coeff. ^o	t-Stat.	Coeff. ^o	t-Stat.	Coeff. ^o	t-Stat.	Coeff. ^o	t-Stat.	Coeff. ^o	t-Stat.
const.	-0.015	***	-8.147	0.012	***	26.982	-0.568	***	-8.582	***	-0.401	***
\bar{Y}_t	0.014	***	20.887	0.001	***	4.926	0.638	***	24.849	***	0.431	***
M_2	0.002		0.734	0.001	***	2.634	0.140	*	1.790			
M_3	0.006	**	2.569	0.004	***	7.313	0.279	***	3.599	***	0.174	**
M_4	0.010	***	4.725	0.007	***	12.297	0.419	***	5.393	***	0.349	***
M_5	0.015	***	7.023	0.010	***	17.128	0.549	***	7.076	***	0.504	***
M_6	0.021	***	9.358	0.012	***	21.616	0.668	***	8.601	***	0.639	***
M_7	0.026	***	11.652	0.015	***	25.775	0.774	***	9.965	***	0.758	***
M_8	0.031	***	13.909	0.017	***	29.655	0.868	***	11.180	***	0.862	***
M_9	0.035	***	16.016	0.019	***	33.511	0.949	***	12.217	***	0.950	***
M_{10}	0.037	***	16.261	0.021	***	37.339	0.968	***	12.095	***	0.981	***
AR terms	yes		yes	yes		yes	yes		yes		yes	
adj. R ²	0.957		0.980	0.980		0.923	0.923		0.912		0.912	
Obs.	600		600	600		600	600		540		540	

^o Significance levels are 10 % = *, 5 % = ** and 1 % = ***.

Finally, each regression specification is controlled for autoregressive (AR) disturbances. Relying on the *Breusch* (1978) and *Godfrey* (1978) serial correlation LM test, a sufficient number of AR terms are applied. Table 9 shows the results of maturity structures for return and risk (corresponding to Table 4) and RO-RAC and SR (corresponding to Table 5, Sample I) as dependent variables. Risk as dependent variable comprises 10 years of preceding returns. The regression results of Table 9 coincide with the results illustrated in Table 4 and Table 5, Sample I. The results are qualitatively comparable for all regression specifications, if AR terms are included. This also holds for a varying number of AR terms.

V. Conclusion

The first articles appeared on bond portfolio strategies in the seventies. One seminal example is *Fogler/Groves/Richardson* (1977), who analysed ladders, dumbbell and long maturity portfolios. The authors find that dumbbell strategies are outperformed by long maturity portfolios with respect to both risk and return. Summarising alternative fixed-income strategies, *Bierwag/Kaufman* (1978) conclude that the results vary with respect to dumbbell and ladder portfolios. However, studies in the context of bond portfolio management are lacking, which is stated in *Judd/Kubler/Schmedders* (2011) and *Ferson/Henry/Kisgen* (2006). The analysis of *Judd/Kubler/Schmedders* (2011) provides valuable insights into bond ladder performance. Their results are based on a dynamic general equilibrium asset-pricing model in which the combination of bond ladders and a market portfolio are nearly optimal strategies. According to their study, welfare increases with the length of bond ladders. Whereas their paper is based on an infinite model, the present study relies on a discrete, specifically chosen time interval.

This paper is the first study to show empirical evidence of bond ladder performance during a period of low interest rates by systematically analysing maturity structures. The period of low interest rates is of special interest to the institutional sector due to reduced earnings opportunities. In addition, *Rehm* (2008) illustrates that financial institutions face high intensity competition. The data set is based on German government bond yields which can be regarded as free from default risk. Relying on *Fama/French* (1993) and *Gultekin/Rogalski* (1985), the absence of default risk allows for a pure focus on maturity characteristics. Hence, the paper provides relevant information for all kinds of fixed-income portfolios, independent from a portfolio's exposition to default risk. To determine return, risk and performance differences of maturity structures, a monthly rolling window of annual returns for bond ladders from 1 to 10 years is constructed. The regression results show that return and risk significantly increase with the maximum maturity of bond ladders. Similar results can be observed for

the RORAC and SR performance measures, where long maturities dominate short maturities. This finding is in line with *Judd/Kubler/Schmedders* (2011), but for a specifically chosen time interval. The findings provide evidence for the application of the REX as an efficient portfolio benchmark during a period of low interest rates. The identification of dominant maturity structures can help bond portfolio managers optimise earnings opportunities. Diverse robustness checks are incorporated in the analysis.

In contrast to *Judd/Kubler/Schmedders* (2011), limitations of the unique dominance of long maturity structures can be illustrated by separately analysing upward and downward moving yield levels. As expected, long maturities significantly dominate short maturities during downward movements. For upward movements, significant performance increases can be found for 8-, 9- and 10-year ladders for RORAC and for 6- to 10-year ladders for SR. Long maturity strategies still dominate short maturity strategies, but to a considerably weaker extent. These results provide valuable insights for bond portfolio managers in the context of future yield expectations. As long as yield levels remain stable or increases are moderate, long maturity bond ladders can be regarded as a dominant strategy. For large increases in yield, performance outcomes might reverse. Hence, the results of this study imply that both performance characteristics and expectations of future yield changes should be incorporated in management decisions.

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